STATIC EQUILLBRIUM IN THE ATMOSPHERS.

In theoretical considerations and in order of magnitude estimations it is often advantageous to use rather simple distributions of the atmospheric Variables, in particular the variations with height of pressure, tempreature, density, etc.
A number of simple atmospheric models have been constructed for such purposes. They are characterized by the fact that the Hydrostatic equation can be integrated exactly using simple functions. So we shall consider some example of atmospheric models.

A- The Homogenous atmosphere:

The density is equal to a constant ρ_0 in this atmosphere and independent on space & time.

$$\rho = \rho_0 = \text{constant}$$

From Hydrostatic Equation $\frac{dP}{dz} = -\rho_0 g$

$$\int_{P_0}^{P} dP = -\rho_0 g \int_{z_0}^{z} dz$$

$$P \Big|_{P_0}^{P} = -\rho_0 g \, Z \Big|_{0}^{z}$$

$$P - P_0 = -\rho_0 g Z.$$

$$p = p_0 - \rho_0 g z \quad \text{———— } ①$$

where p_0 is pressure for $z = 0$.

The homogenous atmosphere has a finite height H

when $p = 0 \quad z = H$

$$0 = p_0 - \rho_0 g H$$

$$\therefore H = \frac{p_0}{\rho_0 g} \qquad p_0 = \rho_0 R T_0$$

$$\therefore H = \frac{\cancel{\rho_0} R T_0}{\cancel{\rho_0} g} \qquad T_0 = 283\,°K$$

$R = 287$

$g = 9.8$

$$\boxed{H \approx 8000 \text{ meter.}}$$

we may define a temperature in the homogenous atmosphere from gas equation.

$$p = \rho_0 R T \Rightarrow \therefore T = \frac{p}{\rho_0 R} \quad \text{———} ②$$

put ~~xxxxx~~ equation ① in ②

$$T = \frac{p_0 - \rho_0 g z}{\rho_0 R} \Rightarrow T = \frac{p_0}{\rho_0 R} - \frac{\cancel{\rho_0} g z}{\cancel{\rho_0} R}$$

$$\boxed{T = T_0 - \frac{g}{R} z}$$

this equation shows that T decreases linearly with height in a homogenous atmosphere.

Q/ From the atmospheric model (homogenous atmosphene) show that the lapse rate γ is:

$$\gamma = \frac{dT}{dz} = -3.4\ ^{\circ}K/100m.$$

B - The isothermal atmosphere:

in this model we have $T = T_0 = \text{const.}$

From the Hydrostatic Equation we get:

$$dP = -\rho g\, dz$$

Recall that $\rho = \dfrac{P}{R T_0}$

$$dP = -\frac{P}{R T_0} g\, dz$$

$$\int_{P_0}^{P} \frac{dP}{P} = -\frac{g}{R T_0} \int_0^z dz$$

$$\ln P \Big|_{P_0}^{P} = -\frac{g}{R T_0} z$$

$$\ln \frac{P}{P_0} = -\frac{g}{R T_0} z$$

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$$\left(\frac{P}{P_0}\right) = e^{\left(-\frac{g}{R T_0} z\right)}$$

This equation shows that the isothermal atmosphere is of infinite extent because $P \to 0$ when $z \to \infty$

$$\boxed{P = P_0\, e^{\left(\frac{-g}{R T_0} z\right)}}$$

The scale height for an isothermal atmosphere is often defined as the height at which the pressure has decreased to e^{-1} of the surface pressure.

$$z = H_s \qquad p = p_0 \, e^{\left(-\frac{g}{RT_0} H_s\right)}$$

$$p = p_0 \, e^{-1}$$

$$\cancel{p_0} \cancel{e}^{\left(-\frac{g}{RT_0} H_s\right)} = \cancel{p_0} \cancel{e}^{-1}$$

$$-\frac{g}{RT_0} H_s = -1 \qquad \therefore H_s = \frac{RT_0}{g} = 8000\,m$$

or, that the scale height is equal to the height of the homogenouse atmosphere having the same surface Tempreature as the isothermal atmosphere.

The density in the isothermal atmosphere can be calculated from gas equation. $p_0 = \rho_0 R T_0$, $p = \rho R T_0$

$$p = p_0 \, e^{\left(-\frac{g}{RT_0} z\right)}$$

$$\rho \cancel{R} \cancel{T_0} = \rho_0 \cancel{R} \cancel{T_0} \, e^{\left(\frac{-g}{RT_0} z\right)}$$

$$\boxed{\therefore \rho = \rho_0 \, e^{\left(-\frac{g}{RT_0} z\right)}}$$