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ADVANCED TRAFFIC ENGINEERING

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LECTURE 5:

CAR-FOLLOWING

- A car following model describes the lateral action of the vehicle as function of it’s leader(s).
- That can be, it describes its position, speed or acceleration.
- There are many different forms, and all have their advantages and disadvantages.
- They all aim to describe driving behavior, and human behavior is inconsistent and hence difficult to capture in models.

NEWELL’S CAR FOLLOWING MODEL

Newell’s model is a kinematic relationship that describes how a following vehicle adjusts its trajectory based on the motion of its leader.

It assumes that the follower’s trajectory in the space–time (x–t) plane is simply a translated copy of the leader’s trajectory — shifted by: a time lag (τ), and a space gap (s_i).

The basic form is:

$$x_{i+1}(t) = x_i(t-\tau) - s_j$$

where:

- xi(t) = position of the leader at time *t*
- xi+1(t) = position of the follower
- τ = reaction or delay time (how long it takes the follower to respond)

s_j = minimum spacing between the vehicles (jam spacing)

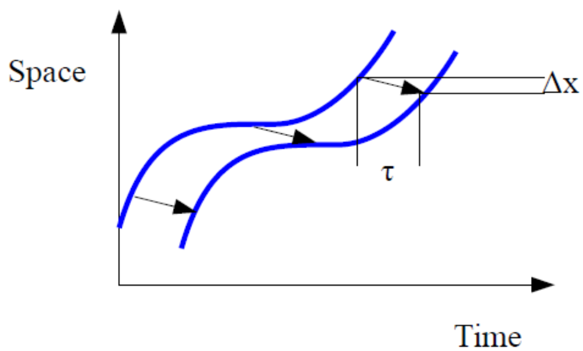
The ratio between these two parameters gives the backward wave speed in congested traffic:

$$w = \frac{s_j}{\tau}$$

- *w* is the slope of the congested branch in the fundamental diagram (negative since waves propagate upstream).
- It represents the speed at which traffic disturbances travel backward through a queue.

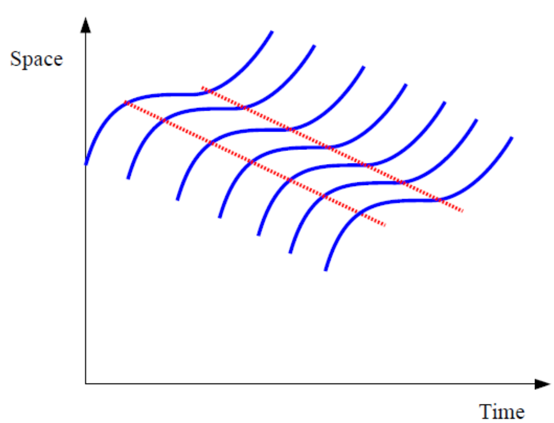
sj and τ parameters differ from one driver to another, reflecting variability in behavior (some react faster, others maintain larger gaps).

Despite these differences, when many drivers' trajectories are averaged, their combined behavior exhibits a consistent shock wave speed.



In the figure:

- The **horizontal axis** is **time**,
- The **vertical axis** is **space**.
- Each curve represents the **trajectory of a vehicle**.
- The translation vector (τ,sj) shifts the leader's trajectory to obtain the follower's.
- The slope w=sj/τ illustrates how congestion waves move upstream.



- This figure shows Shock Wave from Averaged Car-Following Behavior
- Bue curves → individual vehicle trajectories (each driver follows Newell's relation but with unique τ and s_j values).
- Red dotted line → the shock wave representing the average propagation of a traffic disturbance through the vehicle stream.

The slope of the red line defines the shock wave speed (w)**,** given approximately by:

$$w = \frac{\Delta s}{\Delta \tau}$$

This is equivalent to the average backward propagation speed of congestion in the fundamental diagram.

Key Insights

1. Each driver follows a microscopic rule (individual τ and s_j).
2. Collectively, they form a macroscopic pattern (a steady shock wave).
3. This provides a powerful bridge between microscopic behavior and macroscopic traffic phenomena such as congestion waves.

CHARACTERISTICS OF CAR-FOLLOWING MODELS

DEPENDENCIES

Car-following behavior depends on how a follower reacts to the leader's motion.
Key influencing elements include:

1. **Leader's Acceleration**: When the leader accelerates, the follower tends to close the gap.
2. **Speed**: Higher speeds require larger safe headways or spacings.
3. **Speed Difference (Δv)**: A large closing speed requires timely braking to avoid collision.
4. **Spacing (s)**: Large spacing may cause reduced interaction or delayed response; small spacing increases sensitivity to leader actions.
5. **Desired Speed (v_0)**: The faster a driver wishes to travel, the stronger their tendency to close gaps or overtake.

These dependencies vary among models: some use speed and distance as input variables, while others base behavior on speed difference and spacing.

REACTION TIME

- Human drivers have a **reaction delay**, typically **0.1–1.0 seconds**.
- Models either: Explicitly include this delay (τ), or approximate it using the simulation time step.

MULTI-LEADER CAR-FOLLOWING MODELS

Real drivers anticipate not only their immediate leader but also vehicles further ahead.

- Anticipation behavior has been shown empirically.
- Models may include multiple leaders, each influencing the follower with a decaying weight (less influence from distant leaders).

However, two common drawbacks exist:

1. **Including irrelevant leaders**: Only nearby vehicles within the same platoon should influence behavior. Distant leaders' effects are negligible.
2. **Averaging all leader accelerations**: Using a mean of all leaders' accelerations may yield unrealistic results; a minimum operator (most critical constraint) is more appropriate.

INSENSITIVITY DEPENDING ON DISTANCE

Drivers react only when the speed difference exceeds a perceptual threshold that depends on spacing.

- At large spacings, small speed differences go unobserved.
- At short spacings, even small speed differences trigger response.

- The driver is insensitive within a threshold zone:
  - Maintains speed (no acceleration).
  - Reacts only when Δv crosses perceptual bounds.

- Outside these bounds:

  - Follower accelerates when leader pulls away.
  - Follower decelerates when closing in too fast.

- In the relative speed–spacing plane, this causes cyclic or oscillatory patterns (shown as circular loops).
- The wider the spacing, the larger these "observation circles."
- Empirical validation: observed these oscillations in real traffic trajectories.

- Newell's model does not include observation thresholds — trajectories are deterministic and translation-based.
- Therefore, no circles appear in the Δv–spacing plane for Newell-type models.

Aspect	Behavioral Meaning	Effect on Flow
Dependencies	Follower response to leader acceleration, speed, and spacing	Defines model realism
Reaction Time	Delay between perception and action	Longer delay → instability
Multi-Leader Models	Anticipation of multiple leaders	Can improve realism if well-calibrated
Insensitivity to Distance	Threshold for perceiving speed differences	Explains oscillatory driver behavior

EXAMPLES

This section introduces three classic car-following models beyond Newell's: Helly, Optimal Velocity Model (OVM), and Intelligent Driver Model (IDM).

Let's illustrate them using a short numerical example comparing how each model computes acceleration at a given moment.

Assume a following vehicle and a leader:

Parameter	Symbol	Value	Unit
Leader speed	v_l	20	m/s
Follower speed	v_f	18	m/s
Gap (spacing)	s	30	m
Desired time headway	T	1.5	s
Minimum gap	s_0	2	m

1. Helly Model (1959)

Helly model prescribes a desired spacing *s* as function of the speed *v*:

$$s^* = s_0 + Tv_f$$

Note that this could be considered as a spacing at standstill (jam spacing) plus a dynamic part where T is the net time headway (substracting the jam spacing from desired spacing).
Now the acceleration is determined by a desire to drive at the same speed as

$$a(t) = \alpha\left(\Delta v(t-\tau)\right) + \gamma\left(s(t-\tau) - s^*\right)$$

$\alpha\ (\Delta v\ (t-\tau))$ = Speed difference term= Since the leader is faster (Δv = +2 m/s), the follower accelerates to close the gap.

$\gamma\left(s(t-\tau) - s^*\right)$ = Spacing difference term

In this equation, Δv is the speed difference, *t* is a moment in time and τ a reaction time.
α = Sensitivity to speed difference
γ = Sensitivity to **spacing error**
Assume:

- α=0.5
- γ=0.1
- Reaction time ignored (τ=0)

Desired Spacing:

$$s^* = s_0 + Tv_f$$

$$s^* = 2 + 1.5(18) = 29 \text{ m}$$

Compute acceleration:

$$a(t) = \alpha\left(\Delta v(t-\tau)\right) + \gamma\left(s(t-\tau) - s^*\right)$$

$\alpha\ (\Delta v\ (t-\tau))$ = Speed difference term =0.5(20−18)=1 m/s^2, it means that the follower accelerates by 1 m/s^2 to close the gap.

$\gamma\left(s(t-\tau) - s^*\right)$ = Spacing difference term=0.1(30−29) =0.1 m/s^2 (The follower's actual spacing (30 m) is slightly larger than desired (29 m), so a small positive acceleration (0.1) is added to reduce it)

a=0.5(20−18) +0.1(30−29) =0.5(2) +0.1(1) =1.0+0.1=1.1 m/s^2
The follower accelerates at **1.1 m/s^2**, closing the speed gap and slightly reducing distance.

- The follower is slower than the leader (needs to catch up), and
- It's slightly farther than desired (needs to reduce spacing).

Thus, the **Helly model** ensures the vehicle smoothly adjusts both **speed** and **distance** to maintain stable following behavior.

2. Optimal Velocity Model (OVM, Bando et al., 1995)

$$a = a_0(v^* - v)$$

In this equation, v is the speed of the vehicle, and a_0 a reference acceleration (tunable parameter, constant for a specific vehicle-driver combination). v^* is determined as follows:

$$v* = 16.8(\tanh(0.086(s-25)+0.913))$$

s is the spacing (in meters) between the vehicle and its leader, giving the speed in m/s.

Given:
a_0=1.0 m/s^2
s=30 m, vf=18

$$v^* = 16.8[\tanh(0.086(30-25)) + 0.913]$$

$$v^* = 16.8[\tanh(0.43) + 0.913] = 16.8[0.405 + 0.913] = 16.8(1.318) = 22.15 \text{ m/s}$$

$$a = 1.0(22.15 - 18) = \boxed{4.15 \text{ m/s}^2}$$

The OVM gives a stronger acceleration since the current spacing allows a higher "optimal speed."

3. **Intelligent Driver Model (IDM, Treiber et al., 2000)**

$$\frac{\mathrm{d}v}{\mathrm{d}t} = a_0\left(1 - \left(\frac{v}{v_0}\right)^4 - \left(\frac{s^*(v,\Delta v)}{s}\right)^2\right)$$

$$s^*(v,\Delta v) = s_0 + vT + \frac{v\Delta v}{2\sqrt{a\,b}}$$

- $a_0 = 1.0, b = 2.0, v_0 = 33.3 \text{ m/s} \ (\approx 120 \text{ km/h})$

Step 1: Desired spacing

$$s^* = 2 + 18(1.5) + \frac{18(2)}{2\sqrt{1 \times 2}} = 2 + 27 + \frac{36}{2.828} = 2 + 27 + 12.73 = 41.73 \text{ m}$$

Step 2: Acceleration

$$a = 1.0 \left[1 - (18/33.3)^4 - (41.73/30)^2\right]$$

$$a = 1 - (0.54)^4 - (1.39)^2 = 1 - 0.085 - 1.93 = \boxed{-1.02 \text{ m/s}^2}$$

The IDM predicts **braking** because the spacing (30 m) is smaller than the desired safe distance (41.7 m).

Model	Equation Base	Computed (a) (m/s²)	Driver Behavior
Helly	Linear response	**+1.1**	Smooth acceleration
OVM	Nonlinear optimal speed	**+4.2**	Strong acceleration toward optimal flow
IDM	Nonlinear + safety-based	**−1.0**	Braking to maintain safe headway

References

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