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POSTGRADUATE/MSC
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ADVANCED TRAFFIC ENGINEERING

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LECTURE 4:

SHOCK WAVE THEORY

- Shock wave theory is described to capture queuing dynamics.
- This differs from cumulative curves in the way that the spatial extent of the queue is considered

4.1 FIXED BOTTLENECKS

At fixed bottleneck some lanes of a highway are (temporarily) blocked.

4.1.1. THEORY AND DERIVATION OF EQUATIONS

Let us consider a situation with two different states:

- State A (downstream): $(q_A,\ k_A,\ v_A)$
- State B (upstream): $(q_B,\ k_B,\ v_B)$

The states are plotted in the space-time diagram 4.1

A discontinuity between A and B moves through the space–time diagram as a **shockwave** with speed *w* (positive downstream).

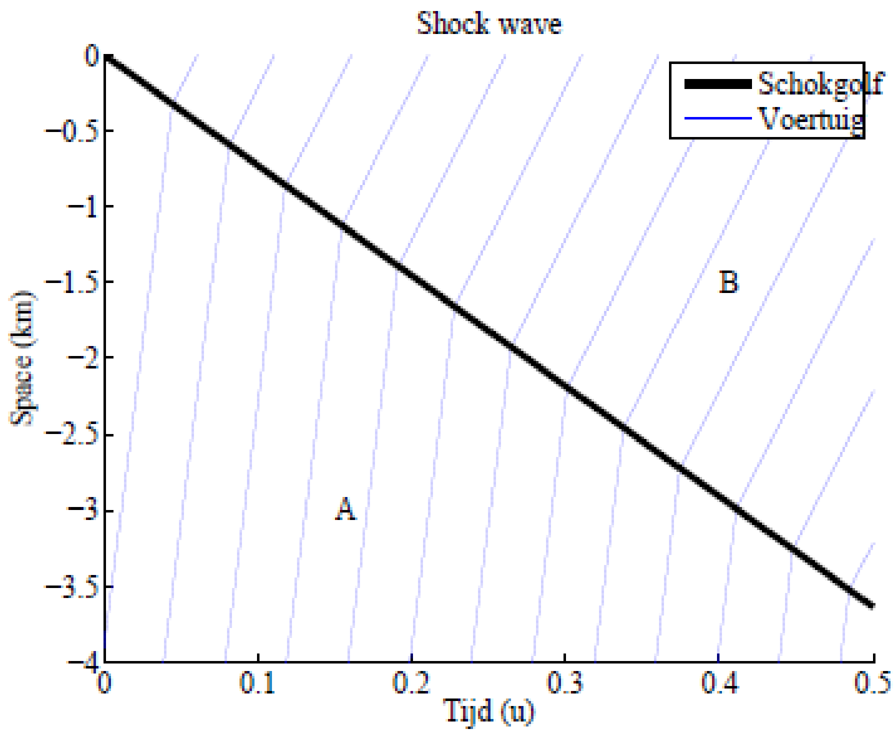


Figure 4.1: A shockwave where traffic speed changes from high to low.

- Assuming that the shockwave moves through the point t=0 at x=0.
- The boundary between (thick black line) is the **shockwave** (the discontinuity between states A and B).
- Speed change is instantaneous at the boundary (Its **slope** in the X–T plane gives the wave speed $w=\Delta x/\Delta t$)…...> the speed *w* is the slope of the shock wave between A and B
- From the figure (approx.): the line goes from (t,x)≈(0 h, 0 km) to (0.5 h, −3.6 km) so $w \approx \frac{-3.6-0}{0.5-0} = -7.2\ \mathrm{km/h}$ (about $-2.0\ \mathrm{m/s}$).
- Negative means the front moves **upstream**.
- The wave itself does not have a physical length.
- or we can state that rate of vehicles entering the wave must be equal to the rate of vehicles exiting the wave.
- No vehicles "inside" the wave → vehicles entering the wave per unit time = vehicles leaving it (in the wave's moving frame).
- This principle, in combination with the following equation is used to calculate the speed of the wave:

$$q = k\,v$$

- The speed of the wave is indicated by *w*
- Downstream (exit) side: the density is k_B, vehicles' speed relative to the wave is $v_B - w$

$$q_{\text{exit}} = k_B\,(v_B - w)$$

- Upstream (attachment) side: vehicles' speed relative to the wave is $v_A - w$

$$q_{\text{attachment}} = k_A \left(v_A - w \right)$$

- Conservation at the wave:

$$q_{\text{exit}} = q_{\text{attachment}} \;\Rightarrow\; k_B(v_B - w) = k_A(v_A - w)$$

Using $q_A = k_A v_A$ and $q_B = k_B v_B$, solve for w:

$$\boxed{w = \frac{q_A - q_B}{k_A - k_B} = \frac{\Delta q}{\Delta k}}$$

- The right-hand side is the ratio between the difference in flow and the difference in density of states A and B.
- This is also the slope of a line segment between A and B in the flow-density plot.
- This becomes very useful when constructing the traffic states.
- $w>0$: wave propagates downstream (with traffic).
- $w<0$: wave propagates upstream (e.g., queue growth).
- Magnitude |w| is the slope of the discontinuity in the space–time diagram
- **Queue growth at a bottleneck:** $k_B > k_A$ and $q_B \approx q_{cap} < qA$ → $w<0$.
- **Queue dissipation:** A improves (higher q_A, lower k_A) → $w>0$.
- **Stationary front:** $q_B - q_A = 0$ or $k_B - k_A \to \infty$ locally → $w \approx 0$

<u>Example</u>

Given
A: q_A=1800 veh/h, k_A=18 veh/km → v_A=100 km/h

B: q_B=900 veh/h, k_B=45 veh/km → v_5=20 km/h

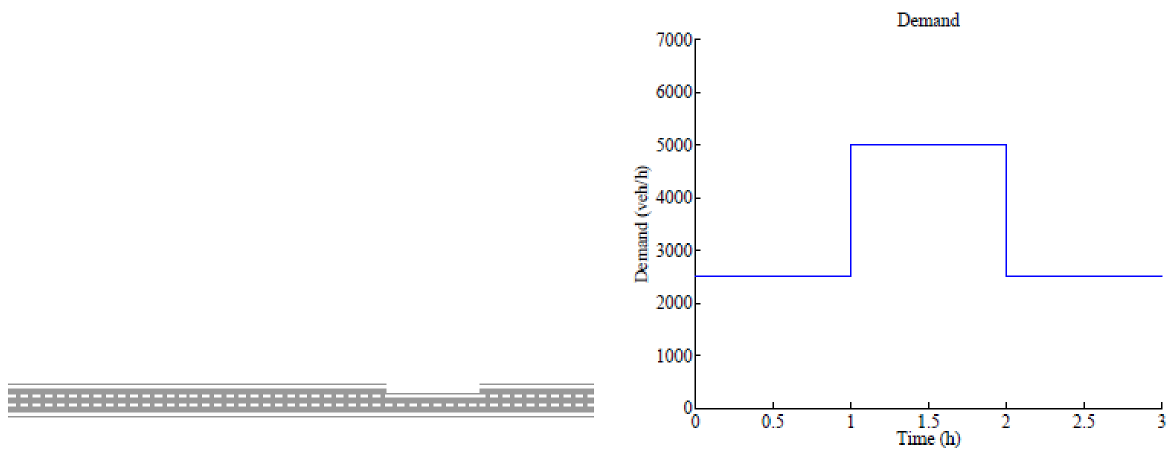
Shockwave speed:

$$w = \frac{900 - 1800}{45 - 18} = \frac{-900}{27} = -33.3 \text{ km/h}$$

The queue front propagates upstream at 33.3 km/h. (Queue growth at a bottleneck)

4.1.2 EXAMPLE: TEMPORAL INCREASE IN DEMAND AT A ROAD WITH A LANE DROP

Let's consider a 3-lane road with a reduction to 2 lanes over a 1 km section between x=10 and x=12.5 (see figure 4.2(b)). For the road, we assume lanes with equal characteristics, described by a triangular fundamental diagram with a free speed of 80 km/h, a capacity of 2000 veh/h/lane and a jam density of 150 veh/km/lane. At the start of the road, there is a demand of 2500 veh/h which temporarily increases to 5000 veh/h between t=1h and t=2h (see the demand profile in figure 4.2(b)). What are the resulting traffic conditions?



(a) The road (b) Demand pattern

Figure 4.2 Situation

Solution

Per lane: v_f=80 km/h, q_{max}=2000veh/h/lane, k_j=150 veh/km/lane.

Critical density (per lane): $k_c=q_{max}/v_f$=2000/80=25 veh/km.

3 lanes: $q_{max,3}$=6000 veh/h, $k_{c,3}$=75 veh/km, $k_{j,3}$=450 veh/km.

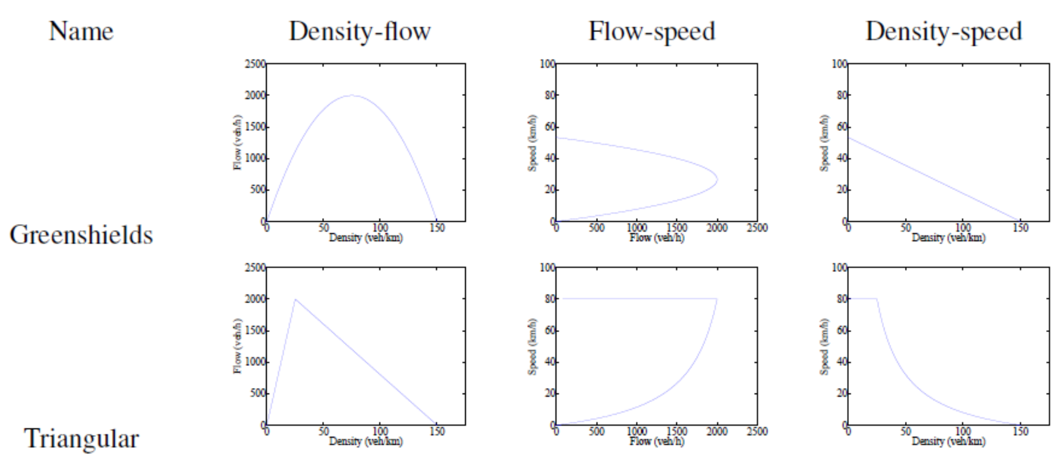
2 lanes: $q_{max,2}$=4000 veh/h, $k_{c,2}$=50 veh/km, $k_{j,2}$=300 veh/km.

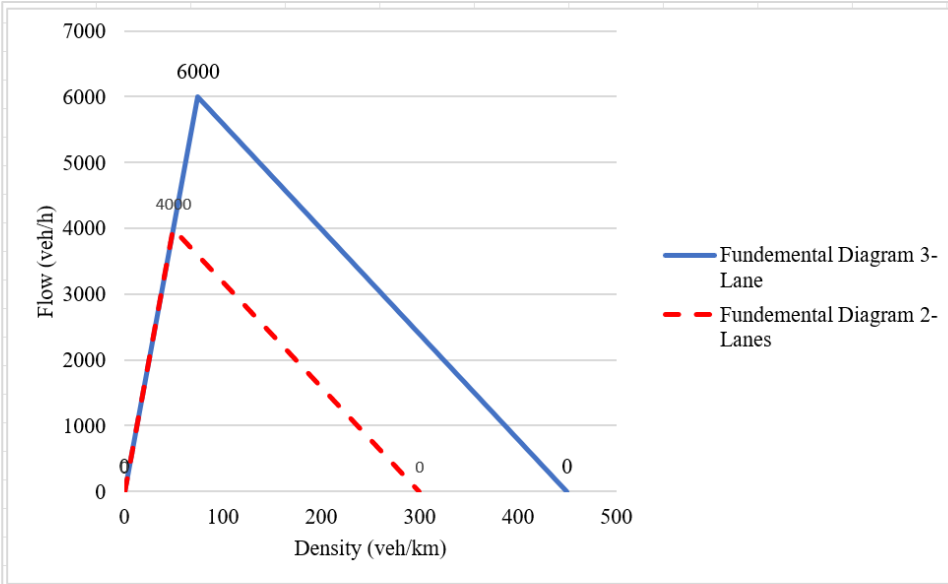
Slope of the congested branch (same for any lane count for a **triangular fundamental diagram**):

The congested branch is the line through (k_c,q_{max})=(25,2000) and (k_j,0)=(150,0)

$$w_c = \frac{0 - q_{\max}}{k_j - k_c} = -\frac{2000}{150 - 25} = -16\ \text{km/h}.$$

(Units check: Δq/Δk = veh/h ÷ (veh/km) = km/h.)





Identify the traffic states

Use $q=vk$ on the free branch ($v=v_f$=80 km/h) and the linear congested branch on the **2-lane** diagram:

Congested branch (2 lanes):

At **jam** (q=0): k=300 veh/km

At **capacity** (q=4000): k=300−4000/16=50 veh/km

Using point–slope form with the jam point (k_j,0) (k,q):

$$\omega_c = -16 = \frac{q-0}{k-300}$$

q=−16 (k−300) ⇒

$$\boxed{k = 300 - \frac{q}{16}}$$

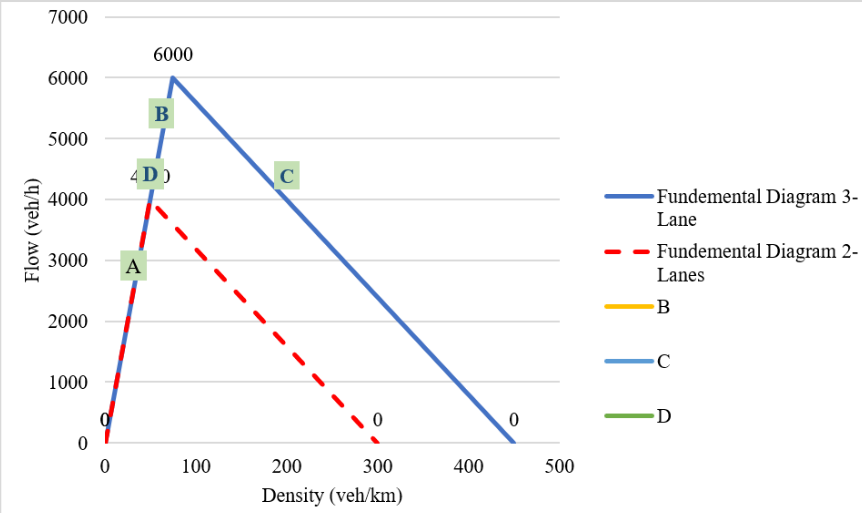
A (initial demand 2500 on 3 lanes, free):
q_A=2500, v_A=80 ⇒ $k_A=q_A/v_A$=31.25 veh/km.

B (between t=1h and t=2h, flow 5000 on 3 lanes, free):
q_B=5000, v_B=80 ⇒ k_B=62.5 veh/km.

C (bottleneck discharge/queue state on 2 lanes, congested):
Capacity limited to q_C=4000.
On the congested branch:
k_C=300−4000/16=300−250=200 veh/km;
$v_C=q_C/k_C$=4000/200=20 km/h.

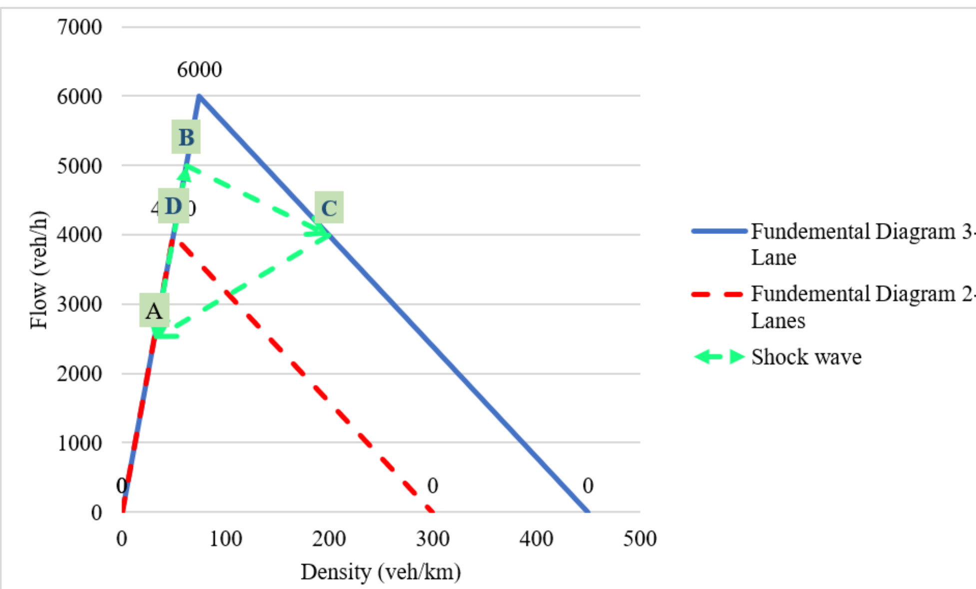
D (free at 4000 on 3 lanes, after the surge propagates):
q_D=4000, v_D=80 ⇒ k_D=50 veh/km.

These are exactly the values in **Table 4.1**:
A:(2500,31.25,80), B:(5000,62.5,80), C:(4000,200,20), D:(4000,50,80).



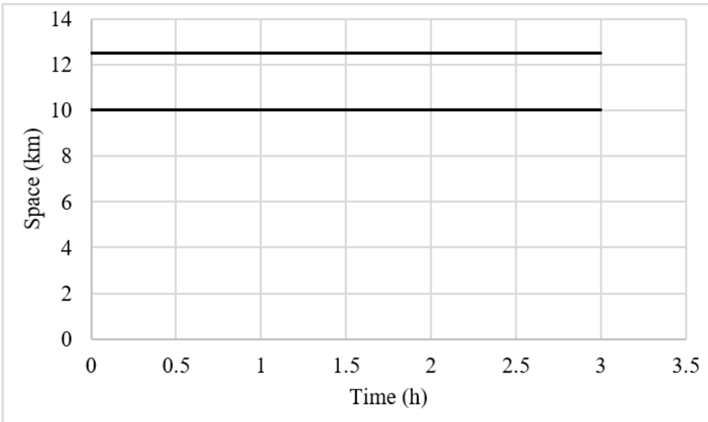
<u>Shockwave speeds (w=Δq/Δk)</u>

- **A→B (arrival of surge):** $w = \dfrac{5000 - 2500}{62.5 - 31.25} = +80$ km/h (downstream with traffic).
- **B→C (queue growth at the drop):** $w = \dfrac{4000 - 5000}{200 - 62.5} = -7.3$ km/h (upstream).
- **C→A (queue dissipation when demand falls):** $w = \dfrac{2500 - 4000}{31.25 - 200} = +8.9$ km/h (downstream).



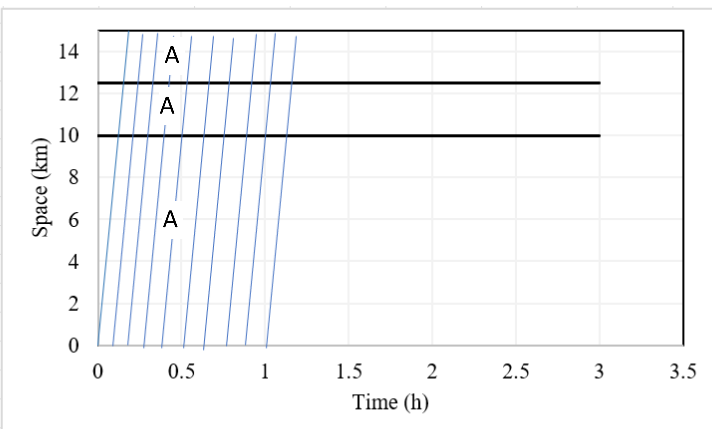
Space–time story (Figure 4.3b)

mark the 2-lane segment between **x = 10 km** and **x = 12.5 km** (horizontal lines) at the space -time diagram.

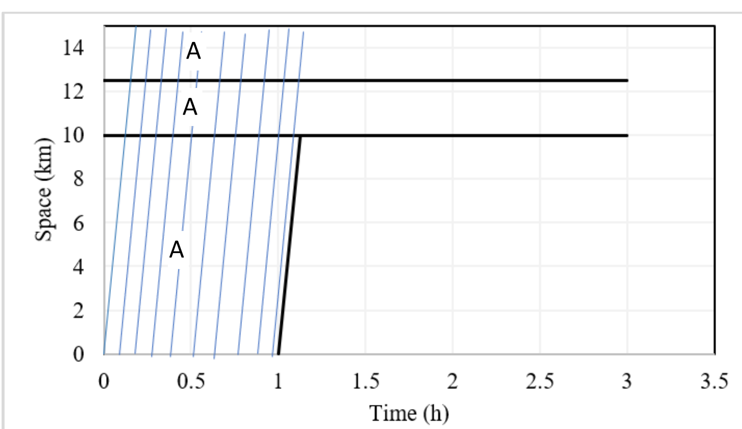


t<1h: **state A** everywhere (free, 3 lanes).

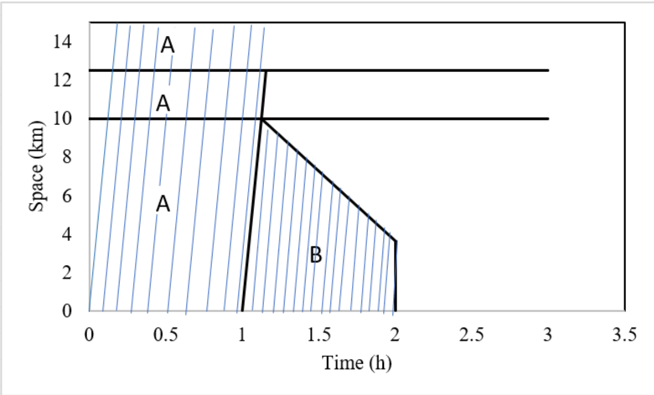
(Draw light vehicle trajectories with slope v=80 km/h.)



At t=1h the 5000 veh/h the **higher-demand B state** forms at the upstream boundary and travels downstream at 80 km/h.; reaching the lane drop at x=10 after 10/80=0.125 h ⇒ at **t=1.125h**



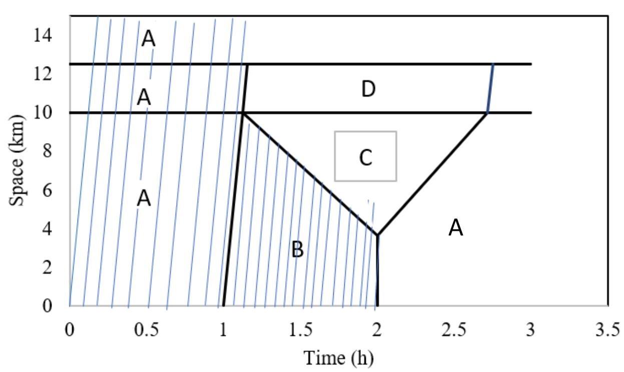
From that moment a **B→C** shock is born at x=10 and moves **upstream** at −7.3 km/h, growing the queue.
$x_{tail}(2)=10-7.3(2-1.125)=10-6.39\approx3.61$ km.



At t=2h demand falls to 2500; a **C→A** dissipation wave starts at the queue tail and moves downstream at +8.9 km/h, clearing the queue; upstream becomes **D** then **A** as waves pass.

Time to reach the drop (clear the queue):
$T_{\text{clear}} = \dfrac{10-3.61}{8.9} \approx 0.718$ h ≈ **43 min.**
So, the queue clears around **t≈2.72 h**.

The triangular region **C** in the spacetime diagram is the queued section between the upstream and downstream fronts.

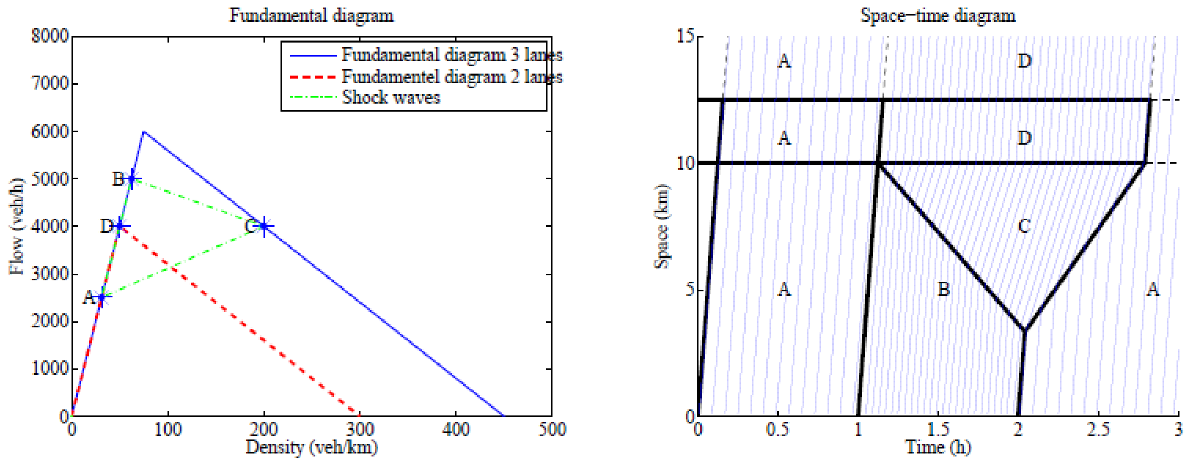


Downstream of the bottleneck the road stays in **D** (4000 veh/h, 80 km/h) while the queue exists; after clearance the whole road returns to free conditions consistent with demand (**A** upstream, **D** on the 2-lane section and downstream).

Max queue length at t=2 h: ≈6.39km (from x≈3.61 to x=10).

Max vehicles in queue (excess density over free flow):
$(k_C-k_A)\times L=(200-31.25)$ veh/km×6.39 km≈1,080 vehicles.

With a **triangular** FD, any shock connecting two **free-flow** points travels at the free-flow speed (its line in the q–k plot has the same slope as the free branch through the origin). That's why A↔B and A↔D shocks all move at ~80 km/h when both states are on the free side; only shocks involving a congested state use other slopes (e.g., −7.3, +8.9).



(a) Position of traffic states at the fundamental diagram

(b) Shock waves in space-time

Figure 4.3 Situation

Table 4.1: The states on the road

State	Flow (veh/h)	Density (veh/l=km)	Speed (km/h)
A	2500	31.25	80
B	5000	62.5	80
C	4000	200	20
D	4000	50	80

Table 4.2: The shock waves present on the road

State 1	State 2	shock wave speed w (km/h)
A	B	80
B	C	-7.3
A	A	8.9

EXAMPLE 2: TEMPORAL CAPACITY REDUCTION

Another typical situation is a road with a temporal local reduction of capacity, for instance due to an accident. The case is as follows:
Consider a three-lane freeway, with a triangular fundamental diagram. The free flow speed is 80 km/h, the capacity is 2000 veh/h/lane and the jam density is 150 veh/km/lane. The demand is constant at 2500 veh/h.
From t=1h to t=2h, an incident occurs at x=10, limiting the capacity to 1000 veh/h. Calculate the traffic states and the shock waves, and draw them in the space-time diagram. Also draw several vehicle trajectories.

Solution

3-lane freeway with a **triangular FD** (per lane):
v_f=80 km/h, $q_{max,lane}$=2000 veh/h, $k_{j,lane}$=150 veh/km.

Scale to 3 lanes: q_{max}=6000 veh/h, k_c=6000/80=75 veh/km, kj=450 veh/km.

Congested-branch slope in the q-k plane: $m = \frac{0 - q_{\max}}{k_j - k_c} = -\frac{6000}{450 - 75} = -16\ \mathrm{km/h}.$

Demand is constant at 2500 veh/h.

Incident at x=10 km between t=1–2h, limiting the **throughput to 1000 veh/h** (total).

Traffic states

Use $q=kv$ (free branch $q=v_f k$; congested branch $q=-16(k-450)$).

A (normal free flow, upstream before the incident):
$q_A=2500$, $v_A=80 \Rightarrow k_A=2500/80=31.25$ veh/km.

C (downstream during the incident—free, limited to 1000):
$q_C=1000$, $v_C=80 \Rightarrow k_C=1000/80=12.5$ veh/km.

B (upstream during the incident—congested with the same flow as the bottleneck):
solve $q=1000=-16(k-450) \Rightarrow k_B=450-1000/16=387.5$ veh/km,
$v_B=q_B/k_B=1000/387.5 \approx 2.58$ km/h.

D (after the incident is cleared, the **head of the queue** discharges at capacity of the 3-lane road):
$q_D=3\times2000=6000$ veh/h, so $k_D=q_D/v_f=6000/80=75$ veh/km.

Shockwave speeds

Shockwave speeds

A ↔ C (start of incident, downstream side switches A→C):

$$w_{AC} = \frac{2500-1000}{31.25-12.5} = \frac{1500}{18.75} = +80 \text{ km/h}$$

(Downstream with the traffic—on a triangular FD, any free–free shock has slope v_f).

A ↔ B (start of incident, upstream side switches A→B):

$$w_{AB} = \frac{2500-1000}{31.25-387.5} = \frac{1500}{-356.25} \approx -4.2 \text{ km/h}$$

(Upstream—this is the **queue growth** front).

B ↔ C (across the incident location while it is active):

$$w_{BC} = \frac{1000-1000}{387.5-12.5} = 0 \text{ km/h}$$

(The B–C interface is **stationary** at the fixed bottleneck location x=10 km).

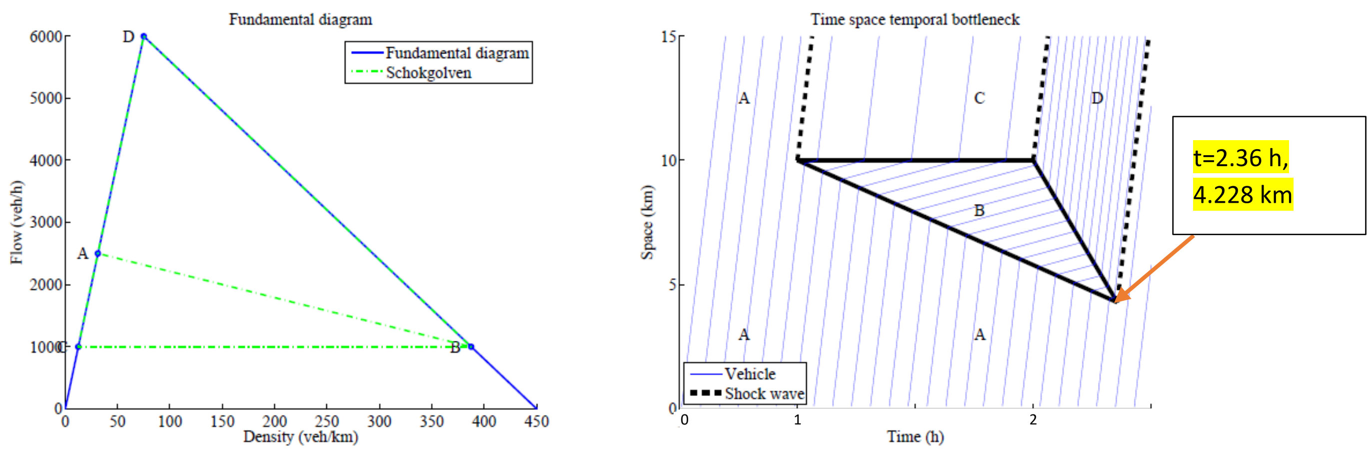
B ↔ D (when the incident ends at t=2h):

$$w_{BD} = \frac{1000-6000}{387.5-75} = \frac{-5000}{312.5} = -16 \text{ km/h}$$

(Upstream at 16 km/h). This is the **queue-dissipation** front moving back through the queued vehicles as the head accelerates.

(10-y)/(1-t)=-4.2…>(10-y)=-4.2+4.2t
(10-y)/(2-t)=-16…..>(10-y)=-32+16t
27.8-11.8t=0…..>t=2.36h
y=10+4.2-4.2(2.36)=4.228

Figure 4.4(a) shows the fundamental diagram and the occurring states, 4.4(b) shows how the states move in space and time. The details of the states can be found in table 4.3, and the details of the shock waves can be found in table 4.4.



(a) Position of traffic states at the fundamental diagram

(b) Shock waves in space-time

Figure 4.4: The situation

Table 4.3: The states on the road with a temporal bottleneck

Number	Flow	Density	Speed
A	2500	31.25	80
B	1000	387.5	2.58
C	1000	12.5	80
D	6000	75	80

Table 4.4: The shock waves present on the road with a temporal bottleneck

State 1	State 2	shock wave speed w (km/h)
A	C	80
B	C	0
A	B	-4.2
B	D	16

4.2 MOVING BOTTLENECK

This section describes what happens if the road is blocked, either completely or not completely, by a moving bottleneck.

THEORY OF "MOVING BOTTLENECK"

This can be a slow-moving truck or agricultural vehicle, a funeral or wedding procession.

A slow vehicle (no overtaking) moves **downstream** at speed u_{mb}

Only Q_p veh/h can pass it (its **passing capacity**).

So **downstream of it** the traffic is uncongested with flow Q_p; **upstream of it** a queue may form.

Same shockwave logic as a fixed bottleneck, except the discontinuity moves with the slow vehicle.

The **shockwave theory** is exactly the same as for fixed bottlenecks:

$$w = \frac{q_2 - q_1}{k_2 - k_1}$$

but now the **moving bottleneck's speed** v_B plays the role of the wave speed — the *shock moves with the bottleneck*.

Key points:

- Upstream (behind the slow vehicle): traffic is **congested**.
- Downstream (ahead of it): traffic is **free**.
- The bottleneck itself defines the boundary between them, and **its speed = shockwave speed**.

Different compared to the regular, fixed bottlenecks, is the position of the congested state. For fixed bottlenecks, the flow upstream of the bottleneck equals the flow downstream of the bottleneck. In case of moving bottlenecks this differs. Consider a bottleneck which moves downstream without any overtaking opportunities. The downstream flow is zero, but vehicles can accumulate in the growing area between the considered point and the moving bottleneck, so the upstream flow is not zero.

to find the upstream (congested) state: the capacity of vehicles passing the moving bottleneck is usually an input. This gives the downstream point at the fundamental diagram. Then

1. Draw the **free-flow branch** (slope = v_f) and the **congested branch** (slope = $-w_c$) of the triangular FD.
2. Locate the downstream point (for example D)(the flow that can pass the slow vehicle).
3. Draw a line through D with slope = the **bottleneck speed** v_b (downstream vehicle's speed).
4. The **intersection** of that line with the **congested branch** gives the upstream congested state B.

That intersection ensures:

speed of moving bottleneck v_b=speed of shock w_{BD}.

EXAMPLE 3: MOVING TRUCK, NO OVERTAKING POSSIBILITIES

Consider a three-lane road, where at all three lanes a triangular fundamental diagram holds. The capacity is 2000 veh/h/lane, the free flow speed 80 km/h and the jam density 150 veh/km. The demand is 3000 veh/h. A truck enters the road at t=0.5h and x=10 km, and leaves the road at t=1h and x=15 km, hence driving 10 km/h. There are no overtaking opportunities. What is the traffic state at the road?

Solution

Inputs and triangular FD (3 lanes)

State A (free upstream before the truck)

q_A=3000, v_A=80⇒k_A=37.5 veh/km.

State C (downstream of a full blockage with no overtaking)

"No vehicles downstream" ⇒ q_C=0, k_C=0

State B (congested just upstream of the moving bottleneck)

How to find **B** is the key idea for a moving bottleneck:

In the q–k plane, draw a line **through C** with slope equal to the **bottleneck speed** v_b (because the shock attached to the bottleneck must move with it):

$$q_1 = q_C + v_b\,(k - k_C) = 10\,k.$$

Intersect that with the **congested branch** (slope −16):

$$q_2 = -16\,\big(k - 450\big).$$

Set $q_1 = q_2$: $10k = -16(k-450) \Rightarrow 26k = 7200 \Rightarrow \boxed{k_B = 277 \text{ veh/km}}$.

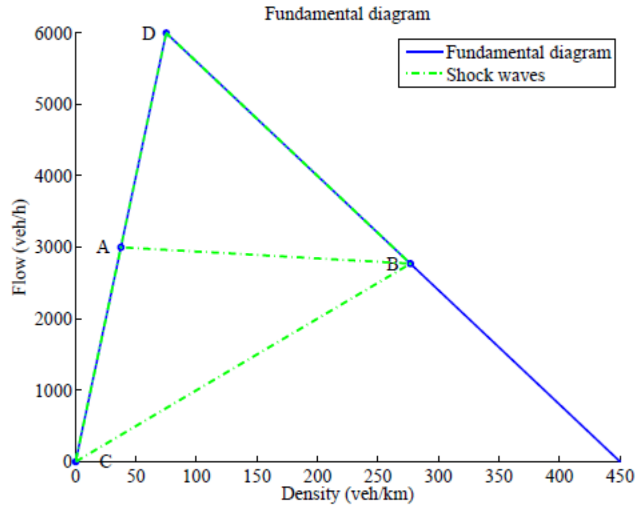
Then $q_B = 10k_B = \boxed{2769 \text{ veh/h}}$ and $v_B = q_B/k_B = \boxed{10 \text{ km/h}}$ (same as the truck).

D (after the truck leaves, the head of the jam discharges at capacity)

$$q_D = 6000 \text{ veh/h}, \quad k_D = 6000/80 = \boxed{75 \text{ veh/km}}, \quad v_D = 80.$$

Table 4.5: The states on the road for a moving bottleneck without overtaking opportunities

Name	Flow	Density	Speed
A	3000	37.5	80
B	2769	277	10
C	0	0	NaN
D	6000	75	80



(a) Position of traffic states at the fundamental diagram

Shockwave speeds $w = \dfrac{q_2 - q_1}{k_2 - k_1}$

A → B (queue tail while truck is present):

$$w_{AB} = \frac{3000 - 2769}{37.5 - 277} = \boxed{-0.96\ \mathrm{km/h}}\ \text{(slow upstream)}.$$

B → C (boundary attached to the truck): must equal the truck speed

$$\boxed{w_{BC} = +10\ \mathrm{km/h}}.$$

A → C, **C → D**, **A → D** (free–free connections on the triangular FD): all move at the **free-flow speed**,

$$\boxed{w = +80\ \mathrm{km/h}}$$

B → D (clearance after the truck leaves): slope of congested branch,

$$w_{BD} = \frac{1000? - 6000}{\text{(not needed)}} = \boxed{-16\ \mathrm{km/h}}.$$

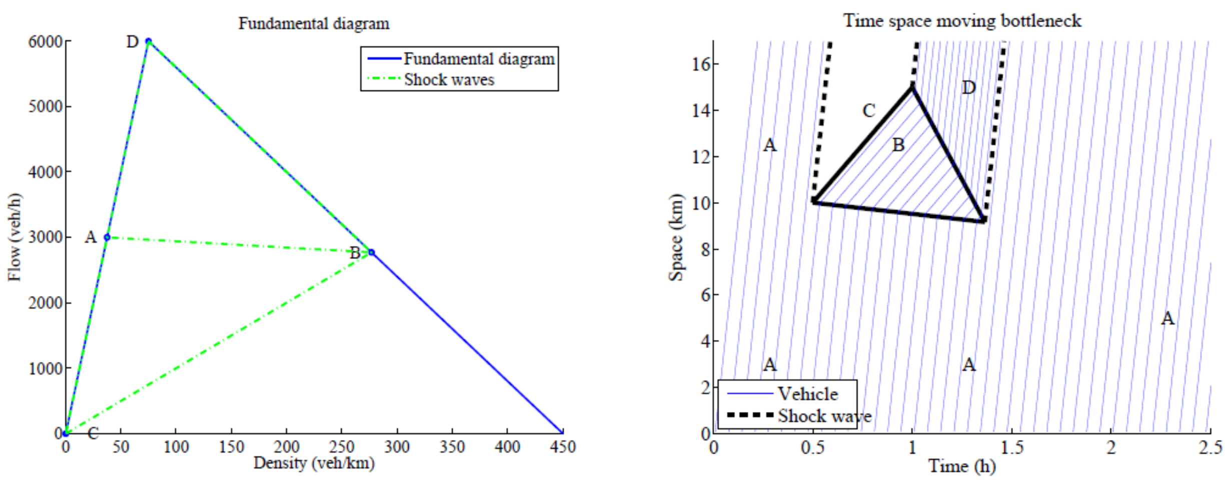
Table 4.6: The shock waves present on the road for a moving bottleneck without overtaking opportunities

State 1	State 2	shock wave speed w (km/h)
A	B	-0.96
B	C	10
A	C	80
D	C	80
B	D	-16
A	D	80

<u>Space–time story</u>

At t=0.5h and x=10 km the truck enters.
A triangular **congestion wedge B** forms behind it. The **downstream boundary** B→C rides with the truck at 10km/h; the **upstream tail** A→B drifts upstream very slowly at −0.96 km/h.

At t=1.0 h and x=15 km the truck leaves.
A **clearance wave B→D** is launched that moves **upstream** at −16 km/h, emptying the wedge. Downstream becomes D then A.



(a) Position of traffic states at the fundamental diagram

(b) Shock waves in space-time

EXAMPLE 4: MOVING TRUCK WITH OVERTAKING POSSIBILITIES

Now, let's consider a different situation, where overtaking of the moving bottleneck is possible. We change the conditions as follows. Consider a three-lane road, where at all three lanes a triangular fundamental diagram holds. The capacity is 2000 veh/h/lane, the free flow speed 80 km/h and the jam density 150 veh/km. The demand is 2500 veh/h.

A truck enters the road at t=0.5h and x=10 km, and leaves the road at t=1h and x=15 km, hence driving 10 km/h. There are overtaking opportunities, such that downstream of the bottleneck the flow is 1000 veh/h. What is the traffic state at the road?

Solution

Given:

- 3-lane road, triangular fundamental diagram (FD) on each lane
- Per-lane parameters: free-flow speed u_f=80 km/h, capacity=2000 veh/h, jam density k_j=150 veh/km
- Demand upstream q_A=2500 veh/h (per lane)
- Downstream of the truck there is an overtaking opportunity → downstream flow set to q_C=1000 veh/h (free flow)
- Moving bottleneck (truck): enters at t=0.5h, x=10 km and leaves at t=1.0 h, x=15 km → speed v=10 km/h

Derived FD items (per lane):

- Critical density $k_{crit}=q_{cap}/u_f$=2000/80=25 veh/km
- Backward wave speed of the congested branch

$$w = \frac{0 - q_{\text{cap}}}{k_j - k_{\text{crit}}} = \frac{-2000}{150 - 25} \approx -16 \text{ km/h}$$

(This is the slope of the congested side of the triangular FD.)

Traffic states

State C (downstream of truck): free flow at q_C=1000 → $k_C=q_C/u_f$=1000/80=12.5 veh/km.

State A (upstream demand, before interacting with the truck):
q_A=2500 →$k_A=q_A/u_f$=2500/80=31.25 veh/km (free flow, because it's upstream before the queue forms).

State B (just upstream of the truck while it is on the road):
Two conditions must simultaneously hold:

1. Vehicles that catch the truck join a moving queue whose **front** travels with the truck. In the FD, that gives a straight line through (k_C,q_C) with slope v (the truck speed):

$$q_1(k) = q_C + v\,(k - k_C)$$

2. State B must lie on the **congested branch** of the triangular FD (line through (k_{crit},q_{cap}) and (k_j,0), which can be written as a line through (k_C,q_C) with slope *w*:

$$q_2(k) = q_D + w\,(k - k_C)$$

Intersecting q1=q2 gives the density k_B.

k_B=243 veh/km, q_B=3307 veh/h

State D (after the truck leaves, the head of the jam discharges at capacity)

$$q_D = 6000 \text{ veh/h}, \quad k_D = 6000/80 = \boxed{75 \text{ veh/km}}, \quad v_D = 80.$$

The states on the road at a moving bottleneck with overtaking possibilities

State	Flow (veh/h)	Density(veh/km)	Speed (km/h)
A	2500	31.25	80
B	3307	243	13.6
C	1000	12.5	80
D	6000	75	80

Shockwaves

$$w_{XY} = \frac{q_X - q_Y}{k_X - k_Y}.$$

- **Between B and C** (at the truck):

$$w_{BC} = \frac{q_B - q_C}{k_B - k_C} = \frac{3307 - 1000}{243 - 12.5} \approx 10 \text{ km/h}.$$

It matches the truck's speed—as it should—because that shock is attached to the moving bottleneck.

- **Between A and B** (front of the upstream queue):

$$w_{AB} = \frac{q_A - q_B}{k_A - k_B} = \frac{2500 - 3307}{31.25 - 243} \approx 3.8 \text{ km/h (downstream)}.$$

So the **queue front** creeps **downstream** slowly while the truck is present.

- After the truck exits, the queued platoon discharges at **capacity** (= state D, $q_D = 6000$ veh/h for 3 lanes, i.e., 2000 per lane). The trailing shock between D and B then moves **upstream** with

$$w_{BD} = w \approx -16 \text{ km/h},$$

equal to the FD's congested-branch wave speed.

The shock waves present on the road at a moving bottleneck with overtaking possibilities

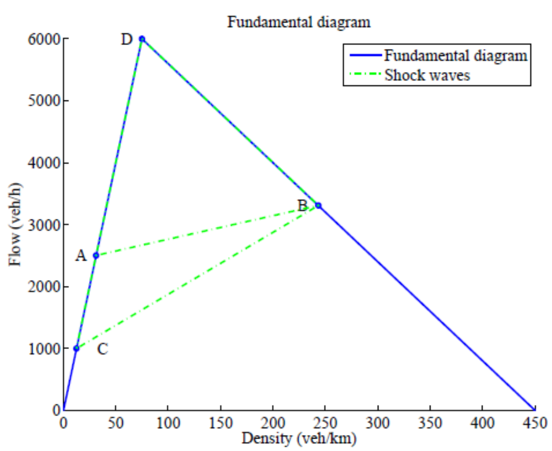
State 1	State 2	shock wave speed w (km/h)
A	B	3.8
A	C	80
D	C	80
B	D	-16
A	D	80

Draw Figure 4.6(a) (states on the FD)

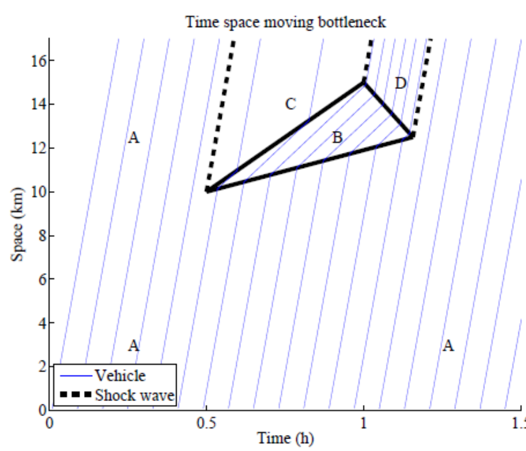
- Sketch the triangular FD (free-flow line through origin with slope u_f; congested line with slope w).
- Mark **C** at (k_C=12.5, q_C=1000) on the free-flow branch.
- Draw a line through C with slope v=10 km/h; its intersection with the congested branch is **B** (k_B,q_B).
- Mark **A** on the free-flow branch at (k_A=31.25, q_A=2500).
- Mark **D** at capacity (k_{crit},q_{cap}) (per lane) or the 3-lane total

draw Figure 4.6(b) (space–time shock diagram)

1. Draw the truck trajectory from (t,x)=(0.5 h,10 km); its slope is 10 km/h.
2. From the truck line, draw the **attached shock BC** along the same line (same speed).
3. From the truck's **entry point**, draw the **queue front AB** moving **downstream** with slope w_{AB}=3.8 km/h until the truck leaves.
4. From the **exit point** of the truck, draw the **release shock BD** moving **upstream** with slope −16km/h (vehicles behind accelerate to capacity and the backward clearing wave propagates).
5. Overlay families of characteristics: free-flow characteristic lines at u_f=80 km/h (right-leaning) and congested characteristics at w=−16 km/h (left-leaning) to indicate regimes A (free flow), B (queue), and C (free flow behind truck).



(a) Position of traffic states at the fundamental diagram



(b) Shock waves in space-time

- Because overtaking is allowed, downstream of the truck stays in free flow (state C).

- Upstream, vehicles stack into state B on the congested branch; the **interface to C** travels with the truck.
- The upstream boundary of the queue advances slowly downstream (3.8 km/h) while the truck is on the road.
- When the truck leaves, the jam tail recedes upstream at the natural backward wave speed (−16 km/h) as the queue discharges at capacity (state D).

EXAMPLE 5: MOVING TRUCK AND HIGH DEMAND

Consider a three-lane road, where at all three lanes a triangular fundamental diagram holds. The capacity is 2000 veh/h/lane, the free flow speed 80 km/h and the jam density 150 veh/km. The demand is increased to 4500 veh/h. A truck enters the road at t=0.5h and x=10 km, and leaves the road at t=1h and x=15 km, hence driving 10 km/h, limiting the flow downstream of the moving bottleneck to 1000 veh/h. What are the conditions on the road?

Solution

Given

- 3 lanes; triangular FD per lane with u_f=80 km/h, q_{cap}=2000 veh/h/lane, k_j=150 veh/km.
- k_{crit}=q_{cap}/u_f=25 veh/km;
- congested-branch wave speed w=−2000/(150−25)≈−16 km/h (per lane FD).
- Truck (moving bottleneck): enters (t,x)=(0.5 h,10 km), leaves (1.0 h,15 km) → speed v=10 km/h.
- Downstream of the truck stays free (overtaking) with given flow q_C=1000 veh/h → k_C=1000/80=12.5 veh/km.
- New upstream demand: q_A=4500 veh/h (for the 3-lane total used in the figure/tables) → k_A=4500/80=56.25 veh/km.

States **C** and the **congested state just upstream of the truck, B**, are **determined by the moving bottleneck**, so they are the same as in Example 2:

- From the "line through C" with slope v=10 km/h intersecting the congested FD, we again get k_B=243veh/km and q_B=3307 veh/h (truck-attached queue state).

Key difference vs Example 2

Because q_A is now much larger, the **shock between A and B** moves **upstream**:

$$w_{AB} = \frac{q_A - q_B}{k_A - k_B} = \frac{4500 - 3307}{56.25 - 243} \approx -6.3\ \text{km/h}$$

So, while the truck is on the road, the queue **grows backward**.

Other waves are unchanged from Example 2:

- **B–C** (attached to the truck):

$$w_{BC} = \frac{q_B - q_C}{k_B - k_C} = \frac{3307 - 1000}{243 - 12.5} \approx 10\ \text{km/h (same as the truck).}$$

After the truck exits, discharge at capacity (**state D**: q_D=6000 veh/h, k_D=6000/80=75 veh/km), and the clearing shock **B–D** moves upstream with the congested-branch speed w_{BD}≈−16 km/h.

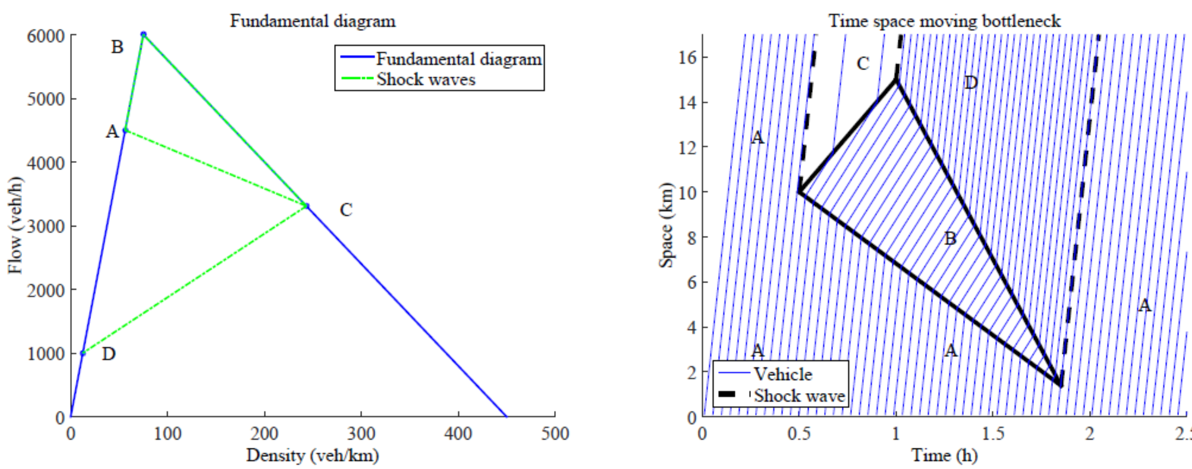
Any free–free boundaries (**A–C**, **D–C**, **A–D**) travel at u_f=80 km/h.

Draw Figure 4.7(a) — states on the FD

- Sketch the triangular FD (free branch slope u_f; congested branch slope w).
- Plot **C** at (12.5,1000) on the free branch.
- Draw a line through C with slope v=10 km/h; its intersection with the congested branch is **B** (243,3307).
- Plot **A** at (56.25,4500) on the free branch.
- Plot **D** at capacity (75,6000) (3-lane total).

Draw Figure 4.7(b) — space–time diagram

1. Draw the truck path from (0.5,10) to (1.0,15) (slope 10 km/h).
2. The **B–C** interface (shock) is attached to the truck → same line.
3. From the truck's **entry point**, draw the **A–B** shock **upstream** with slope −6.3km/h (queue expands backward).
4. At the truck's **exit point**, draw the **B–D** clearing shock **upstream** with slope −16 km/h.
5. Overlay free-flow characteristics at 80 km/h (right-leaning hatching) to indicate regions A, C, D; the queued region (B) sits between the A–B and B–C boundaries while the truck is present.



(a) Position of traffic states at the fundamental diagram

(b) Shock waves in space-time

Figure 4.7: The situation for the moving bottleneck with overtaking opportunities and a high demand.

- Overtaking keeps the **downstream** side in free flow (C).

- With higher demand, the **upstream** free-flow state A is so heavy that the A–B boundary is forced to move **upstream**, i.e., the queue grows back while the truck is on the link.
- After the truck leaves, the queue discharges at capacity and the tail recedes upstream at the natural backward wave speed of the FD.

Homework

A **three-lane freeway** follows a **triangular fundamental diagram** per lane with the following parameters:

- Free-flow speed, v_f=80 km/h
- Jam density, k_j=150 veh/km/lane
- Capacity per lane, $q_{max,lane}$=2000 veh/h/lane
- From t=0 to 1.0 h, the freeway operates under normal conditions with a uniform demand of 4000 veh/h (total).
- Between t=1.0 h and 2.0 h, a fixed bottleneck (lane drop from 3 lanes to 2 lanes) occurs at x=10 km.
- At t=1.5 h, a slow-moving truck (speed v_b=10 km/h enters the freeway at x=5 km and exits at x=15 km, further limiting the passing flow to q_p=1000 veh/h downstream of the truck.

a) Derive **the** fundamental diagram parameters for 3-lane and 2-lane segments.
b) Identify all traffic states before, during, and after the events.
c) Compute the shockwave speeds for:
- Queue formation at the fixed bottleneck,
- Dissipation after the lane reopens,
- Queue formation and clearance caused by the moving bottleneck.

d) Determine the maximum queue length and clearance time.
e) Sketch the space–time diagram showing the shockwaves, the bottleneck location, and the truck trajectory.

References

1. Knoop, V. L. (2017). Introduction to traffic flow theory: An introduction with exercises. *Delft University of Technology: Delft, The Netherlands*.
2. Manual, H. C. (2000). Highway capacity manual. *Washington, DC*, *2*(1), 1.
3. Traffic engineering / Roger P. Roess, Elena S. Prassas, William R. McShane. -4th ed.