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HIGHWAY AND TRANSPORTATION ENGINEERING DEPARTMENT
POSTGRADUATE/MSC
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ADVANCED TRAFFIC ENGINEERING

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LECTURE 1:

MICROSCOPIC AND MACROSCOPIC TRAFFIC DESCRIPTION

Traffic can be described at different levels, depending on the focus of analysis.

The **macroscopic traffic description** does not account for individual vehicles. Instead, it considers aggregated traffic characteristics over a group of vehicles or, more commonly, along a road segment. Typical m (see Section 1.1.2).

In a **microscopic traffic description**, every vehicle–driver combination is represented individually. The smallest unit of analysis is the vehicle–driver pair, allowing for detailed modeling of interactions between drivers and vehicles.

Other levels of traffic description also exist and are presented in the last subsection

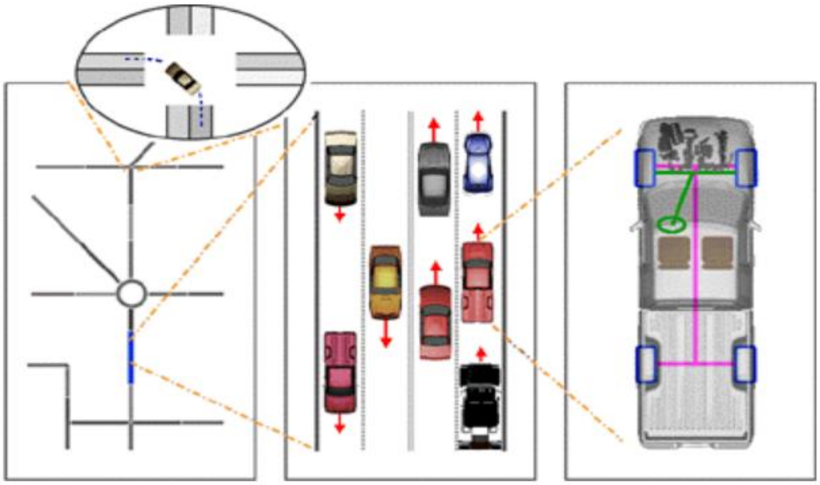


Figure: The different simulation granularities; from left to right: macroscopic, microscopic, sub-microscopic (within the circle: mesoscopic)

1.1 MACROSCOPIC TRAFFIC DESCRIPTION

In a **macroscopic traffic description**, individual vehicles are not described separately. Instead, the analysis focuses on **aggregated variables** defined for each road section.

- **Density (k):** describes how closely vehicles are spaced in a given section of the roadway.
- **Flow (q):** represents the number of vehicles passing a reference point per unit of time. Alternative terms sometimes used for this concept are *throughput*, *volume*, or *intensity*. In this work, however, we will consistently use the term **flow**.
- **Average speed (u):** indicates the mean travel speed of vehicles along the road section.

These three variables—**density (k), flow (q), and average speed (u)**—form the core of macroscopic traffic flow theory and provide a simplified yet powerful way to represent overall traffic behavior on a roadway. These variables are interconnected, and understanding their relationships is essential for analyzing roadway performance.

Figure 1 shows a **combined diagram of the three fundamental relationships** in traffic flow theory:

1. **Speed–Density (u–k):** speed decreases linearly as density increases.
2. **Flow–Density (q–k):** flow rises with density up to capacity, then falls as congestion sets in.
3. **Flow–Speed (q–u):** flow increases with speed until reaching capacity, then declines as vehicles move too slowly.

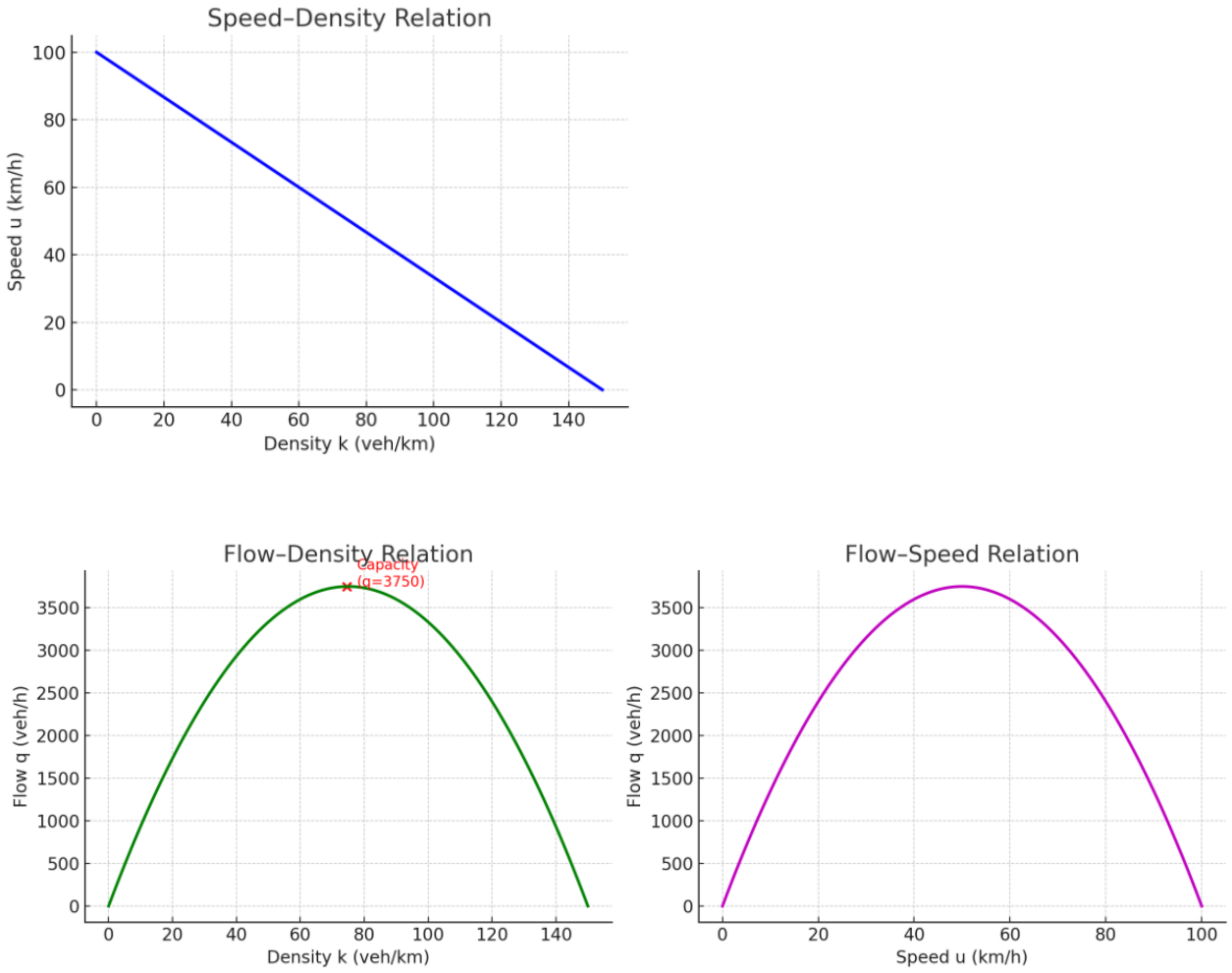


Figure 1 Fundamental relationships in traffic flow theory

1. Speed–Density Relation (u–k)

- At **zero density**, vehicles can travel at the **free-flow speed (u_0)** without hindrance.
- As density increases, the average speed decreases due to interactions among vehicles.
- At **jam density (k_j)**, vehicles come to a complete stop (u = 0).

2. Flow–Density Relation (q–k)

- Flow increases with density in the **uncongested regime**, reaching a maximum at the **critical density (k_a)**.
- The peak flow at this point is called the **capacity (q_a)** of the road.
- Beyond critical density, congestion dominates: flow decreases as density continues to rise.

3. Flow–Speed Relation (q–u)

- At low speeds, flow is small because vehicles move slowly.
- As speed increases, flow rises until reaching capacity.
- At very high speeds, flow is again limited, since density becomes very low.

Fundamental Equations

The three variables are linked by the basic traffic flow relationship:

$$q = k \times u$$

This equation shows that flow is the product of density and speed.

Each diagram gives a different perspective on the same system. Taken together, they explain why traffic flow improves as vehicles spread out (low density), peaks at an optimal point (capacity), and then collapses when congestion builds (high density).

(Please refer to your undergraduate traffic flow lecture notes/textbook to review the fundamental models and basic traffic variables (flow, speed, and density). This will give you the background and A clear understanding of these foundations)

Applications of Macroscopic variables (Examples)

Example (1): Highway Capacity Analysis (road design / widening check):

A **freeway segment** has the following observed conditions during peak hour:

- **Number of lanes:** 3 lanes (one direction)
- **Average speed (u):** 85 km/h
- **Density (k):** 35 veh/km/lane

Using the fundamental relation:

$$q = k \times u$$

$$q = 35 \times 85 = 2975 \text{ veh/h/lane}$$

Capacity benchmark (HCM, typical): The Highway Capacity Manual (HCM) suggests that practical capacity per freeway lane is about **2000–2400** veh/h/lane. the calculated flow per lane (≈ 2975 veh/h/lane) is above typical capacity, which indicates the section is overloaded and is operating over capacity.

If q (observed) > q at capacity → oversaturated; expect breakdown/queues.

Total flow across 3 lanes:

$$Q = 3 \times 2975 = 8925 \text{ veh/h}$$

Very high demand compared to capacity

Conclusion: This macroscopic analysis shows that the freeway segment is overloaded during peak hour. Without intervention, congestion will persist or worsen.

Recommendation:

-The road may require widening (adding lanes)

- traffic management (e.g., ramp metering, using ITS, demand management) to restore stable conditions

Example (2): Urban Arterial Performance (Peak hour)

On a busy urban arterial road during the morning peak:

Arterial length $L = 1.5\ \mathrm{km}$
Observed flow $q = 1800\ \mathrm{veh/h}$
Average travel speed $u = 25\ \mathrm{km/h}$
Assume free-flow speed $u_f = 60\ \mathrm{km/h}$

Performance Benchmarking (LOS by HCM)

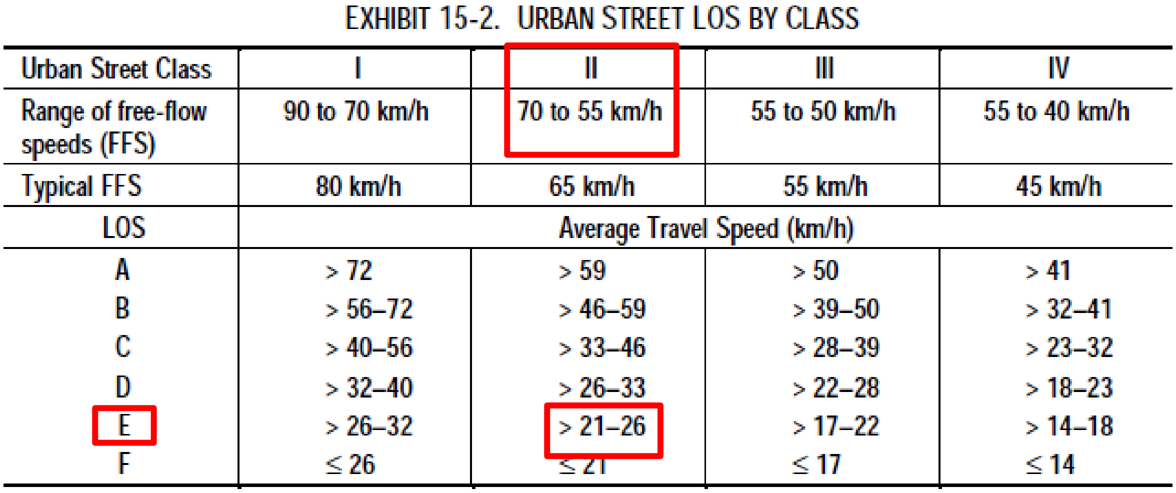


EXHIBIT 15-2. URBAN STREET LOS BY CLASS

Urban Street Class	I	II	III	IV
Range of free-flow speeds (FFS)	90 to 70 km/h	70 to 55 km/h	55 to 50 km/h	55 to 40 km/h
Typical FFS	80 km/h	65 km/h	55 km/h	45 km/h
LOS	Average Travel Speed (km/h)			
A	> 72	> 59	> 50	> 41
B	> 56–72	> 46–59	> 39–50	> 32–41
C	> 40–56	> 33–46	> 28–39	> 23–32
D	> 32–40	> 26–33	> 22–28	> 18–23
E	> 26–32	> 21–26	> 17–22	> 14–18
F	≤ 26	≤ 21	≤ 17	≤ 14

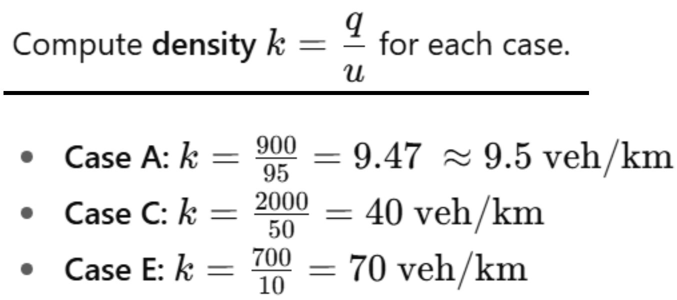
Conclusion Congested arterial, LOS E, near-capacity operations.

Recommendation: signal optimization, access control, or capacity enhancements.

Example (3): Traffic State Classification

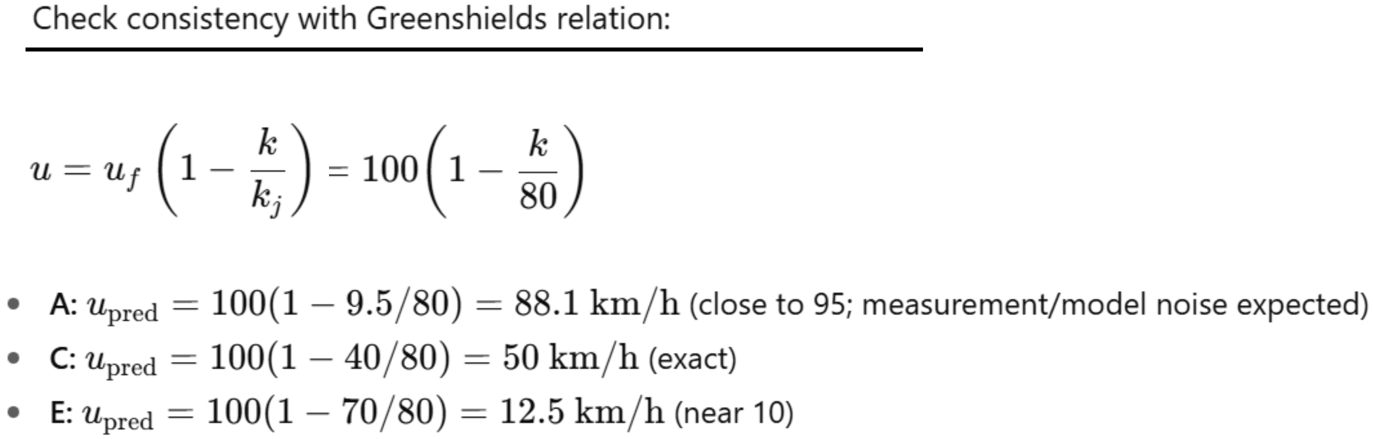
Given:

- Greenshields model parameters:
  - Free-flow speed $u_f = 100\ \mathrm{km/h}$
  - Jam density $k_j = 80\ \mathrm{veh/km}$
  - Critical density $k_c = k_j/2 = 40\ \mathrm{veh/km}$
  - Capacity $q_{\max} = u_f \cdot k_j/4 = 2000\ \mathrm{veh/h}$
- Three traffic cases with observed values:
  - **Case A:** $u = 95\ \mathrm{km/h},\ q = 900\ \mathrm{veh/h}$
  - **Case C:** $u = 50\ \mathrm{km/h},\ q = 2000\ \mathrm{veh/h}$
  - **Case E:** $u = 10\ \mathrm{km/h},\ q = 700\ \mathrm{veh/h}$



Compute **density** $k = \dfrac{q}{u}$ for each case.

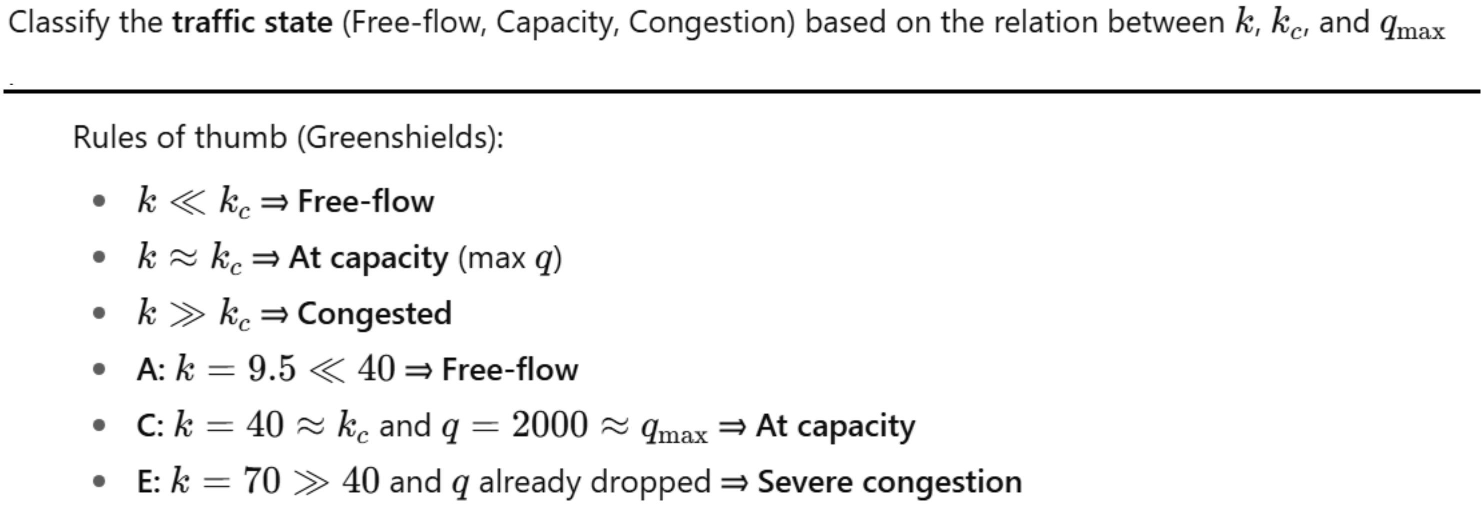
- **Case A:** $k = \frac{900}{95} = 9.47 \ \approx 9.5 \text{ veh/km}$
- **Case C:** $k = \frac{2000}{50} = 40 \text{ veh/km}$
- **Case E:** $k = \frac{700}{10} = 70 \text{ veh/km}$



Check consistency with Greenshields relation:

$$u = u_f\left(1 - \frac{k}{k_j}\right) = 100\left(1 - \frac{k}{80}\right)$$

- **A:** $u_{\text{pred}} = 100(1 - 9.5/80) = 88.1 \text{ km/h}$ (close to 95; measurement/model noise expected)
- **C:** $u_{\text{pred}} = 100(1 - 40/80) = 50 \text{ km/h}$ (exact)
- **E:** $u_{\text{pred}} = 100(1 - 70/80) = 12.5 \text{ km/h}$ (near 10)



Classify the **traffic state** (Free-flow, Capacity, Congestion) based on the relation between k, k_c, and $q_{\max}$

Rules of thumb (Greenshields):

- $k \ll k_c \Rightarrow$ **Free-flow**
- $k \approx k_c \Rightarrow$ **At capacity** (max q)
- $k \gg k_c \Rightarrow$ **Congested**
- **A:** $k = 9.5 \ll 40 \Rightarrow$ **Free-flow**
- **C:** $k = 40 \approx k_c$ and $q = 2000 \approx q_{\max} \Rightarrow$ **At capacity**
- **E:** $k = 70 \gg 40$ and q already dropped $\Rightarrow$ **Severe congestion**

Conclusion

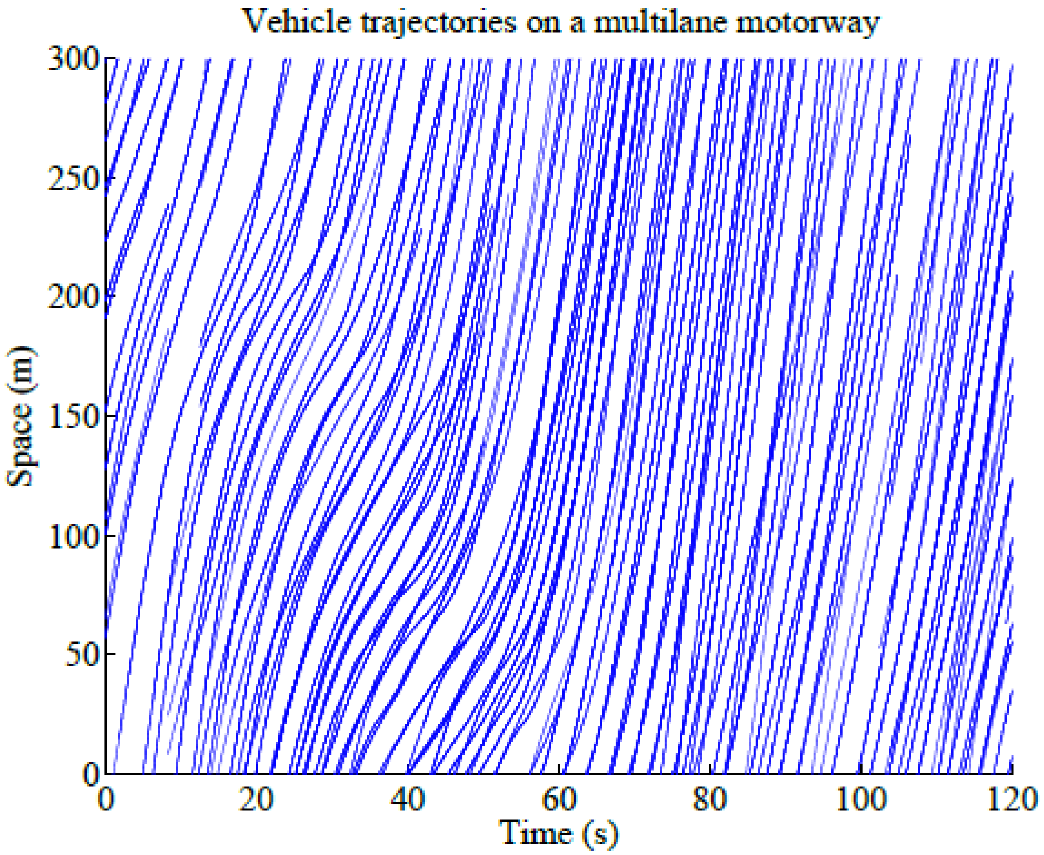
Case A interprets Light volumes, high speeds, stable operations

Case C interprets at margins situation (near capacity)

Case E interprets sever congestion, high density, low speed

1.2 MICROSCOPIC TRAFFIC DESCRIPTION

- In a microscopic traffic description, microscopic description treats each vehicle–driver unit individually.
- A vehicle complete information is described by its trajectory, i.e. the specification of the position of the vehicle at all times.
- Vehicle trajectories are shown in a space–time diagram:
  - **x-axis:** time
  - **y-axis:** space (distance along the road)
  - **Slope = speed** (steeper line → slower vehicle, flatter line → faster vehicle).
  - **Vertical line:** impossible (infinite speed).
  - **Horizontal line:** possible at zero speed (stopped vehicle).



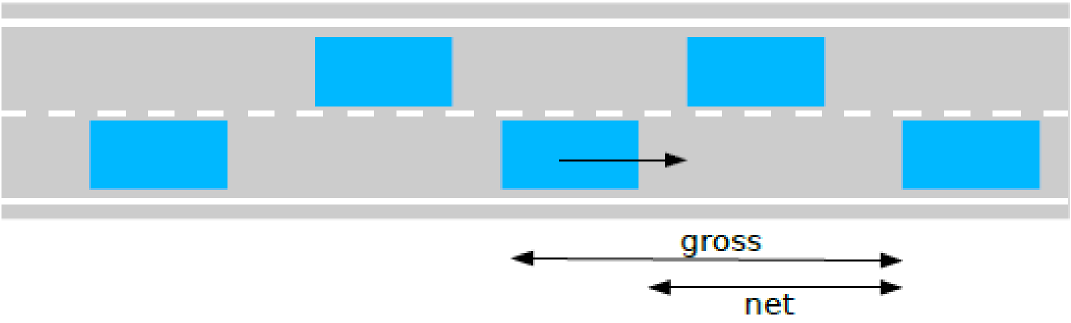
- Vehicle trajectories can never go back in time.
- Trajectories might move back in space if the vehicles are going in the opposite direction, for instance on a two-lane bidirectional rural road. This is not expected on motorways.

Basic Variables in Microscopic Traffic Flow

1. **Speed:** distance covered by a vehicle per unit of time. (Units: km/h or m/s.)

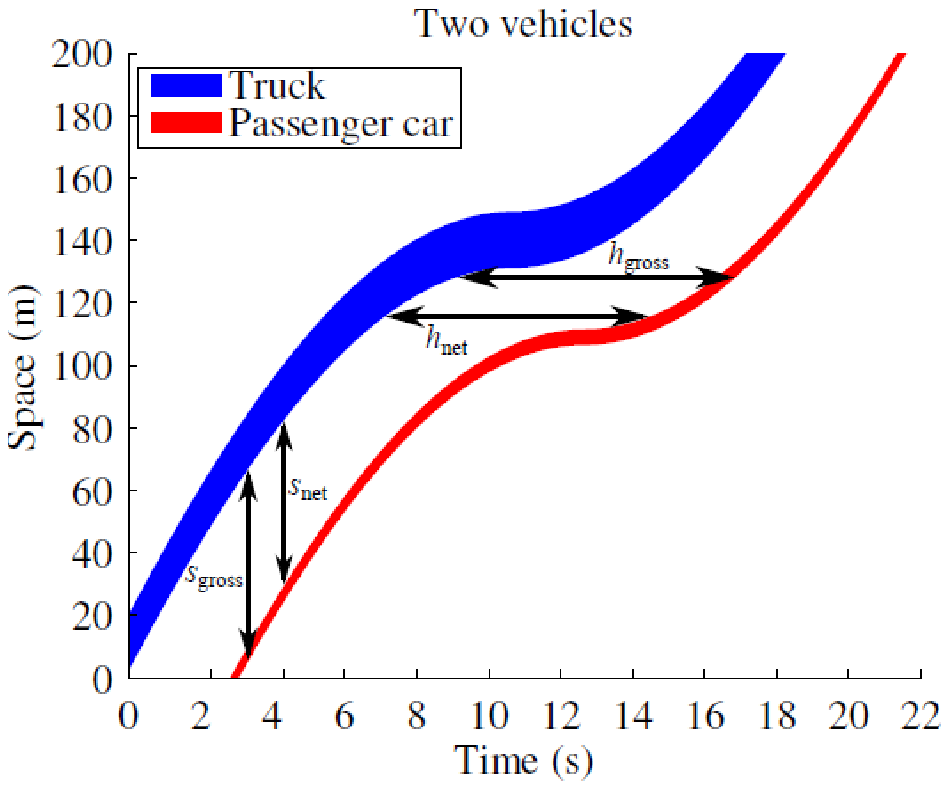
   Related measure: **Pace = 1/v**, i.e., time needed to cover one unit of distance.

2. **Space Headway:** Distance between a vehicle and its leader.
   - **Net space headway (gap):** distance between the rear bumper of the leader and the front bumper of the follower.
   - **Gross space headway:** net spacing + the vehicle length (i.e., rear bumper of leader to rear bumper of follower).



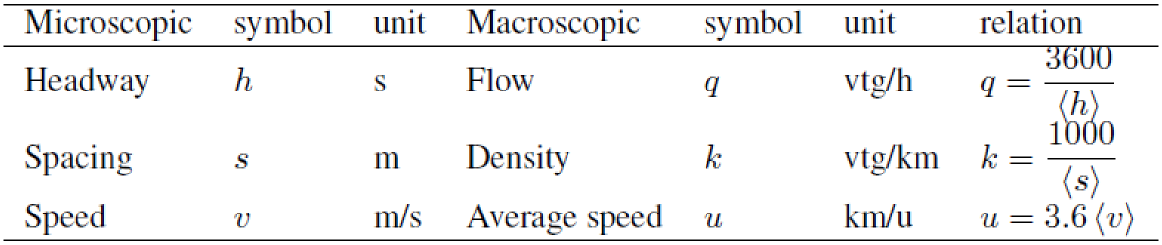
3. **Time Headway:** Time difference between successive vehicles passing a reference point.
   - **Net time headway:** time for the front bumper of the follower to reach the position of the rear bumper of the leader.
   - **Gross time headway:** net time headway + the time required to travel one vehicle length.

Figure below shows the variables graphically.



- **Space Headway**
  - s_{gross}: distance from rear bumper of the truck to rear bumper of the car (includes vehicle length).
  - s_{net}: distance from rear bumper of the truck to front bumper of the car (gap only).
  - Since vehicle length is fixed, **difference between s_{gross} and s_{net} = vehicle length**.
- **Time Headway**
  - h_{gross}: time difference measured from rear bumper of leader to rear bumper of follower.
  - h_{net}: time difference measured from rear bumper of leader to front bumper of follower.
  - **Difference between h_{gross} and h_{net} depends on vehicle speed**:
    - At low speed, the extra time to cover vehicle length is larger.
    - At high speed, this difference is smaller.
- **Trajectories**
  - Blue (truck) and red (car) lines show **space vs. time**.
  - **Slope of trajectory = speed**:
    - Flat slope → high speed (car accelerating later in plot).
    - Steeper slope → low speed (truck moving slower).
  - **Curvature = acceleration**:
    - Curved upward → vehicle accelerating.
    - Curved downward → vehicle decelerating.

RELATION BETWEEN MACROSCOPIC AND MICROSCOPIC LEVEL



Microscopic	symbol	unit	Macroscopic	symbol	unit	relation
Headway	h	s	Flow	q	vtg/h	$q = \frac{3600}{\langle h \rangle}$
Spacing	s	m	Density	k	vtg/km	$k = \frac{1000}{\langle s \rangle}$
Speed	v	m/s	Average speed	u	km/u	$u = 3.6\,\langle v \rangle$

- **Average Speed**
    a) **Space mean speed** = average speed of vehicles across a road segment.
    b) **Time mean speed** = average of vehicle speeds at a point (detector).
    c) They differ because of measurement principles.

Another concept for a traffic flow, in particular in relation to a detector

- **Occupancy *(o)*:** the **fraction of time** a detector is occupied by vehicles.

$$o = \frac{\tau_{\text{occupied}}}{\tau_{\text{occupied}} + \tau_{\text{not occupied}}}$$

dimensionless (ratio or %).

higher occupancy → higher density.

- **Occupancy Time τ_{occupied} :** A vehicle occupies the detector for the time it takes to cover

The occupation time can be derived from the distances and the speed. The distance the vehicle has to cover from the moment it starts occupying the detector up to the time it leaves the detector is its own length plus the length of the detector

$$L_i + L_{det}$$

where L_i = vehicle length, L_{det} = detector length.

$$\tau_{\text{occupied}} = \frac{L_i + L_{det}}{v}$$

where v = vehicle speed.

- **Gap Time ($\tau_{\text{not occupied}}$):** The time between vehicles depends on the **net spacing minus detector length**. It is the time gap before the next vehicle reaches the detector.

Once the first vehicle drives off the detector, the distance for the following vehicle to reach the detector is the gap (i.e., the spacing minus the length of the vehicle) between the vehicles minus the length of the detector.

$$\tau_{\text{not occupied}} = \frac{s - L_i - L_{det}}{v}$$

s = space headway (gross spacing, front-to-front)
L_i = vehicle length
L_{det} = detector length
v = speed

$$o = \frac{\tau_{\text{occupied}}}{\tau_{\text{occupied}} + \tau_{\text{not occupied}}}$$

Substitute:

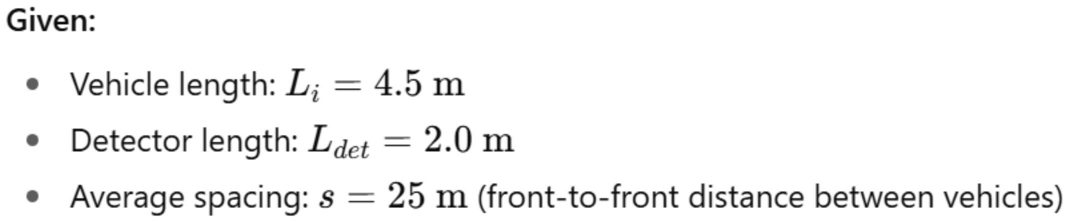
- $\tau_{\text{occupied}} = \frac{L_i + L_{det}}{v}$
- $\tau_{\text{not occupied}} = \frac{s - L_i - L_{det}}{v}$

Simplify

$$o = \frac{L_i + L_{det}}{s}$$

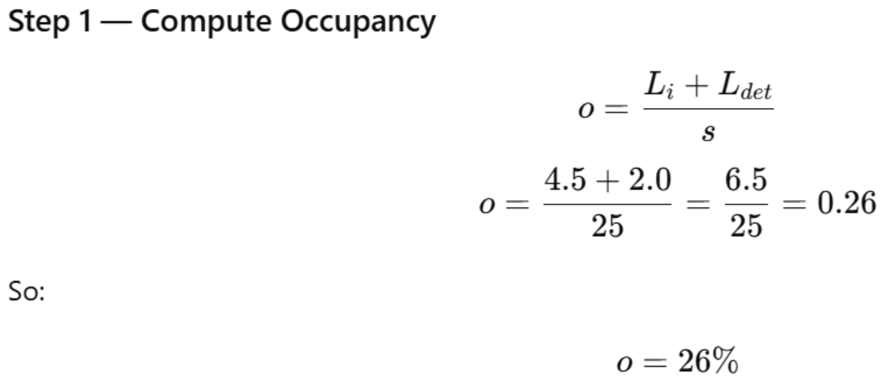
- Short vehicles at high speed → **low occupancy**, even if flow is high.
- Long vehicles or congestion (low speed) → **high occupancy**, approaching 100%.

Example1 — Occupancy and Density from Detector Data



Given:

- Vehicle length: $L_i = 4.5 \text{ m}$
- Detector length: $L_{det} = 2.0 \text{ m}$
- Average spacing: $s = 25 \text{ m}$ (front-to-front distance between vehicles)



Step 1 — Compute Occupancy

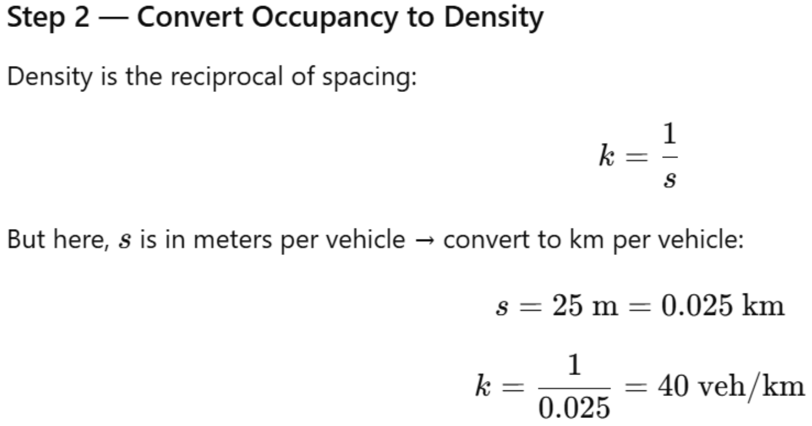
$$o = \frac{L_i + L_{det}}{s}$$

$$o = \frac{4.5 + 2.0}{25} = \frac{6.5}{25} = 0.26$$

So:

$$o = 26\%$$

Occupancy = 26%: detector is occupied about 1/4 of the time.



Step 2 — Convert Occupancy to Density

Density is the reciprocal of spacing:

$$k = \frac{1}{s}$$

But here, s is in meters per vehicle → convert to km per vehicle:

$$s = 25 \text{ m} = 0.025 \text{ km}$$

$$k = \frac{1}{0.025} = 40 \text{ veh/km}$$

Density = 40 veh/km: moderately high density, approaching typical critical density for many urban roads.

Example2 — Occupancy and Density from Detector Data

Occupancy vs. Density for Different Spacings

Assumptions:

- Vehicle length $L_i = 4.5 \text{ m}$
- Detector length $L_{det} = 2.0 \text{ m}$

Spacing, s (m)	Occupancy, o = (L+Ldet)/s	Occupancy (%)	Density, k = 1/s (veh/km)	Traffic State (Benchmark)
15 m	6.5/15=0.433	43.3 %	1000/15 ≈ 66.7 veh/km	**Oversaturated / Heavy Congestion**
25 m	6.5/25=0.260	26.0 %	1000/25 = 40.0 veh/km	**At Capacity (Critical Density)**
40 m	6.5/40=0.163	16.3 %	1000/40 = 25.0 veh/km	**Stable (Near Free-flow)**
60 m	6.5/60=0.108	10.8 %	1000/60 ≈ 16.7 veh/km	**Free-flow**
100 m	6.5/100=0.065	6.5 %	1000/100 = 10.0 veh/km	**Free-flow (Very Low Density)**

MEASURING PRINCIPLES

three principles of measuring the traffic flow

1. Local Measurements (Point-based, over time)
2. Instantaneous Measurements
3. Spatio-temporal measurements

Local Measurements

- Point-based, over time, measure traffic at a fixed location (cross-section of road).
- Often with inductive loop detectors embedded in pavement.
- Loops detect when a vehicle enters and leaves → gives occupancy.
- Using eq., occupancy can be translated to density.

$$o = \frac{L_i + L_{\text{det}}}{s}$$

- Flow is measured by vehicle counts per unit time.
- Dual loops (two loops ~1 m apart) → measure speed from travel time between loops.
- With speed + occupancy → can also estimate vehicle length.

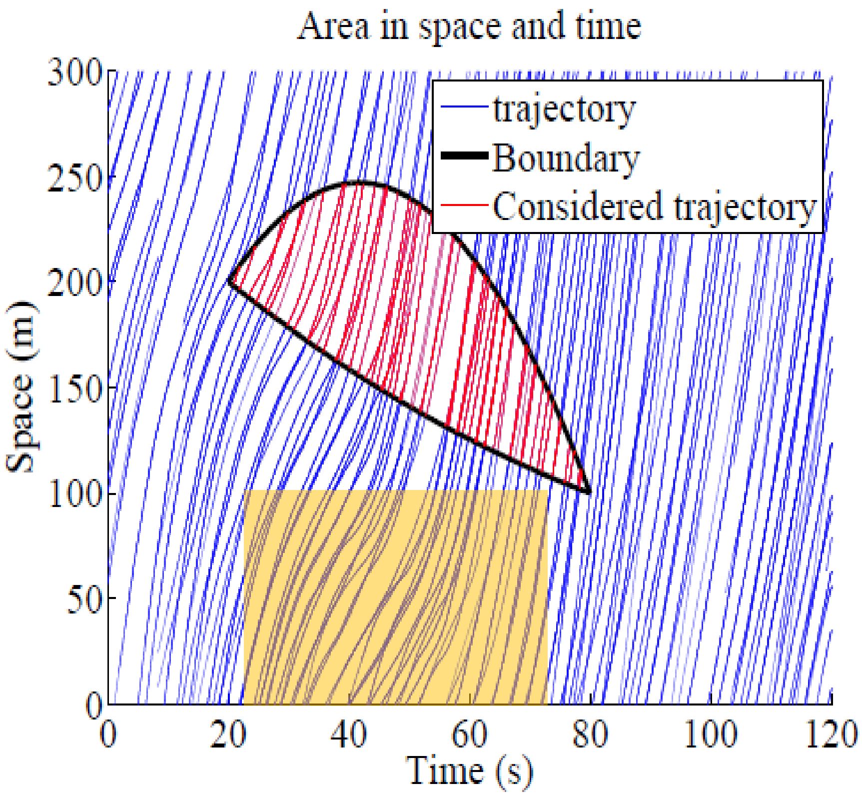
- Advantage: Continuous, reliable flow & occupancy data.
- Needs calibration for vehicle mix & detector size.

Instantaneous Measurements

- Snapshot in space, at one time, capture traffic state at one moment over a stretch of road.
- Aerial photograph or drone image.
- Spatial distribution (e.g., density, spacing).
- Vehicle positions and trajectories at that instant.
- Cannot provide Flow (since no time element).
- Good for research, calibration, validation.
- No dynamic info about how traffic evolves.

Spatio-temporal measurements

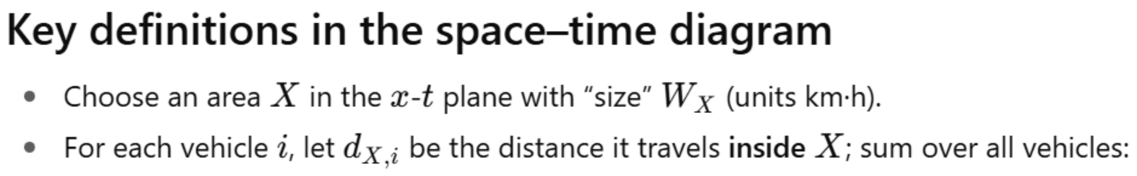
- Use measurements which stretch over a period of time and a stretch of road (Across space & time)
- This section will introduce **Edie's definitions of flow**, density and speed for an area in space and time.
- A combination of instantaneous measurements and local measurements can be found in remote sensing observations.
- These are observations which stretch in both space and time. For instance, the trajectories can be observed using a camera mounted on a high point or a helicopter. One can see a road stretch, and observe it for a period of time.
- Measuring average speed by definition requires an observation which stretches over time and space
- Local (point) measurements → flow & occupancy at one spot.
- Instantaneous (snapshot) measurements → density at one time.
- But to measure all three (flow, density, speed) consistently, we need to look at vehicles over an area in space and time.
- Can be collected using remote sensing (camera on a tall mast, drone, or helicopter).
- observe a road section for a period of time, not just at one point or instant.
- Figure below shows an area is selected in a trajectory. Note that a selected area is not necessarily square. It is even possible to have a convex area, or boundaries moving backwards and forwards in time.



• The figure is used to **analyze interactions between vehicles**, such as:

- **Car-following behavior**: How one vehicle reacts to the trajectory of the vehicle ahead.
- **Traffic density estimation**: Number of vehicles within a space–time region.
- **Flow analysis**: Calculating flow by counting trajectories crossing a boundary.

• The shaded area essentially shows the **"area in space and time" occupied by the considered vehicle and its interaction domain** with others.



Key definitions in the space–time diagram

- Choose an area X in the x-t plane with “size” W_X (units km·h).
- For each vehicle i, let $d_{X,i}$ be the distance it travels **inside** X; sum over all vehicles:

Let us consider the area X. We indicate its size by W_X, which is expressed in km-h, or any other unit of space times time. For all vehicles, we consider the distance they drive in area X, which we call $d_{X;i}$. Adding these for all vehicles i gives the total distance covered in area X, indicated by TD:

$$TD = \sum_i d_{X,i} \qquad \text{(total distance within X; veh·km)} \qquad (1.5)$$

For a rectangular region in the space–time diagram, the distance a vehicle covers **within the region** is often the full roadway length from the upstream to the downstream boundary. However, a trajectory may enter or leave through a **side** boundary at some time. In that case, the vehicle's distance **inside the region** is **less** than the full roadway length.

Similarly, we can define the time a vehicle spends in area X, $t_{X,i}$, which we can sum for all vehicles i to get the total time spent in area X, indicated by TT.

- Let $t_{X,i}$ be the time vehicle i spends **inside** X; sum over all vehicles:

$TT = \sum_i t_{X,i}$ (total time within X; veh·h) (1.6)

Obviously, both quantities grow in principle with the area size. Therefore, the traffic flow is best characterized by the quantities TD/W_X and TT/W_X. This gives the flow and the density respectively:

$q = TD / W_X$ (flow; veh/h) (1.7)

$k = TT / W_X$ (density; veh/km) (1.8)

Intuitively, the relationship is best understood reasoning from the known relations of density and flow. Starting with a situation of 1000 veh/h at a cross section, and an area of 1 h and 2 km. In 1 hour, 1000 vehicles pass by, which all travel 2 kilometres in the area (There the vehicles which cannot cover the 2 km because the time runs out, but there are just as many which are in the section when the time window starts). So, the total distance is the flow times the size of the area:

$TD = qW_X = 1000\times(2km\backslash h) = 2000\ veh\backslash km.$

(Replacing vehicles that exit early are those already present when the time window begins.)

$TD = qW_X$. This can be simply rewritten to equation 1.7.

A similar relation is constructed for the density, considering again the rectangular area of 1-hour times 2 kilometers.

With a density k=10 veh/km and a rectangular region of length 2km and duration 1 h, the number of vehicles present in the region is

$N = k\times length = 10\ veh/km \times 2\ km = 20$ vehicles.

If we follow these vehicles for one hour, the total time spent in the region is

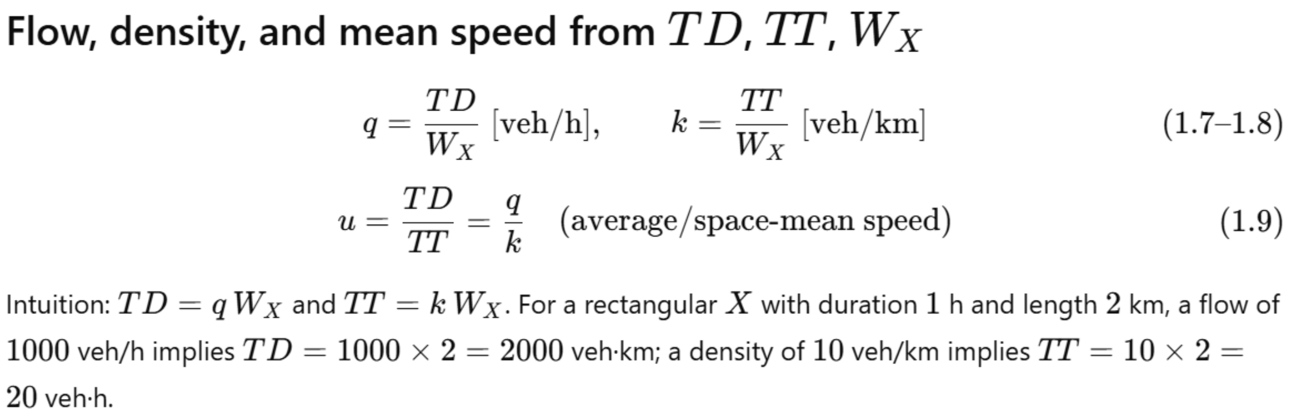
$TT = N\times 1\ h = 20\ veh/h$

Equivalently, using the area size

W_X=(2 km)×(1 h)=2 km\h

TT=k W_X= (10 veh/km) × (2 km\h) =20 veh\h

This can be rewritten to equation 1.8.



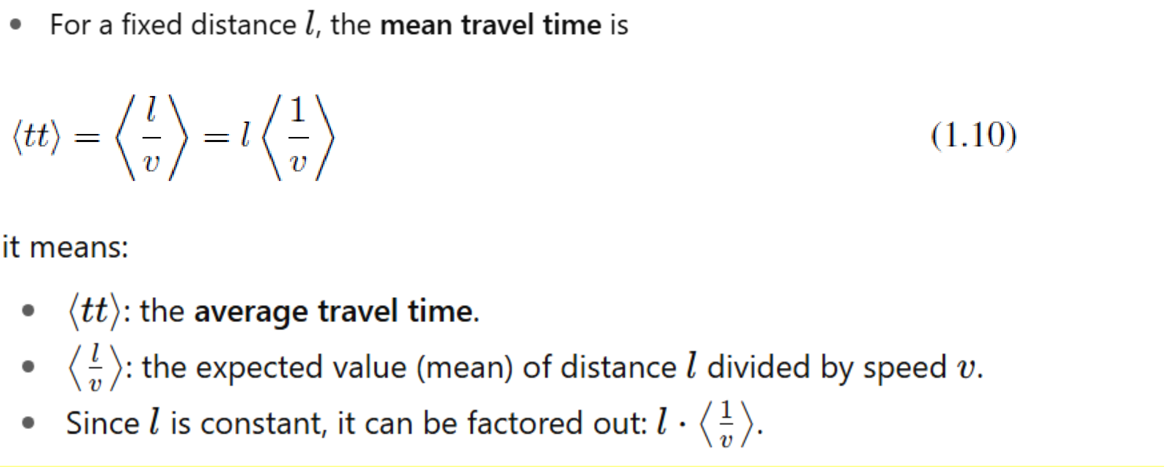
Flow, density, and mean speed from TD, TT, W_X

$$q = \frac{TD}{W_X}\ [\text{veh/h}], \qquad k = \frac{TT}{W_X}\ [\text{veh/km}] \tag{1.7–1.8}$$

$$u = \frac{TD}{TT} = \frac{q}{k} \quad (\text{average/space-mean speed}) \tag{1.9}$$

Intuition: $TD = q\,W_X$ and $TT = k\,W_X$. For a rectangular X with duration 1 h and length 2 km, a flow of 1000 veh/h implies $TD = 1000 \times 2 = 2000$ veh·km; a density of 10 veh/km implies $TT = 10 \times 2 = 20$ veh·h.

Averaging travel time correctly



- For a fixed distance l, the **mean travel time** is

$$\langle tt \rangle = \left\langle \frac{l}{v} \right\rangle = l \left\langle \frac{1}{v} \right\rangle \tag{1.10}$$

it means:

- $\langle tt \rangle$: the **average travel time**.
- $\left\langle \frac{l}{v} \right\rangle$: the expected value (mean) of distance l divided by speed v.
- Since l is constant, it can be factored out: $l \cdot \left\langle \frac{1}{v} \right\rangle$.

In this equation, *tt* indicates the travel time and the pointy brackets indicate the mean. This can be measured for all vehicles passing a road stretch, for instance at a local detector. Note that the mean travel time is not equal to the distance divided by the mean speed:

$$\langle tt \rangle = l \left\langle \frac{1}{v} \right\rangle \neq l \frac{1}{\langle v \rangle} \tag{1.11}$$

In fact, it can be proven that in case speeds of vehicles are not the same, the average travel time is underestimated if the mean speed is used

$$\langle tt\rangle = l\left\langle \frac{1}{v}\right\rangle \leq l\frac{1}{\langle v\rangle} \tag{1.12}$$

The harmonically averaged speed (i.e., 1 divide by the average of 1 divided by the speed) does provide a good basis for the travel time estimation. In an equation, we best first define the pace, pi:

$$p_i = \frac{1}{v_i} \tag{1.13}$$

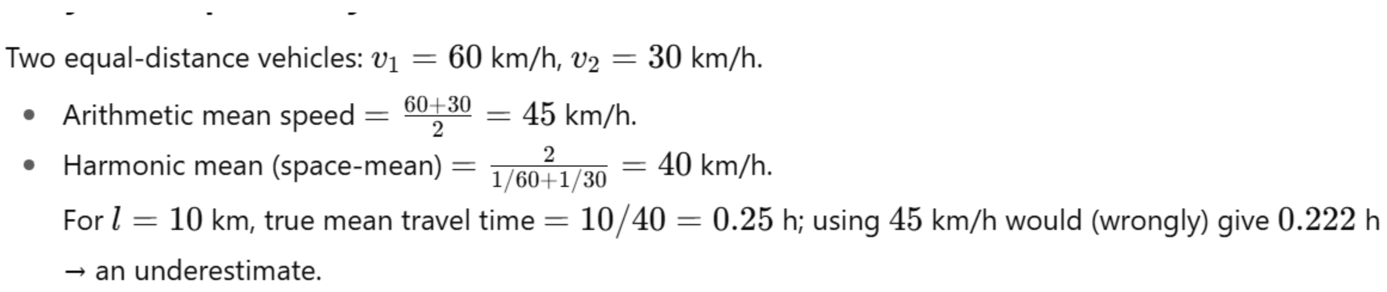
The **harmonic mean speed** is

$$\langle v\rangle_{\text{harmonically}} = \frac{1}{\langle p\rangle} = \frac{1}{\left\langle \frac{1}{v_i}\right\rangle} \tag{1.14}$$

- Therefore, the best estimator for mean travel time over distance l is

$$\boxed{\langle tt\rangle = \frac{l}{\langle v\rangle_{\text{harm}}}}$$

Example:



Two equal-distance vehicles: $v_1 = 60$ km/h, $v_2 = 30$ km/h.

- Arithmetic mean speed $= \frac{60+30}{2} = 45$ km/h.
- Harmonic mean (space-mean) $= \frac{2}{1/60+1/30} = 40$ km/h.

  For $l = 10$ km, true mean travel time $= 10/40 = 0.25$ h; using 45 km/h would (wrongly) give 0.222 h → an underestimate.

Example

Over a 1 h × 3 km rectangle, detectors report q = 1200 veh/h and k = 8 veh/km. Compute TD, TT, and u.

Answer:

TD = q·W_X = 1200×3 = 3600 veh·km;

TT = k·W_X = 8×3 = 24 veh·h;

u = TD/TT = 3600/24 = 150 km/h

Example

Two equal-distance vehicles at 80 and 40 km/h. Find arithmetic vs harmonic mean speeds.

Answer:

Arithmetic $= (80+40)/2 = 60$ km/h.
Harmonic $= 2/(1/80+1/40) = 53.33$ km/h.

Example

For *l* = 5 km and speeds 50, 60, 90 km/h, compute ⟨tt⟩ using ⟨1/v⟩ and compare with 5/⟨v⟩.

Answer

$$\langle 1/v\rangle = \frac{1/50+1/60+1/90}{3} = \frac{0.020000+0.016667+0.011111}{3} = 0.015926 \text{ h/km.}$$

So

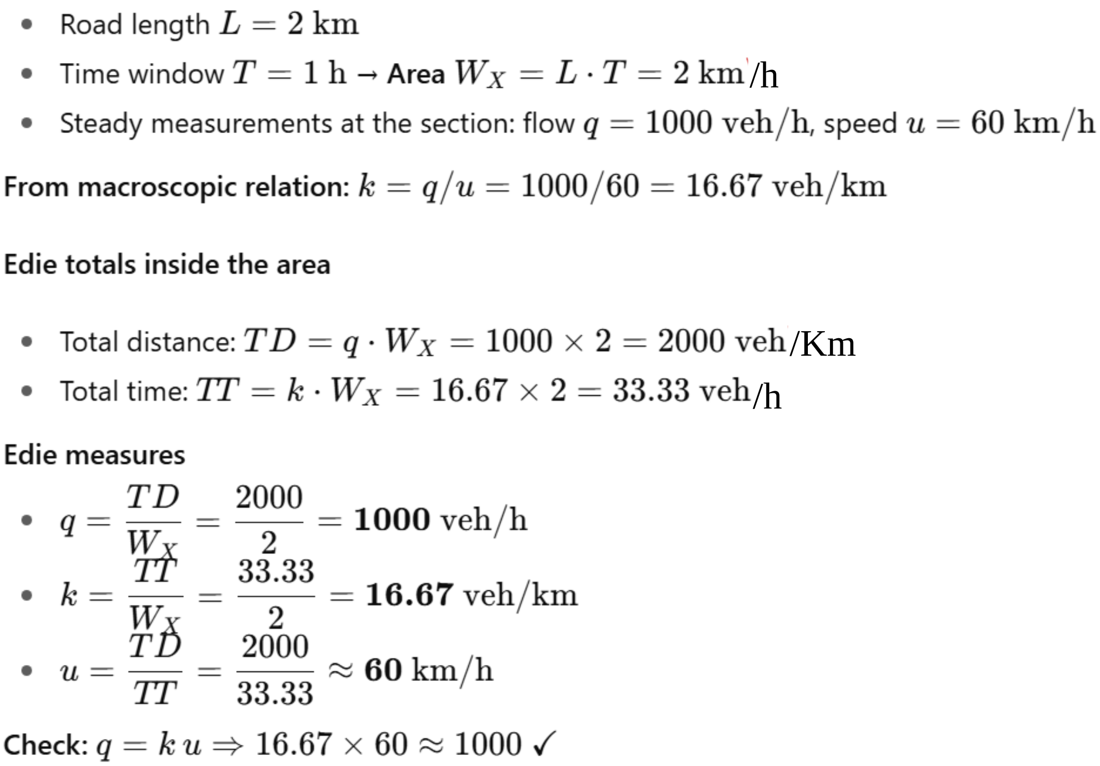
$$\langle tt\rangle = 5 \times 0.015926 = 0.07963 \text{ h} = 4.78 \text{ min},$$

while

$$\langle v\rangle = \frac{50+60+90}{3} = 66.67 \text{ km/h} \Rightarrow \frac{5}{\langle v\rangle} = 0.07500 \text{ h} = 4.50 \text{ min.}$$

Tip: Use harmonic mean for space-mean speed and travel-time estimation whenever speeds vary.

Example



- Road length $L = 2\ \mathrm{km}$
- Time window $T = 1\ \mathrm{h}$ → **Area** $W_X = L \cdot T = 2\ \mathrm{km}$/h
- Steady measurements at the section: flow $q = 1000\ \mathrm{veh/h}$, speed $u = 60\ \mathrm{km/h}$

From macroscopic relation: $k = q/u = 1000/60 = 16.67\ \mathrm{veh/km}$

Edie totals inside the area

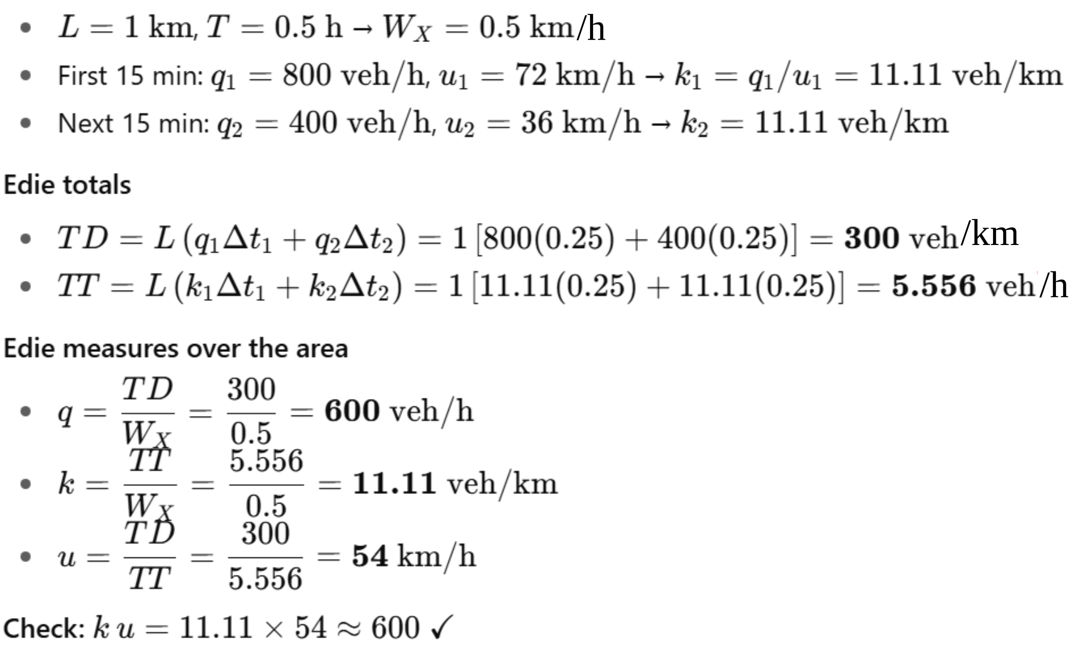
- Total distance: $TD = q \cdot W_X = 1000 \times 2 = 2000\ \mathrm{veh}$/Km
- Total time: $TT = k \cdot W_X = 16.67 \times 2 = 33.33\ \mathrm{veh}$/h

Edie measures

- $q = \dfrac{TD}{W_X} = \dfrac{2000}{2} = \mathbf{1000}\ \mathrm{veh/h}$
- $k = \dfrac{TT}{W_X} = \dfrac{33.33}{2} = \mathbf{16.67}\ \mathrm{veh/km}$
- $u = \dfrac{TD}{TT} = \dfrac{2000}{33.33} \approx \mathbf{60}\ \mathrm{km/h}$

Check: $q = k\,u \Rightarrow 16.67 \times 60 \approx 1000$ ✓

Example



- $L = 1\ \mathrm{km}$, $T = 0.5\ \mathrm{h}$ → $W_X = 0.5\ \mathrm{km}$/h
- First 15 min: $q_1 = 800\ \mathrm{veh/h}$, $u_1 = 72\ \mathrm{km/h}$ → $k_1 = q_1/u_1 = 11.11\ \mathrm{veh/km}$
- Next 15 min: $q_2 = 400\ \mathrm{veh/h}$, $u_2 = 36\ \mathrm{km/h}$ → $k_2 = 11.11\ \mathrm{veh/km}$

Edie totals

- $TD = L\,(q_1\Delta t_1 + q_2\Delta t_2) = 1\,[800(0.25) + 400(0.25)] = \mathbf{300}\ \mathrm{veh}$/km
- $TT = L\,(k_1\Delta t_1 + k_2\Delta t_2) = 1\,[11.11(0.25) + 11.11(0.25)] = \mathbf{5.556}\ \mathrm{veh}$/h

Edie measures over the area

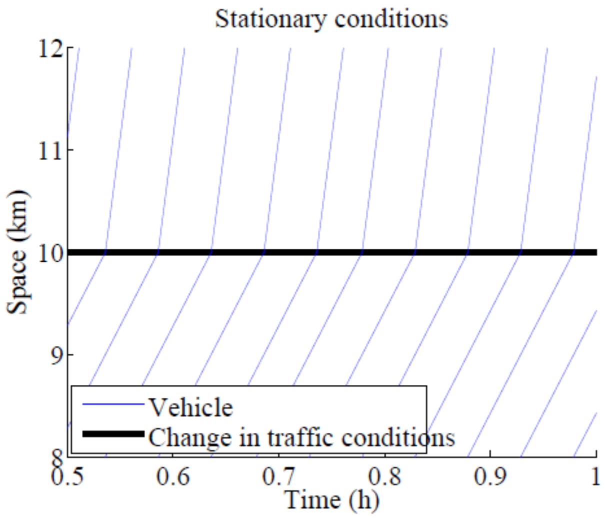
- $q = \dfrac{TD}{W_X} = \dfrac{300}{0.5} = \mathbf{600}\ \mathrm{veh/h}$
- $k = \dfrac{TT}{W_X} = \dfrac{5.556}{0.5} = \mathbf{11.11}\ \mathrm{veh/km}$
- $u = \dfrac{TD}{TT} = \dfrac{300}{5.556} = \mathbf{54}\ \mathrm{km/h}$

Check: $k\,u = 11.11 \times 54 \approx 600$ ✓

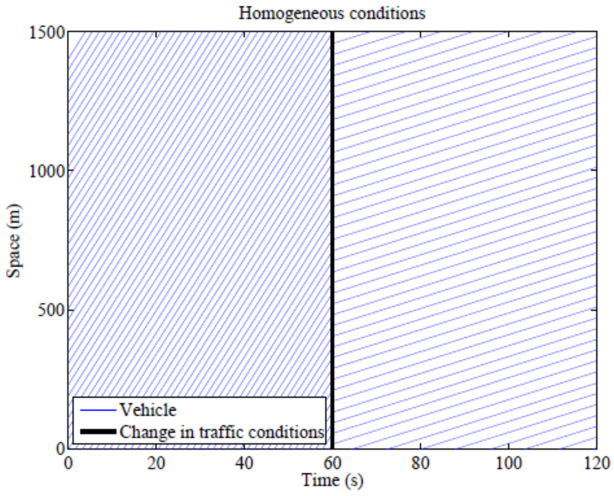
.3 Stationarity vs. Homogeneity (what they mean)

- **Stationary traffic (over time):**
  Traffic characteristics (e.g., q, k, u) are **constant in time** at a fixed location, but may differ from place to place.
  Example: At km 2 the mean speed stays ~70 km/h for 30 minutes; at km 5 it's ~45 km/h the whole time.
- **Homogeneous traffic (over space):**
  Traffic characteristics are **constant in space** along a road section at a given time, but may change over time.
  Example: At 08:00, speeds are ~60 km/h everywhere from km 0–5; at 08:15 they drop to ~40 km/h everywhere (e.g., new speed limit or rain).

In practice, **stationarity** is common (conditions steady over time at one point). **Homogeneity** is rarer; it often requires an external, corridor-wide change (speed limit, signal plan, weather front, incident clearance).



(a) Stationary conditions



(b) Homogeneous conditions

HOW TO TEST THEM WITH DATA

A) Test for Stationarity (time-consistency at one point)

Goal: At a *single detector/location*, check if traffic conditions are roughly constant over time.

Steps

1. Pick one detector (fixed location).
2. Split the observation period into equal windows (e.g., 5, 10, or 15 min).
3. For each window compute: flow q, speed u, and density k (or $k \approx q/u$ if needed).
4. Compare across windows:
   - Quick screen: coefficient of variation (CV).
   - Formal test: one-way ANOVA (or Kruskal–Wallis if non-normal).

Rule of thumb
If

$$\mathrm{CV}(u) < 10\% \quad \text{and} \quad \mathrm{CV}(q) < 15\%$$

across the windows, treat the period as **approximately stationary**.

B) Test for Homogeneity (space-consistency at one time)

Goal: At a *single timestamp*, check if conditions are similar along the section.

Steps

1. Choose several detectors along the corridor (same timestamp or narrow time slice).
2. Collect q, u, k from each detector at that time.
3. Compare across detectors:
   - Quick screen: spatial CV.
   - Formal test: one-way ANOVA/Kruskal–Wallis across locations.

Rule of thumb
If

$$\mathrm{CV}_{\text{space}}(u) < 10\% \quad \text{and} \quad \mathrm{CV}_{\text{space}}(q) < 15\%$$

treat the section at that time as **approximately homogeneous**.

$$\mathrm{CV}(x) = \frac{\mathrm{std}(x)}{\mathrm{mean}(x)} \times 100\%.$$

- Use shorter windows in unstable traffic; longer in steady free-flow.
- Missing data/outliers: Winsorize or median-smooth before testing.
- Arterials vs freeways: Arterial corridors (signals, side-friction) usually need looser thresholds than freeways.

Example:

Suppose there are two classes of vehicles, fast vehicles driving 120 km/h and slow vehicles moving at 90 km/h. At the entrance to the road (x=0), an equal number of each type of vehicles is measured under a total flow of 1000 veh/h (so 500 veh/h each).

a) Calculate the density on the road. Base your answer on the density of the slow vehicles and the fast vehicles.
b) Calculate the space mean speed.
c) Calculate the time mean speed.
d) Give the equation for Edie's generalised definition of density.

Solution

a. The partial densities can be calculated by
k=q/v
k1=500/120=4.2 veh/km
k2=500/90=5.6 veh/km .

The total density is the sum of these two = 9.7 veh/km

b. For the space mean speed,
Option 1: average the paces
1/((1/90+1/120)/2) = 102.8 km/h

or option 2 Put weight on the speed values the speeds using the densities
4.2 * 120+ 5.6 * 90 /(4.2+5.6) = 102.8 km/h

c. For the time mean speed, one observes as many slow as fast vehicles (equal flows).
the mean speed is (90+120)/2 =105 km/h

d.

$$k_{\text{Edie}} = \frac{\text{Total time spent}}{\text{Area in space-time}}$$

References

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2. Manual, H. C. (2000). Highway capacity manual. *Washington, DC*, *2*(1), 1.

3. Traffic engineering / Roger P. Roess, Elena S. Prassas, William R. McShane. -4th ed.