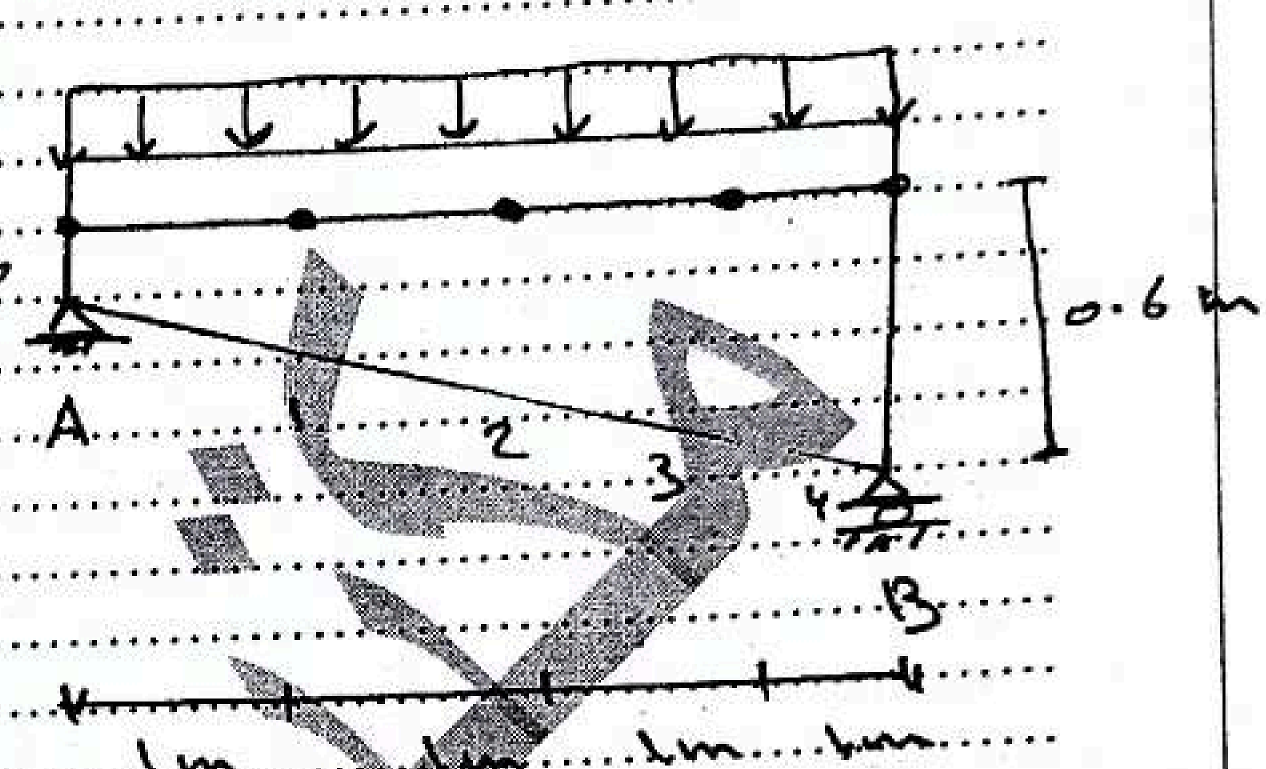
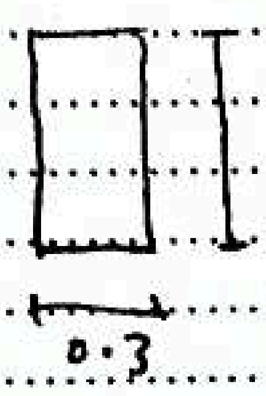
Engineering Analysis & Numerical Methods

Example :- Solve the beam shown in fig. and find the deflection at the points.



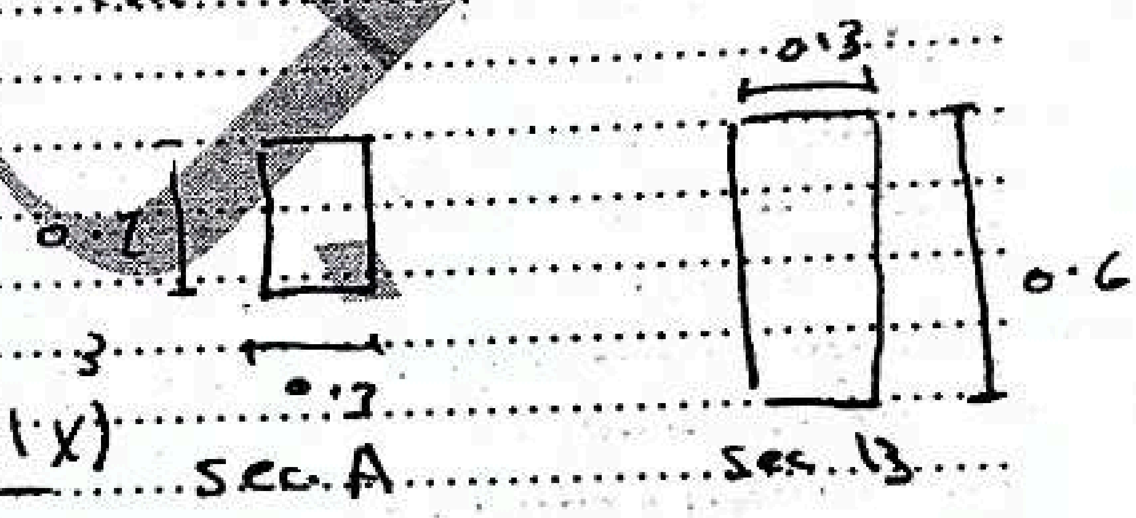


$$\frac{h'(x)}{x} = \frac{0.4}{4}$$

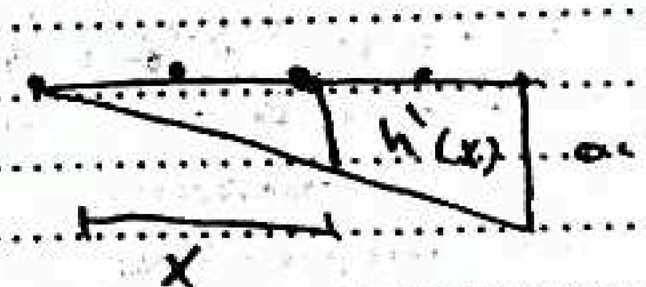
$$h'(x) = 0.1\,x$$

$$h(x) = 0.2 + h'(x)$$

$$= 0.2 + 0.1\,x$$



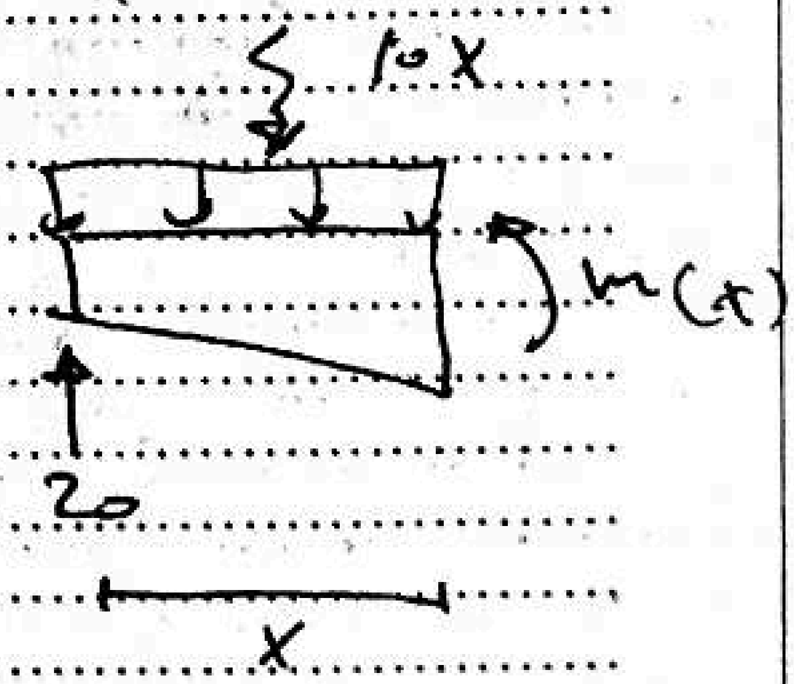
$$\therefore I_x = \frac{b\,h^3(x)}{12} = \frac{0.3\,(0.2+0.1x)^3}{12}$$



$$\frac{d^2y}{dx^2} = \frac{m(x)}{EI(x)}$$

$$m(x) = 20\,x - 10\,x\left(\frac{x}{2}\right)$$

$$= 20\,x - 5\,x^2$$



$$\frac{d^2y}{dx^2} = \frac{20\,x - 5\,x^2}{E\left(\frac{0.3(0.2+0.1x)^3}{12}\right)}$$

$$\frac{d^2y}{dx^2} = \frac{4\,(20\,x - 5\,x^2)}{E\,(0.1\,(0.2+0.1\,x)^3)}$$

$$\frac{d^2y}{dx^2} = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$$

$$y_{i-1} - 2y_i + y_{i+1} = \frac{h^2 4(20x_i - 5x_i^2)}{E(0.1(0.2+0.1x_i)^3)}$$

$i = 1$

$$y_0 - 2y_1 + y_2 = \frac{1^2 * 4(20*1 - 5*1^2)}{E(0.1(0.2+0.1*1)^3)}$$

$$-2y_1 + y_2 = \frac{4(15)}{E\,0.0027} = \frac{600\,000}{E\,27} = \frac{200\,000}{E\,9} \quad \text{---} (1)$$

$i = 2$

$$y_1 - 2y_2 + y_3 = \frac{4(20*2 - 5*2^2)}{E(0.1(0.2+0.1(2))^3)}$$

$$= \frac{80}{E\,0.0064} = \frac{12500}{E} \quad \text{---} (2)$$

$i = 3$

$$y_2 - 2y_3 + \cancel{y_4}^{0} = \frac{4(20*3 - 5*3^2)}{E(0.1(0.2+0.1x3)^3)}$$

$$= \frac{60}{E\,0.0125} \quad \text{---} (3)$$

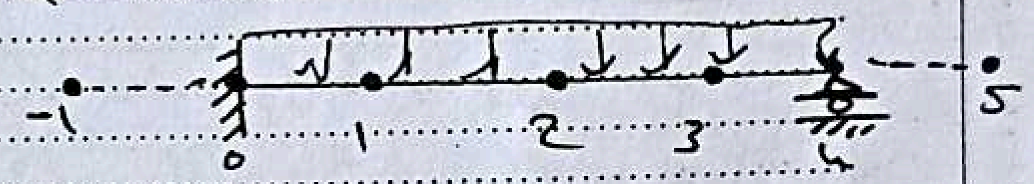
$$\begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \frac{1}{E} \begin{Bmatrix} \frac{200\,000}{9} \\ 12500 \\ 4800 \end{Bmatrix}$$

Solution of Higher-Order Problems by Central Differences

Example: Solve the beam shown in fig. and find (EI = constant)

① The deflection at the points.
② The moment at " "
③ The shear at points

Solution:-



$\theta_0 = 0$

$$\left.\frac{dy}{dx}\right|_0 = 0 \Rightarrow \frac{-y_{i-1} + y_{i+1}}{2h} = \frac{dy}{dx}$$

$$\therefore -y_{-1} + y_1 = 0 \Rightarrow \boxed{y_1 = y_{-1}}$$

$y_1 \quad y_0 \quad y_{-1}$

$h \quad h$

$$EI\left.\frac{d^2y}{dx^2}\right|_4 = 0 \Rightarrow \frac{y_3 - \cancel{2y_4} + y_5}{h^2} = 0$$

$$y_3 + y_5 = 0 \Rightarrow \boxed{y_5 = -y_3}$$

$$\frac{d^4y}{dx^4} = \frac{w(x)}{EI}$$

$$y_{i-2} - 4y_{i-1} + 6y_i - 4y_{i+1} + y_{i+2} = \frac{h^4}{EI}(w(x))$$

$i = 1$

$$y_{-1} - \cancel{4y_0}^{\,0} + 6y_1 - 4y_2 + y_3 = \frac{1.0}{EI}$$

$$y_1 + 6y_1 - 4y_2 + y_3 = \frac{1.0}{EI}$$

$$7y_1 - 4y_2 + y_3 = \frac{1.0}{EI} \quad ---- \text{①}$$

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$i=2$

$$\cancel{y_0}^{\,0} - 4y_1 + 6y_2 - 4y_3 + \cancel{y_4}^{\,0} = \frac{10}{EI}$$

$$-4y_1 + 6y_2 - 4y_3 = \frac{10}{EI} \quad ----- (2)$$

$i=3$

$$y_1 - 4y_2 + 6y_3 - 4\cancel{y_4}^{\,0} + y_5 = \frac{10}{EI}$$

$$y_1 - 4y_2 + 6y_3 - 4\cancel{y_4}^{\,0} - y_3 = \frac{10}{EI}$$

$$y_1 - 4y_2 + 5y_3 = \frac{10}{EI} \quad ----- (3)$$

$$\begin{bmatrix} 7 & -4 & 1 \\ -4 & 6 & -4 \\ 1 & -4 & 5 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \frac{10}{EI} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \qquad \frac{4}{7}r_1 + r_2$$

$$\begin{bmatrix} 7 & -4 & 1 \\ 0 & 26/7 & -24/7 \\ 1 & -4 & 5 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \frac{10}{EI} \begin{Bmatrix} 1 \\ 11/7 \\ 1 \end{Bmatrix} \qquad -1/7\, r_1 + r_3$$

$$\begin{bmatrix} 7 & -4 & 1 \\ 0 & 26/7 & -24/7 \\ 0 & -24/7 & 34/7 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \frac{10}{EI} \begin{Bmatrix} 1 \\ 11/7 \\ 6/7 \end{Bmatrix} \qquad \begin{matrix} 24/26\, r_2 + r_3 \\ \text{or} \\ 12/13\, r_1 + r_3 \end{matrix}$$

$$\begin{bmatrix} 7 & -4 & 1 \\ 0 & \frac{26}{7} & \frac{-24}{7} \\ 0 & 0 & 154/91 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \frac{10}{EI} \begin{Bmatrix} 1 \\ 11/7 \\ 210/91 \end{Bmatrix}$$

$$\frac{154}{91} y_3 = \frac{2100}{91\,EI} \Rightarrow y_3 = \frac{2100}{154\,EI} = \frac{1050}{77\,EI}$$

$$y_3 = \frac{150}{11\,EI}$$

$$0.y_1 + \frac{26}{7} y_2 - \frac{24}{7} y_3 = \frac{110}{7EI} \quad * 7$$

$$26 y_2 - 24 y_3 = \frac{110}{EI}$$

$$26 y_2 - 24\left(\frac{1050}{77\,EI}\right) = \frac{110}{EI}$$

$$y_2 = \frac{1}{26}\left[\frac{110}{EI} + \frac{24(1050)}{77\,EI}\right] = \frac{2405}{143\,EI}$$

$$7y_1 - 4y_2 + y_3 = \frac{10}{EI}$$

$$7y_1 - 4\left[\frac{2405}{143\,EI}\right] + \frac{1050}{77\,EI} = \frac{10}{EI}$$

$$y_1 = \frac{1300}{143\,EI} = \frac{9.09}{EI}$$

$$y_2 = \frac{2405}{143\,EI} = \frac{16.818}{EI} , \quad y_3 = \frac{13.636}{EI}$$

② The moment

Point 1

$$\frac{d^2y}{dx^2} = \frac{m(x)}{EI} \quad oR \quad m(x) = EI \frac{d^2y}{dx^2}$$

$$m(x) = EI\left[\frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}\right] , h = 1$$

$$m(1) = EI\left[y_0^{\,0} - 2y_1 + y_2\right] = EI\left[-2\frac{9.09}{EI} + \frac{16.818}{EI}\right]$$

$$= -1.362 \text{ kN}$$

$$m(2) = EI(y_1 - 2y_2 + y_3) = -10.91 \text{ kN}$$

$$m(3) = -10.454 \text{ kN}$$

③ The Shear at points

$$\frac{d^3y}{dx^3} = -\frac{V(x)}{EI} \quad \text{OR} \quad V(x) = -EI\,\frac{d^3y}{dx^3}$$

$$V(x_i) = -EI\left[\frac{-y_{i-2} + 2y_{i-1} - 2y_{i+1} + y_{i+2}}{2h^3}\right] =$$

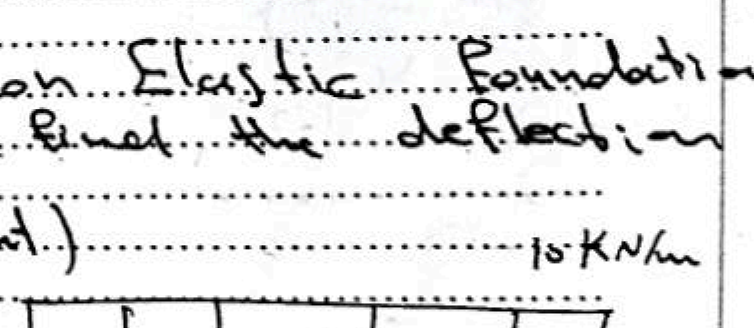
$$V(1) = \frac{-EI}{2}\left[-y_{-1} + 2\cancel{y_0}^{0} - 2y_2 + y_3\right] = +14.54\,kN$$

$$V(2) = \frac{-EI}{2}\left[-\cancel{y_0}^{0} + 2y_1 - 2y_3 + \cancel{y_4}^{0}\right] = 4.541\ kN.$$

$$V(3) = \frac{-EI}{2}\left[-y_1 + 2y_2 - 2\cancel{y_4}^{0} + y_5\right] = -5.455\,kN$$

Example:- Solve the beam on Elastic foundation shown in fig. and find the deflection at the points.
(EI = constant , K = constant)

$$\frac{d^4y}{dx^4} + \frac{K}{EI}\,y = \frac{q(x)}{EI}$$



B.C.

$y_0 = y_4 = 0$

$$\theta_0 = \frac{dy}{dx}\Big|_0 = 0 \Rightarrow \frac{-y_{-1} + y_1}{2h} = 0$$

$$\boxed{y_1 = y_{-1}}$$

$$m_4 = \frac{d^2y}{dx^2}\Big|_4 = 0 \Rightarrow \frac{y_3 - 2y_4 + y_5}{h^2} = 0$$

$$\boxed{y_5 = -y_3}$$

$$\frac{d^4y}{dx^4} = \frac{y_{i-2} - 4y_{i-1} + 6y_i - 4y_{i+1} + y_{i+2}}{h^4}$$

$$\frac{d^4y}{dx^4} + \frac{K}{EI}y = \frac{q(x)}{EI}$$

$$y_{i-2} - 4y_{i-1} + \left(6 + \frac{Kh^4}{EI}\right)y_i - 4y_{i+1} + y_{i+2} = \frac{q(x_i)h^4}{EI}$$

$i = 1$, $x_i = L/4$

$$y_{-1} - 4y_0 + \left(6 + \frac{Kh^4}{EI}\right)y_1 - 4y_2 + y_3 = \frac{qh^4}{EI}$$

From $y_{-1} = y_1$ (B.C)

$$\left(7 + \frac{Kh^4}{EI}\right)y_1 - 4y_2 + y_3 = \frac{qh^4}{EI} \quad \text{--- (1)}$$

$i = 2$, $x_i = L/2$

$$\cancel{y_0} - 4y_1 + \left(6 + \frac{Kh^4}{EI}\right)y_2 - 4y_3 + \cancel{y_4} = \frac{qh^4}{EI}$$

$$-4y_1 + \left(6 + \frac{Kh^4}{EI}\right)y_2 - 4y_3 = \frac{qh^4}{EI} \quad \text{--- (2)}$$

$i = 3$, $x_i = 3L/4$

$$y_1 - 4y_2 + \left(6 + \frac{Kh^4}{EI}\right)y_3 - 4\cancel{y_4} + y_5 = \frac{qh^4}{EI}$$

$y_5 = -y_3$ (B.C)

$$y_1 - 4y_2 + \left(5 + \frac{Kh^4}{EI}\right)y_3 = \frac{qh^4}{EI} \quad \text{--- (3)}$$

$$\begin{bmatrix} \left(7 + \frac{Kh^4}{EI}\right) & -4 & 1 \\ -4 & \left(6 + \frac{Kh^4}{EI}\right) & -4 \\ 1 & -4 & \left(5 + \frac{Kh^4}{EI}\right) \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{Bmatrix} qh^4/EI \\ qh^4/EI \\ qh^4/EI \end{Bmatrix}$$