Ex 5 :- Find F(2) from the data shown below.

| X | F(x) | D1 | D2 | D3 | D4 | D5 |
|---|---|---|---|---|---|---|
| -1 | 3.00 | | | | | |
| | | * -5 | | | | |
| 0 | -2.0 | | * 5.5 | | | |
| | | 3.250 | | -1 | | |
| 0.5 | 0.375 | | 3.5 | | 0 | |
| | | 6.75 | | -1 | | 0 |
| 1.0 | 3.00 | | 1.00 | | | |
| | | 8.75 | | | 0 | |
| | | | | -1 | | |
| 2.5 | 16.125 | | -1.500 | | | |
| | | 5.75 | | | | |
| 3 | 19.00 | | | | | |

$6-1=5 \Rightarrow D5$

* D1

$$\frac{-2-(3)}{0-(-1)}=-5$$

* $D_2$

$$\frac{3.25-(-5)}{0.5-(-1)}=5.5$$

Newton's Divided Difference Formual

$$F(X)=f(X_0)+(X-X_0)\,f(X_0,X_1)+(X-X_0)(X-X_1)\,f(X_0,X_1,X_2)---$$

$$f(2)=-2+(2-0)*3.25+(2-0)(2-0.5)(3.5)+(2-0)(2-0.5)(2-1)(-1)=12$$

Ex 6:- From the data shown below derive expresion of f(x) and then used this formula to find f(22).

| X | F(x) | D1 | D2 |
|---|---|---|---|
| 20 | 0.34202 | | |
| | | * 0.01579 | |
| 30 | 0.5 | | * −0.0007 |
| | | 0.01472 | |
| 35 | 0.5735 | | |

* D2

$$\frac{0.01472 - 0.01574}{35 - 20} = -0.00007$$

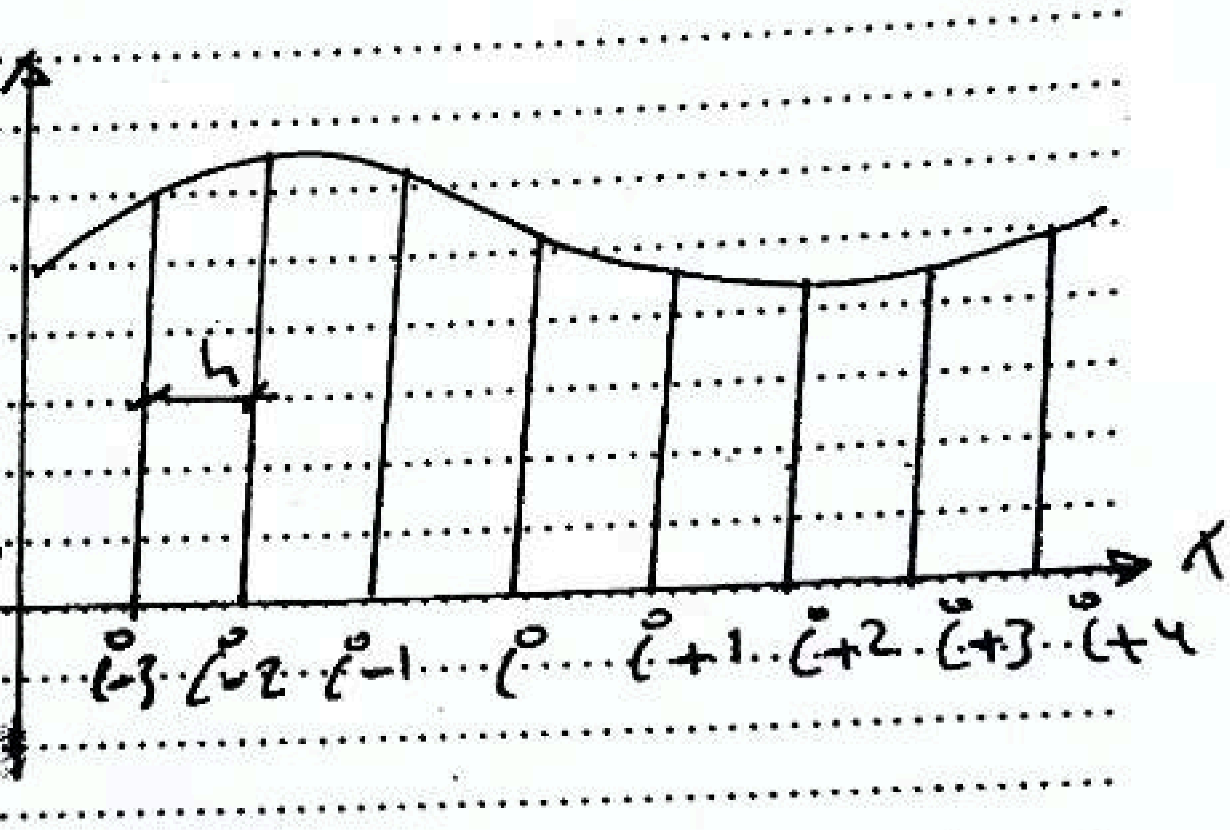
* D1

$$\frac{0.5 - 0.34202}{30 - 20} = 0.01579$$

$F(22) = 0.39202 + (22-20) * 0.01579 + (22-20)(22-30)(0.00007) = 0.37472$

## 2 Forward Differences:

$$\frac{df}{dx} = \Delta y_i = \frac{y_{i+1} - y_i}{h}$$




$$\frac{d^2f}{dx^2} = \Delta^2 y_i$$

$$= \frac{\frac{y_{i+2} - y_{i+1}}{h} - \frac{y_{i+1} - y_i}{h}}{h}$$

$$\Delta^2 y_i = \frac{y_{i+2} - 2y_{i+1} + y_i}{h^2}$$

| | $i$ | $i+1$ | $i+2$ | $i+3$ | $i+4$ |
|---|---|---|---|---|---|
| $hD =$ | $-1$ | $1$ | | | |
| $h^2D^2 =$ | $1$ | $-2$ | $1$ | | |
| $h^3D^3 =$ | $-1$ | $3$ | $-3$ | $1$ | |
| $h^4D^4 =$ | $1$ | $-4$ | $6$ | $-4$ | $1$ |

مؤثرات الفروق الامامية

Note:

$$\frac{dy}{dx} = \theta \qquad \frac{d^2y}{dx^2} = \frac{m(x)}{EI}$$

$$\frac{d^2m}{dx^2} = w(x) \qquad \frac{d^3y}{dx^3} = \frac{-V(x)}{EI}$$

$$\frac{d^4y}{dx^4} = \frac{w(x)}{EI}$$

3. Central Differences

$$\frac{df}{dx} = \Delta y_i = \frac{y_{i+1} - y_{i-1}}{2h}$$

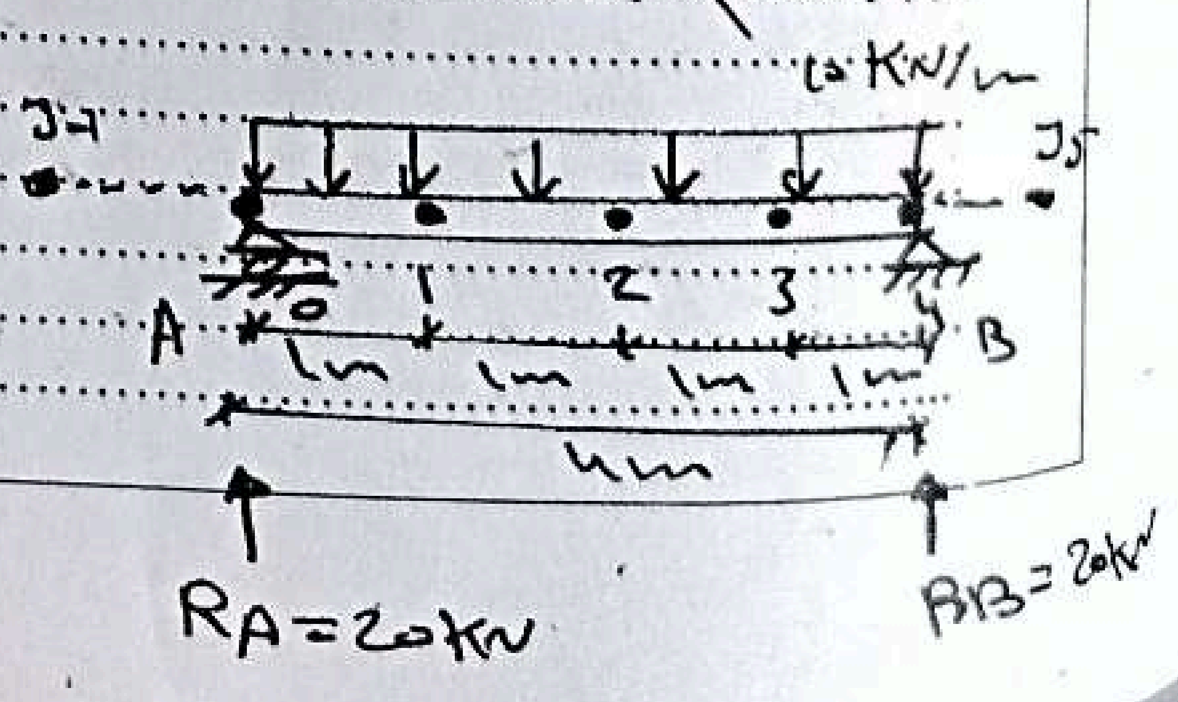
$$\frac{d^2f}{dx^2} = \Delta^2 y_i = \frac{\frac{y_{i+1}-y_i}{h} - \frac{y_i - y_{i-1}}{h}}{h} = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

| | $i-2$ | $i-1$ | $i$ | $i+1$ | $i+2$ |
|---|---|---|---|---|---|
| $2hD =$ | | $-1$ | $0$ | $1$ | |
| $h^2D^2 =$ | | $1$ | $-2$ | $1$ | |
| $2h^3D^3 =$ | $-1$ | $2$ | $0$ | $-2$ | $1$ |
| $h^4D^4 =$ | $1$ | $-4$ | $6$ | $-4$ | $1$ |

مؤثرات الفروق المركزية

Solution of Second - Order Problems by Central Differences.

Example 5. Solve the beam shown in fig. and find the deflection at the points. $EI$ = constant, $h = 1$ m



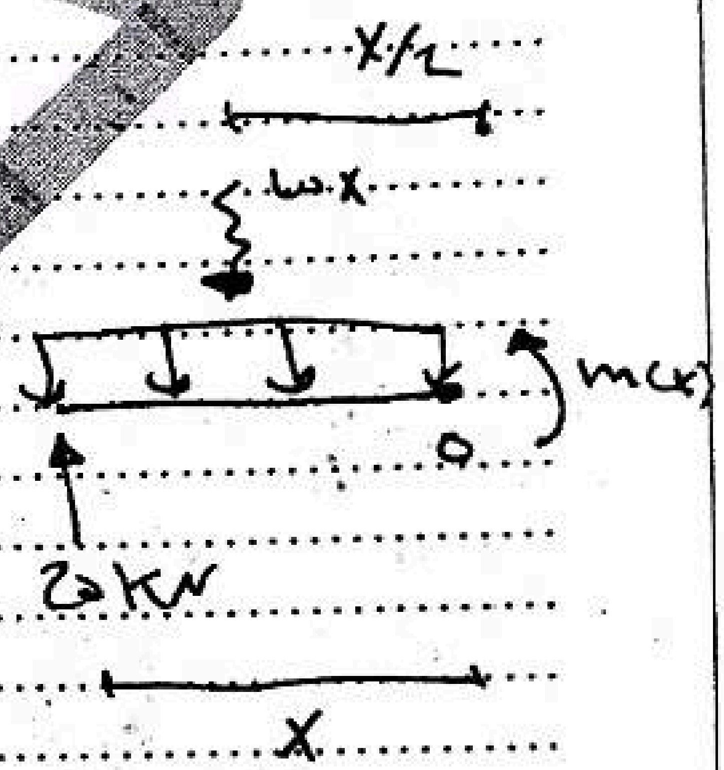
B.C.:- $y_0 = y_4 = 0$

$M_0 = M_4 = 0$

$\therefore \left.\frac{d^2y}{dx^2}\right|_0 = 0 \quad , \quad \left.\frac{d^2y}{dx^2}\right|_4 = 0$

Soln.:-

$M = 0$ $\quad \boxed{\frac{d^2y}{dx^2}} = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$




$y_{-1} - 2*0 + y_1 = 0 \Rightarrow y_{-1} = -y_1$

$y_3 - 2*0 + y_5 = 0 \Rightarrow y_3 = -y_5$

$\sum M_o = 0 \; +\circlearrowleft$

$M(x) = 20x - 10x\left(\frac{x}{2}\right)$

$= 20x - 5x^2$

$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$

$\frac{d^2y}{dx^2} = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$

$y_{i-1} - 2y_i + y_{i+1} = \frac{h^2}{EI}(20x_i - 5x_i^2)$

$i = 1$

$\cancel{y_0}^{\,0} - 2y_1 + y_2 = \frac{1}{EI}(20*1 - 5*1^2)$

$-2y_1 + y_2 = \frac{1}{EI}(15)$ ------ ①

$i = 2$

$y_1 - 2y_2 + y_3 = \frac{1}{EI}(20*2 - 5*2^2)$

$y_1 - 2y_2 + y_3 = \frac{20}{EI}$ ---- (2)

$i = 3$

$y_2 - 2y_3 + \cancel{y_4} = \frac{1}{EI}(20*3 - 5*3^2)$

$y_2 - 2y_3 = \frac{15}{EI}$ ---- (3)

$$\begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \frac{1}{EI} \begin{Bmatrix} 15 \\ 20 \\ 15 \end{Bmatrix} \quad \frac{1}{2}r_1 + r_2$$

$$\begin{bmatrix} -2 & 1 & 0 \\ 0 & -1.5 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \frac{1}{EI} \begin{Bmatrix} 15 \\ 27.5 \\ 15 \end{Bmatrix} \quad \frac{1}{1.5}r_2 + r_3$$

$$\begin{bmatrix} -2 & 1 & 0 \\ 0 & -3/2 & 1 \\ 0 & 0 & -4/3 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \frac{1}{EI} \begin{Bmatrix} 15 \\ 27.5 \\ \frac{100}{3} \end{Bmatrix}$$

$0y_1 + 0y_2 - \frac{4}{3}y_3 = \frac{100}{3EI} \Rightarrow y_3 = \frac{-100}{4EI} = \frac{-25}{EI}$

$0y_1 - \frac{3}{2}y_2 + \frac{-25}{EI} = \frac{27.5}{EI} \Rightarrow \frac{-3}{2}y_2 = \frac{52.5}{EI} \Rightarrow y_2 = \frac{-5}{3}$

$-2y_1 - \frac{52.5*2}{3EI} = \frac{15}{EI} \Rightarrow -2y_1 = \frac{52.5}{3EI} + \frac{45}{3EI}$

$y_1 = \frac{-1}{2EI}[2*52.5+45]*\frac{1}{3} = \frac{-1}{6EI}[150] = \frac{-25}{EI}$

$\therefore \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \frac{-1}{EI} \begin{Bmatrix} 25 \\ 35 \\ 25 \end{Bmatrix}$