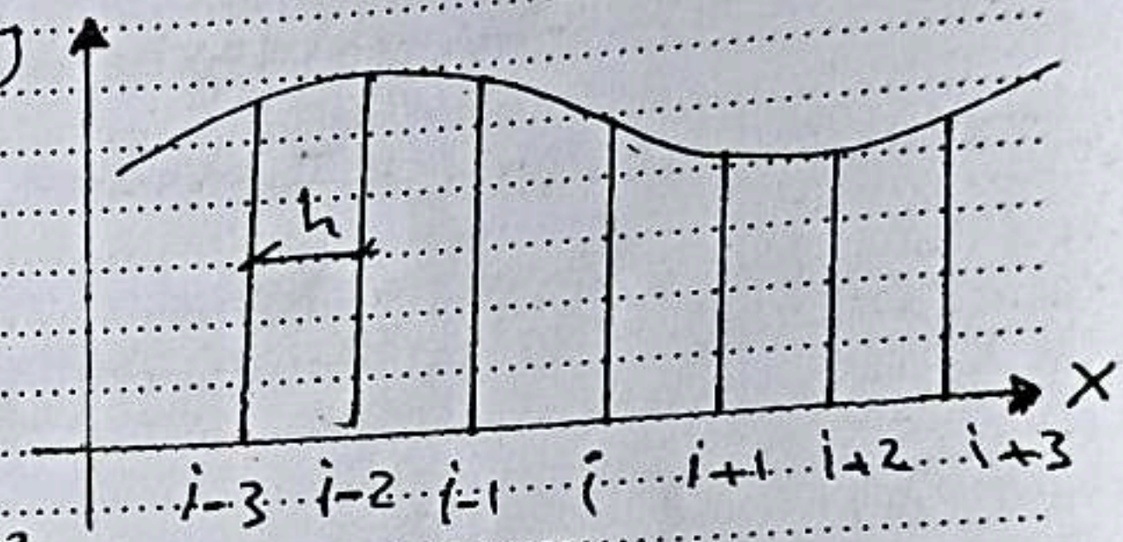
Highway & Transp. Eng. Dep.
Assist Lec. Israa Kh. Muhsen

Engineering Analysis & Numerical Methods

Finite Differences and their Applications

1. Backward Differences

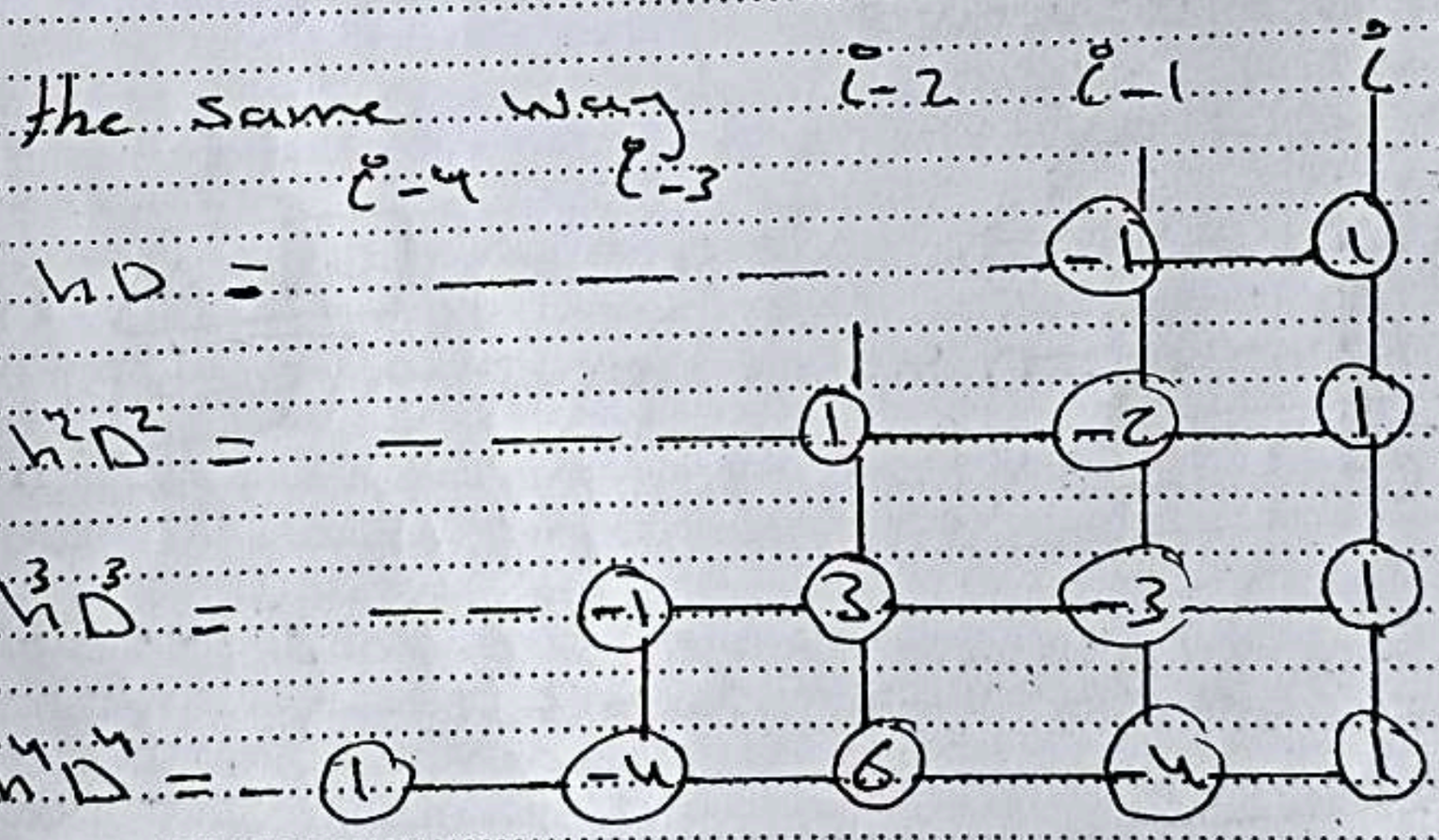


$$\frac{df}{dx} = \nabla y_i = \frac{y_i - y_{i-1}}{h}$$

$$\frac{d^2f}{dx^2} = \nabla^2 y_i$$

$$= \frac{\frac{y_i - y_{i-1}}{h} - \frac{y_{i-1} - y_{i-2}}{h}}{h} = \frac{y_i - 2y_{i-1} + y_{i-2}}{h^2}$$

In the same way



موضوعات المفروض الكلمة

$F(x) = 1 + 2X - \frac{1}{3}X^2$

Example :- Find the polynomial of min. degree which fit the given data & find the value of X_0.

* لدينا n=3 لكن هناك قيمة
X_0 مجهولة تمثل عدد النقاط
نقطة ايضا

X	X_0	2
$F(x)$	3	4
$\bar{F}(x)$		1

$n = 3$

$n - 1 - 1 = 1$

$F(x) = a_0 + a_1 X$

$F'(x) = a_1$

$F'(2) = 1 \Rightarrow 1 = a_1 \Rightarrow \boxed{a_1 = 1}$

$F(2) = 4 \Rightarrow 4 = a_0 + a_1 * 2 \Rightarrow \boxed{a_0 = 2}$

$\boxed{F(x) = 2 + X}$

$F(X_0) = 3$

$3 = 2 + X_0 \Rightarrow X_0 = 3 - 2 = 1$

$\boxed{X_0 = 1}$

Example 1:- Constracte The table for finite Diffrechas from the data shown below.

X	f(x)	D_1	D_2	D_3	D_4
0	0				
		1			
1	1		2		
		3		0	
2	4		2		0
		5		0	
3	9		2		
		7			
4	16				

$5-1=4$ use D_4

$1-0=1$ $2-1=1$ $\therefore$ equal

$=$ $=$

Example 2:- Constracte the table for finite Diff. from the data shown below.

X	f(x)	D_1	D_2
0	1		
		1	
1	2		$\frac{1-1}{3-0}=0$
		1	
3	4		$\frac{1-1}{4-1}=0$
		1	
4	5		$\frac{1-1}{7-3}=0$
		1	
7	8		

$\therefore$ equal

$$+\frac{(X/60)(X/60-1)(X/60-2)(X/60-3)(X/60-4)}{5!}(0.4643$$

if $X = 15$

$\therefore$ $F(150) = 0.4364$

EX 4/8- From the data shown below find $f(2.3)$, $f'(3.1)$, $f''(3.1)$.

X	f(x)	D_1	D_2	D_3	D_4
0	-4	1			
1	-3	3	2	6	
2	0	5	2	0	0
3	5	7	2		
4	12				

$$x = x_0 + rh$$

$$r = \frac{x - x_0}{h}$$

$h = 1$

$x_0 = 0$

$\therefore \boxed{r = X}$

$$F(x) = -4 + \frac{r(1)}{1!} + \frac{2r(r-1)}{2!} + 0 + 0$$

$$f(x) = r^2 - 4$$

$$f(x) = x^2 - 4$$

$$f(2.3) = (2.3)^2 - 4 = 1.24$$

$$f'(x) = 2x = 2(3.1) = 6.2$$

EX 3:- from the data shown below derive expression of f(x) and then used this formula to find f(150), ~~and~~ a

Time (sec)	Speed (m/s)	D1	D2	D3	D4	D5
0	0					
		0.0824				
60	0.0824		0.1019			
		0.1923		0.0733		
120	0.2744		0.1832		0.1024	
		0.3755		0.1762		0.4644
180	0.6502		0.3594		0.5673	
		0.7349		0.7433		
240	1.3851		1.1029			
		1.8378				
300	3.2229					

6-1 ⇒ ⇒ use D5

Gregory - Newton formula

$$f(x) = f_0 + r\Delta f_0 + \frac{r(r-1)}{2!}\Delta^2 f_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 f_0 + \cdots$$

$$X = X_0 + rh$$

$$X = 0 + r(60)$$

$$\therefore r = \frac{X}{60}$$

$$f(X) = 0 + \left(\frac{X}{60}\right)(0.0824) + \frac{\left(\frac{X}{60}\right)\left(\frac{X}{60}-1\right)}{2!}(0.1099) + \frac{\left(\frac{X}{60}\right)\left(\frac{X}{60}-1\right)\left(\frac{X}{60}-2\right)}{3!}(0.0733) + \frac{\left(\frac{X}{60}\right)\left(\frac{X}{60}-1\right)\left(\frac{X}{60}-2\right)\left(\frac{X}{60}-3\right)}{4!} * 0.1024$$

Ex 5 :- Find F(2) from the data shown below.

X	F(x)	D_1	D_2	D_3	D_4	D_5
-1	3.00	-5 *				
0	-2.0		5.5 *			
		3.250		-1		
0.5	0.375		3.5		0	
		6.75		-1		0
1.0	3.00		1.00			
		8.75		-1	0	
2.5	16.125		-1.500			
		5.75				
3	19.00					

6-1 = 5 ⇒ D_5

* D_1

$$\frac{-2-(3)}{0-(-1)} = -5$$

* D_2

$$\frac{3.25-(-5)}{0.5-(-1)} = 5.5$$

Newton's Divided Difference Formula

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) \cdots$$

$$f(2) = -2 + (2-0) * 3.25 + (2-0)(2-0.5)(3.5) + (2-0)(2-0.5)(2-1)(-1) = 12$$