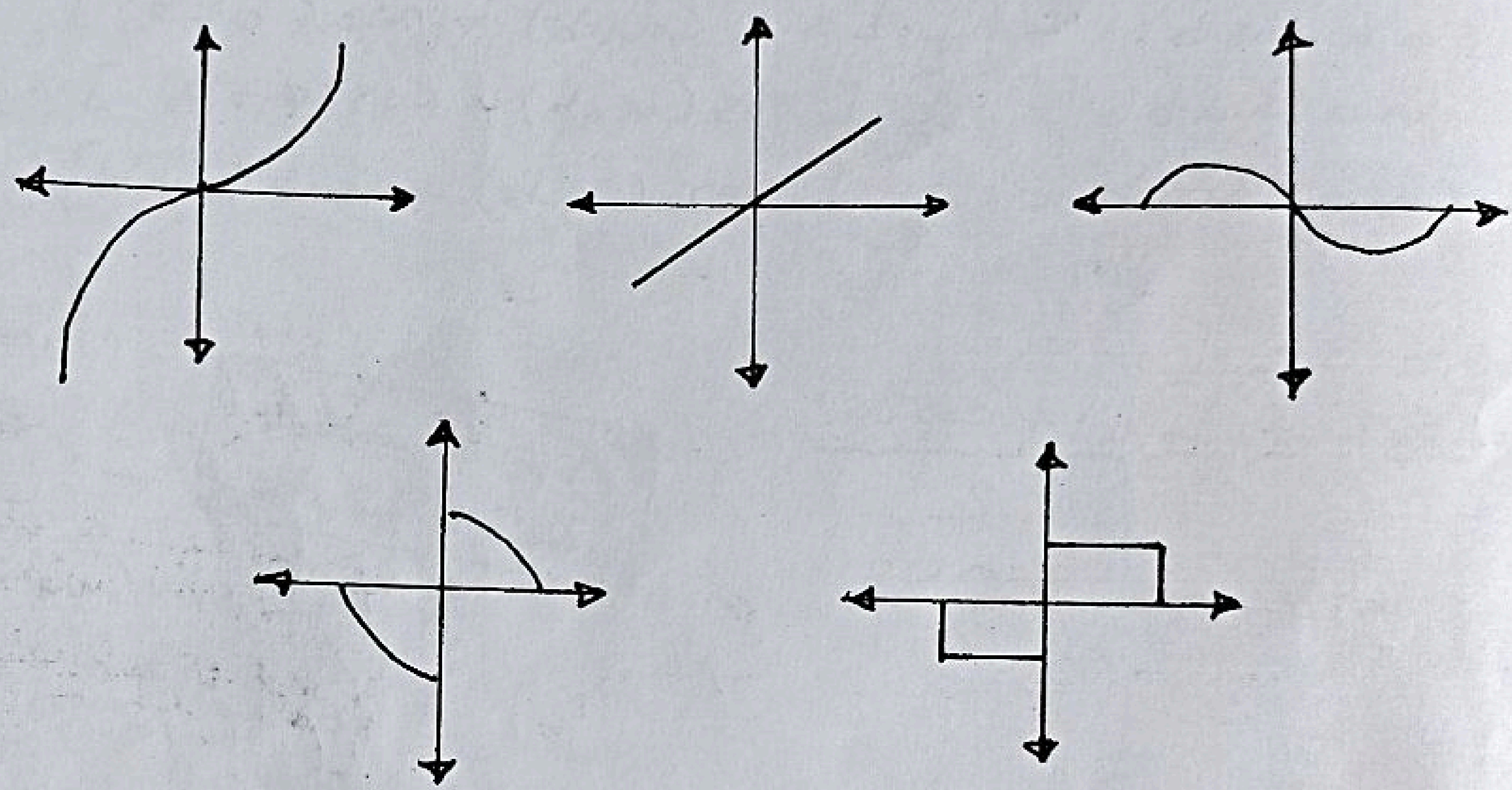
Engineering Analysis & Numerical Methods

Odd Function :- (الدوال الفردية)

symm. @ the origin

$f(t) = -f(-t)$



* Period (T)

$T = 3-(-2) = 5$ $\qquad -2 < x < 3$

$P = \frac{T}{2} = \frac{5}{2}$

$T = 4-0 = 4$ $\qquad 0 < x < 4$

$P = \frac{T}{2} = 2$

$a < x < b$ للدالة $P = \frac{b-a}{2}$ دائماً *

تمثل (half range.)

Example:- What is the fourier expansion of the periodic function whose definition in one period?

$$f(t) = \begin{cases} t-1 & -1 < t < 0 \\ t+1 & 0 < t < 1 \end{cases}$$

Solu:- $P = \frac{T}{2} = \frac{1-(-1)}{2} = 1$

$$a_o = \frac{1}{1} \int_{-1}^{0} (t-1)\,dt + \int_{0}^{1} (t+1)\,dt$$

$$= \left[\frac{t^2}{2} - t\right]_{-1}^{0} + \left[\frac{t^2}{2} + t\right]_{0}^{1}$$

$$= \left[(0) - \left(\cancel{\tfrac{1}{2}} + \cancel{1}\right)\right] + \left[\left(\cancel{\tfrac{1}{2}} + \cancel{1}\right) - 0\right]$$

$$\boxed{a_o = 0}$$

$$a_n = \int_{-1}^{0} (t-1)\cos n\pi t\,dt + \int_{0}^{1} (t+1)\cos n\pi t\,dt$$

مشتقة	تكامل		
$t-1$ (+)	$\cos n\pi t$	$t+1$	$\cos n\pi t$
1 (−)	$\frac{1}{n\pi}\sin n\pi t$	1	$\frac{1}{n\pi}\sin n\pi t$
0	$\frac{-1}{n^2\pi^2}\cos n\pi t$	0	$\frac{-1}{n^2\pi^2}\cos n\pi t$

$$\int_a^b u v\,dx = \left[u\int v\,dx - \int u'\left(\int v\,dx\right)dx\right]_a^b$$

$$a_n = \left[\frac{1}{n\pi}(t-1)\sin n\pi t + \frac{1}{n^2\pi^2}\cos n\pi t\right]_{-1}^{0} + \left[\frac{1}{n\pi}(t+1)\sin n\pi t + \frac{1}{n^2\pi^2}\cos n\pi t\right]_{0}^{1}$$

$$= \left[\cancel{\frac{1}{n^2\pi^2}}\right] - \left[\cancel{\frac{(-1)^n}{n^2\pi^2}}\right] + \left[\cancel{\frac{(-1)^n}{n^2\pi^2}}\right] - \left[\cancel{\frac{1}{n^2\pi^2}}\right]$$

$$\boxed{a_n = 0}$$

$$b_n = \int_{-1}^{0}(t-1)\sin \pi n t \, dt + \int_{0}^{1}(t+1)\sin n\pi t \, dt$$

$$b_n = \left[\frac{-1}{n\pi}(t-1)\cos n\pi t + \frac{1}{n^2\pi^2}\sin n\pi t\right]_{-1}^{0} + \left[\frac{-1}{n\pi}(t+1)\cos n\pi t + \frac{1}{n^2\pi^2}\sin n\pi t\right]_{0}^{1}$$

$$= \left[\frac{1}{n\pi}\right] - \left[\frac{2}{n\pi}(-1)^n\right] + \left[\frac{-2(-1)^n}{n\pi} + \frac{1}{n\pi}\right]$$

$$= \frac{2}{n\pi} - \frac{4(-1)^2}{n\pi} = \frac{2}{n\pi}\left[1 - 2(-1)^n\right]$$

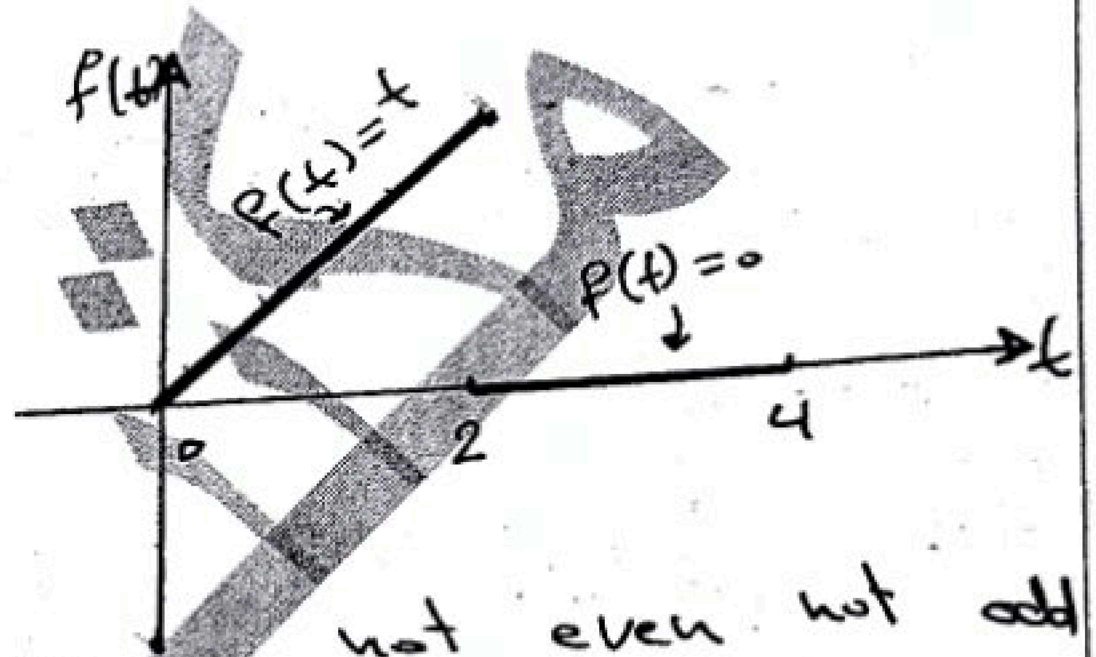
$$\therefore f(t) = \cancel{\frac{a_0}{2}}^{0} + \sum_{n=1}^{\infty} \cancel{a_n}^{0} \cos\frac{n\pi t}{p} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi t}{p}$$

$$p = 1$$

$$f(t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left[1 - 2(-1)^n\right]\sin n\pi t$$

Example: Find the Fourier expansion of the function whose definition.

$$F(t) = \begin{cases} t & 0 < t \le 2 \\ 0 & 2 < t < 4 \end{cases}$$



Sol:-

The period $= \frac{4-0}{2} = 2$

find a_0, a_n, b_n

not even not odd

$$a_0 = \frac{1}{P}\int_0^2 t\, dt + \frac{1}{P}\int_2^4 0\, dt$$

$$a_0 = \frac{1}{2}\int_0^2 t\, dt = \frac{1}{2}\left[\frac{t^2}{2}\right]_0^2$$

$$a_0 = \frac{1}{2}\left(\frac{4}{2} - 0\right) \Rightarrow \boxed{a_0 = 1}$$

$$a_n = \frac{1}{P}\int_0^2 t\cos\frac{n\pi t}{P}\, dt + \frac{1}{P}\int_2^4 0\cdot\cos\frac{n\pi t}{P}\, dt$$

$$a_n = \frac{1}{2}\int_0^2 t\cos\frac{n\pi t}{2}\, dt$$ integration by part

$$a_n = \frac{1}{2}\left[t\cdot\frac{2}{n\pi}\sin\frac{n\pi}{2}t - \frac{4}{n^2\pi^2}\cos\frac{n\pi t}{2}\right]_0^2$$

$$a_n = \frac{1}{2}\left[\left(\frac{2(2)}{n\pi}\sin\frac{n\pi}{2}(2) + \frac{4}{n^2\pi^2}\cos\frac{n\pi}{2}(2)\right) - \left(\frac{2(0)}{n\pi}\sin\frac{n\pi}{2}(0) + \frac{4}{n^2\pi^2}\cos\frac{n\pi}{2}(0)\right)\right]$$

$$a_n = \frac{1}{2}\left(\frac{4}{n^2\pi^2}\cos n\pi + (-)\frac{4}{n^2\pi^2} \right)$$

$$a_n = \frac{2}{n^2\pi^2}\cos n\pi - \frac{2}{n^2\pi^2}$$

$$\therefore \boxed{a_n = \frac{2}{n^2\pi^2}(\cos n\pi - 1)}$$

$$b_n = \frac{1}{p}\int_0^2 t \sin\frac{n\pi}{p} t \, dt + \frac{1}{p}\int_2^4 0 \sin\frac{n\pi}{p} t \, dt$$

$$b_n = \frac{1}{2}\int_0^2 t \sin\frac{n\pi}{2} t \, dt$$

$$b_n = \frac{1}{2}\left[\frac{-2t}{n\pi}\cos\frac{n\pi}{2} t + \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2} t \right]_0^2$$

$$b_n = \frac{1}{2}\left[\left(\frac{-2(2)}{n\pi}\cos\frac{n\pi}{2}(2) + \frac{4}{n^2\pi^2}\cancelto{0}{\sin\frac{n\pi}{2}(2)} \right) - \left(\cancelto{0}{\frac{-2(0)}{n\pi}\cos\frac{n\pi}{2}(0)} + \frac{4}{n^2\pi^2}\cancelto{0}{\sin\frac{n\pi}{2}(0)} \right) \right]$$

$$b_n = \frac{1}{2}\left(\frac{-4}{n\pi}\cos n\pi \right)$$

$$\therefore \boxed{b_n = \frac{-2}{n\pi}\cos n\pi}$$

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{p} t + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{p} t$$

$$\therefore f(t) = \frac{1}{2} + \left(\frac{-4}{\pi^2}\cos\frac{\pi}{2} t + 0 - \frac{4}{4\pi^2}\cos\frac{3\pi}{2} t + \cdots \right)$$
$$+ \left(\frac{2}{\pi}\sin\frac{1}{2}\pi t - \frac{1}{\pi}\sin\frac{2\pi}{2} t + \cdots \right)$$

Fourier Expansion :-

even fct $\rightsquigarrow$ Fourier series contain's (cos) term.

odd fct $\rightsquigarrow$ " " " (sin) " .

Theorm 1:- IF $f(t)$ is an even periodic fct. the Euler-Fouriers coeff. are given by:-

$$a_0 = \frac{2}{P}\int_0^P f(t)\,dt \quad ; \quad a_n = \frac{2}{P}\int_0^P f(t)\cos\frac{n\pi t}{P}\,dt$$

$$b_n = 0 \quad ; \quad f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi t}{P}$$

Theorm 2:- IF $f(t)$ is an odd periodic fct. the Euler-Fouriers coeff. are given by:-

$$a_0 = 0 \quad ; \quad a_n = 0$$

$$b_n = \frac{2}{P}\int_0^P f(t)\sin\frac{n\pi t}{P}\,dt \quad ; \quad f(t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi t}{P}$$

Half-Range Expansion of Fourier Series:-

IF a fct. F is considerd only in closed $[0, P]$, we can extened in 2 ways:-

Fourier Expansion :-

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Theorm 2:- IF $f(t)$ is an odd periodic fct. the Euler - Fourier's coeff. are given by:-

$$a_0 = 0 \quad ; \quad a_n = 0$$

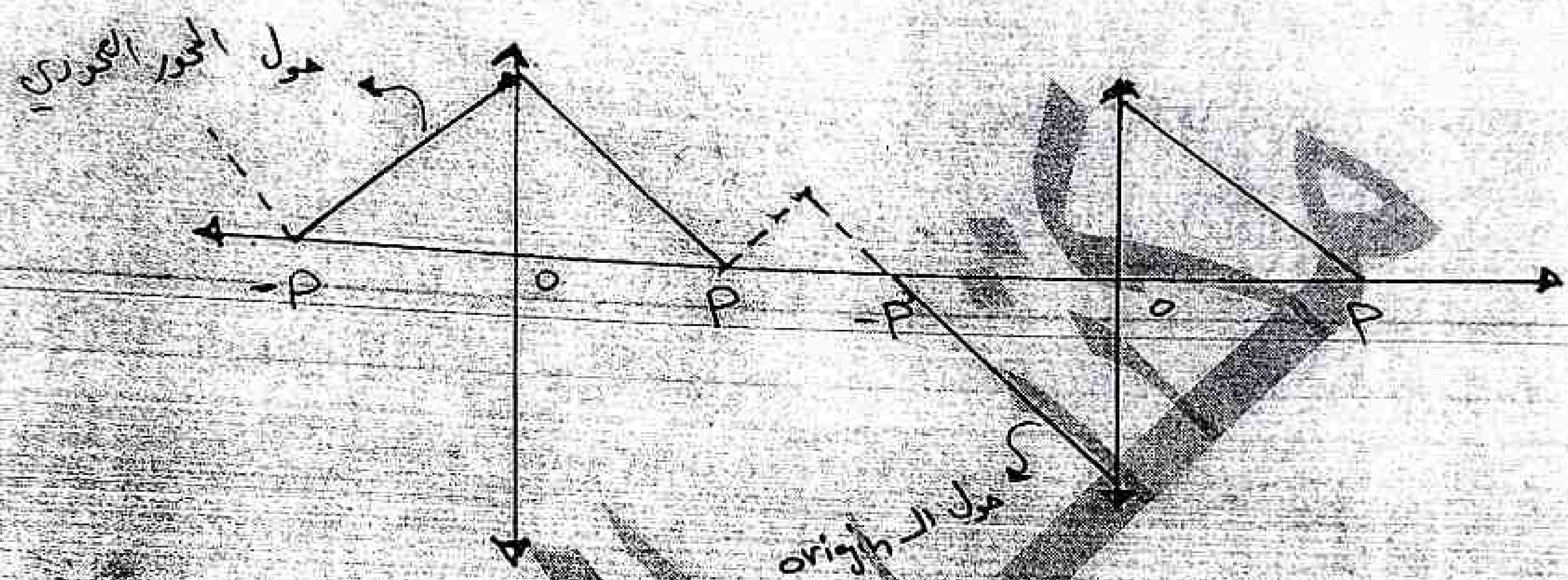
$$b_n = \frac{2}{p}\int_0^p f(t)\sin\frac{n\pi t}{p}\,dt \quad ; \quad f(t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi t}{p}$$

Half - Range Expansion of Fourier Series:-

IF a fct. F is considerd only in closed $[0, p]$, we can extened in 2 ways:-

1- Reflecting F in the vertical axis (even exte).

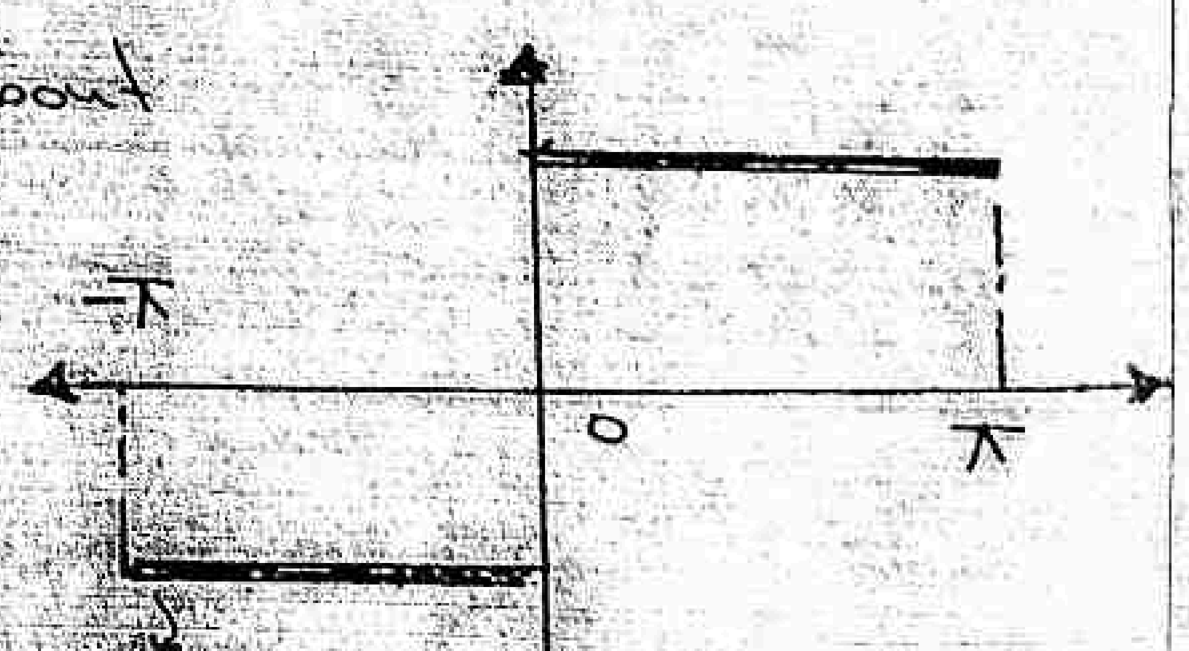
2- Reflecting F in the origin (odd exte.).



Example :- Find the Fourier expansion of the periodic function whose definition in one periodic is :-

$$f(x) = \begin{cases} -\frac{\pi}{4} & -\pi < x < 0 \\ \frac{\pi}{4} & 0 < x < \pi \end{cases}$$

odd function symm. about the origion



$a_0 = a_n = 0$

$$b_n = \frac{2}{P}\int_0^{\pi} \frac{\pi}{4} \sin\frac{n\pi}{P}x\,dx \quad f(x)$$

$$P = \frac{\pi-(-\pi)}{2} = \pi$$