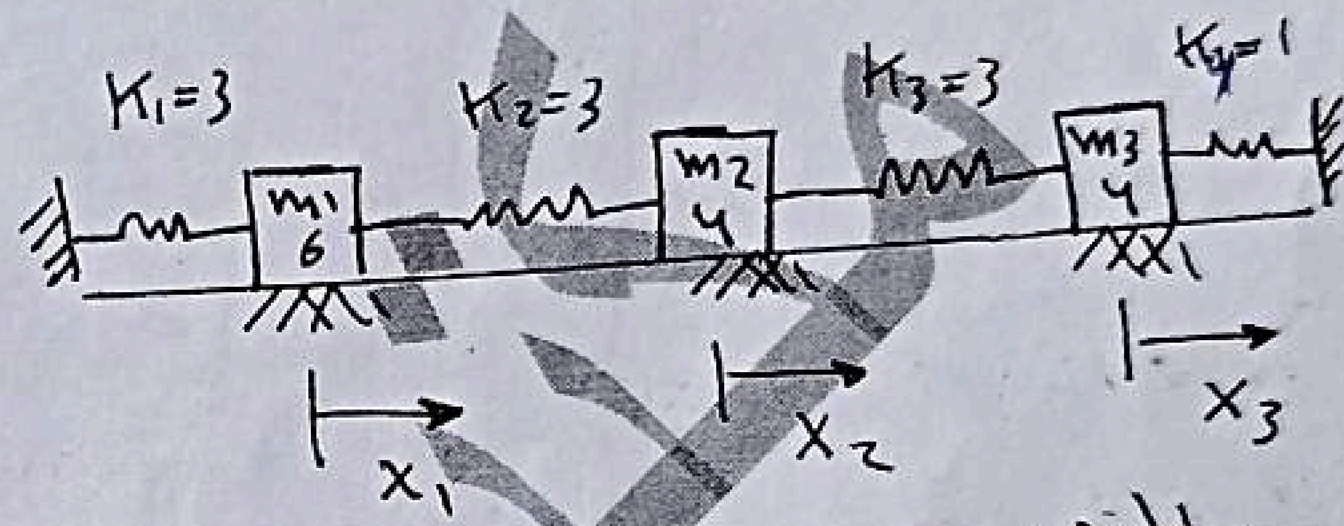
Example :- The three masses shown in fig. are initially displaced so that.

$(x_1)_o = 2$, $(x_2)_o = 1$, $(x_3)_o = 1$

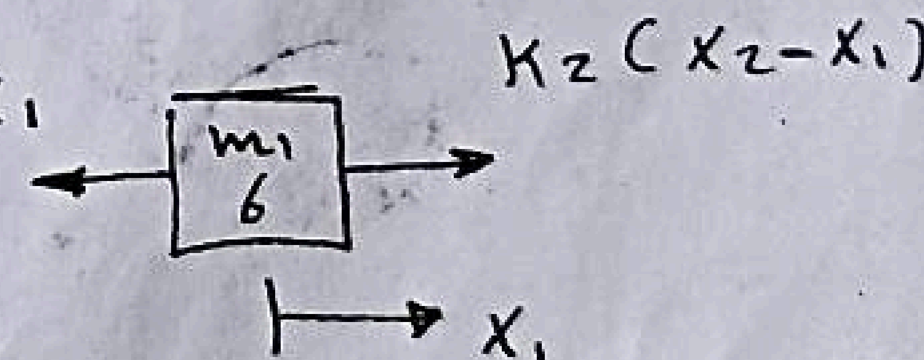


From these positions they begin to move with
$(\dot{x}_1)_o = 1$, $(\dot{x}_2)_o = 2$, $(\dot{x}_3)_o = 0$

Assuming that there is no friction in the system, determine the subsequent motion of each mass.

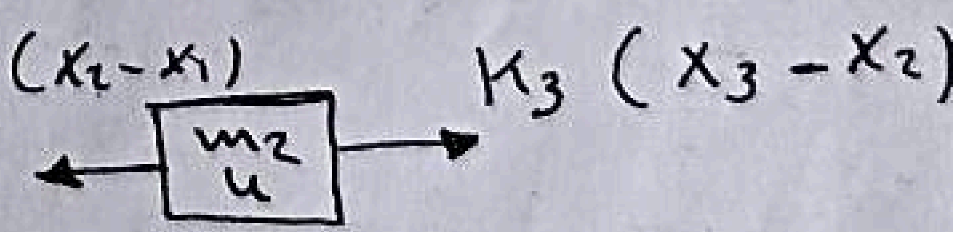
$ma = \Sigma F$

mass 1:-



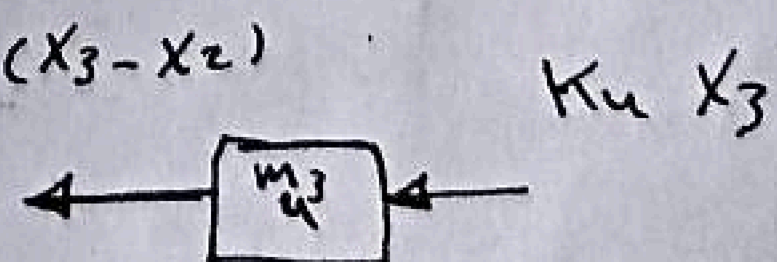
$$6 \frac{d^2x_1}{dt^2} = -3x_1 + 3(x_2 - x_1)$$

mass 2 :-



$$4 \frac{d^2x_2}{dt^2} = -3(x_2 - x_1) + 3(x_3 - x_2)$$

mass 3 :-



$$4 \frac{d^2x_3}{dt^2} = -3(x_3 - x_2) - x_3$$

$$6\ddot{X}_1 + 6X_1 - 3X_2 = 0$$
$$4\ddot{X}_2 + 6X_2 - 3X_1 - 3X_3 = 0$$
$$4\ddot{X}_3 + 4X_3 - 3X_2 = 0$$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{Bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \ddot{X}_3 \end{Bmatrix} + \begin{bmatrix} 6 & -3 & 0 \\ -3 & 6 & -3 \\ 0 & -3 & 4 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$[M]\{\ddot{X}\} + [K]\{X\} = \{0\}$$

$$\begin{bmatrix} (6D^2+6) & -3 & 0 \\ -3 & (4D^2+6) & -3 \\ 0 & -3 & (4D^2+4) \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\Delta = \begin{vmatrix} 6D^2+6 & -3 & 0 \\ -3 & 4D^2+6 & -3 \\ 0 & -3 & 4D^2+4 \end{vmatrix}$$

$$\Delta X_1 = \begin{vmatrix} 0 & -3 & 0 \\ 0 & 4D^2+6 & -3 \\ 0 & -3 & 4D^2+4 \end{vmatrix} = 0$$

$$\Delta X_2 = 0 \quad , \quad \Delta X_3 = 0$$

$$(4D^2-1)(D^2-1)(4D^2-9)X_1=0$$
$$(4D^2-1)(D^2-1)(4D^2-9)X_2=0$$
$$(4D^2-1)(D^2-1)(4D^2-9)X_3=0$$

$m_{1,2}=\mp\frac{1}{2}i$, $m_{3,4}=\mp i$, $m_{5,6}=\mp\frac{3}{2}i$

$$X_1=c_1\cos\frac{t}{2}+c_2\sin\frac{t}{2}+c_3\cos t+c_4\sin t+c_5\cos\frac{3}{2}t+c_6\sin\frac{3}{2}t.$$
$$X_2=c_7\cos\frac{t}{2}+c_8\sin\frac{t}{2}+c_9\cos t+c_{10}\sin t+c_{11}\cos\frac{3}{2}t+c_{12}\sin\frac{3}{2}t.$$
$$X_3=c_{13}\cos\frac{t}{2}+c_{14}\sin\frac{t}{2}+c_{15}\cos t+c_{16}\sin t\;c_{17}\cos\frac{3}{2}t+c_{18}\sin\frac{3}{2}t.$$

$$\begin{Bmatrix}X_1\\X_2\\X_3\end{Bmatrix}=c_1\begin{Bmatrix}2\\3\\3\end{Bmatrix}\cos\frac{t}{2}+c_2\begin{Bmatrix}2\\3\\3\end{Bmatrix}\sin\frac{t}{2}+c_3\begin{Bmatrix}1\\0\\-1\end{Bmatrix}\cos t+c_4\begin{Bmatrix}1\\0\\-1\end{Bmatrix}\sin t+c_5\begin{Bmatrix}2\\-5\\3\end{Bmatrix}\cos\frac{3}{2}t+c_6\begin{Bmatrix}2\\-5\\3\end{Bmatrix}\sin\frac{3}{2}t$$

at $t=0$ $(X_1)_0=2$, $(X_2)_0=-1$, $(X_3)_0=1$

$$\begin{Bmatrix}2\\-1\\1\end{Bmatrix}=\begin{bmatrix}2&1&2\\3&0&-5\\3&-1&3\end{bmatrix}\begin{Bmatrix}c_1\\c_3\\c_5\end{Bmatrix}$$

$c_1=\frac{1}{4}$, $c_3=\frac{4}{5}$, $c_5=\frac{7}{20}$

Fourier Series

Periodic Function :-

A function $f(t)$ can be expressed as a series of sine and cosine terms.

Specifically, since, for all integral values of n.

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{l} x + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{l} x$$

formulas for the coefficients a_0, a_n and b_n in the expansion

$$a_0 = \frac{1}{p} \int_{d}^{d+2p} f(t)\, dt$$

$$a_n = \frac{1}{p} \int_{d}^{d+2p} f(t) \cos \frac{n\pi t}{p}\, dt$$

$$b_n = \frac{1}{p} \int_{d}^{d+2p} f(t) \sin \frac{n\pi t}{p}\, dt$$

Note 1:-

$$\int \cos x \, dx = \sin x$$

$$\int \sin x \, dx = -\cos x$$

$$\int 2\cos 2x \, dx = \sin 2x$$

$$\int 2\sin 2x \, dx = -\cos 2x$$

$$\int \cos 2x \, dx = \int \frac{2}{2} \cos 2x \, dx$$

Note 2:-

$$\sin(a \mp b) = \sin a \cos b \mp \cos a \sin b$$

$$\cos(a \mp b) = \cos a \cos b \mp \sin a \sin b$$

$$\sin a \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

$$\sin a \sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$$

$$\cos a \cos b = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$$

Note 3:-

$$\cos^2 a = \frac{1 + \cos 2a}{2}$$

$$\sin^2 a = \frac{1 - \cos 2a}{2}$$

Eeven Function:- (الدوال الزوجية)
symm. @ the vertical axis.

$$f(t) = f(-t)$$