Highway & Transp. Eng. Dep.
Assist Lec. Israa Kh. Muhsen

Engineering Analysis & Numerical Methods

Systems of Simultaneous Linear Equations:-

$$\frac{dx}{dt} + 3x + 2\frac{d^2y}{dt^2} = e^{-t} \quad \ldots\ldots (1)$$

$$\frac{d^2x}{dt^2} - \frac{dx}{dt} + x - \frac{d^2y}{dt^2} + \frac{dy}{dt} + y = \sin t \quad \ldots (2)$$

$$Dy = \frac{dy}{dt} \quad , \quad Dx = \frac{dx}{dt}$$

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$$(D+3)x + 2D^2y = e^{-t} \quad \ldots\ldots (1')$$

$$(D^2 - D+1)x - (D^2 + D + 1)y = \sin t \quad \ldots (2')$$

$$\frac{dx}{dt} + \frac{dy}{dt} + y = 1 \quad \ldots (1)$$

$$\frac{dx}{dt} - \frac{dz}{dt} + 2x + z = 1 \quad \ldots (2)$$

$$\frac{dy}{dt} + \frac{dz}{dt} + y + 2z = 0 \quad \ldots (3)$$

$$Dz = \frac{dz}{dt} \quad , \quad Dy = \frac{dy}{dt} \quad , \quad Dx = \frac{dx}{dt}$$

$$Dx + (D+1)y = 1 \quad \ldots (1')$$

$$(D+2)x - (D-1)z = 1 \quad \ldots (2')$$

$$(D+1)y + (D+2)z = 0 \quad \ldots (3')$$

Example :- Solve the following simultaneous diff. equations.

$(D-1)x + (D-2)y = 0$ ----①   multi. by $(D-5)$

$(D-5)x + (2D-7)y = e^{-t}$ ----②   multi. by $(D-1)$

$\cancel{(D-5)(D-1)}x + (D-2)(D-5)y = 0$

$\mp\cancel{(D-1)(D-5)}x \mp (2D-7)(D-1)y = \mp(D-1)e^{-t}$

$[(D-2)(D-5) - (2D-7)(D-1)]y = -(-e^{-t} - e^{-t})$ *

* Note :- $-(D-1)e^{-t} = -(De^{-t} - e^{-t}) = -(-e^{-t} - e^{-t}) = +2e^{-t}$

$(D^2 - 7D + 10 - 2D^2 + 9D - 7)y = 2e^{-t}$

*D هو عامل يدخل على المعادلة يعمل على إشتقاقها

$(-D^2 + 2D + 3)y = 2e^{-t}$ (multi. by -1)

$(D^2 - 2D - 3)y = -2e^{-t}$

$y = y_h + y_p$ to find y_h put $(D^2 - 2D - 3)y = 0$

or $(m^2 - 2m - 3) = 0 \Rightarrow (m-3)(m+1) = 0$

$m_1 = -1$, $m_2 = 3$

$\therefore y_h = c_1 e^{-t} + c_2 e^{3t}$

$y_p = Ae^{-t}$ (not o.k.)

So $y_p = Ate^{-t}$

$y_p' = -Ate^{-t} + Ae^{-t}$

$y_p'' = Ate^{-t} - Ae^{-t} - Ae^{-t} = Ate^{-t} - 2Ae^{-t}$

نعوض بالمعادلة الاصلية

$Ate^{-t} - 2Ae^{-t} + 2Ate^{-t} - 2Ae^{-t} - 3Ate^{-t} = 2e^{-t}$

$-4Ae^{-t} = 2e^{-t} \Rightarrow A = -\frac{1}{2}$

∴ $y_p = -\frac{1}{2}te^{-t}$

∴ $y = c_1e^{-t} + c_2e^{3t} - \frac{1}{2}te^{-t}$

$(D-1)x + (D-2)(c_1e^{-t} + c_2e^{3t} - \frac{1}{2}te^{-t}) = 0$

$(D-1)x + [(-c_1e^{-t} + 3c_2e^{3t} + \frac{1}{2}te^{-t} - \frac{1}{2}e^{-t}) - (2c_1e^{-t} + 2c_2e^{3t} - te^{-t})] = 0$

$(D-1)x = (3c_1 + \frac{1}{2})e^{-t} - c_2e^{3t} - \frac{3}{2}te^{-t}$

$\frac{dx}{dt} - x = (\frac{6c_1+1}{2})e^{-t} - c_2e^{3t} - \frac{3}{2}te^{-t}$

$\frac{dx}{dt} + P(t)x = Q(t)$ *

*بعد ان وجدنا حل المعادلة y(t) يجب استخراج الحل x(t) وبعد ذلك بتعويض y في احدى المعادلات ① او ② ايهما السهل اي الاقل درجة

* معادلة من الدرجة الاولى ح ع linear

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Example: Solve the following simultaneous diff. equations.

$(D-1)x + Dy = 2t+1 \quad \text{----} (1)$

$(2D+1)x + 2Dy = t \quad \text{----} (2)$

multip. by 2

$$(2D-2)x + 2Dy = 4t+2$$
$$\mp(2D\mp 1)x \mp 2Dy = \mp t$$

$$-3x = 3t+2$$
$$x = -t - \frac{2}{3}$$

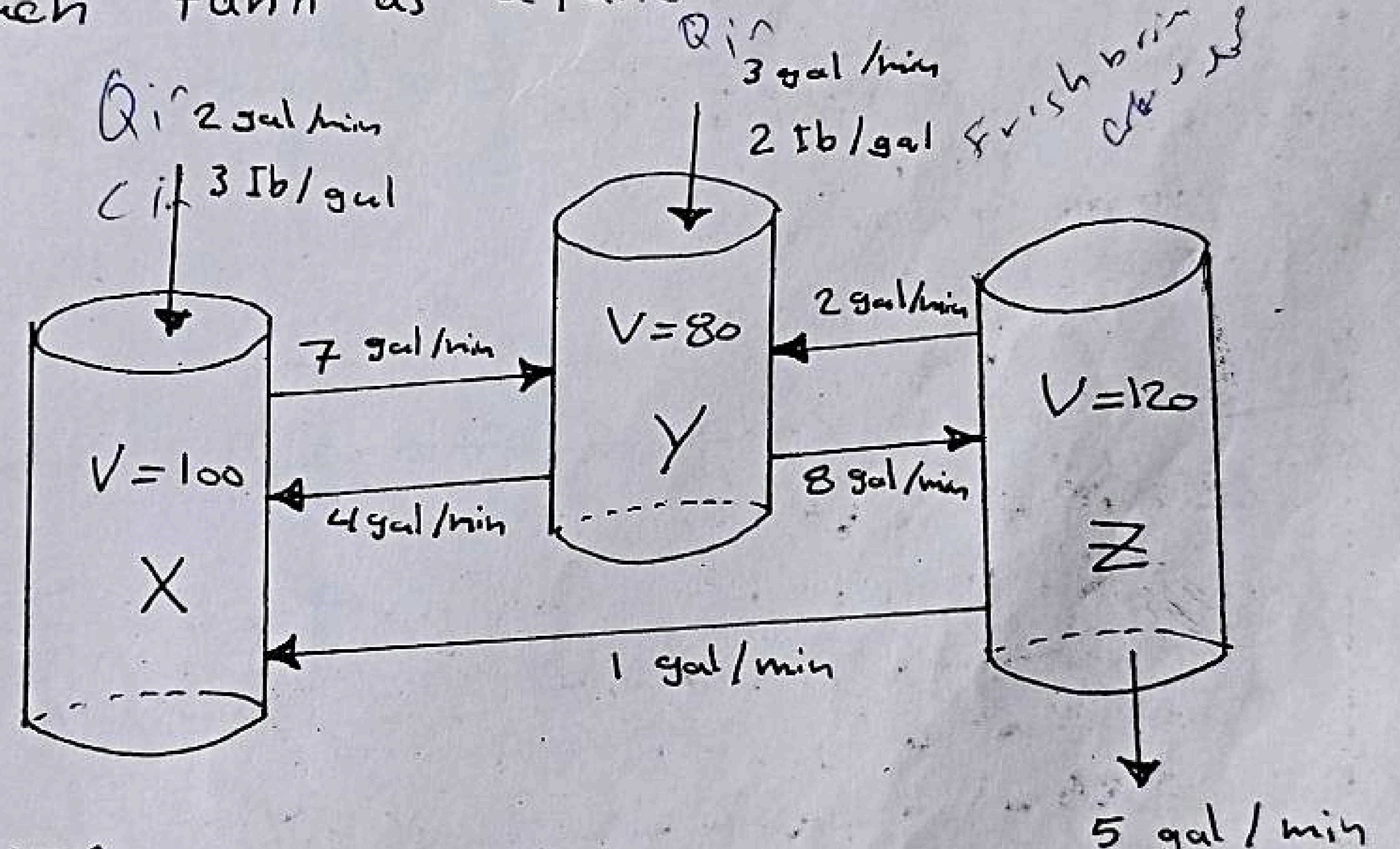
$$Dy = 2t+1 - (D-1)x$$
$$= 2t+1 - (D-1)(-t-\tfrac{2}{3})$$
$$= 2t+1 - [D(-t-\tfrac{2}{3}) - (-t-\tfrac{2}{3})]$$
$$= 2t+1 - [-1+t+\tfrac{2}{3}]$$
$$= 2t+1 - [t-\tfrac{1}{3}]$$
$$= t+1+\tfrac{1}{3}$$
$$= t+\tfrac{4}{3}$$

$$Dy = t+\frac{4}{3} \Rightarrow \frac{dy}{dt} = t+\frac{4}{3}$$

$$y = \frac{t^2}{2} + \frac{4}{3}t + c_1$$

Applications

Example:- Three tanks are conected as shown in the fig., liquid circulate through the tanks at rates as shown, find the amount of salt in each tank as a function of (t).



Solution:-

x = amount of salt in first tank.
y = 〃 〃 〃 〃 second tank.
z = 〃 〃 〃 〃 third tank.

$$\frac{ds}{dt} = C_{in}\,Q_{in} - C_{out}\,Q_{out}$$

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First tank :-

$$C_{in} = \frac{y}{80} \quad , \quad C_{in} = \frac{z}{120}$$

$C_{out} = \frac{x}{100}$

$\therefore \frac{dx}{dt} = 2*\cancel{3}^{6} + 4 * \frac{y}{80} + 1 * \frac{z}{120} - 7 * \frac{x}{100}$

$Dx + \frac{7}{100} x - \frac{y}{20} - \frac{z}{120} = 6$

$(D + \frac{7}{100}) x - \frac{1}{20} y - \frac{1}{120} z = 6 \quad \ldots\ldots (1)$

2nd tank :-

$\frac{dy}{dt} = (3 * 2 + 2 * \frac{z}{120} + 7 * \frac{x}{100}) - (4 * \frac{y}{80} + 8 * \frac{y}{80})$

$\therefore \frac{dy}{dt} + \frac{\cancel{12}^{3}}{\cancel{80}_{20}} y - \frac{2}{\cancel{100}_{50}} z - \frac{7}{100} x = 6$

$(D + \frac{3}{20}) y - \frac{1}{50} z - \frac{7}{100} x = 6 \quad \ldots\ldots (2)$

3rd tank :-

$\frac{dz}{dt} = 8 \frac{x}{80} - (2 * \frac{z}{120} + 5 * \frac{z}{120} + 1 * \frac{z}{120})$

$\therefore Dz + \frac{8}{120} z - \frac{1}{10} y = 0$

$(D + \frac{1}{15}) z - \frac{1}{10} y = 0 \quad \ldots\ldots (3)$

$$\begin{bmatrix} D+\frac{7}{100} & \frac{-1}{20} & \frac{1}{120} \\ \frac{-7}{100} & D+\frac{3}{20} & \frac{-1}{50} \\ 0 & \frac{-1}{10} & \left(D+\frac{1}{15}\right) \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} 6 \\ 6 \\ 0 \end{Bmatrix}$$

$$\Delta = D+\frac{7}{100}\begin{vmatrix} D+\frac{3}{20} & \frac{-1}{50} \\ \frac{-1}{10} & D+\frac{1}{15} \end{vmatrix} + \frac{1}{20}\begin{vmatrix} \frac{-7}{100} & \frac{-1}{50} \\ 0 & \left(D+\frac{1}{15}\right) \end{vmatrix}$$

$$+\frac{1}{120}\begin{vmatrix} \frac{-7}{100} & D+\frac{3}{20} \\ 0 & \frac{-1}{10} \end{vmatrix}$$

$$\Delta = D^3 + \frac{7}{100}D^2 + \frac{39.2}{120000}$$ ⟹ تكملة الحل H.W

$$\Delta_x = \begin{vmatrix} 6 & \frac{-1}{20} & \frac{-1}{120} \\ 6 & \left(D+\frac{3}{20}\right) & \frac{-1}{50} \\ 0 & \frac{-1}{10} & \left(D+\frac{1}{15}\right) \end{vmatrix}$$

$$\Delta_y = \begin{vmatrix} \left(D+\frac{7}{100}\right) & 6 & \frac{-1}{120} \\ \frac{-7}{100} & 6 & \frac{-1}{50} \\ 0 & 0 & \left(D+\frac{1}{15}\right) \end{vmatrix}$$

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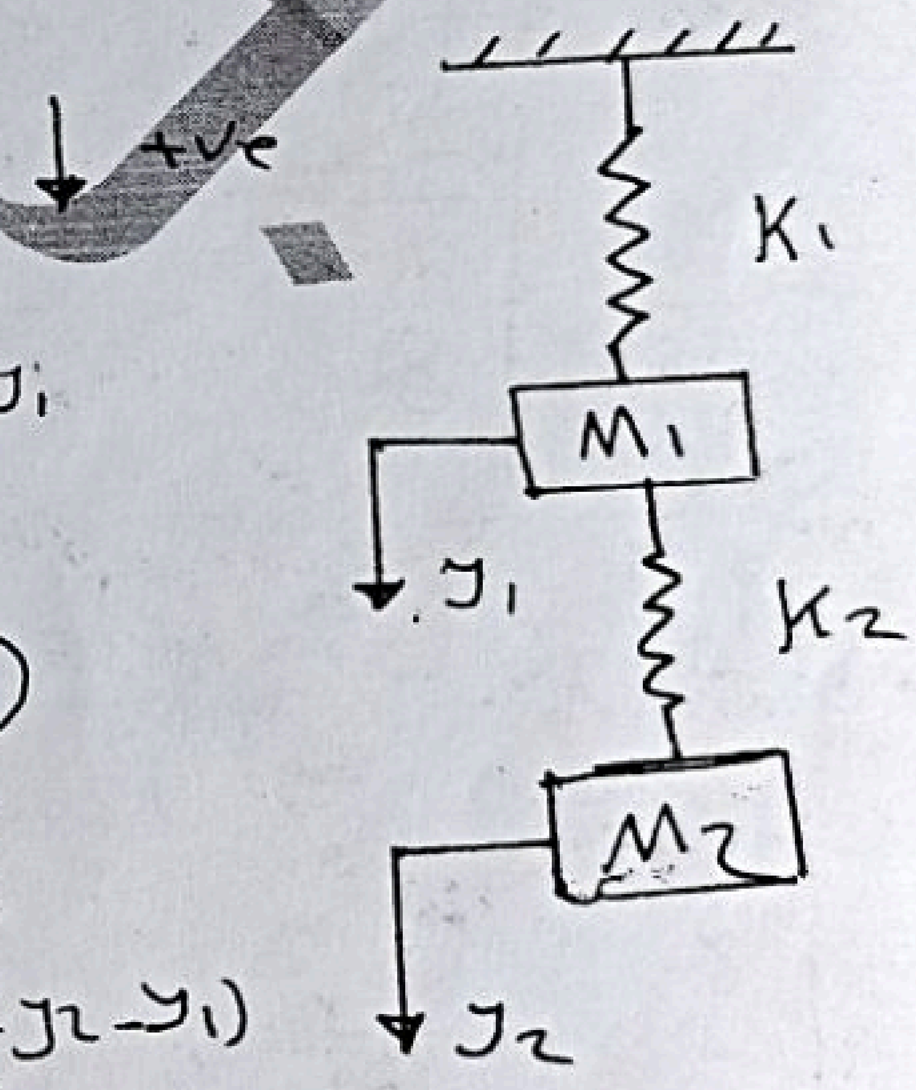
$$\Delta z = \begin{vmatrix} D+\frac{7}{100} & \frac{-1}{20} & 6 \\ \frac{-7}{100} & D+\frac{3}{20} & 6 \\ 0 & \frac{-1}{10} & 0 \end{vmatrix}$$

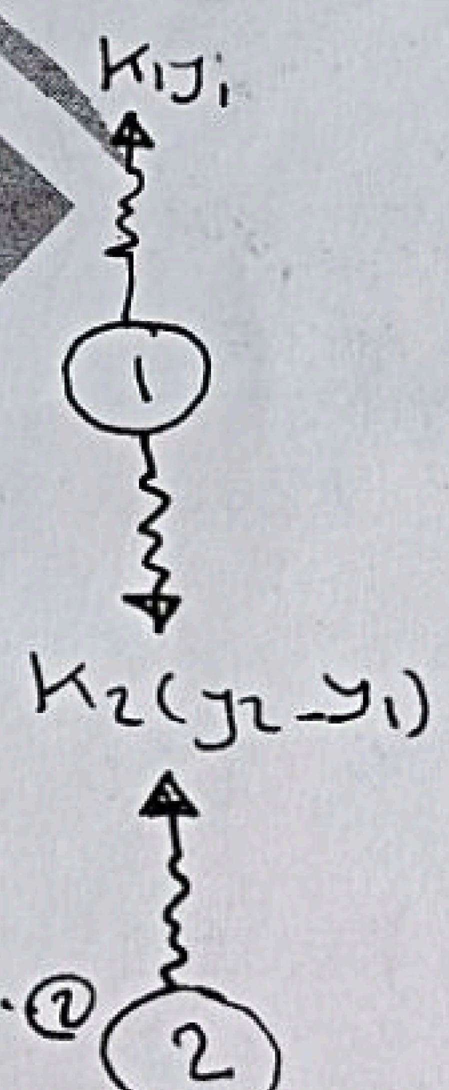
Example :- Set up the equations for the system shown in fig.

$m_1 = 2\ Kg$, $M_2 = 3\ Kg$

$K_1 = 1$, $K_2 = 2$



$+\downarrow \sum F_y = ma$



$m_1 a = K_2(y_2 - y_1) - K_1 y_1$ ....①

$m_2 a = -K_2(y_2 - y_1)$ ..②

$2\ddot{y}_1 = 2(y_2 - y_1) - y_1$ .... ①

$3\ddot{y}_2 = -2(y_2 - y_1)$ ... ②

$(2D^2 + 3)y_1 - 2y_2 = 0$

$-2y_1 + (3D^2 + 2)y_2 = 0$