

Engineering Analysis & Numerical Methods

$$= \left[\frac{x^2}{2} + xy \right]_2^x + \left[xy + \frac{y^2}{2} \right]_2^y$$

$$= \frac{x^2}{2} + xy - 2 - 2y + 2y + y^2 - 6 - 9$$

$$\therefore \frac{x^2}{2} + xy + y^2 = 17$$

4) Linear First-Order Eq.s

The general form $\frac{dy}{dx} + P(x)y = Q(x)$

i. I.F = $e^{\int P(x) dx}$

ii. I.F * y = $\int I.F. Q(x) dx$

$$\frac{d}{dx} (y e^{\int P(x) dx}) = \frac{dy}{dx} e^{\int P(x) dx} + y P(x) e^{\int P(x) dx}$$

$$= e^{\int P(x) dx} \left(\frac{dy}{dx} + y P(x) \right)$$

$e^{\int P(x) dx}$ is an integrating factor

$$y e^{\int P(x) dx} = \int Q(x) \cdot e^{\int P(x) dx} dx + C$$

Example: Solve the D.E. $\frac{dy}{dx} + 2xy = 4x$

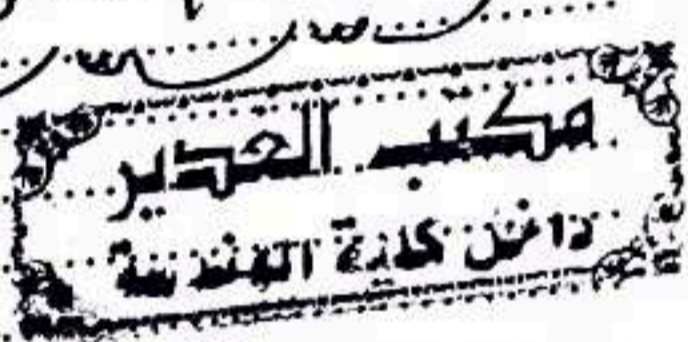
$$\int P(x) dx = 2x dx = x^2 \Rightarrow e^{\int P(x) dx} = e^{x^2}$$

$$y e^{x^2} = \int 4x e^{x^2} dx \Rightarrow x^2 2 e^{x^2} + C$$

~~$$y = 2x + C$$~~

$$u = x^2 \Rightarrow du = 2x dx$$

$$\int 4x e^{x^2} dx = \int 2 \cdot (2x dx) e^{x^2} = \int 2e^u du$$



$y = 2x + C$

$$2e^{x^2} + C$$

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Examples - Solve the D.E. $y' + \cos x \cdot y = \cos 2x \div (\sin x)$

$$\frac{dy}{dx} + \frac{\cos x}{\sin x} y = \frac{\cos 2x}{\sin x}$$

$$\therefore P(x) = \frac{\cos x}{\sin x} \quad Q(x) = \frac{\cos 2x}{\sin x}$$

$$I.F. = e^{\int P(x) dx} = e^{\int \frac{\cos x}{\sin x} dx} = e^{\ln \sin x} = \sin x$$

$$I.F. \cdot y = \int I.F. \cdot Q(x) dx$$

$$\sin x \cdot y = \int \sin x \cdot \frac{\cos 2x}{\sin x} dx$$

$$y \sin x = \frac{1}{2} \sin 2x + C$$

[5] Special First Order Bernoulli's Eq. -

A non-l. D.E. which can be transformed into l. D.E. by change of the dependent variables.

$$\frac{dy}{dx} + y P(x) = y^n Q(x)$$

or

$$y^{-n} \frac{dy}{dx} + y^{-n+1} P(x) = Q(x)$$

Note: $n = \text{any real no. } n > 0$

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$$z = y^{1-n}$$

$$\frac{dz}{dy} = (1-n)y^{-n} \quad ; \quad \frac{dz}{dx} = (1-n)y^{-n} \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{dz}{dx} \frac{y^n}{1-n} \quad \text{sub.s.} \left(\frac{dz}{dx} \frac{y^n}{1-n} \right) + P(x)y = Q(x)y^n$$

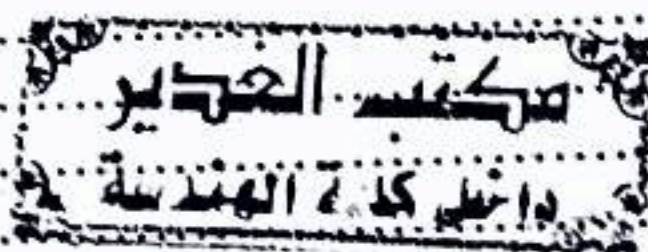
$$\frac{dz}{dx} + P(x)(1-n)y = Q(x)(1-n)$$

$$\underbrace{z' + (1-n)P(x)z = (1-n)Q(x)}_{\text{L.D.E. of } z}$$

i. let $z = y^{1-n}$

ii. $\frac{dz}{dx} = (1-n)y^{-n} \frac{dy}{dx}$

iii. مع المعادلة من الخطية وليست كذلك



Example 8. Solve the D.E. $3xy' + y + x^2y^4 = 0$

$$y' + \frac{1}{3x}y = -\frac{x}{3}y^4 \quad * (-3)$$

∴ $n=4$

$$z = y^{1-n} = y^{-3} \quad \frac{dz}{dx} = (1-n)y^{-4} \frac{dy}{dx}$$

$$\frac{dz}{dx} = -3y^{-4} \frac{dy}{dx} \implies \frac{dz}{dx} - \frac{1}{x}z = x$$

$$P(x) = \frac{-1}{x} \quad Q(x) = x$$

$$I.F = e^{\int \frac{dx}{x}} = \frac{1}{x} - x$$

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$$\frac{z}{x} = \int x - \frac{1}{x} dx + C$$

$$\Rightarrow \frac{z}{x} = x + C \Rightarrow z = x^2 + Cx$$

$$y^3 = z \Rightarrow y^3 = \frac{1}{x^2 + Cx}$$

H.W. :- Solve i. $y \ln y dx + (x - \ln y) dy = 0$

ii. $\frac{dy}{dx} + \frac{1}{3}y = \frac{1}{3}(1-2x)y^4$

Examples

Ex 1:- Solve the D.E. $(2y + x^2) dx = x dy$

$$M = 2y + x^2, N = -x \quad \therefore \text{non-homog.}$$

$$\frac{\partial M}{\partial y} = 2, \quad \frac{\partial N}{\partial x} = -1 \quad \therefore \text{not exact.}$$

$$\frac{2y + x^2}{x} = \frac{dy}{dx} \quad \therefore (x dx)$$

$$\frac{dy}{dx} - \frac{2}{x}y = x \quad \therefore \text{linear}$$

$$P(x) = \frac{-2}{x} \quad Q(x) = x$$

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Ex. 4: Solve the D.E. $(\cos 2x)y' + (2\sin 2x)y = 2$

$$\cos 2x \frac{dy}{dx} + 2\sin 2x y - 2 = 0$$

$$\cos 2x dy + (2\sin 2x y - 2) dx = 0 \quad \text{is non-homog.}$$

$$M = 2\sin 2x y - 2, \quad N = \cos 2x$$

$$\frac{\partial M}{\partial y} = 2\sin 2x, \quad \frac{\partial N}{\partial x} = -2\sin 2x$$

is not exact

$$\frac{dy}{dx} + 2\tan 2x y = \frac{2}{\cos 2x}$$

is Linear with $p(x) = 2\tan 2x$ & $q(x) = \frac{2}{\cos 2x}$

$$\int 2\tan 2x dx = 2 \int \frac{\sin 2x}{\cos 2x} dx$$

$$I.F. = e^{\int p(x) dx} = e^{\int 2\tan 2x dx}$$

$$= e^{\int \frac{2\sin 2x}{\cos 2x} dx} = e^{-\ln \cos 2x} = \frac{1}{\cos 2x}$$

$$\frac{y}{\cos 2x} = \int \frac{2}{\cos 2x \cos 2x} dx + C$$

$$= 2 \int \sec^2 2x dx + C$$

$$\frac{y}{\cos 2x} = 2 \tan 2x + C$$

$$y = \cos 2x (2 \tan 2x + C)$$

