Engineering Analysis & Numerical Methods

Ordinary Differential Equation of The First Order

O.D.E. :- is relation which involves one or several derivatives.

$y' = \frac{dy}{dx}$

x = independent variable.

y = dependent variable.

Examples :-

$x^2 y'' + x y' + (x^2 - 4) = 0$ (2nd order Linar)

$\partial^4 u/\partial x^4 + 2\, \partial^4 u/\partial x^2 \partial y^2 + \partial^4 u/\partial y^4 = 0$ (4th order partial)

$y'' + \sin y = 0$ (2nd order non-Linar)

[1] Separable first - Order Eq.s :-

i- $f(x)\, dx = g(y)\, dy$

$\int f(x)\, dx = \int g(y)\, dy + c$

ii- $f(x)\, G(y)\, dx = F(x)\, g(y)\, dy$

we find integrating factor which equal to $\frac{1}{G(y) \cdot F(x)}$ and multiply the eq

$$\frac{f(x)}{F(x)}\, dx = \frac{g(y)}{G(y)}\, dy$$

Example:- Solve the differential equation

$$dx + xy\,dy = y^2\,dx + y\,dy$$

Solution:- $(1-y^2)\,dx = y\,(1-x)\,dy$

$\therefore$ integration factor $= \dfrac{1}{(1-x)(1-y^2)}$

$$\frac{dx}{(1-x)} = \frac{y\,dy}{(1-y^2)} \quad * \;(-2) \qquad = -\ln(1-y) = -\frac{1}{2}\ln(1-y \ldots$$

$$2\ln(1-x) = \ln(1-y^2) + C \qquad + C$$

Example:- Solve the D.E. $y\sqrt{2x^2+3}\,dy + x\sqrt{4-y^2} \ldots$

$\therefore$ integration factor $= \dfrac{1}{\sqrt{(2x^2+3)(4-y^2)}}$

$$\frac{y}{\sqrt{4-y^2}}\,dy + \frac{x}{\sqrt{2x^2+3}}\,dx = 0$$

$$y(4-y^2)^{-1/2}\,dy + x(2x^2+3)^{-1/2}\,dx = 0$$

$$-\frac{1}{2}\,\frac{(4-y^2)^{1/2}}{1/2} + \frac{1}{4}\,\frac{(2x^2+3)^{1/2}}{1/2} = C$$

H.W.:- Solve $\dfrac{dy}{dx} = \dfrac{4y}{x(y-3)}$

2 Homogenous First_Order Eq_s:-

i- A function is called homog. if it has the property that the subs.

$x \to \lambda x \quad , \quad y \to \lambda y \quad , \quad \lambda > 0$ reproduce the original form multiplyed by : λ^n

$M(x,y) = M(\lambda x, \lambda y) = \lambda^n M(x,y)$.

ii- Standred D. form $M(x,y)\,dx + N(x,y)\,dy = 0$ is homog. if & only if both M & N are homog. of the same degree.

iii- Solved by the subs $y = ux$. This will reduce the homog. Eq. into separable eq_n

Example: Solve the D.E. $(x^2 + 3y^2)\,dx = 2xy\,dy$

$(\lambda^2 x^2 + 3\lambda^2 y^2)\,dx = 2\lambda x \lambda y\,dy$

$\lambda^2 (x^2 + 3y^2)\,dx = 2\lambda^2 xy\,dy$

$\therefore$ homog. 2°

subs. $y = ux \;;\; dy = u\,dx + x\,du$

$(x^2 + 3u^2x^2)\,dx = 2xux\,(u\,dx + x\,du)$

$x^2(1 + u^2)\,dx = 2ux^3\,du$ (separable eq_n)

let integration factor $= \dfrac{1}{x^3(1+u^2)}$

$(x^2 + 3u^2x^2 - 2u^2x^2)\,dx = \boxed{2x^3u\,du}$

$\therefore \frac{dx}{x} = \frac{2u}{1+u^2}\,du \quad , \ln x = \ln(1+u^2) + c$

H.W :- Solve $(x^3 + y^3)\,dx - 3xy^2\,dy = 0$

3. Exact First order Eq.s :-

$\underbrace{f(x,y)}_{M}\,\partial x + \underbrace{f(x,y)}_{N}\,\partial y = 0$

if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ then the eq. be exact.

Example :- Solve $(2x+3y-2)\,dx + (3x-4y-1)\,dy = 0$

solution :- $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 3 \quad \therefore$ exact

$\Longrightarrow \int_a^x (2x+3y-2)\,dx + \int_b^y (3x-4y-1)\,dy$

$\Longrightarrow [x^2+3yx-2x]_a^x + [3xy - 2y^2 - y]_b^y$

$\Longrightarrow x^2+3yx-2x - a^2 - \cancel{3ya} + 2a + \cancel{3ay}$

$-2y^2 - y \boxed{-3ab+2b^2+b}$

$a=0, b=0$ use

$\therefore c = x^2+3yx-2x-2y^2-y$

Example :- Solve the D.E. $(x+y)\,dx + (x+2y)\,dy$ with the initial condition $y(2)=3$

$\frac{\partial M}{\partial y} = 1 = \frac{\partial N}{\partial x} \quad \therefore$ exact D.E.

$\int_2^x (x+y)\,dx + \int_3^y (\cancel{x}+2y)\,dy$