Bandwidth of the Angle modulation waves:

FM

$$\emptyset_{FM}(t) = A_c \cos\left[\omega_c t + k_f \int_{-\infty}^{t} f(\propto) d\propto\right]$$

Let $\alpha(t) = \int_{-\infty}^{t} f(\propto) d\propto$

$$\emptyset_{FM}(t) = Re\left\{A_c\, e^{j[\omega_c t + k_f \alpha(t)]}\right\}$$

$$\emptyset_{FM}(t) = Re\left\{A_c \left[1 - jk_f\alpha(t) - \frac{{k_f}^2}{2!}\alpha^2(t) + \cdots + j^n \frac{{k_f}^n}{n!}\alpha^n(t)\right] e^{j\omega_c t}\right\}$$

$$\emptyset_{FM}(t) = A_c \begin{bmatrix} \cos\omega_c t - k_f\alpha(t)\sin\omega_c t - \frac{{k_f}^2}{2!}\alpha^2(t)\cos\omega_c t \\ -\frac{{k_f}^3}{3!}\alpha^3(t)\sin\omega_c t + \cdots \end{bmatrix}$$

It yields infinite number of sidebands, for practical purposes, an angle-modulating signal can be considered band limited.

If the BW of $f(t)$ is B Hz, then

The BW of $\alpha(t)$ is B Hz,

The BW of $\alpha^2(t)$ is 2B Hz,

The BW of $\alpha^n(t)$ is nB Hz.

Since $n \to \infty$, then the BW of FM and PM is infinite, but most of the modulated signal power resides in a finite bandwidth.

Narrow Band FM (NBFM)

If $|k_f\alpha(t)| \ll 1$ in the equation then we could write,

$$\emptyset_{NBFM}(t) \cong A_c[\cos\omega_c t - k_f\alpha(t)\sin\omega_c t] \quad \ldots(5\text{-}9)$$

This is a linear modulation similar to AM/(DSB-LC)

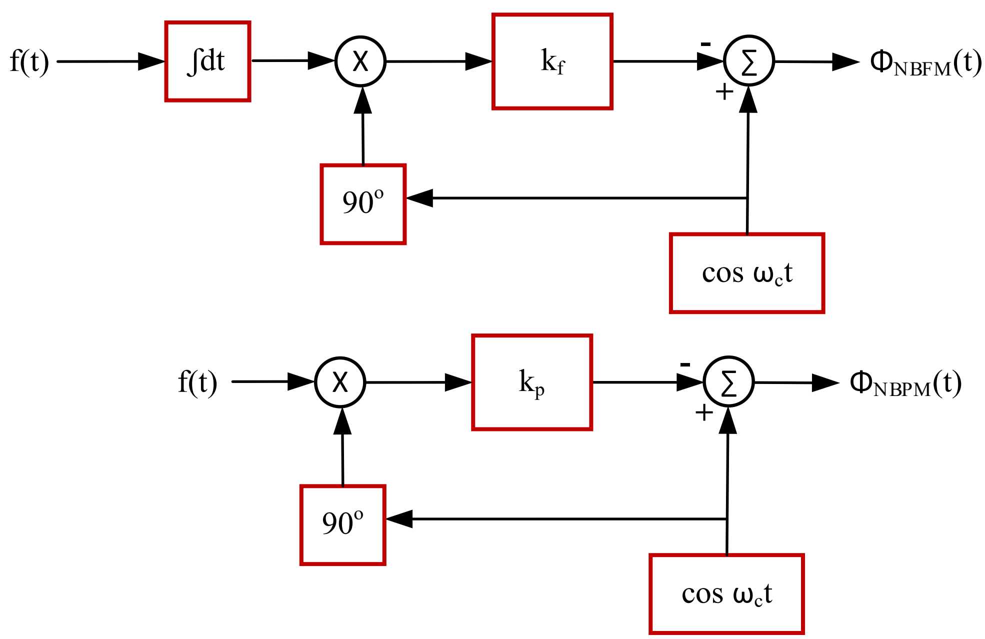
$$BW_{NBFM} = 2B$$ ...(5-10) similarly for PM

$$\emptyset_{NBPM}(t) \cong A_c\left[\cos\omega_c t - k_p\alpha(t)\sin\omega_c t\right]$$ ...(5-11)

H.W

Write relation of sideband power, carrier power and total power for narrowband FM &PM modulation.

Generation of NBFM, NBPM Signals



NBFM and NBPM Generators

Wideband FM (WBFM):

If the condition $|k_f\alpha(t)| \ll 1$ is not satisfied, many sidebands would occur and increasing the BW.

The instantaneous frequency is $\omega_i(t) = \omega_c + k_f f(t)$, it varies in the range $(\omega_c - k_f f_p)$ to $(\omega_c + k_f f_p)$, where $f_p = |f(t)|_{min} = |f(t)|_{max}$

Then, the BW of WBFM would be $2k_f f_p$, $k_f f_p$ is called the maximum deviation of ω_c (Δf)

$$\Delta f = \frac{k_f}{2\pi} f_p$$ Hz ….. (5-12)

$$BW = 2\Delta f$$ Hz ….. (5-13)

Carson's Rule

It is a general rule to compute the BW of FM (and PM) signal regardless of it is narrowband or wideband.

$$BW_{FM} = 2(\Delta f + B)$$ … (5-14)

If $|k_f \alpha(t)| \ll 1$, then $\Delta f \ll B$, then, $BW_{FM} \cong 2B$ [narrowband case]

If $|k_f \alpha(t)|$ $not \ll 1$, then $\Delta f \gg B$, then, $BW_{FM} \cong 2\Delta f$ [wideband case]

Therefore, we define deviation ratio,

$$\beta = \frac{\Delta f}{B}$$

if $\beta \ll 1$(usually $\beta < 0.2$) → NBFM

if $\beta \gg 1$(usually $\beta > 5$) → WBFM …. (5-15)

$$\beta = \frac{k_f f_p}{B}$$ Unit less … (5-16)

For PM

The instantaneous frequency depends on the derivative of f(t), i.e,

$$\Delta f = \frac{k_p}{2\pi} f_p' \quad \ldots (5\text{-}17)$$

$$\beta_{PM} = \frac{k_p f_p'}{B} \text{ rad} \quad \ldots (5\text{-}18)$$

Since β_{PM} have unit "Rad", sometimes it's called phase deviation in PM

$$\Delta\theta = \beta \quad \ldots (5\text{-}19)$$

Ex 5-2:

A 10 MHz carrier is frequency modulated by a sinusoidal signal such that the peak frequency deviation is 50 kHz, determine the BW of FM signal if the frequency of modulating sinusoid is (a) 500 kHz (b) 500Hz (c) 10 KHz.

Solution:

a) $\beta = \frac{\Delta f}{B} = \frac{\Delta f}{f_m} = \frac{50}{500} = 0.1$ "NBFM"

   $BW=2B=2f_m=1$ MHz

b) $\beta = 100$ "WBFM" , $BW \cong 2\Delta f = 100\ kHz$

c) $\beta = 5$ "using Carson's rule" , $BW \cong 2(\Delta f + f_m) \cong 120$ kHz

Single Tone FM:

$f(t) = A_m cos\omega_m t$

$$\alpha(t) = \int_{-\infty}^{t} f(\alpha) d\propto = \frac{A_m}{f_m} \sin\omega_m t$$

NBFM:

$$\emptyset_{NBFM/ST}(t) = A_c[cos\omega_c t - k_f\alpha(t)sin\omega_c t]$$

$$= A_c\left[cos\omega_c t - \frac{k_f A_m}{\omega_m} sin\omega_m t\ sin\omega_c t\right]$$

$$= A_c\left[cos\omega_c t - \frac{\Delta\omega}{\omega_m} sin\omega_m t\ sin\omega_c t\right]$$

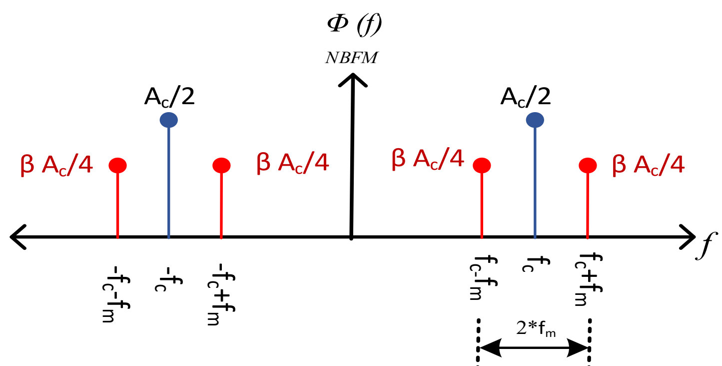
$$\emptyset_{NBFM/ST}(t) = A_c[cos\omega_c t - \beta sin\omega_m t\ sin\omega_c t] \quad \ldots(5\text{-}20)$$

ST-Single Tone

β *is the modulation index of FM*

$$\beta = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m}$$

BW=2f_m Hz



WBFM: Standard FM

$$\emptyset_{FM}(t) = A_c\cos\left[\omega_c t + k_f\int_{-\infty}^{t} f(\alpha)d\alpha\right]$$

$$= A_c cos\left[\omega_c t + \frac{k_f A_m}{\omega_m} sin\omega_m t\right]$$

$$\emptyset_{FM/ST}(t) = A_c\, cos[\omega_c t + \beta sin\omega_m t] \quad \ldots(5\text{-}21)$$

$$\emptyset_{FM/ST}(t) = Re\{A_c e^{j[\omega_c t + \beta sin\omega_m t]}\}$$

$$= Re\{A_c e^{j\omega_c t} e^{j\beta sin\omega_m t}\}$$

F.S expansion

$$e^{j\beta sin\omega_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta)e^{jn\omega_m t}$$

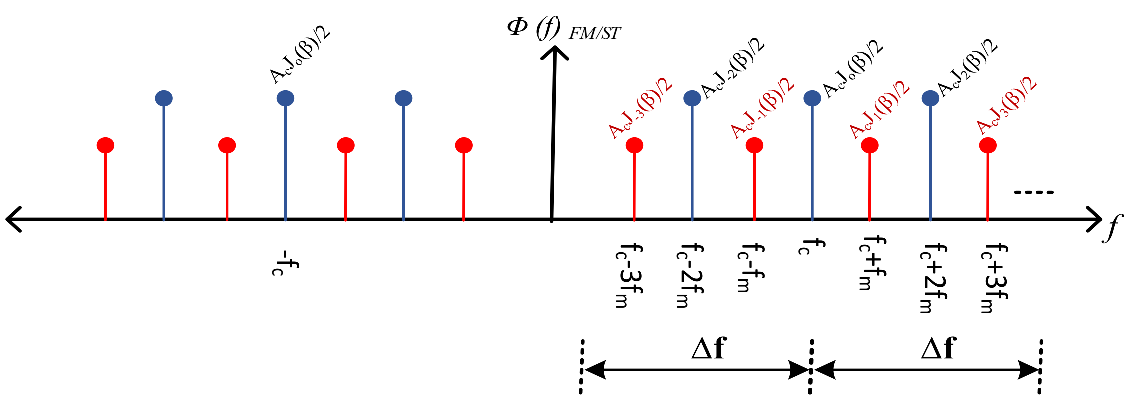
Where $J_n(\beta)$is the **Bessel function** of first kind and nth order.

$$\emptyset_{FM/ST}(t) = Re\left\{A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j(\omega_c t + n\omega_m t)}\right\}$$

$$\emptyset_{FM/ST}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t \quad \dots (5\text{-}21)$$

$$= A_c J_o(\beta)\cos(\omega_c)t + A_c J_1(\beta)\cos(\omega_c + \omega_m)t + A_c J_{-1}(\beta)\cos(\omega_c - \omega_m)t$$
$$+ A_c J_2(\beta)\cos(\omega_c + 2\omega_m)t + A_c J_{-2}(\beta)\cos(\omega_c + 2\omega_m)t$$

(Coefficient of series for Bessel function)



$$BW = 2\Delta f = 2k_f A_m \quad \dots(5\text{-}23)$$

$$for\ PM, BW = 2\Delta f = 2k_p A_m \omega_m$$

Or

$$BW = 2nf_m \quad \dots(5\text{-}24)$$

Where n is the number of significant sidebands (depend on the value of β).

Properties of Bessel function:

1-$J_n(\beta)$ Are real valued function.

2-$J_n(\beta)$=$J_{-n}(\beta)$, for n even.

3-$J_n(\beta) = -J_{-n}(\beta)$, for n odd.

4-$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

Average Power in the Single Tone FM:

Total power

$$P_t = \frac{A_c^2}{2}$$ Watt, if R=1Ω from eqn. (5-21)

$$P_t = \frac{A_c^2}{2}\sum_{n=-\infty}^{\infty} J_n^2(\beta) = \frac{A_c^2}{2}$$ watt from equation (5-22)

sideband power

$$P_n = 2\left[\frac{A_c^2}{2} J_n^2(\beta)\right] \quad \ldots(5\text{-}25)$$

carrier power

$$P_c = \frac{A_c^2}{2} J_o^2(\beta) \quad \ldots(5\text{-}26)$$

The value of β is chosen such that the power is minimized at any desired component (carrier or sidebands), [carrier term $J_0(\beta)$ can be made zero for β=2.405, 5.52, 8.65,….].