Car-Following Models

Introduction
Car following models are the most important representatives of microscopic traffic flow models. They describe traffic dynamics from the perspective of individual driver-vehicle units. In a strict sense, car-following models describe the driver's behavior only in the presence of interactions with other vehicles while free traffic flow is described by a separate model. In a more general sense, car-following models include all traffic situations such as car-following situations, free traffic, and also stationary traffic. In this case, we say that the microscopic models are complete:

A car-following model is complete if it is able to describe all situations including acceleration and cruising in free traffic, following other vehicles in stationary and non-stationary situations, and approaching slow or standing vehicles, and red traffic lights.

The first car-following models were proposed more than fifty years ago by Reuschel (1950), and Pipes (1953). These two models already contained one essential element of modern microscopic modeling: The minimum bumper-to-bumper distance to the leading vehicle (also known as the "safety distance") should be proportional to the speed. This can be expressed equivalently by requiring that the time gap should not be below a fixed safe time gap. We emphasize that, for obvious reasons, the relevant spatial or temporal distances are the net, i.e., rear-bumper-to-front bumper, distances. In contrast, the commonly used term time headway generally refers to the time interval between the passage times of the front bumpers of two consecutive vehicles, i.e., including the occupancy time interval needed for a vehicle to move forward its own length. Unfortunately, this distinction (which is essential for vehicular traffic) is often ignored. To avoid confusion and in order to be consistent, we will refer to "gaps" if net quantities are meant and define gaps and headways as follows (the modifiers in parentheses will be omitted if the meaning is clear from the context):

Distance headway = (distance) gap + length of the leading vehicle, (time) headway = time gap + occupancy time interval of the leading vehicle.

In this lecture, we will describe minimal models for the longitudinal dynamics that do not describe realistic driving behavior. Particularly, they yield unrealistic acceleration values. Nevertheless, they capture many essential features at a qualitative level and can be implemented and simulated easily (sometimes even allowing an analytical solution).

Examples of minimal models include the first-ever car-following models of Reuschel and Pipes in which the speed is varied instantaneously as a function of the actual distance to the leading vehicle. Another class of minimal models is the General Motors (GM) based car-following models in which the acceleration depends on the speed difference and the distance gap according to a power law while the driver's own speed is not considered as an influencing factor. These models are not complete since they cannot describe either free traffic or approaches to standing obstacles. In this lecture, we will therefore focus on other models.

Mathematical Description

Each driver-vehicle combination α is described by the state variables location $x_\alpha(t)$ (position of the front bumper along the arc length of the road, increasing in driving direction), and speed $v_\alpha(t)$ as a function of the time t, and by the attribute "vehicle length" l_α.

Depending on the model, additional state variables are required, for example, the acceleration $\dot{v}_\alpha$= dv/dt, or binary activation-state variables for brake lights or indicators. We define the vehicle index α such that vehicles pass a stationary observer (or detector) in ascending order, i.e., the first vehicle has the lowest index (cf. Fig. 1).

Notice that this implies that the vehicles are numbered in descending order with respect to their location x.

From the vehicle locations and lengths, we obtain the (bumper-to-bumper) distance gaps

$$x_\alpha = x_{\alpha-1} - l_{\alpha-1} - x_\alpha = x_l - l_l - x_\alpha \qquad [1]$$

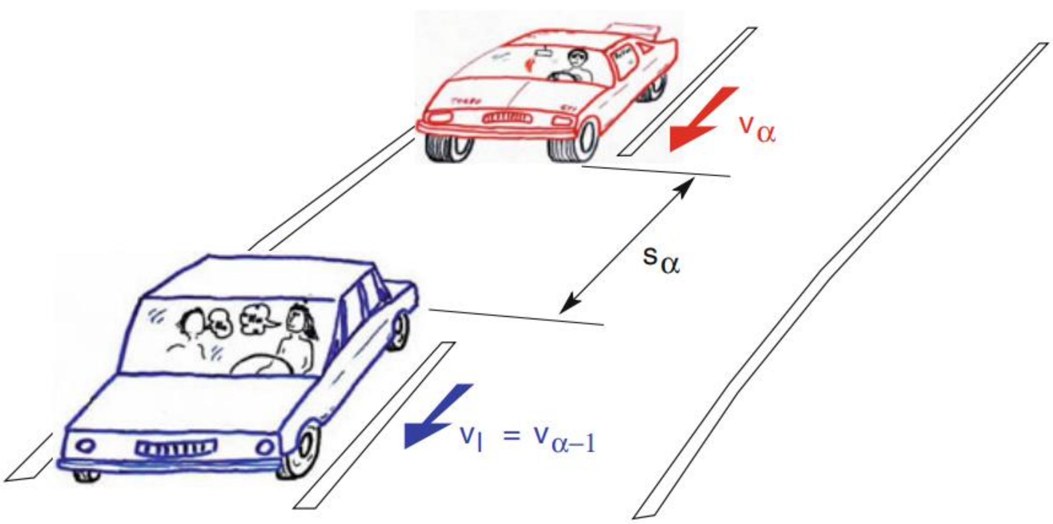

Fig. 1 defines the state variables of car-following models.

Which (together with the vehicle speeds) constitute the main input of the microscopic models. For ease of notation, we sometimes denote the index α − 1 of the leading vehicle with the symbol l (see Fig. 1).

The minimal models (and many of the more realistic models) describe the response of the driver as a function of the gap s_α to the lead vehicle, the driver's speed v_α, and the speed v_l of the leader. In continuous-time models, the driver's response is directly given in terms of an acceleration function $a_{mic}(s, v, v_l)$ leading to a set of coupled ordinary differential equations of the form:

$$\dot{x}(t) = \frac{dx_\alpha(t)}{dt} = v_\alpha(t) \qquad [2]$$

$$\dot{v}_\alpha(t) = \frac{dv_\alpha(t)}{dt} = a_{mic}(s_\alpha, v_\alpha, v_l) = \acute{a}_{mic}(s_\alpha, v_\alpha, \Delta v_\alpha) \quad [3]$$

In most acceleration functions, the speed v_l of the leader enters only in form of the speed difference (approaching rate):

$$\Delta v_\alpha = v_\alpha - v_{\alpha-1} = v_\alpha - v_l \quad [4]$$

The corresponding models can be formulated more concisely in terms of the alternative acceleration function

$$\acute{a}_{mic}(s, v, \Delta v) = a_{mic}(s, v, v - \Delta v) \quad [5]$$

Taking the time derivative of Eq. 1, one can reformulate Eq.2 by:

$$\dot{s}_\alpha(t) = \frac{ds_\alpha(t)}{dt} = v_l(t) - v_\alpha(t) = -\Delta v_\alpha(t) \quad [6]$$

The set of Eqs. (3) and (6) can be considered the generic formulation of most time-continuous car-following models. In this formulation, the coupling between the gap s_α and the speed v_α as well as the coupling between the speed v_α and the speed v_l of the leader becomes explicit.

There are also discrete-time car-following models, where time is not modeled as a continuous variable but discretized into finite and generally constant time steps. Instead of differential equations, one obtains iterated coupled maps of the general form

$$v_\alpha(t + \Delta t) = v_{mic}(s_\alpha(t), v_\alpha(t), v_l(t)) \quad [7]$$

$$x_\alpha(t + \Delta t) = x_\alpha(t) + \frac{v_\alpha(t) + v_\alpha(t + \Delta t)}{2} \Delta t \quad [8]$$

The driver's response is no longer modeled by an acceleration function but by a speed function v_{mic} (s, v,v_l) indicating the speed that will be reached at the end of the next time step.

Compared to continuous models, discrete-time car-following models are generally less realistic and less flexible but require less computing power for their numerical integration. Most discrete-time car-following models have been proposed at times when computing was more expensive. Nowadays, hundreds of thousands of vehicles can be simulated with time-continuous models on a PC in real-time, so this numerical advantage becomes less relevant. Most commercial traffic simulation software uses time-continuous models.

We emphasize that the Eqs. (2) and (6) represent kinematic facts that are valid a priori—in analogy to the continuity equations of the macroscopic models. Therefore, a specific time-continuous model is uniquely characterized by its acceleration function a_{mic}. Similarly, a specific discrete-time model is completely characterized by its speed function v_{mic}. When simulating heterogeneous traffic consisting of a variety of driving styles and vehicle classes (such as cars and trucks), each driver-vehicle combination is described by different acceleration functions a^{α}_{mic} (s, v, v_l) or speed functions v^{α}_{mic} (s, v, v_l), respectively.

Numerical integration. In general, time-continuous models cannot be solved analytically and an integration scheme is necessary for an approximate numerical solution of the system of Eqs. (3) and (6). For traffic flow applications, only explicit update schemes with a fixed time step are practical. Furthermore, the performance of the standard fourth-order Runge-Kutta scheme is generally inferior to simpler lower-order update methods.

Furthermore, the proposed scheme (9), (10) has an intuitive meaning in the context of car-following models: It corresponds to drivers that act only at the beginning of each time step but do nothing in between.

Assuming a constant update time step Δt, a simple but efficient explicit update method is given by the "ballistic" assumption of constant accelerations during each time step,

$$v_{\alpha}(t+\Delta t) = v_{\alpha}(t) + a_{mic}\big(s_{\alpha}(t), v_{\alpha}(t), v_l(t)\big)\Delta t \quad [9]$$

$$x_{\alpha}(t+\Delta t) = x_{\alpha}(t) + \frac{v_{\alpha}(t)+v_{\alpha}(t+\Delta t)}{2}\Delta t \quad [10]$$

Consequently, the combination of a continuous-time model with the ballistic update scheme (9), (10) is mathematically equivalent to discrete-time models if one set

$$a_{mic}(s, v, v_l) = \frac{v_{mic}(s,v,v_l)-v}{\Delta t} \quad [11]$$

However, there is a conceptual difference: For discrete-time models, the time step Δt plays the role of a model parameter typically describing the reaction time, the time headway, or the speed adaptation time. For time-continuous models, the update time Δt is an auxiliary variable of the approximate numerical solution which preferably should be small as the true solution is obtained in the limit Δt → 0 (at least if the numerical method is consistent and stable).

Steady State Equilibrium and the Fundamental Diagram

Since the driver-vehicle units of microscopic models are equivalent to driven particles of physical systems, there is no equilibrium in the strict sense. Instead, there is a stationary state where the forces and the entering and exiting energy fluxes are balanced. Strictly physically, this can be interpreted in terms of a balance of the forces (the sum of friction, wind drag, and the engine driving force equal to zero) or energy fluxes (engine power equals the change in potential and kinetic energy plus energy dissipation rate by friction and wind drag). More relevant for traffic flow, however, is the concept of balancing the social forces: The desire to go ahead generates a positive (accelerating) social force while the interactions with other vehicles generally lead to negative social forces in order to avoid critical situations and crashes.

In any case, such a balanced state is denoted as steady-state equilibrium. For microscopic models, a consistent description of the steady-state equilibrium requires identical driver-vehicle units on a homogeneous road. Technically, this implies that the model parameters are the same for all drivers and vehicles, i.e., the acceleration or speed functions characterizing the respective model do not depend on the vehicle index, a^{α}_{mic} (s, v, v_l) = a_{mic} (s, v, v_l), and v^{α}_{mic} (s, v, v_l) = v_{mic} (s, v, v_l), respectively. From the modeling point of view, the steady-state equilibrium is characterized by the following two conditions:

• **Homogeneous traffic**: All vehicles drive at the same speed (v_α = v) and keep the same gap behind their respective leaders (s_α = s).
• **No accelerations**:
$\dot{v}_\alpha$ = 0 or
v_α (t + Δt) = v_α (t) for all vehicles α.

For time-continuous models with acceleration functions of the form a_{mic} or $\acute{a}_{mic}$ this implies

$$a_{mic}(s, v, v) = 0 \quad or \quad \acute{a}_{mic}(s, v, 0) = 0 \qquad [12]$$

Respectively, while the condition

$$v_{mic}(s, v, v) = v \qquad [13]$$

is valid for discrete-time models with the speed function (7). Depending on the model, the microscopic steady-state relations (12) or (13) can be solved for

- The equilibrium speed v_e (s) as a function of the gap (microscopic fundamental diagram, see below),
- The equilibrium gap s_e(v) for a given speed.

Microscopic fundamental diagram. Eqs. (12) and (13) allow for a one-dimensional manifold of possible steady states that can be parameterized by the distance gap s and described by the equilibrium speed function v_e (s) which is also termed the microscopic fundamental diagram.

Transition to macroscopic relations. In order to obtain a micro–macro relation between the distance gap s and the density ρ we directly apply the definition of density as the number of vehicles per road length. For a given vehicle length l, we obtain:

$$s_\alpha = s = \frac{1}{\rho} - l \qquad [14]$$

Furthermore, the steady-state equilibrium implies that the speed of all vehicles is the same and equal to the macroscopic speed

$$V(x, t) = \langle v_\alpha(t) \rangle = v_e(s) \qquad [15]$$

With these relations, we can derive the macroscopic steady-state speed-density diagram and the macroscopic fundamental diagram:

$$V_e(\rho) = v_e\left(\frac{1}{\rho} - l\right),\ Q_e(\rho) = \rho v_e\left(\frac{1}{\rho} - l\right) \qquad [16]$$

Heterogeneous Traffic

Microscopic models play out their advantages when describing different drivers and vehicles, i.e., heterogeneous traffic. Including different drivers and vehicles is crucial when modeling the effects of active traffic management such as variable message signs, speed limits, or ramp metering, or when simulating traffic-related effects of new driver-assistance systems. Heterogeneous traffic can be microscopically modeled in two ways:

1. All driver-vehicle units are described by the same model using different parameter values. The heterogeneity can be applied on the level of vehicle classes (e.g., different parameters for cars and trucks), individually (distributed parameters), or both (different parameter distributions for cars and trucks). The last combined approach has the advantage that it automatically leads to realistic correlations between the parameters.

2. Different driver-vehicle classes can also be described with different models. This allows us to directly represent qualitatively different driving characteristics between, e.g., cars and trucks or between human driving and semi-automated driving with the help of adaptive cruise control (ACC) systems.

We emphasize that simulating heterogeneous traffic is only sensible in the context of multi-lane traffic models. Otherwise, a single long queue will eventually form behind the slowest vehicle, which is unrealistic. Finally, when parameterizing heterogeneous traffic, it is favorable if the model parameters have an intuitive meaning.

Optimal Velocity Model

The Optimal Velocity Model (OVM) is a time-continuous model whose acceleration function is of the form $a_{mic}(s, v)$, i.e., the speed difference exogenous variable is missing. The acceleration equation is given by:

$$\dot{v} = \frac{v_{opt}(s) - v}{\tau} \qquad \text{Optimum Velocity Model} \qquad [17]$$

This equation describes the adaption of the actual speed v = v_α to the optimal velocity v_{opt} (s) on a time scale given by the adaptation time τ. See Fig. 2

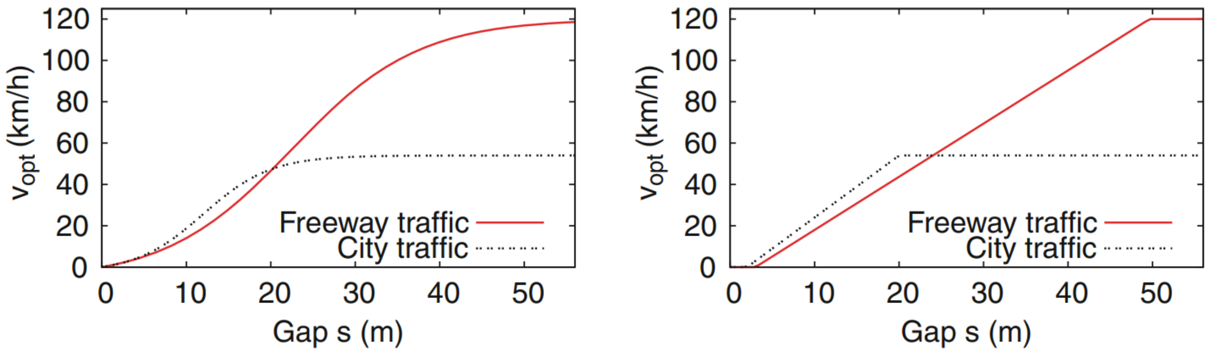

Fig. 2 Optimal velocity functions (left) and (right) for the parameter values of Table 1.

Table 1 Parameter of two variants of the Optimal Velocity Model (OVM).

Parameter	Typical value highway	Typical value city traffic
Adaptation time τ	0.65 s	0.65 s
Desired speed v_0	120 km/h	54 km/h
Transition width Δs [v_{opt} according to Eq. (10.21)]	15 m	8 m
Form factor β [v_{opt} according to Eq. (10.21)]	1.5	1.5
Time gap T [v_{opt} according to Eq. (10.22)]	1.4 s	1.2 s
Minimum distance gap s_0 [v_{opt} according to Eq. (10.22)]	3 m	2 m

Full Velocity Difference Model
By extending the OVM with an additional linear stimulus for the speed difference, one obtains the Full Velocity Difference Model (FVDM):

$$\dot{v} = \frac{v_{opt}(s)-v}{\tau} - \gamma\Delta v \quad \text{Full Velocity Difference Model} \qquad [18]$$

However, in contrast to the OVM, the Full Velocity Difference Model is not complete, i.e., it is not able to describe all traffic situations. The reason is that the term γ Δv describing the sensitivity to speed difference does not depend on the gap. Consequently, a slow vehicle (or a red traffic light corresponding to a standing virtual vehicle) leads to a significant decelerating contribution even if it is miles away

Newell's Car-Following Model
Newell's car-following model is the arguably simplest representative of time-discrete models of the type (7). Its speed function is directly given by the optimal speed corresponding to the triangular fundamental diagram with s_0= 0,

$$v(t+T) = v_{opt}(s(t)), \quad v_{opt}(s) = \min\left(v_0, \frac{s}{T}\right) \quad \textit{Newells Model} \qquad [19]$$

When restricting to the car-following regime, Newell's model has two parameters:

- The time gap or reaction time T, and

- The (effective) vehicle length l_{eff}.

In this regime, the kinematic wave velocity is constant and given by:

$$w = C_{cong} = -l_{eff}/T$$

The set of model parameters can alternatively be expressed by {T,w} or by {l_{eff},w}. The standard value for the time gap is T = 1 s while the wave speed should be within the observed range w ∈ [−20 km/h, −15 km/h] corresponding to a plausible effective vehicle length l_{eff} of about 5 m. The minimum condition of the optimal velocity function makes the model complete by defining a free-flow regime and introducing the desired speed v_0 as a third model parameter. It is straightforward to generalize Newell's model by replacing Eq. of optimum speed (triangular diagram) with other microscopic fundamental diagrams.

Newell's model can also be considered as a continuous-in-time model with a time delay assuming that the drivers have a constant reaction time Tr = T. In this interpretation, Eq. (19) has the mathematical form of a delay-differential equation.

Figure 3 shows that this equivalence only applies to the triangular fundamental diagram and only in the bound traffic regime, i.e., for gaps s satisfying $v_e(s) < v_0$ or $s < s_0 + v_0 T$. Otherwise, discretization errors are present.
Generally, the OVM is updated with time steps significantly smaller than the adaptation time. However, this does not invalidate the reasoning above, at least, qualitatively. In any case, the steady-state equilibria of the two models are equivalent.

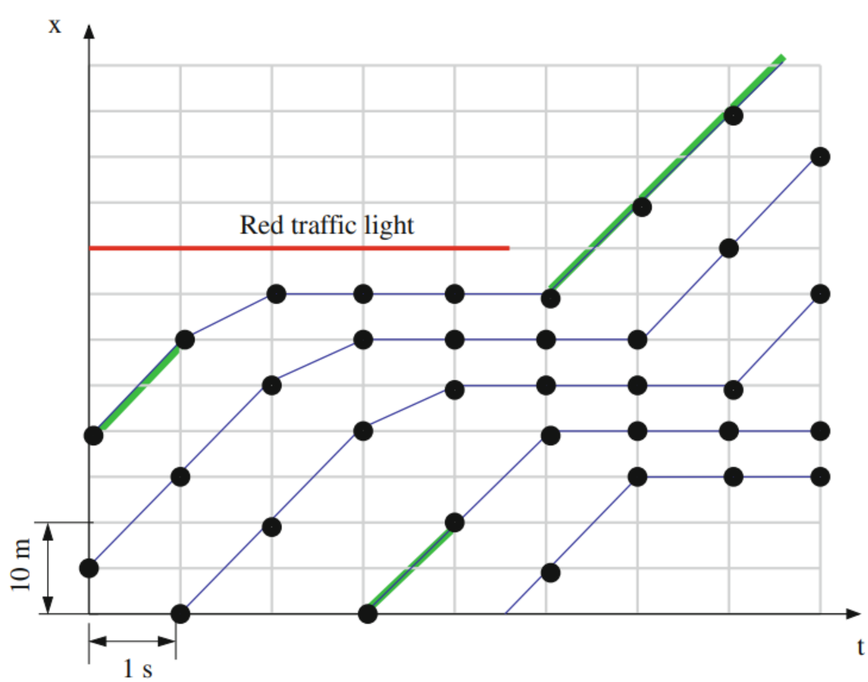

Fig. 3 Trajectory plot of the OVM with the triangular fundamental diagram (l_{eff} = 5 m, v_0 = 10 m/s, T = τ = 1 s) with an update time Δt = 1 s