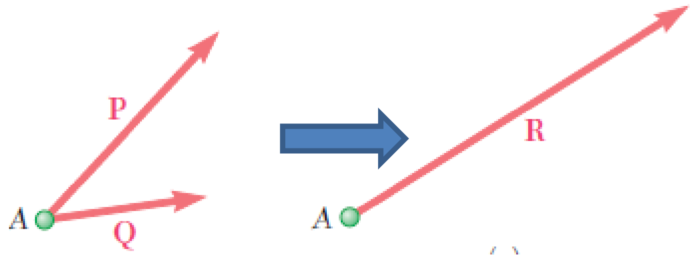
Chapter One

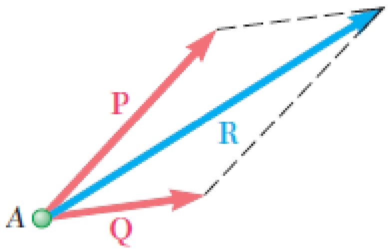
1. Resultant Force

Experimental evidence shows that two forces P and Q acting on a particle A can be replaced by a single force R which has the same effect on the particle. This force is called the resultant of the forces P and Q



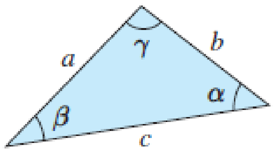
1.2 Parallelogram Law and Triangle Law

The *resultant* of the forces **P** and **Q** and can be obtained, as shown in Fig., by constructing a parallelogram, using **P** and **Q** as two adjacent sides of the parallelogram. *The diagonal that passes through A represents the resultant.* This method for finding the resultant is known as the *parallelogram law* for the addition of two forces.

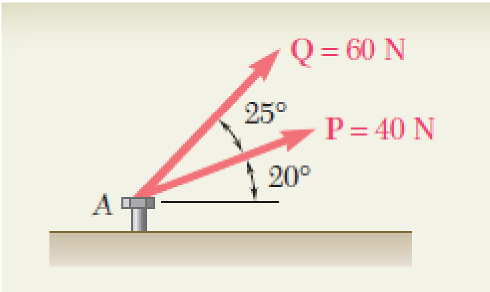


1.2.1 Sines and Cosines Law

Because of the geometric nature of the parallelogram law and the triangle law, vector addition can be accomplished graphically. A second technique is to determine the relationships between the various magnitudes and angles analytically by applying the *laws of sines and cosines* to a sketch of the **parallelogram**

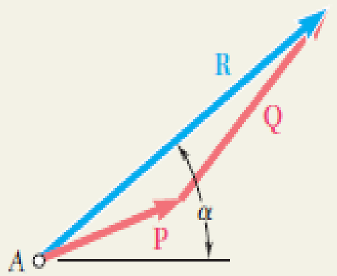


Law of sines	$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$
Law of cosines	$a^2 = b^2 + c^2 - 2bc\cos\alpha$ $b^2 = c^2 + a^2 - 2ca\cos\beta$ $c^2 = a^2 + b^2 - 2ab\cos\gamma$



Example 1. The two forces **P** and **Q** act on a bolt *A* . Determine their resultant.

Solution:



Trigonometric Solution. The triangle rule is again used; two sides and the included angle are known. We apply the law of cosines.

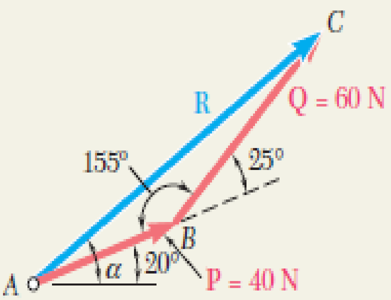
$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$R^2 = (40\text{ N})^2 + (60\text{ N})^2 - 2(40\text{ N})(60\text{ N})\cos 155^\circ$$
$$R = 97.73\text{ N}$$

Now, applying the law of sines, we write

$$\frac{\sin A}{Q} = \frac{\sin B}{R} \qquad \frac{\sin A}{60\text{ N}} = \frac{\sin 155^\circ}{97.73\text{ N}} \qquad (1)$$

Solving Eq. (1) for sin *A*, we have

$$\sin A = \frac{(60\text{ N})\sin 155^\circ}{97.73\text{ N}}$$



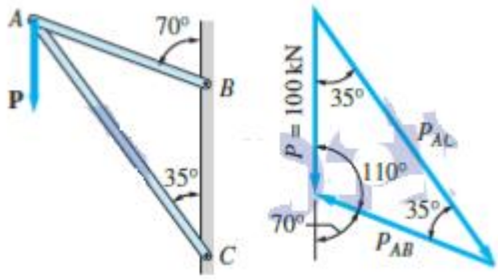
Using a calculator, we first compute the quotient, then its arc sine, and obtain

$$A = 15.04^\circ \qquad \alpha = 20^\circ + A = 35.04^\circ$$

We use 3 significant figures to record the answer (cf. Sec. 1.6):

$$\mathbf{R} = 97.7\text{ N} \measuredangle 35.0^\circ$$

Example 2: The vertical force P of magnitude 100 kN is applied to the frame shown in Fig. Resolve P into components that are parallel to the members AB and AC of the truss.

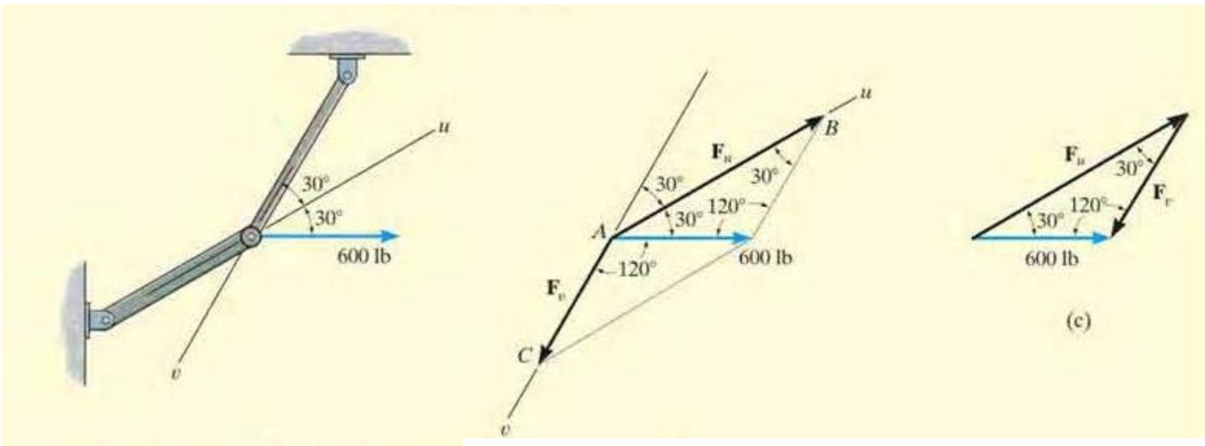


$$\frac{100}{\sin 35^\circ} = \frac{P_{AB}}{\sin 35^\circ} = \frac{P_{AC}}{\sin 110^\circ}$$

which yields for the magnitudes of the components

$$P_{AB} = 100.0\,\text{kN} \qquad P_{BC} = 163.8\,\text{kN}$$

Example 3: Resolve the horizontal 600-lb force in Fig. 2–12*a* into components acting along the *u* and *v* axes and determine the magnitudes of these components.



Example 4:

It is required that the resultant force acting on the eyebolt in Fig. 2–14*a* be directed along the positive *x* axis and that $\mathbf{F}_2$ have a *minimum* magnitude. Determine this magnitude, the angle θ, and the corresponding resultant force.

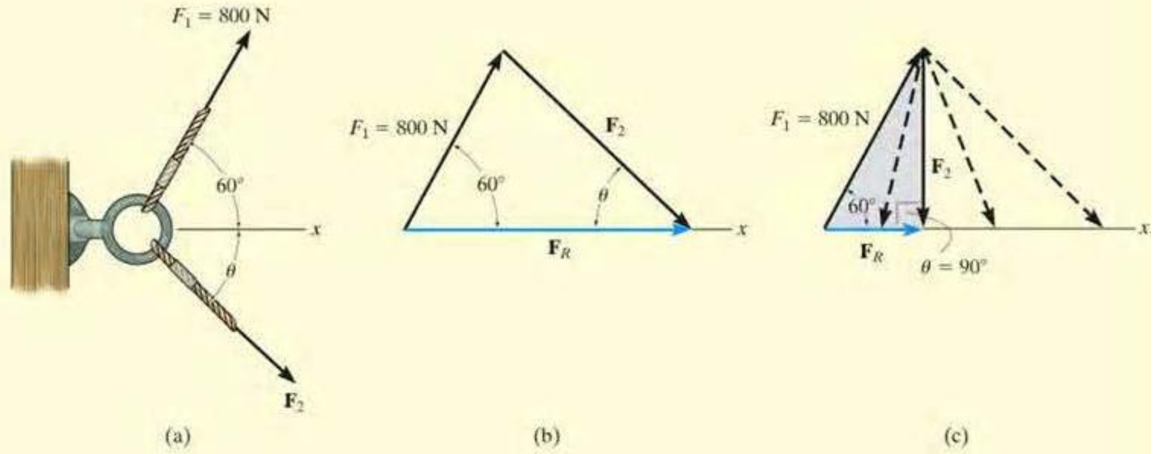


Fig. 2–14

SOLUTION

The triangle rule for $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ is shown in Fig. 2–14*b*. Since the magnitudes (lengths) of $\mathbf{F}_R$ and $\mathbf{F}_2$ are not specified, then $\mathbf{F}_2$ can actually be any vector that has its head touching the line of action of $\mathbf{F}_R$, Fig. 2–14*c*. However, as shown, the magnitude of $\mathbf{F}_2$ is a *minimum* or the shortest length when its line of action is *perpendicular* to the line of action of $\mathbf{F}_R$, that is, when

$$\theta = 90^\circ$$

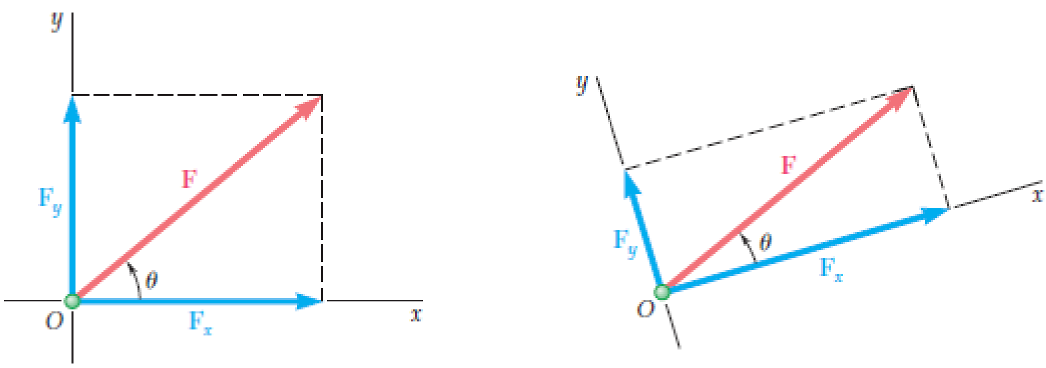
Since the vector addition now forms a right triangle, the two unknown magnitudes can be obtained by trigonometry.

$$F_R = (800\text{ N})\cos 60^\circ = 400\text{ N}$$

$$F_2 = (800\text{ N})\sin 60^\circ = 693\text{ N}$$

1.3 Rectangular Component of a Force

In many problems it will be found desirable to resolve a force into two components which are perpendicular to each other. In Fig. , the force **F** has been resolved into a component **F** *x* along the *x* axis and a component **F** *y* along the *y* axis. The parallelogram drawn to obtain the two components is a *rectangle* , and **F** *x* and **F** *y* are called *rectangular components.*



$$F_x = F \cos\theta \qquad F_y = F \sin\theta$$

EXAMPLE 1. A force of 800 N is exerted on a bolt *A* as shown in Fig. 2.22 *a* . Determine the horizontal and vertical components of the force. Addition of Forces By Summing X and Y Components.

Solution:

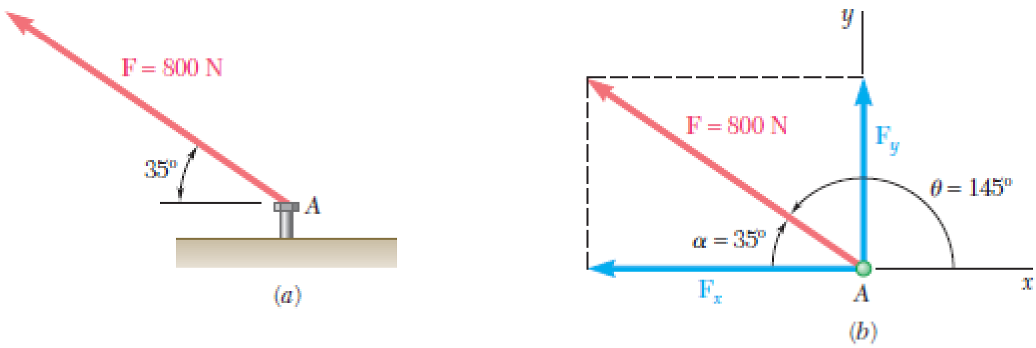


Fig. 2.22

$$F_x = -F\cos\alpha = -(800\ \text{N})\cos 35^\circ = -655\ \text{N}$$
$$F_y = +F\sin\alpha = +(800\ \text{N})\sin 35^\circ = +459\ \text{N}$$

1.3.1 Addition of Forces By Summing X and Y Components

It was seen in Sec. 2.2 that forces should be added according to the parallelogram law. From this law, two other methods, more readily applicable to the *graphical* solution of problems, were derived in Secs. 2.4 and 2.5: the triangle rule for the addition of two forces and the polygon rule for the addition of three or more forces. It was also seen that the force triangle used to define the resultant of two forces could be used to obtain a *trigonometric* solution. When three or more forces are to be added, no practical trigonometric solution can be obtained from the force polygon which defines the resultant of the forces. In this case, an *analytic* solution of the problem can be obtained by resolving each force into two rectangular components. Consider, for instance, three forces **P**, **Q** ,and **S** acting on a particle *A* (Fig. 2.25 *a*). Their resultant **R** is defined by the relation

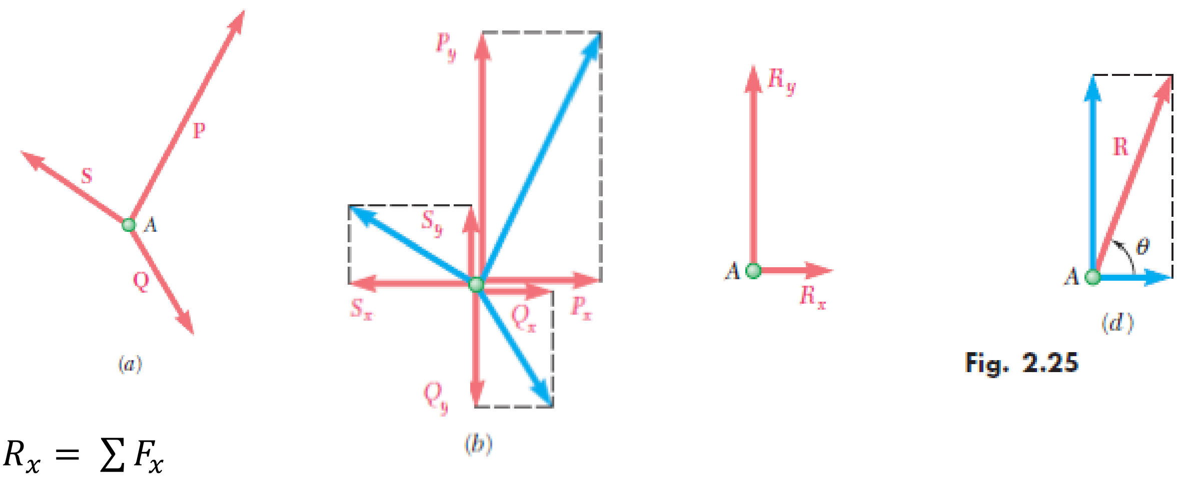


Fig. 2.25

$$R_x = \sum F_x$$

$$R_y = \sum F_y$$

$$R = \sqrt{R_x{}^2 + R_y{}^2}$$

$$\tan\theta = \frac{R_y}{R_x} \qquad ; \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

Example 1: Determine the magnitude of the resultant force and its direction, measured x axis counterclockwise form the positive.

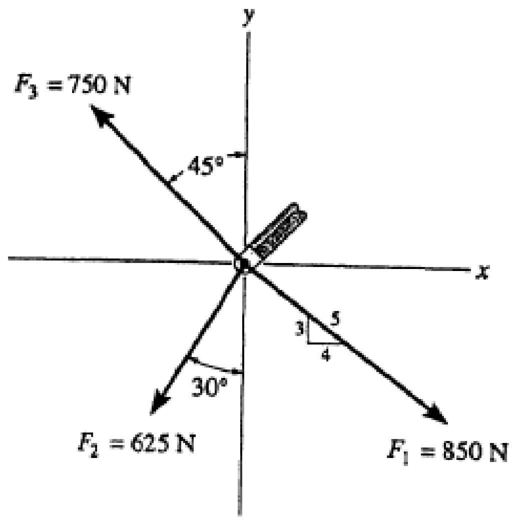
$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x;\quad F_{R_x} = \frac{4}{5}(850) - 625\sin 30^\circ - 750\sin 45^\circ = -162.8\text{ N}$

$+\uparrow F_{R_y} = \Sigma F_y;\quad F_{R_y} = -\frac{3}{5}(850) - 625\cos 30^\circ + 750\cos 45^\circ = -520.9\text{ N}$

$F_R = \sqrt{(-162.8)^2 + (-520.9)^2} = 546\text{ N}$ **Ans**

$\phi = \tan^{-1}\left[\frac{-520.9}{-162.8}\right] = 72.64^\circ$

$\theta = 180^\circ + 72.64^\circ = 253^\circ$ **Ans**



Example 2: Tree forces act on the bracket. Determine the magnitude and direction **θ** of $\mathbf{F_1}$**,** so that the resultant force is directed along the positive $\mathbf{x}'$ axis and has a magnitude of **1 kN**.

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;\quad 1000\cos 30^\circ = 200 + 450\cos 45^\circ + F_1\cos(\theta + 30^\circ)$

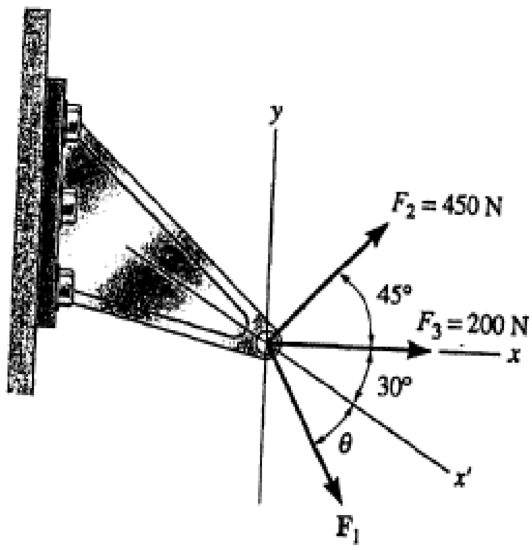
$+\uparrow F_{Ry} = \Sigma F_y;\quad -1000\sin 30^\circ = 450\sin 45^\circ - F_1\sin(\theta + 30^\circ)$

$F_1\sin(\theta + 30^\circ) = 818.198$

$F_1\cos(\theta + 30^\circ) = 347.827$

$\theta + 30^\circ = 66.97^\circ,\quad \theta = 37.0^\circ$ **Ans**

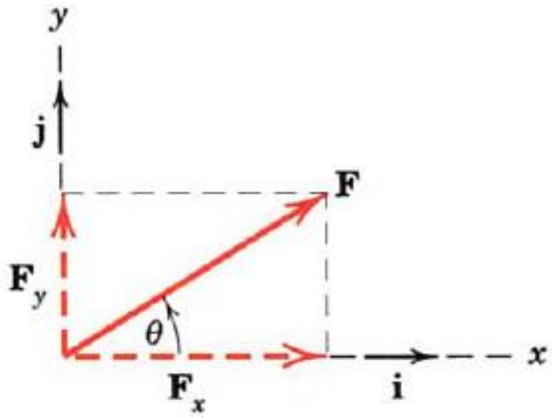
$F_1 = 889\text{ N}$ **Ans**



1.4 Cartesian Vector Notation

It is also possible to represent the x and y components of a force in terms of Cartesian unit vectors i and j. Each of these unit vectors has a dimensionless magnitude of one, and so they can be used to designate the directions of the x and y axes, respectively as shown in Fig.

$$\mathbf{F} = \mathbf{F_x\, i} + F_y\, \mathbf{j}$$



$$F_x = F\cos\theta \qquad F = \sqrt{F_x^{\,2} + F_y^{\,2}}$$

$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

1.4.1 Coplanar Force Resultant

Using *cartesian vector notation,* each force is first represented as a Cartesian vector i.e.

$$\mathbf{F}_1 = F_{1x}\mathbf{i} + F_{1y}\mathbf{j}$$
$$\mathbf{F}_2 = -F_{2x}\mathbf{i} + F_{2y}\mathbf{j}$$
$$\mathbf{F}_3 = F_{3x}\mathbf{i} - F_{3y}\mathbf{j}$$

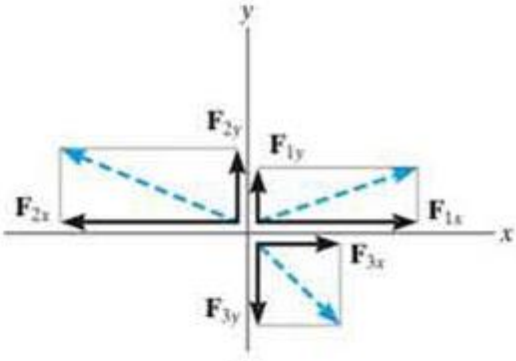
The vector resultant is therfore

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\\ &= F_{1x}\mathbf{i} + F_{1y}\mathbf{j} - F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{3x}\mathbf{i} - F_{3y}\mathbf{j}\\ &= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j}\\ &= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}\end{aligned}$$

If in scalar notation is used, then we have

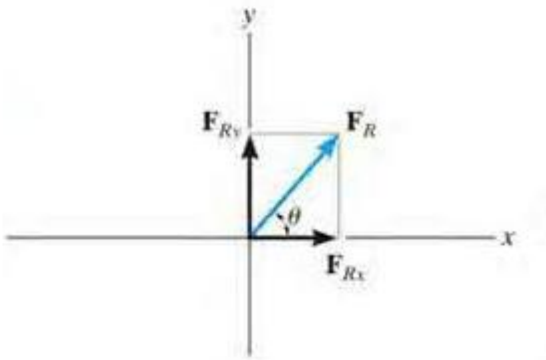
$$(\overset{+}{\rightarrow}) \qquad F_{Rx} = F_{1x} - F_{2x} + F_{3x}$$

$$(+\uparrow) \qquad F_{Ry} = F_{1y} + F_{2y} - F_{3y}$$



(b)

$$\boxed{\begin{aligned} F_{Rx} &= \Sigma F_x \\ F_{Ry} &= \Sigma F_y \end{aligned}}$$

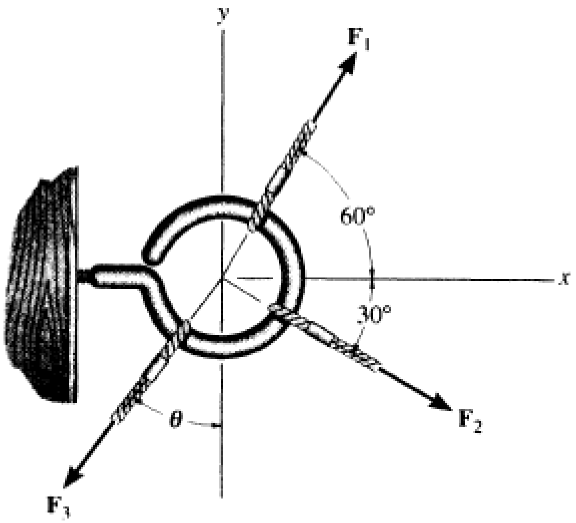


(c)

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

***2-52.** The three concurrent forces acting on the screw eye produce a resultant force $\mathbf{F}_R = 0$. If $F_2 = \frac{2}{3}F_1$ and $\mathbf{F}_1$ is to be 90° from $\mathbf{F}_2$ as shown, determine the required magnitude of $\mathbf{F}_3$ expressed in terms of F_1 and the angle θ.



Cartesian Vector Notation :

$$\mathbf{F}_1 = F_1 \cos 60°\mathbf{i} + F_1 \sin 60°\mathbf{j}$$
$$= 0.50F_1\,\mathbf{i} + 0.8660F_1\,\mathbf{j}$$

$$\mathbf{F}_2 = \frac{2}{3}F_1 \cos 30°\mathbf{i} - \frac{2}{3}F_1 \sin 30°\mathbf{j}$$
$$= 0.5774F_1\,\mathbf{i} - 0.3333F_1\,\mathbf{j}$$

$$\mathbf{F}_3 = -F_3 \sin \theta\mathbf{i} - F_3 \cos \theta\mathbf{j}$$

Resultant Force :

$$\mathbf{F}_R = 0 = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$0 = (0.50F_1 + 0.5774F_1 - F_3 \sin \theta)\,\mathbf{i} + (0.8660F_1 - 0.3333F_1 - F_3 \cos \theta)\,\mathbf{j}$$

Equating **i** and **j** components, we have

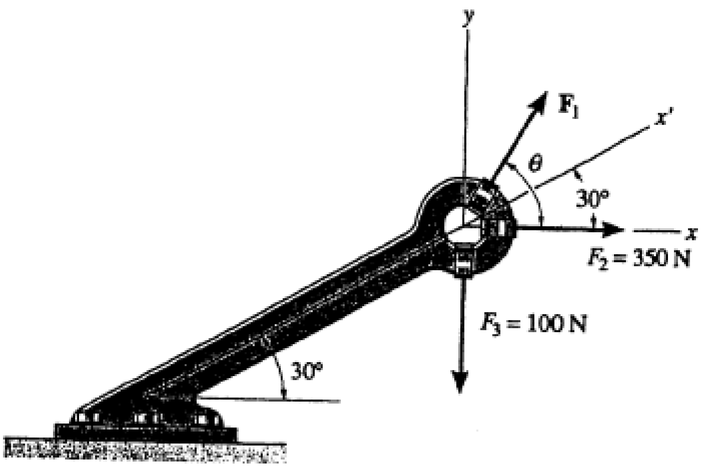
$$0.50F_1 + 0.5774F_1 - F_3 \sin \theta = 0 \qquad [1]$$

$$0.8660F_1 - 0.3333F_1 - F_3 \cos \theta = 0 \qquad [2]$$

Solving Eq.[1] and [2] yields

$$\theta = 63.7° \qquad F_3 = 1.20F_1 \qquad \textbf{Ans}$$

2-54. Express each of the three forces acting on the bracket in Cartesian vector form with respect to the x and y axes. Determine the magnitude and direction θ of $\mathbf{F}_1$ so that the resultant force is directed along the positive x' axis and has a magnitude of $F_R = 600$ N.



$\mathbf{F}_1 = \{F_1 \cos\theta\,\mathbf{i} + F_1 \sin\theta\,\mathbf{j}\}$ N

$\mathbf{F}_2 = \{350\mathbf{i}\}$ N

$\mathbf{F}_3 = \{-100\mathbf{j}\}$ N

Require,

$\mathbf{F}_R = 600 \cos 30°\,\mathbf{i} + 600 \sin 30°\,\mathbf{j}$

$\mathbf{F}_R = \{519.6\mathbf{i} + 300\mathbf{j}\}$ N

$\mathbf{F}_R = \Sigma\mathbf{F}$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components yields :

$519.6 = F_1 \cos\theta + 350$

$F_1 \cos\theta = 169.6$

$300 = F_1 \sin\theta - 100$

$F_1 \sin\theta = 400$

$\theta = \tan^{-1}\left[\frac{400}{169.6}\right] = 67.0°$ **Ans**

$F_1 = 434$ N **Ans**