Lecture 8 Rigid Pavement

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References:

Huang, Y. H. (2004): "Pavement Analysis and Design", 2nd edition, Prentice Hall, Englewood Cliffs, New Jersey).

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Stresses and deflection in Rigid Pavement

Stresses and deflection due to curling

During the day, when the temperature on the top of the slab is greater than that at the bottom of it, the top of the slab tends to expand with respect to the neutral axis, while the bottom tends to contract. However, the weight of the slab restrains it from expansion and contraction; thus, compressive stresses are induced at the top, tensile stresses at the bottom of the slab as shown in Figure 1.

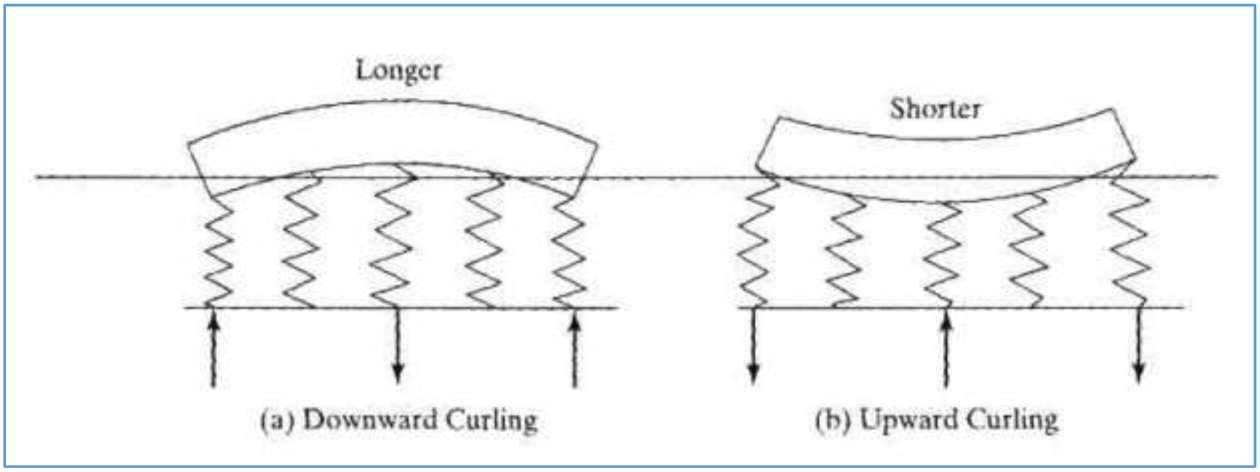

Figure 1: Curling of a concrete slab due to change in temperature.

Stresses and deflection due to Loading

Three types of methods can be used to determine the stresses and deflections in rigid pavements: the closed-form formulas method, the influence charts method, and the finite-element method. In this lecture, we will focus on the first method which is developed by Westergaard.

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Methods of determination of Stresses and Deflections in Rigid Pavement

As mentioned above, there are three methods used to determine stresses and deflections in rigid pavements. The first two methods (Westergaard Method and Influence charts Method) are applicable only to a large slab on a liquid subgrade. If the wheel loads are applied to multiple slabs on a liquid, solid, or layer foundation with load transfer across the joints, the finite-element analysis should be applied. The liquid foundation assumes the subgrade to be a set of springs. In addition, the deflection at any given point is proportional to the force at that point and independent of the forces at all other points. This assumption of course is unrealistic and does not represent subgrade soil reactions.

Closed-Form Formulas (Westergaard Method)

Corner Loading

Using the Westergarrd method, three critical positions were considered. These are the Edge loading, the Interior loading, and the Corner loading, see Figure 2.

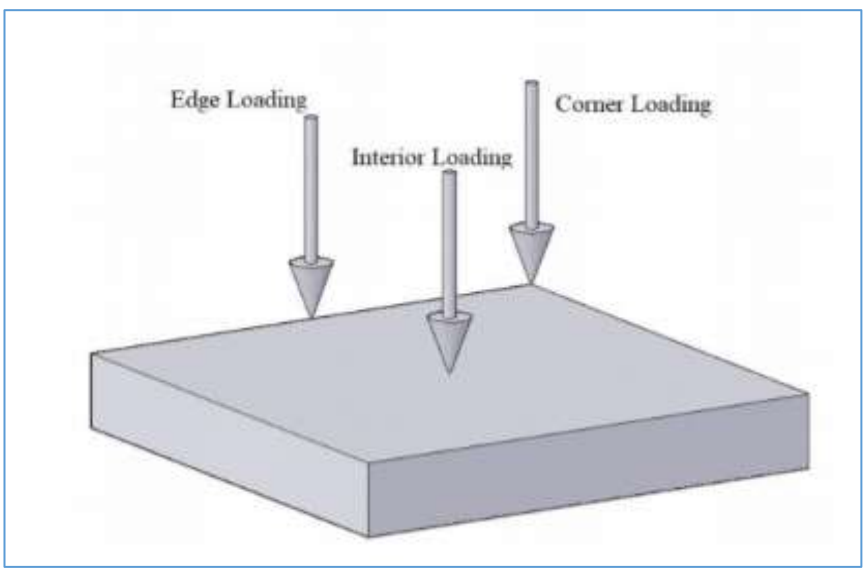

Figure 2: Critical loading of a concrete slab.

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Westergaard (1926) applied a method of successive approximations and obtained the following formulas to determine the stress and deflection due to corner loading (Figure 3).

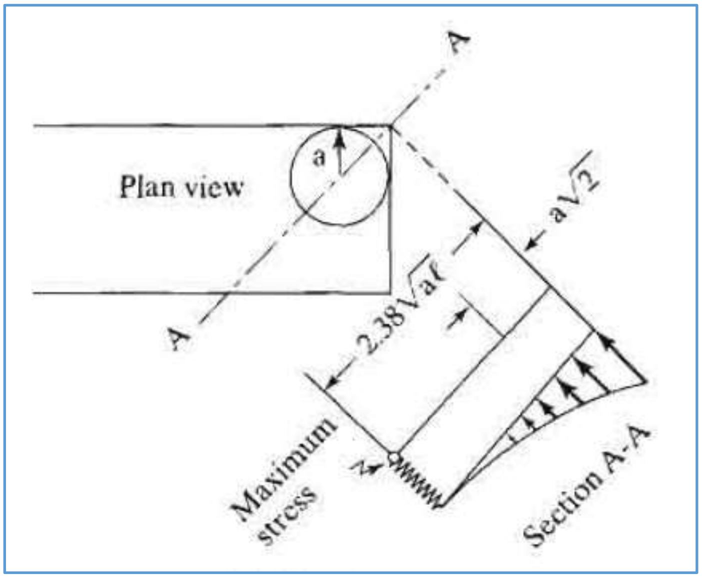

Figure 3: Circular load on Corner

$$\sigma_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{\ell}\right)^{0.6}\right]$$

and

$$\Delta_c = \frac{P}{k\ell^2}\left[1.1 - 0.88\left(\frac{a\sqrt{2}}{\ell}\right)\right]$$

In which:

σc= maximum corner stress (psi)

$\triangle$c= Maximum corner deflection (in)

$$\ell = \left[\frac{Eh^3}{12(1-\upsilon^2)k}\right]^{0.25}$$

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ℓ is the radius of relative stiffness which defined as:

A modulus (E) of 4 x 10^6 psi (27 .6 GPa) and a Poisson ratio (v) of 0 .15 are assumed for the concrete can be used.

k=modulus of subgrade reaction pci

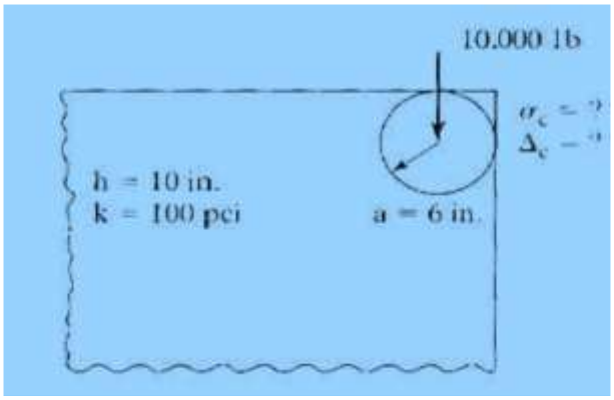
h = thickness of slab (in)

p= wheel load (Ib)

a= radius of equivalent circular contact area (in)

Example 1 (Corner loading)

The Figure below shows a concrete slab subjected to a corner loading. Given k = 100 pci (27 .2 MN/m3), *h* = 10 in. (254 mm), *a* = 6 in. (152 mm), and *P* = 10,000lb (44 .5 kN), determine the maximum stress and deflection due to corner loading.



Solution:

$$\ell = \left[\frac{Eh^3}{12(1 - v^2)k}\right]^{0.25}$$

ℓ = [4 x 10^6*1000/(12*(1-0.15^2)*100)] ^ 0.25 = **42 .97 in**.

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$$\sigma_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{\ell}\right)^{0.6}\right]$$

and

$$\Delta_c = \frac{P}{k\ell^2}\left[1.1 - 0.88\left(\frac{a\sqrt{2}}{\ell}\right)\right]$$

σc = 3 *10,000/100 *[1- (6*√2/42 .97)^0.6] = **186 .6 psi** (1.29 MPa).

△c = 10,000/(100*42.97^2)*[1.1 — 0.88(6*√2/42.97)] = **0 .0502 in** . (1.27 mm).

Interior Loading

The formula developed by Westergaard (1926) for computing the stress in the interior of a concrete slab under a circular loaded area of radius a is:

$$\sigma_i = \frac{0.316P}{h^2}\left[4\log\left(\frac{\ell}{b}\right) + 1.069\right]$$

In which the ℓ is the radius of relative stiffness and

$$b = a \qquad \text{when } a \geq 1.724h$$

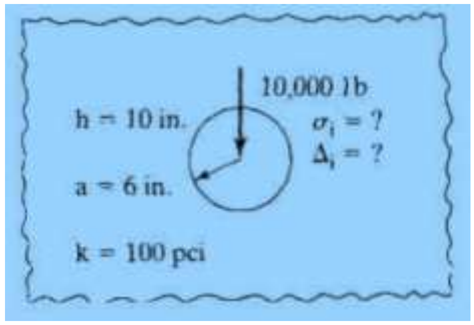
$$b = \sqrt{1.6a^2 + h^2} - 0.675h \quad \text{when } a < 1.724h$$

The formula developed by Westergaard to compute deflection due to interior loading (Westergaard, 1939) is:

$$\Delta_i = \frac{P}{8k\ell^2}\left\{1 + \frac{1}{2\pi}\left[\ln\left(\frac{a}{2\ell}\right) - 0.673\right]\left(\frac{a}{\ell}\right)^2\right\}$$

Example 2 (Interior Loading):

Same as Example 1 but the load is applied in the interior, see the Figure below. Determine the maximum stress and deflection due to interior loading.



Solution: Since a is smaller than 1.724 h. Thus:

b = √(1 .6 x 36 + 100) – 0 .675 * 10 = 5 .804 in . (147 mm).

$$\sigma_i = \frac{0.316P}{h^2}\left[4\log\left(\frac{\ell}{b}\right) + 1.069\right]$$

σi= 0.316 *10,000/100 * [4 log*(42.97/5 .804) + 1 .069] = **143 .7 psi** (992 kPa).

$$\Delta_i = \frac{P}{8k\ell^2}\left\{1 + \frac{1}{2\pi}\left[\ln\left(\frac{a}{2\ell}\right) - 0.673\right]\left(\frac{a}{\ell}\right)^2\right\}$$

△i = 10,000/(8*100*1846 .1) *{1 + (1/2π)* [1n(6/85 .94) – 0 .673]*(6/42.97)^2}

= **0 .0067 in**. (0.17 mm).

Looking at the results of Example 1 and 2, the stress due to interior loading is 77% of that at corner, while the deflection is only 13% of that at corner. This is true only when there is no load transfer across the joint at the corner. If sufficient load transfer is provided, the stress due to corner loading will be smaller than that due to interior loading. However, the deflection at the corner will still be greater than that at the interior.

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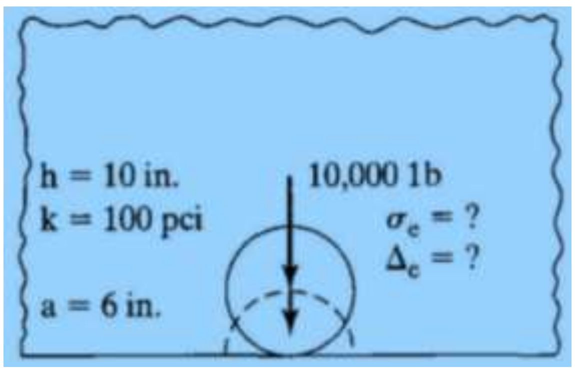
Edge Loading

The maximum stress and deflection at the edge of a concrete slab can be computed as follows:

$$\underset{(\text{circle})}{\sigma_e} = \frac{0.803P}{h^2}\left[4\log\left(\frac{\ell}{a}\right) + 0.666\left(\frac{a}{\ell}\right) - 0.034\right]$$

$$\underset{(\text{circle})}{\Delta_e} = \frac{0.431P}{k\ell^2}\left[1 - 0.82\left(\frac{a}{\ell}\right)\right]$$

Example 3: Same as Example 1 except that the load is subjected at the edge of the slab.



Solution:

$$\underset{(\text{circle})}{\sigma_e} = \frac{0.803P}{h^2}\left[4\log\left(\frac{\ell}{a}\right) + 0.666\left(\frac{a}{\ell}\right) - 0.034\right]$$

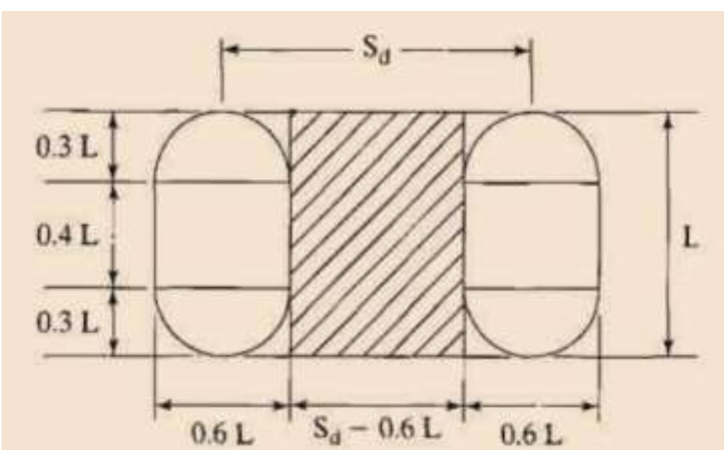
σe = 0.803* 10,000/100* [4 log(42 .97/6) + 0 .666(6/42.97) – 0 .034] = **279 .4 psi** (1.93 MPa).

$$\underset{(\text{circle})}{\Delta_e} = \frac{0.431P}{k\ell^2}\left[1 - 0.82\left(\frac{a}{\ell}\right)\right]$$

△e = 0.431*10,000/(100*1846 .4)* [1 – 0.82(6/42 .97)] = **0 .0207 in** . (0.525 mm). It can be seen that the maximum stress due to edge loading is greater than that due to corner and interior loadings and that the maximum deflection due to edge loading is greater than that due to interior loading but much smaller than that due to corner loading.

Dual Tires

When a load is applied over a set of dual tires, it is necessary to convert it into a circular area, so that the equations based on a circular loaded area can be applied. If the total load is the same but the contact area of the circle is equal to that of the duals, as has been frequently assumed for flexible pavements, the resulting stresses and deflection will be too large. Therefore, for a given total load, a much larger circular area should be used for rigid pavements. The Figure below shows a set of dual tires. It has been found that satisfactory results can be obtained if the circle has an area equal to the contact area of the duals plus the area between the duals, as indicated by the hatched area shown in the figure. If Pd is the load on one tire and q is the contact pressure, the area of each tire is:



$$\frac{P_d}{q} = \pi(0.3L)^2 + (0.4L)(0.6L) = 0.5227L^2$$

$$L = \sqrt{\frac{P_d}{0.5227q}}$$

The area of an equivalent circle can be computed as following:

$$\pi a^2 = 2 \times 0.5227L^2 + (S_d - 0.6L)L = 0.4454L^2 + S_d L$$

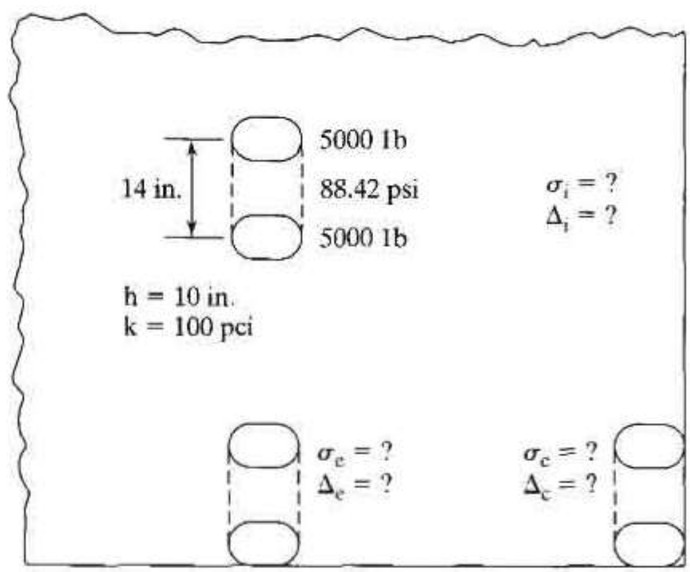
Substituting L value from above equation yields:

$$\pi a^2 = \frac{0.8521P_d}{q} + S_d\sqrt{\frac{P_d}{0.5227q}}$$

Thus, the radius of the contact area is:

$$a = \sqrt{\frac{0.8521P_d}{q\pi} + \frac{S_d}{\pi}\left(\frac{P_d}{0.5227q}\right)^{1/2}}$$

Example 4: Using Westergaard's formulas, determine the maximum stress in Examples 1, 2, and 3 if the 10,000-lb (44.5-kN) load is applied on a set of duals spaced at 14 in. (356 mm) on centers, as shown in Figure, instead of over a 6-in. (152-mm) circular area.



Solution:

With Sd = 14 in . (356 mm), q = 10,000/(36π) = 88 .42 psi (610 kPa), and Pd = 5000 lb (22 .3 kN), therefore;

$$a = \sqrt{\frac{0.8521 \times 5000}{88.42\pi} + \frac{14}{\pi}\left(\frac{5000}{0.5227 \times 88.42}\right)^{1/2}} = 7.85 \text{ in. (199 mm)}$$

So a is greater than 6 in. (152 mm).

$$\sigma_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{\ell}\right)^{0.6}\right]$$

σc= 3*10,000/100*[1 – (7.85 √(2/42 .97)^0.6] = **166 .8 psi** (1 .15MPa), which is about 89% of the stress in Example 1 .

$$b = \sqrt{1.6a^2 + h^2} - 0.675h \quad \text{when } a < 1.724h$$

b=(√1.6×7.85^2+10^2)−0.675×10= 7.34 in .(186 mm)

$$\sigma_i = \frac{0.316P}{h^2}\left[4\log\left(\frac{\ell}{b}\right) + 1.069\right]$$

σi=0.316×10000/100*[4*log**(42.97/7.34)+1.069]=**130.8 psi** (902 kPa) which is about 91% of the stress in Example 2

$$\underset{\text{(circle)}}{\sigma_e} = \frac{0.803P}{h^2}\left[4\log\left(\frac{\ell}{a}\right) + 0.666\left(\frac{a}{\ell}\right) - 0.034\right]$$

σe=0.803×10000/100*[4*log**(42.97/7.85)+0.666(7.85/42.97)−0.034]= **244.2 psi** (1.68 MPa) which is about 87% of the stress in Example 3.