CHAPTER 3

GROUNDWATER MOVEMENT

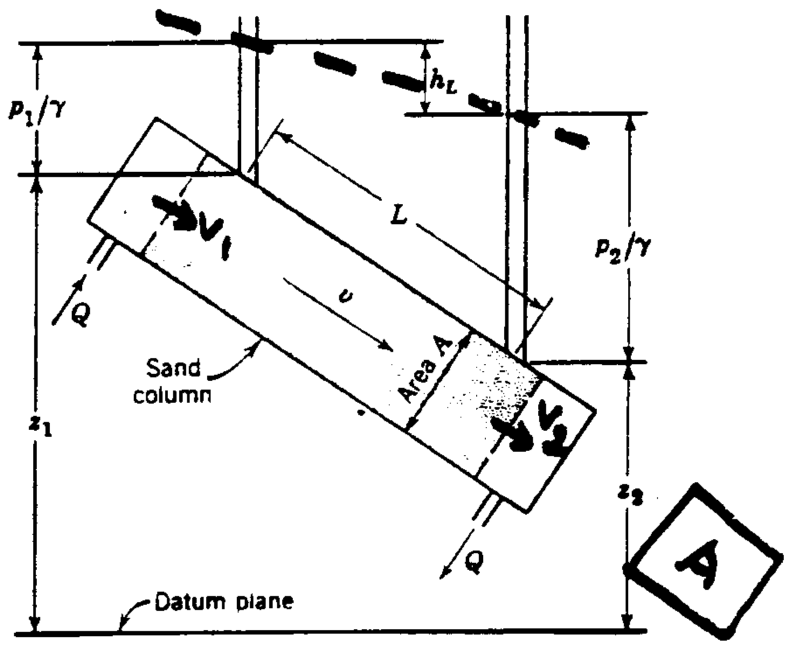
Rate of Groundwater movement related to transmission property of porous media.

What is Groundwater velocity of flow?

Darcy's law

Henry Darcy (1856) investigated water flow thru horiz.

bed of sand.



$\frac{v^2}{2g} \rightarrow$ v. small

Steady

Laminar flow – velocity very small.

Bernaulli Equation

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_L$$

P – pressure

γ – specific weight of water

v – velocity of flow

g – acceleration of gravity

z – elevation

h – head loss

Since v - is very small in linear Groundwater flow,

$$h_L = [(P_1/\gamma) + z_1] - [(P_2/\gamma) + z_2]$$

h_L defined as potential loss within sand column.

This energy lost by frictional resistance dissipated as heat energy.

h_L independent of slope of cylinder or column.

Darcy's experiment showed:

$Q \approx h_L$

$\approx 1/L$

$\approx A$

$Q \propto (h_L / L)\, A$

Q – flux

K – constant

A – area of cross section

L – column length

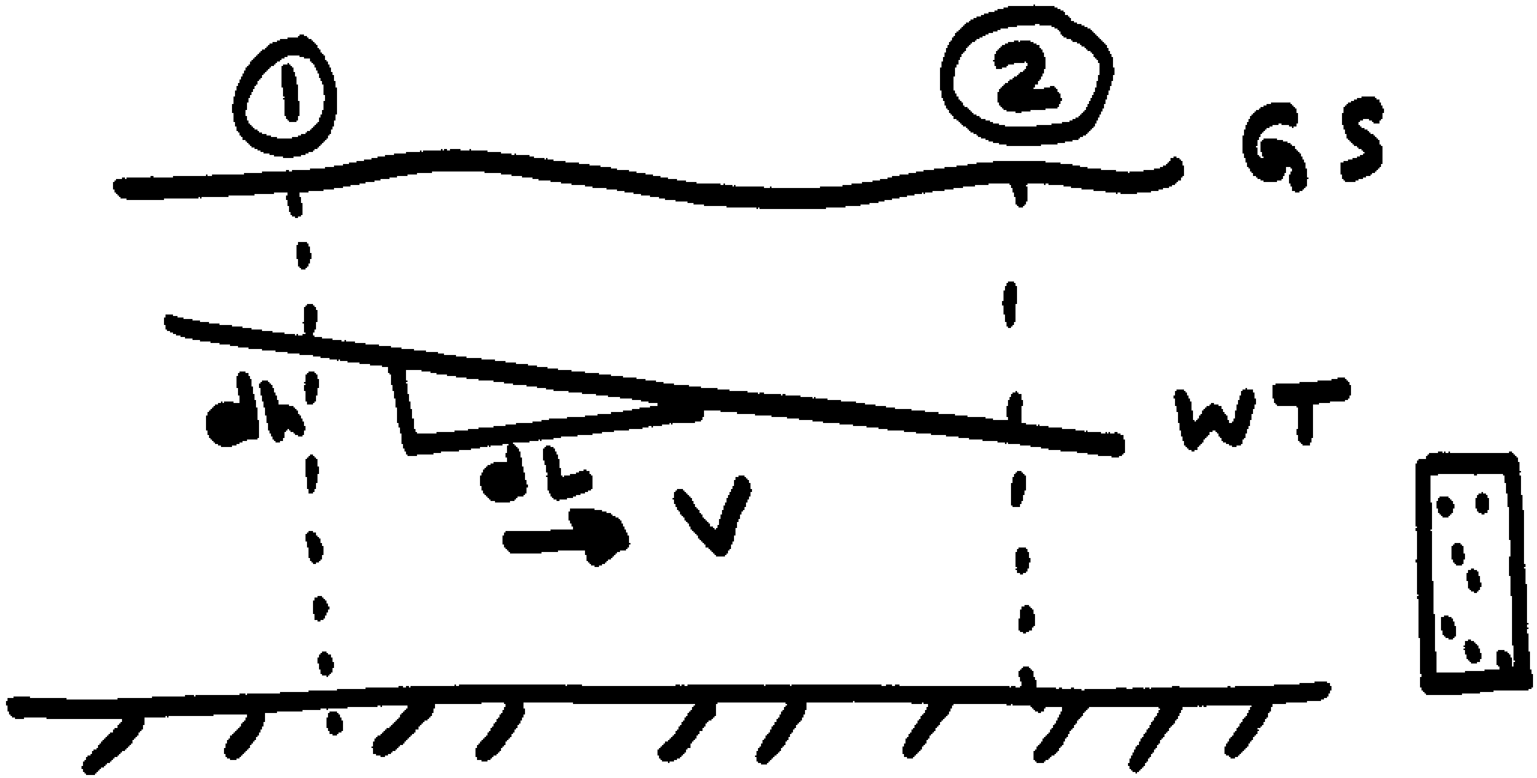
$$Q = -KA \frac{h_L}{L}$$

$$Q = -KA \frac{dh}{dL}$$

$i = dh/dL = h_L/L$ = hyd. gradient

$$V = \frac{Q}{A} = -K\frac{dh}{dL}$$

Hence, $V = -Ki$


1
2
GS
WT
dh
dL
V

Darcy's law states that flow velocity,

V = Product of Constant K, Coefficient of Permeability, and Hydraulic Gradient

Coefficient of Permeability is also known as Permeability or Hydraulic Conductivity.

(1) Darcy velocity or apparent velocity –

$V = Q/A$; Assumes water moving thru solids and pores.

(2) Pore velocity or seepage velocity –

Since water moves thru pores only, actual vel > Darcy vel.

Pore velocity = $(Q/\alpha A) = v/\alpha = - k\, i/ \alpha$

where:

Available area of flow = αA; and α = porosity

(3) Actual velocity –

variation due to pore geometry

more velocity at constriction.

Validity of Darcy's Law:

Darcy's law valid in laminar flow, not turbulent flow.

In laminar flow, flow vel. relatively small;

water molecules travel in smooth path II to solid boundaries of pores by viscous forces of fluid.

Head loss $i = av$

In <u>turbulent flow</u>,

inertial forces due to increased flow vel. dominant.

Water molecules travel in irregular paths forming eddies.

Head loss $i = av + bv^2$ a & b = constant

Criteria between laminar and turbulent flow –

Reynolds number

$$R = \frac{\rho v D}{\mu} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{vD}{\nu}$$

v – flow velocity

ρ – fluid density

D – diameter - (pipe dia. in pipes; grain size or pore dia. in porous med. - grain dia. more convenient and used)

μ – fluid dynamic viscosity

ν – kinematic viscosity =

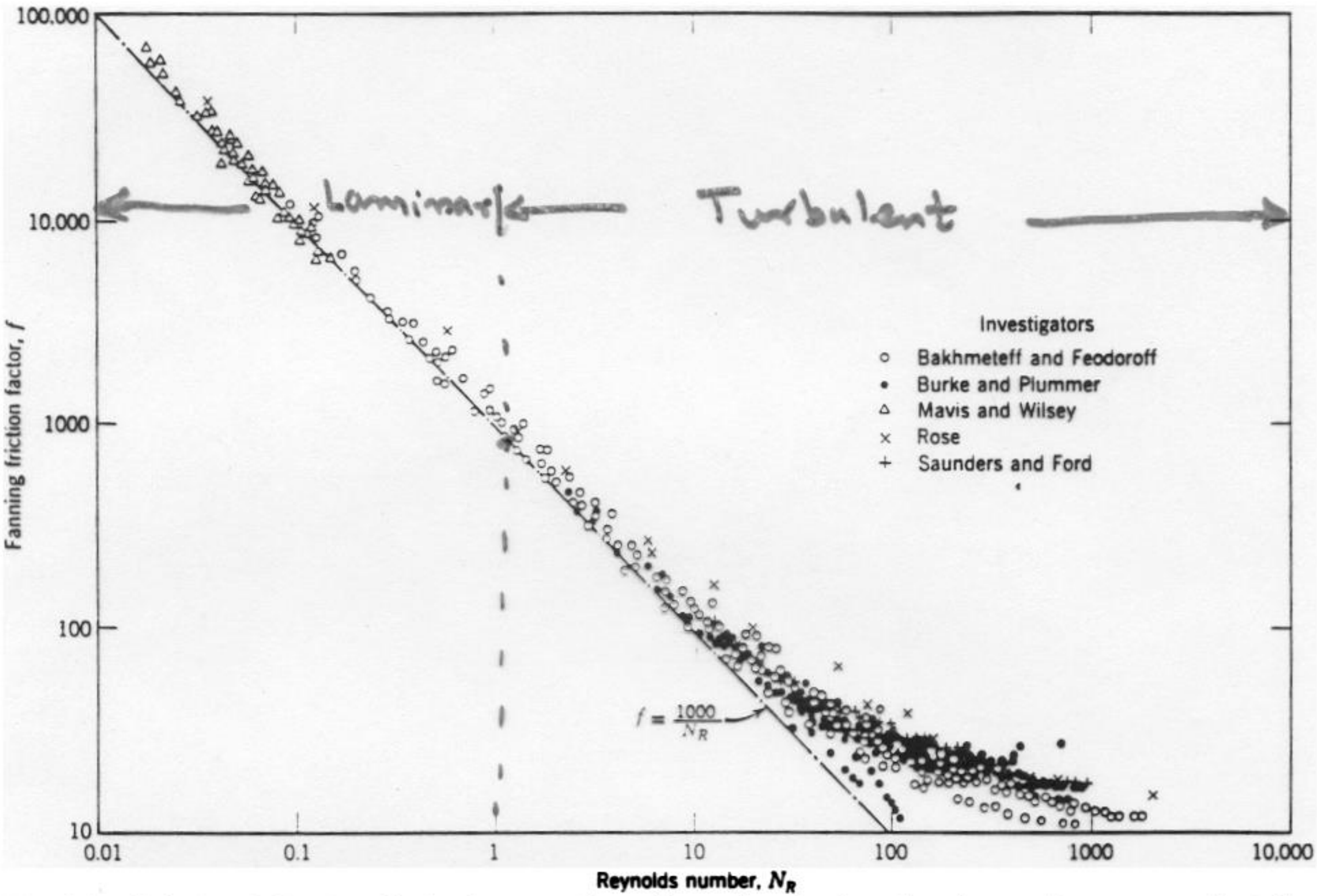


Fig. 3.2. Relation of Fanning friction factor to Reynolds number for flow through granular porous media (after Rose [60]).

Fanning factor

$$f = \frac{d\Delta P}{2\ Lv^2} = \frac{d}{2v^2}\ \frac{\Delta pg}{Lg} = \frac{d}{2v^2}\ \frac{\Delta h}{L}\ g = \frac{d}{4(v^2/2g)}\ \frac{\Delta h}{L} = \frac{di}{4(v^2/2g)}$$

where:

Δp – pressure diff. over L

d – grains size

Plot f vs. R or N_R for porous media

Laminar range –

R - 1 to 10 (<1)

Darcy's Law valid

<u>Turbulent flow</u> – occurs near pumped well casing; porous formations as basalt and limestone.

Permeability

Hydraulic Conductivity (Permeability), K:

$$K = -\frac{Q}{Ai} = -\frac{VA}{iA} = -\frac{V}{i}$$

Dimensions –

$K = -V / (dh/dL)$, m/d or gpd/ft^2 or ft/d

K =1 if Vol = 1 in time = 1

Area = 1

i = 1

A porous medium has unit K if it transmits a unit vol. of water in unit time thru unit area of cross section normal to flows under unit i at prevailing temperature.

Standard (Laboratory) Perm., K_s –

flow of water at 60^0 F in gpd thru a porous media having an area of 1 ft^2 perpendicular to flow under a hyd grad. of 1 ft/ft

K_s – 10 - 5000 gpd/ft^2

K_s – 2000 gpd/ft^2 $\Rightarrow$ good aquifer

Field perm., K_f –

Flow of water in gpd thru an aquifer of 1 ft thickness by 1 mile width perpendicular to flow under a grad. of 1 ft/mile at field temp.

$K_s = K_f$

$$\frac{K_s}{K_f} = \frac{\mu_f}{\mu_{60}} \qquad \left[\; K \propto 1/\mu \;\right]$$

$\because$

Transmissivity; T :

$T = Kb$ gpd/ft or m^2/d b = thickness of aquifer

$$Q = K\,i\,A = K\,i\,(\,b \times 1\,)$$
$$= T \times i \times 1$$

Intrinsic Perm, k:

$$K = \frac{k\gamma}{\mu}$$

K – hydraulic constant

μ – dynamic viscosity

γ – sp. weight of water

K = f (P.M., Fluid)

k is property of porous medium

$k = cd^2$, cm^2 or ft^2

d – grain size

c – constant

c = f(porosity, packing, grain size distribution, shape)

Range of Groundwater velocity:

Low velocity – clay average

$K = 10\ gpd/ft^2$

$i = 10\ ft/mile$

$v = 10(10/5280) = 2.5 \times 10^{-3}\ ft/d$

High velocity – Alluvial average

$K = 5000\ gpd/ft^2$

$i = 100\ ft/mile$

$v = 12.7\ ft/d$

Natural velocity – 5 ft/d to 5 ft/yr

EXAMPLE 3.1.1 A field sample of an unconfined aquifer is packed in a test cylinder (see Figure 3.1.1). The length and the diameter of the cylinder are 50 cm and 6 cm, respectively. The field sample is tested for a period of 3 min under a constant head difference of 16.3 cm. As a result, 45.2 cm^3 of water is collected at the outlet. Determine the hydraulic conductivity of the aquifer sample.

SOLUTION The cross-sectional area of the sample is

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.06\text{ m})^2}{4} = 0.00283\text{ m}^2$$

The hydraulic gradient, dh/dl, is given by

$$\frac{dh}{dl} = \frac{(-16.3\text{ cm})}{50\text{ cm}} = -0.326$$

and the average flow rate is

$$Q = \frac{45.2\text{ cm}^3}{3\text{ min}} = 15.07\text{ cm}^3/\text{min} = 0.0217\text{ m}^3/\text{day}$$

Apply Darcy's law, Equation 3.1.4, to obtain the hydraulic conductivity as

$$Q = -KA\frac{dh}{dl} \rightarrow K = -\frac{Q}{A(dh/dl)} = -\frac{0.0217\text{ m}^3/\text{day}}{(0.00283\text{ m}^2)(-0.326)} = 23.5\text{ m/day}$$

■

EXAMPLE 3.1.2

A confined aquifer with a horizontal bed has a varying thickness as shown in Figure 3.1.2. The aquifer is inhomogeneous with $K = 12 + 0.006x$, where $x = 0$ at section (1), and the piezometric heads at sections (1) and (2) are 14.2 m and 18.8 m, respectively measured above the upper confining layer. Assuming the flow in the aquifer is essentially horizontal, determine the flow rate per unit width.

SOLUTION

Darcy's law for a constant thickness aquifer is given by Equation 3.1.4,

$$Q = -KA\frac{dh}{dl}$$

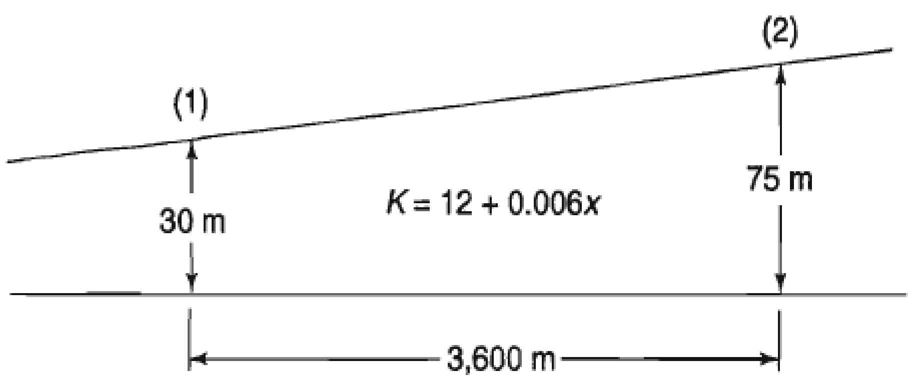


Figure 3.1.2. Aquifer for Example 3.1.2

Since the aquifer thickness is variable in this problem, we must also write the cross-sectional area and the hydraulic gradient as a function of the distance x. Assuming a unit width, $A = b_1 + \frac{(b_2 - b_1)x}{L}$, where $b_1 = 30$ m, $b_2 = 75$ m, and $L = 3{,}600$ m, then we have

$$A = 30 + \frac{(75-30)x}{3{,}600} = 30 + 0.0125x$$

Substituting the expressions for A and K into Darcy's equation yields the expression for Q in following form:

$$Q = -(12+0.006x)(30+0.0125x)\frac{dh}{dx}$$

Rearranging this equation and integrating from section (1) to section (2) yields

$$\int_0^{3600} \frac{1}{(12+0.006x)(30+0.0125x)}\,dx = \int_{14.2}^{18.8} -\frac{1}{Q}\,dh$$

This equation is integrated using partial fraction decomposition to obtain

$$\int_0^{3600}\left[\frac{0.2}{(12+0.006x)} - \frac{0.416}{(30+0.0125x)}\right]dx = \int_{14.2}^{18.8} -\frac{1}{Q}\,dh$$

$$\left[33.333\ \ln(12+0.006x) - 33.28\ \ln(30+0.0125x)\right]_{x=0}^{x=3{,}600} = -\frac{1}{Q}h\Big]_{h_1=14.2}^{h_2=18.8}$$

$$-26.54 - (-30.36) = -\frac{1}{Q}(18.8-14.2)$$

$$Q = -1.20\ (\mathrm{m^3/day/m})$$

The minus sign implies that the flow is from section (2) to (1). ■

EXAMPLE 3.1.3

The following additional information is given for the aquifer sample in Example 3.1.1. The sample has a median grain size of 0.037 cm and a porosity of 0.30. The test is conducted using pure water at 20°C. Determine the Darcy velocity, average interstitial velocity, and assess the validity of Darcy's law.

SOLUTION

Darcy velocity is computed using Equation 3.1.5:

$$v = -K\frac{dh}{dl} = -(23.54\ \text{m/day})(-0.326) = 7.67\ \text{m/day}$$

The average linear velocity is computed using Equation 3.1.6:

$$v_a = \frac{Q}{\alpha A} = \frac{v}{\alpha} = \frac{7.67\ \text{m/day}}{0.30} = 25.6\ \text{m/day}$$

In order to assess the validity of Darcy's Law we must determine the greatest velocity for which Darcy's law is valid using Equation 3.1.7, $N_R = \frac{\rho v D}{\mu}$, knowing Darcy's law is valid for $N_R<1$. For water at

$$v_{max} = \frac{\mu}{\rho D} = \frac{1.005\times10^{-3}\ \text{kg/ms}}{(998.2\ \text{kg/m}^3)(0.00037\ \text{m})} = 0.00272\ \text{m/s} = 235\ \text{m/day}$$

Then Darcy's law will be valid for Darcy velocities equal to or less than 235 m/day for this sample. Thus, the answer we have found in Example 3.1.1 is valid since v = 7.67 m/day < 235 m/day. ■

alid for very slow water flow

the minute pores produce non-

EXAMPLE 3.2.1

A leaky confined aquifer is overlain by an aquitard that is also overlain by an unconfined aquifer. The estimated recharge rate from the unconfined aquifer into the confined aquifer is 0.085 m/year. Piezometric head measurements in the confined aquifer show that the average piezometric head in the confined aquifer is 6.8 m below the water table of the unconfined aquifer. If the average thickness of the aquitard is 4.30 m, find the vertical hydraulic conductivity, K_v, of the aquitard. What type of material could this possibly be?

SOLUTION

Given $v = 0.085$ m/year $= 2.329 \times 10^{-4}$ m/day, Equation 3.2.6 is used to compute the vertical hydraulic conductivity of the aquitard:

$$K = -\frac{v}{dh/dl} = -\frac{2.329\times10^{-4}\ \text{m/day}}{(6.8\ \text{m}/4.30\ \text{m})} = 1.473\times10^{-4}\ \text{m/day}$$

From Table 3.2.1, the aquitard is composed of clay. ∎

Measurement of Permeability:

1. **Formulas**
2. **Lab. Measurement (permeameters)**
3. **Tracers**
4. **Pumping tests of wells** – CH4

Measurement of Permeability:

1. <u>Formulas</u> – derived from analytic or experimental work

   $k = f(\alpha$, packing, grain size)

   Basically, problem reduces to relating factor C to media properties

Measurement of Permeability:

Fair and Hatch formula –

$$k = \frac{1}{n \{[(1-\alpha)/\alpha^3]\}^2 \ [(\theta/100)\Sigma P/d_m)]^2}$$

k = intrinsic perm. : $K = k\,\gamma / \mu$

n = packing factor (found experimentally;)

θ = shape factor (spherical sand 6, angular grains 7.7)

α = porosity

P = % of sand held between adjacent sieves

d_m = geometric mean of the adjacent sieves

$d_m = (d_1.d_2....d_m)^{1/m}$

Measurement of Permeability:

2. Lab. Measurement (permeameters)

   Method of determining K

   (i) constant head

   (ii) falling head

   (iii) non-discharging

$Q = KA\, dh/dL$ $\quad\quad$ $K = Q/A$

Criticism:

(i) representative sample

(ii) undisturbed sample

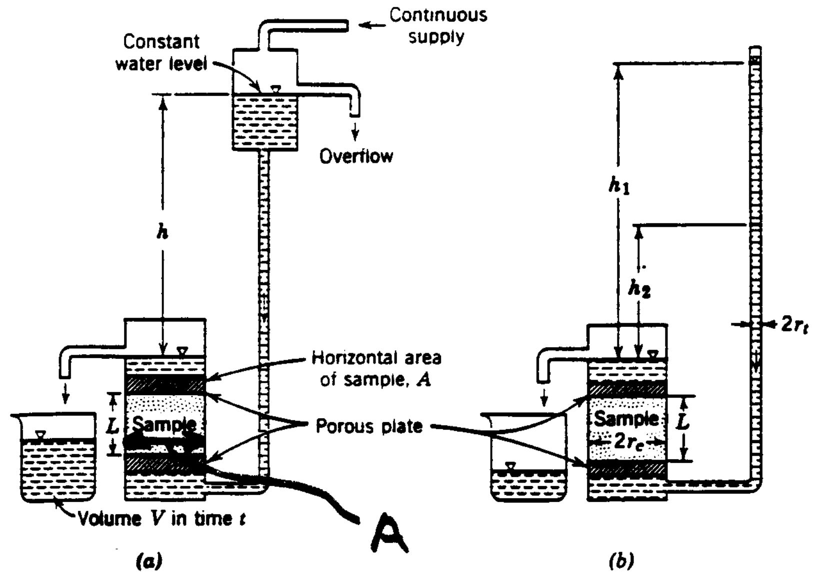


Fig. 3.4 Permeameters for measuring hydraulic conductivity of geologic samples. (a) Constant head. (b) Falling head.

Measurement of Permeability:

3. Tracers: Field method of determining K

Introduce tracer in U/S well.

Observe the time required for it to appear in D/S well.

Estimate Groundwater velocity

Measurement of Permeability:

3. Tracers cont.

Use this vel. and hyd. grad. to determine K.

Since flow occurs only in pores,

$$Q = (A\alpha)\, V_p$$

$$V_p = (K/\alpha)(h/L)$$

$$K = \frac{\alpha V_p L}{h} = \frac{\alpha L^2}{t\, h}$$

where: Vp = L/t ;

t = time of tracer appearance in well B

Measurement of Permeability:

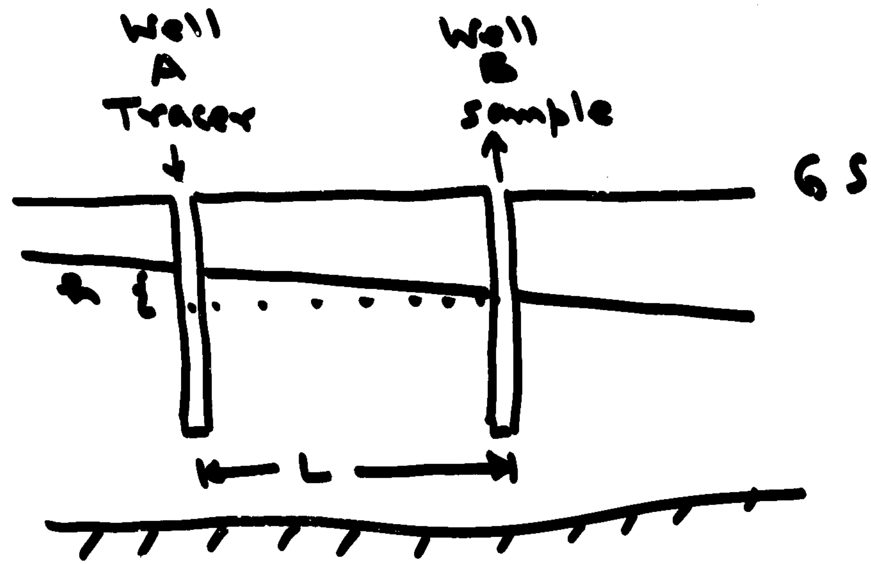
3. Tracers cont.

Criticism:

(i) Direction of flow

(ii) Front moves at unequal vel. due to variation of K

(iii) As tracers miscible with water, there is diffusion & dispersion.


Well
A
Tracer
Well
B
sample
G S
h
L

4. Pumping tests of wells

Anisotropy –

$K_x >> K_z$, anisotropic aquifer

If $K_x = K_z$ at a point, isotropic aquifer

4. Pumping tests of wells

Hetrogeneous (Nonhomogeneous) Aquifer - Layered Aquifer

If K_x or K_z same at various points in aquifer, homogeneous aquifer.

If it varies, nonhomogenous aquifer.

Average K for horizontal and vertical flows. (Prob. Given: see textbook)

EXAMPLE 3.3.1

A field sample of medium sand with a median grain size of 0.84 mm will be tested to determine the hydraulic conductivity using a constant-head permeameter. The sample has a length of 30 cm and a diameter of 5 cm. For pure water at 20°C, estimate the range of piezometric head differences to be used in the test.

SOLUTION

The maximum allowable Darcy velocity (assuming $N_R = 1$) for $d = 0.84$ mm is

$$v_{max} = \frac{\mu}{\rho D} = \frac{1.005 \times 10^{-3}\ \text{kg/ms}}{\left(998.2\ \text{kg/m}^3\right)\left(0.00084\ \text{m}\right)} = 0.0012\ \text{m/s} = 103.6\ \text{m/day}$$

Thus, the Darcy velocity in the test must be equal to or less than 103.6 m/day so that Darcy's law will be valid, so that

$$v = -K\frac{dh}{dl} \leq 103.6\ \text{m/day} \rightarrow |dh| \leq \frac{(103.6\ \text{m/day})(0.30\ \text{m})}{K}$$

For the representative value of hydraulic conductivity for medium sand given in Table 3.2.1,

$$K = 12\ \text{m/day, then } |dh| \leq \frac{(103.6\ \text{m/day})(0.30\ \text{m})}{12\ \text{m/day}} \cong 2.6\ \text{m} = 260\ \text{cm}$$

It should be noted that the K value for clean sand ranges approximately from 0.1 m/day to 4,320 m/day. See Figure 3.2.1. Therefore, the early series of tests must be conducted with relatively low piezometric head differences if possible. After analyzing the results of early test data, a better estimate of the maximum allowable piezometric head difference can be made using the above inequality. ■

EXAMPLE 3.3.2

If the field sample in Example 3.3.1 is tested with a head difference of 5.0 cm and 200 ml of water is collected at the outlet in 15 min, determine the hydraulic conductivity of the sample. What should the maximum allowable piezometric head difference be for a series of tests?

SOLUTION

Equation 3.3.3 is used to compute the hydraulic conductivity in a constant-head permeameter test:

$$K = \frac{VL}{Ath} = \frac{\left(200\ \text{cm}^3\right)\left(30\ \text{cm}\right)}{\left(\frac{\pi\left(5\ \text{cm}\right)^2}{4}\right)\left(15\,\text{min} \times 60\frac{\text{s}}{\text{min}}\right)\left(5.0\ \text{cm}\right)} = 0.0679\ \text{cm/s} = 58.7\ \text{m/day}$$

Based upon this estimate and referring to Example 3.3.1, the maximum allowable piezometric head difference for tests should be approximately

$$|dh| \le \frac{\left(103.6\ \text{m/day}\right)\left(0.30\ \text{m}\right)}{58.7\ \text{m/day}} \cong 0.53\ \text{m} = 53\ \text{cm}$$

■

EXAMPLE 3.3.3

A 20-cm long field sample of silty, fine sand with a diameter of 10 cm is tested using a falling-head permeameter. The falling-head tube has a diameter of 3.0 cm and the initial head is 8.0 cm. Over a period of 8 hr, the head in the tube falls to 1.0 cm. Estimate the hydraulic conductivity of the sample.

SOLUTION

Equation 3.3.6 is used to compute the hydraulic conductivity in a falling-head permeameter test:

$$K = \frac{r_t^2 L}{r_c^2 t} \ln\frac{h_1}{h_2} = \frac{(1.5\text{ cm})^2(20\text{ cm})}{(5.0\text{ cm})^2(8\times 3600\text{ sec})}\ln\frac{8.0\text{ cm}}{1.0\text{ cm}} = 1.3\times 10^{-4}\text{ cm/s} = 0.112\text{ m/day}$$

■

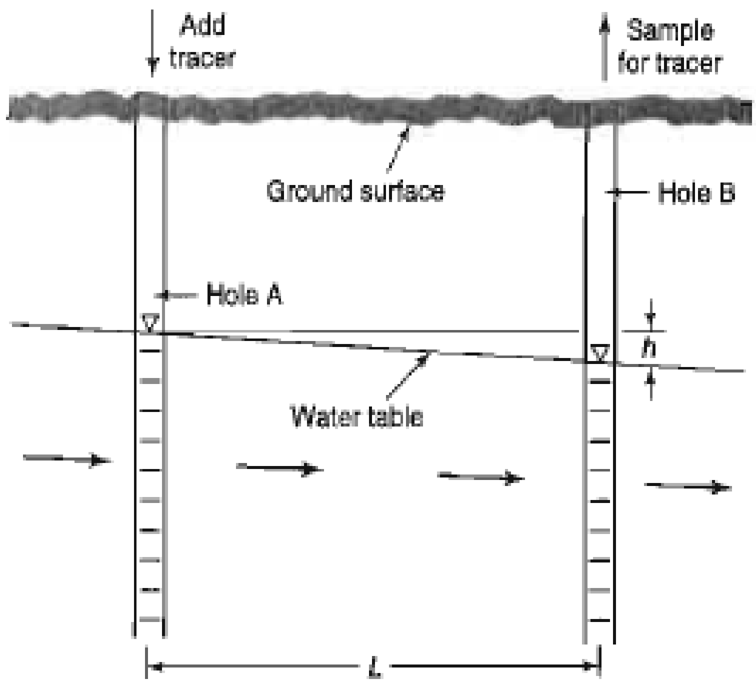


Figure 3.3.2. Cross section of an unconfined aquifer illustrating a tracer test for determining hydraulic conductivity.

Dupuit – Forchheimer Assumption:

Darcy's law –

$$V = K\,(dh/dL) \approx K \tan\theta$$

$$K\sin\theta \approx K \tan\theta$$

Applicable to 1-D horizontal or vertical flow.

In vertical flow, horizontal flow component is neglected and in horizontal flow, vertical flow is neglected.

This is called D–F assumption.

Horizontal flow–

Flux to effluent stream

$$Q = V*A$$

$$= K(dh/dL)\ (b \times 1)$$

$$= \text{flux per unit width, } (m^3/d/m \text{ or gpd/ft})$$

Vertical flow –

Flux from shallow influent stream to aquifer (depth is small and width is large)

Assume vertical leakage

Q = K(dh/dL) (W x 1)

= flux per unit length of stream

Flow Equations:

Darcy's law

V = -K

s - distance along flow direction

Velocity components in x,y,z directions –

V_x = ; V_y = ; V_z =

K_x,K_y,K_z – Perm. in x, y, and z directions

Assume homogeneous and isotropic aquifer,

$K_x = K_y = K_z = K$

V_x = ; V_y = ; V_z =

In hydrodynamics, vel. potential, ϕ, defined as a scalar function of space and time, such that

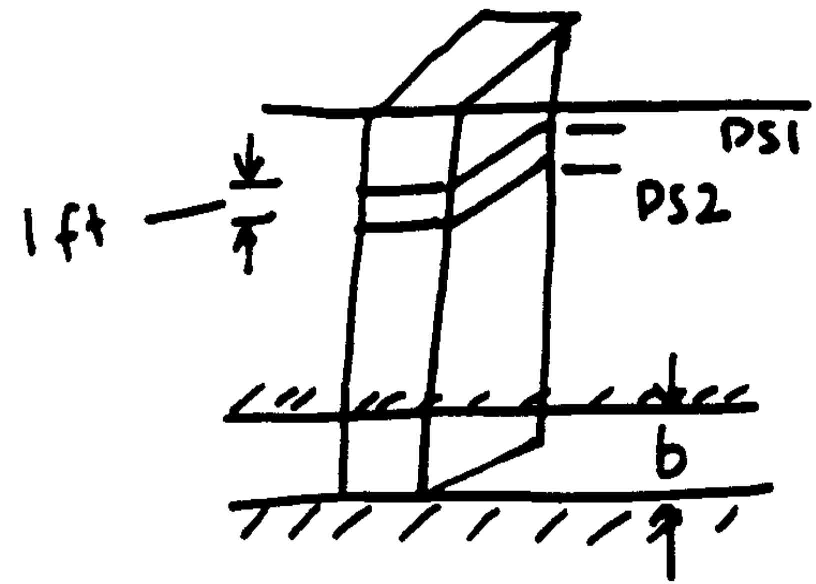
$V_x =$; $V_y =$; $V_z =$

Thus $\phi = Kh$

- Since

- This is general partial differential equation for steady flow of water in homo. & isotropic medium.

- Unsteady Flow:
  - To derive unsteady flow equation, consider storage coefficient and aquifer compressibility in confined aquifer, and yield in unconfined aquifer, S related to aquifer compressibility,



- v – volume
- p – pressure
- E – elastic modulus

- It is assumed the compressive force act in vertical direction and is negligible in horizontal direction. When p.s. lowered by 1 ft., water released = S

- Thus

- Volume of aquifer column
  - v = b.1 = b
- Change in pressure

- is negative because of decline in water level

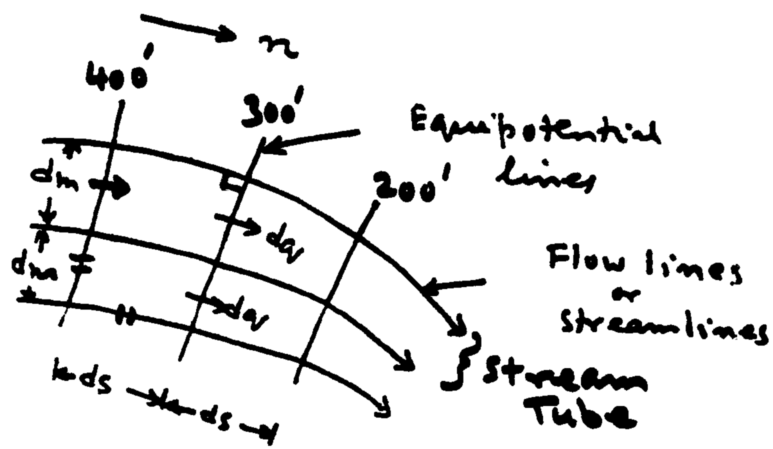
PDE for unconfined aquifer is nonlinear. The confined aquifer P.D. applied to an unconfined aquifer where variations in sat. thickness is small.

Boundary conditions–

1. **Infinite aquifer**
2. **Impermeable boundary – fault**
3. **Permeable boundary – wells, water table, surface water body (lakes, etc.)**

Method of Solution– (steady & unsteady Equations):

1. Analytic – transformations (Hodographs)
2. Flow nets - steady state flow
3. Hydraulic models
4. Analog models – electric network
5. Digital models
6. Hybrid models



n
400'
300'
200'
Equipotential lines
Flow lines or streamlines
Stream Tube
dm
dm
dq
dq
ds
ds

If n squares along a flow line & h is total head loss

n= no. equipotential tubes

$dh = h/n$

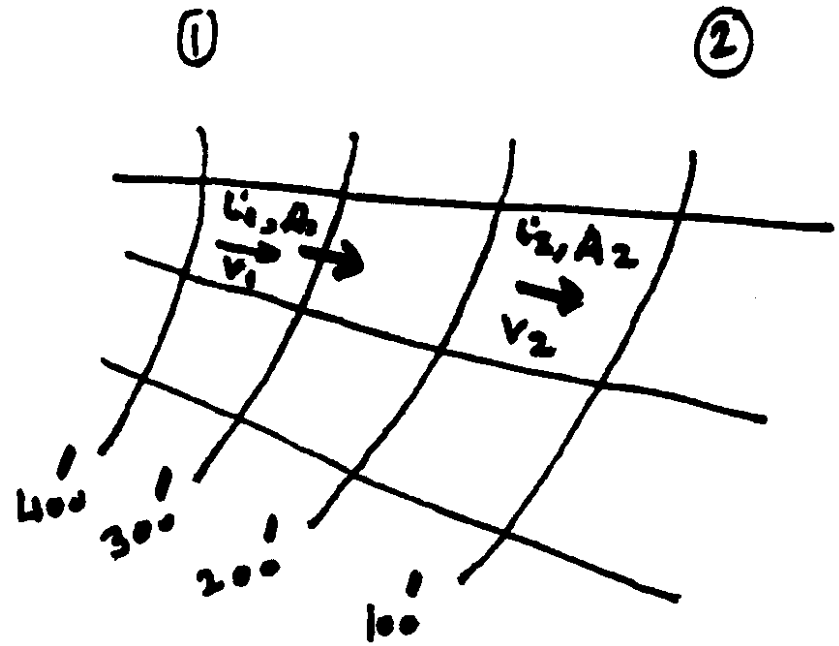
dh = head loss in 1 square

$dq = Kh/m$

If total flows Q divided into m squares (m = no. of flow tubes)

$$Q = mdq = \frac{Kmh}{n}$$

Contour map of W.T or P.S. and flow lines useful for locating new wells. Flow occurs along stream tube, velocity of flow zero across tubes.



$q = A_1V_1 = A_2V_2$

A = flow area perpendicular to flow

EXAMPLE 3.5.1 Determine h_B and the vertical velocity for the situation shown in Figure 3.5.2.

SOLUTION Assume steady-state conditions. Writing Darcy's law from point A to B with the dimensions indicated in Figure 3.5.2, we have

$$v = K\frac{dh}{dl} = 10\frac{27 - h_B}{27} \tag{3.5.3}$$

and from point B to C,

$$v = K\frac{dh}{dl} = 0.2\frac{h_B + 5 - 30}{5}$$

Solving these yields, $h_B = 26.8$ m* and $v = 0.07$ m/day. ■

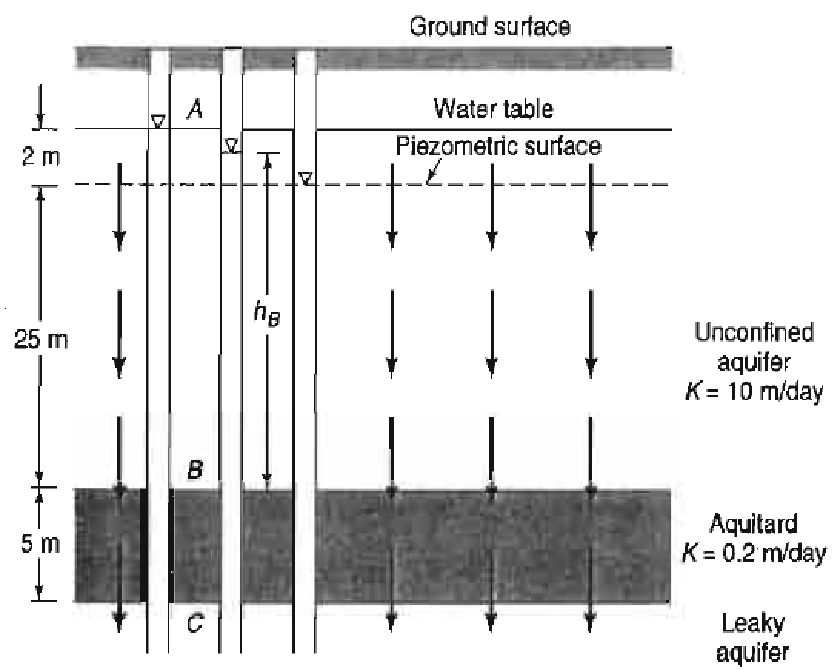


Figure 3.5.2. Diagram illustrating application of Darcy's law for vertically downward flow.

EXAMPLE 3.6.1

Three observation wells are installed to determine the direction of groundwater movement and the hydraulic gradient in a regional aquifer. The distance between the wells and the total head at each well are shown in Figure 3.6.7*a*.

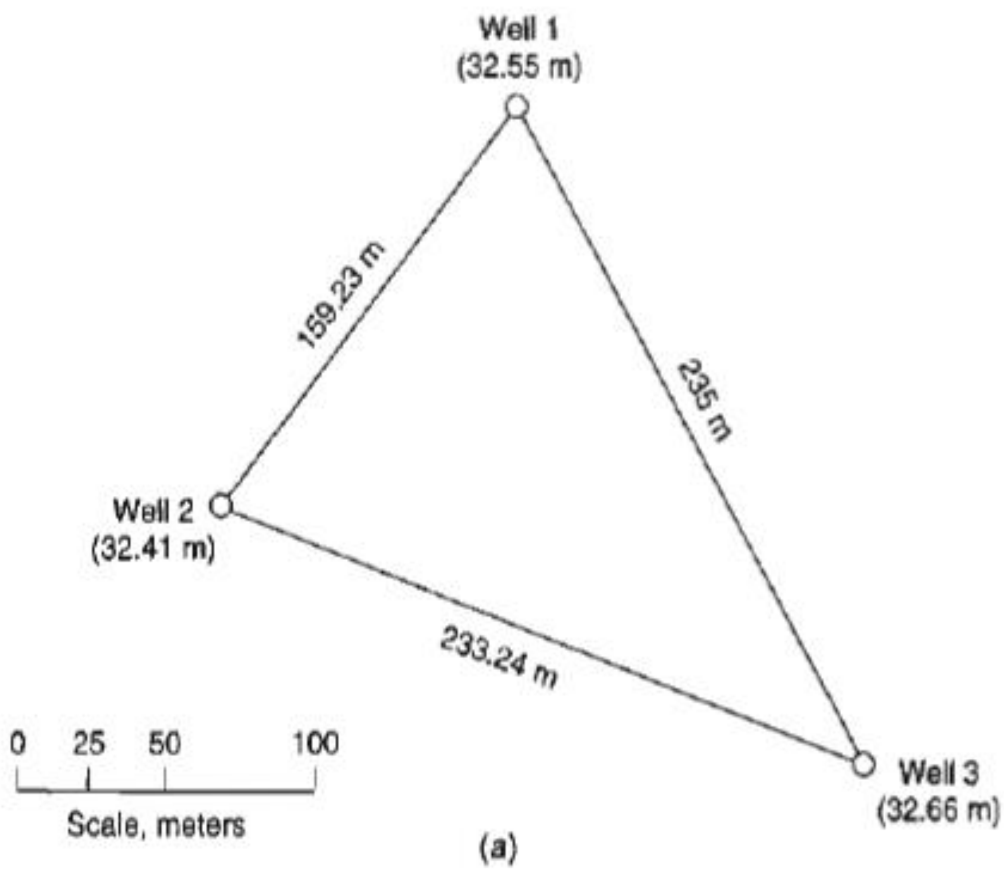


Figure 3.6.7*a*. Configuration of three observation wells in Example 3.6.1

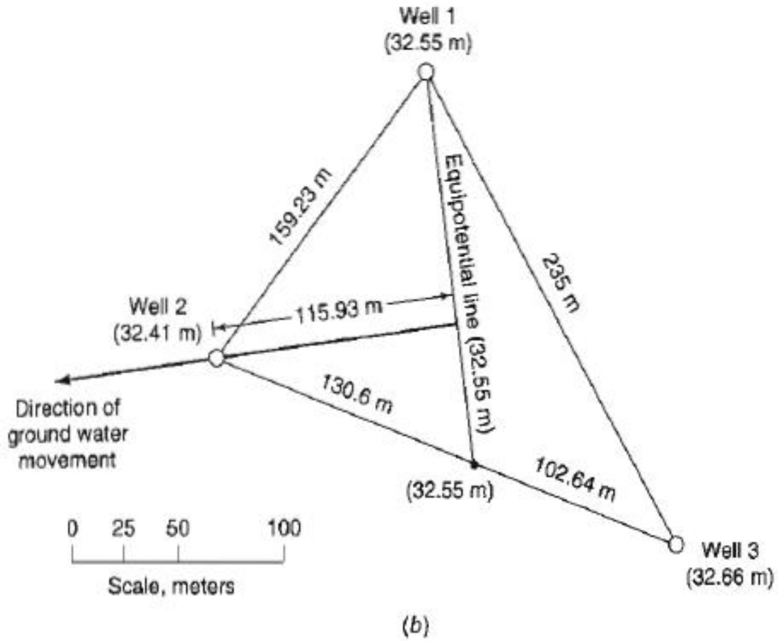


Figure 3.6.7*b*. Illustration of graphical procedure in Example 3.6.1.

SOLUTION

Step 1: Identify the well with the intermediate water level—Well 1 in this case.

Step 2: Along the straight line between the wells with the highest head and the lowest head, identify the location of the same head of the well from Step 1. Note that this is accomplished by locating the elevation of 32.55 m between Well 2 and Well 3 in the graphical solution.

Step 3: Draw a straight line between the intermediate well from Step 1 and the point identified in Step 2. This is a segment of the equipotential line along which the total head is the same as that in the intermediate well (i.e., equipotential line of 32.55 m head in this case).

Step 4: Draw a line perpendicular to the equipotential line passing through the well with the lowest head. The hydraulic gradient is the slope of that perpendicular line. Also, the direction of the line indicates the direction of groundwater movement. The graphical procedure above is illustrated in Figure 3.6.7*b*. The hydraulic gradient is then computed as

$$i = \frac{32.55\text{ m} - 32.41\text{ m}}{115.93\text{ m}} = 0.0012$$

■

EXAMPLE 3.6.2

The average daily discharge from the Patuxent Formation (see Figure 3.6.8) in the Sparrows Point district of Baltimore, Maryland, in 1945 was estimated as 1×10^6 ft^3/day. A flow net of the region is constructed using the available contour lines as shown in Figure 3.6.8. (This example is adapted from Lohman.[66]) Compute the transmissivity of the regional aquifer.

SOLUTION

As shown in the flow net, there are 15 flow channels, hence $m = 15$. There are four equipotential drops from the 60-ft contour line to the 20-ft contour line, so $h = 40$ ft and $n = 4$. Then the overall transmissivity of the district can be computed using Equation 3.6.13:

$$T = \frac{nQ}{mh} = \frac{(4)\left(1\times10^6\ \text{ft}^3/\text{day}\right)}{(15)(40\ \text{ft})} \cong 6700\ \text{ft}^2/\text{day}$$

■

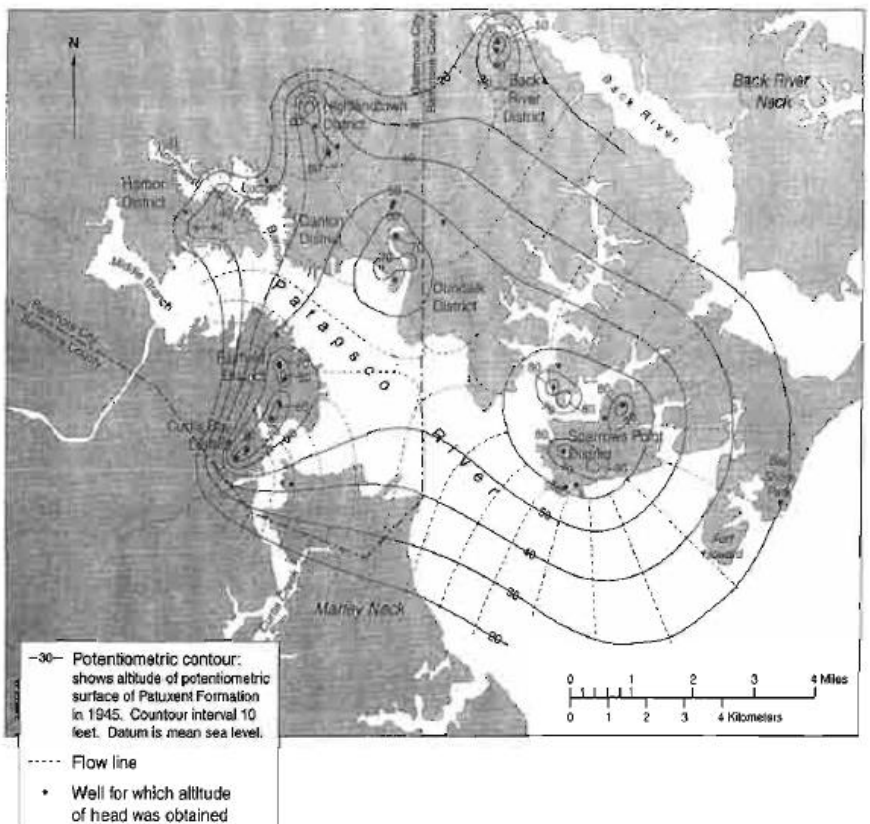
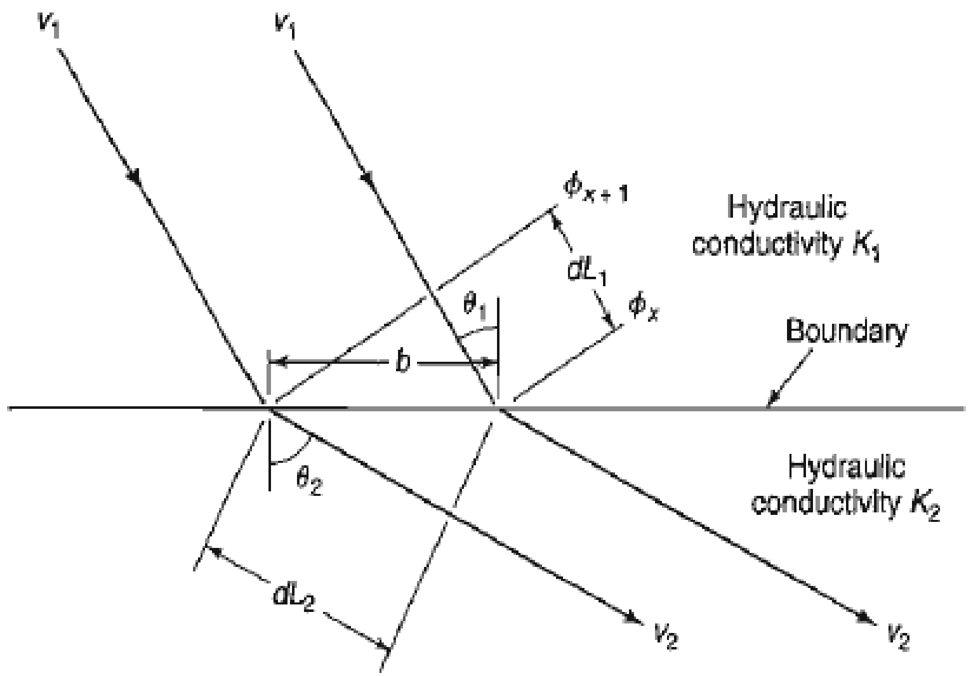
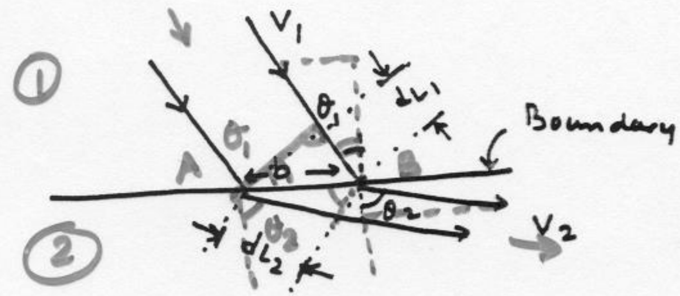


Figure 3.6.8. Map of Baltimore industrial area, Maryland, showing potentiometric surface in 1945 and generalized flow lines in the Patuxent Formation. From Bennett and Meyer[9] (as presented in Lohman[66]).

Flow Across a boundary of different perm.-

From Continuity normal compts. of flow approaching & leaving the boundary must be equal





$$dh_1 = dh_2$$

$$K_1 \frac{dh_1}{dL_2 \frac{\sin\theta_1}{\sin\theta_2}} \cos\theta_1 = K_2 \frac{dh_2}{dL_2} \cos\theta_2$$

$$K_1 \frac{\sin\theta_2}{\cos\theta_2} = K_2 \frac{\sin\theta_1}{\cos\theta_1}$$

$$K_1 \tan\theta_2 = K_2 \tan\theta_1$$

EXAMPLE 3.6.4

Consider a case where a leaky confined aquifer with 4.5 m/day horizontal hydraulic conductivity is overlain by an aquitard with 0.052 m/day vertical hydraulic conductivity. If the flow in the aquitard is in the downward direction and makes an angle of 5° with the vertical (see Figure 3.6.12), determine θ_2.

SOLUTION

Given $K_1 = 0.052$ m/day, $K_2 = 4.5$ m/day, and $\theta_1 = 5°$, Equation 3.6.25 is used to compute θ_2:

$$\frac{K_1}{K_2} = \frac{\tan\theta_1}{\tan\theta_2} \rightarrow \frac{0.052 \text{ m/day}}{4.5 \text{ m/day}} = \frac{\tan\left(5^0\right)}{\tan\theta_2} \rightarrow \theta_2 = 82.5^\circ$$

The flow lines become nearly horizontal as they enter into the confined aquifer. This is a typical case for regional flow systems, as the hydraulic conductivity of a confined aquifer is generally a few orders of magnitude larger than that of the confining layers. ■

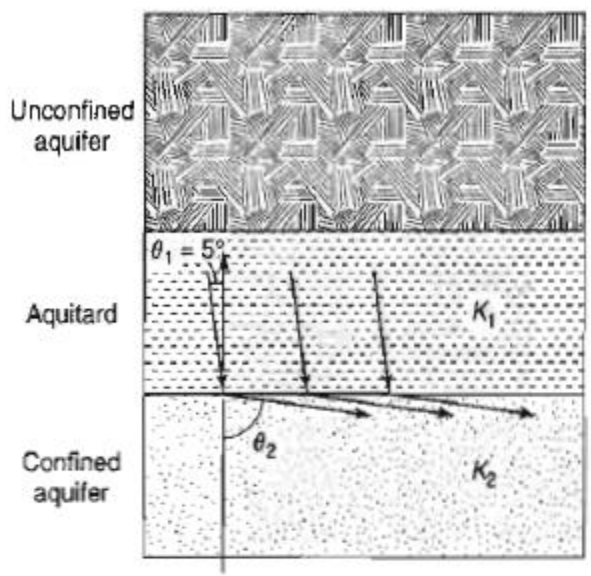


Figure 3.6.12. Example 3.6.4

Perm. in Unsat. Flow –

–

Using Darcy's Law –

$$Q = A(k\gamma/\mu)(dh/dL)$$

$$k = \frac{\mu Q/A}{\gamma(dh/dL)} = \frac{\mu Q/A}{(dp/dL)}$$

Value of k in cm2 or ft2 is very small; so a large unit darcy used in Petroleum eng. Groundwater hydrology.

$$1 \text{ darcy} = \frac{1 \text{ centipoise} \times (1\text{cm}^3/\text{s}/1\text{cm}^2)}{(1 \text{ atm.}/1 \text{ cm})}$$

$$1 \text{ centipoise} = 0.01 \text{ poise} = 0.01 \text{ dyne-sec/cm}^2$$

$$1 \text{ atm} = 1.10132 \times 10^6 \text{ dynes/cm}^2$$

thus,

$$1 \text{ darcy} = 0.987 \times 10^{-8} \text{ cm}^2$$

$$= 1.062 \times 10^{-11} \text{ ft}^2$$