Traffic Flow Theory

Overview

Traffic Flow Theory is a tool that helps transportation engineers understand and express the properties of traffic flow. At any given time, there are millions of vehicles on our roadways. These vehicles interact with each other and impact the overall movement of traffic, or the traffic flow.
Whether the task is evaluating the capacity of existing roadways or designing new roadways, most transportation engineering projects begin with an evaluation of the traffic flow. Therefore, the transportation engineer needs to have a firm understanding of the theories behind Traffic Flow Analysis.

In Traffic flow modeling two extreme situations as shown below:

Low to intermediate density
Low interaction between vehicles;
Everyone travels almost their desired speed

(Almost) complete stoppage
Flow dynamics irrelevant;
simply queuing theory

Levels of description

In a microscopic traffic description, every vehicle-driver combination is described. The smallest element in the description is the vehicle-driver combination. The other often used the level of traffic flow description is the macroscopic traffic description. Different from the microscopic description, this level does not consider individual vehicles. Instead, the traffic variables are aggregated over several vehicles or, most commonly, a road stretch. Typical characteristics of the traffic flow on a road stretch are the average speed, vehicle density, or flow. Other levels of description can also be used, these are described in the last section.

Microscopic

In a microscopic traffic description, the vehicle-driver combinations (often referred to as “vehicles”) are described individually. Full information of a vehicle is given in its trajectory, i.e. the specification of the position of the vehicle at all times. To have full information on these, the positions of all vehicles at all times have to be specified. A graphical representation of vehicle trajectories is given in figure 1.

The trajectories are drawn in a space time plot, with time on the horizontal axis. Note that vehicle trajectories can never go back in time. Trajectories might move back in space if the vehicles are going in the opposite direction, for instance on a two-lane bidirectional rural road. This is not expected on motorways. The slope of the line is the speed of the vehicles. Therefore, the trajectories cannot be vertical – that would mean an infinite speed. Horizontal trajectories are possible at speed zero.

Basic variables in the microscopic representation are speed, headway, and space headway. The speed is the amount of distance a vehicle covers in a unit of time, which is indicated by v. Sometimes, the inverse of speed is a useful measure, the amount of time a vehicle needs for to cover a unit of space; this is called the pace p. Furthermore, there is the space headway or spacing (s) of the vehicle. The net space headway is the distance between the vehicle and its leader. This is also called the gap. The gross space headway of a (following) vehicle the distance including the length of the vehicle, so the distance from the rear bumper of the leading vehicle to the rear bumper of the following vehicle. Similarly, we can time it takes for a follower to get to reach (with its front bumper) the position of its leader’s rear bumper. This is called the net time headway. If we also add the time it cost to cover the distance of a vehicle length, we get the gross time headway. See also figure 2. The symbol used to indicate the headway is h. From now on, in this reader we will use the following conventions:

- ❖ Unless specified otherwise, headway means time headway
- ❖ Unless specified otherwise, headways and spacing are given as gross values

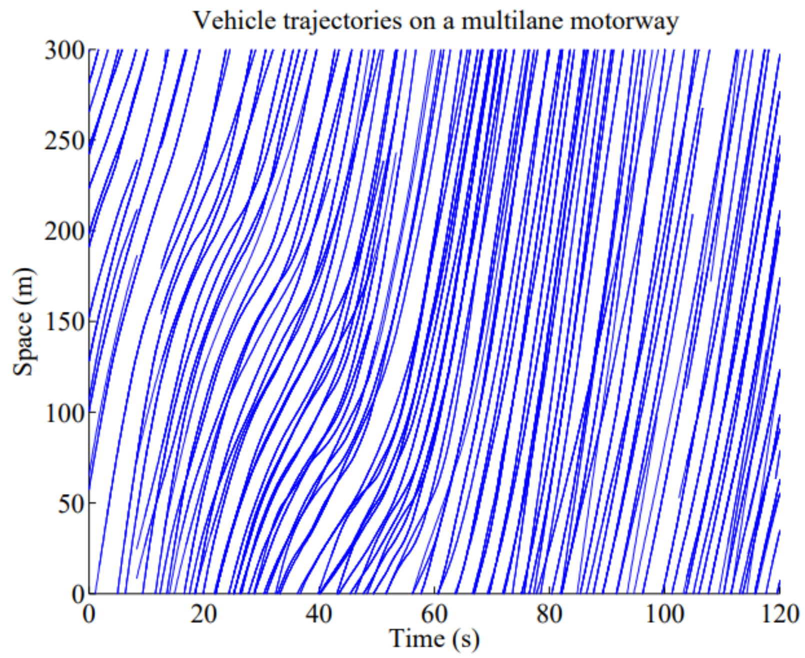


Figure 1: Vehicle trajectories on a multilane motorway.

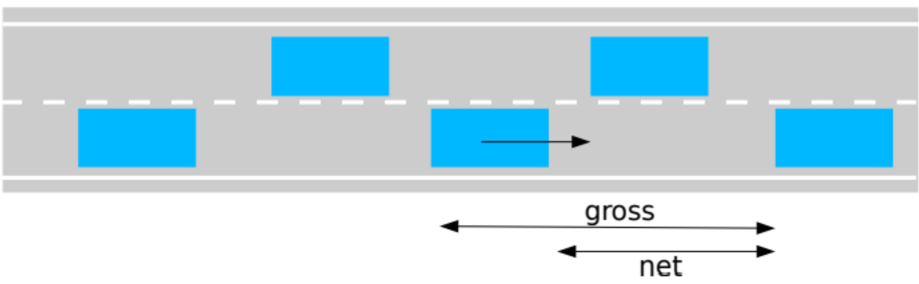


Figure 2: The difference between gross and net spacing (or headway).

Figure 3 shows the variables graphically. The figure shows two vehicles, a longer vehicle, and a shorter vehicle. Note that the length of the vehicles remains unchanged, so the difference between the gross and net spacing is the same, namely the vehicle length. However, the difference between the gross and net time headway changes based on the vehicle speed. In a trajectory plot, the slope of the line is the speed. If this slope changes, the vehicle accelerates or decelerates. So, the curvature of the lines in a trajectory plot shows the acceleration or deceleration of the vehicle. If the slope increases, the vehicle accelerates, if it decreases, it decelerates.

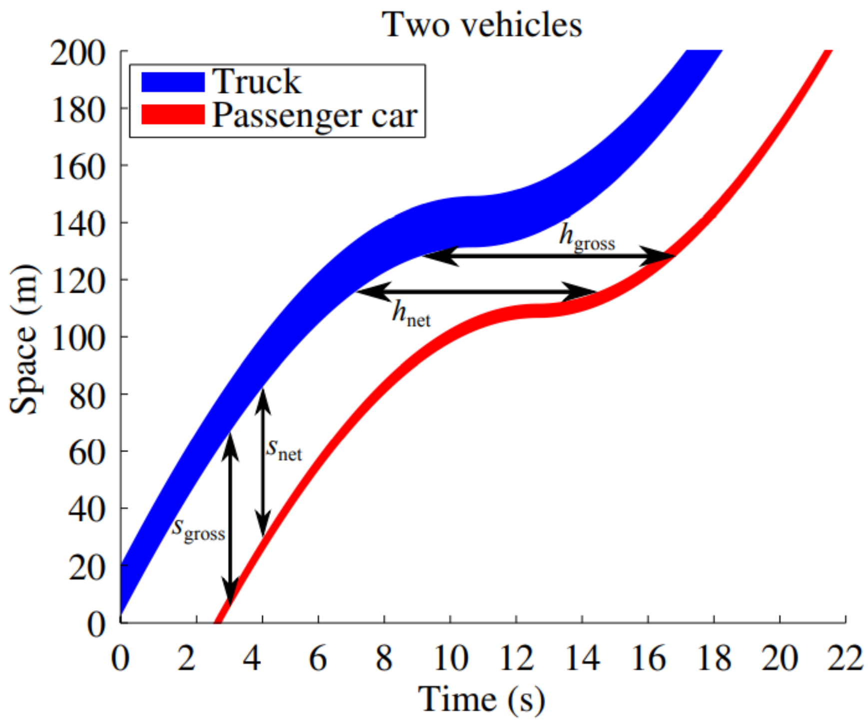


Figure 3: The microscopic variables explained based on two vehicles.

Marcoscopic

In a macroscopic traffic description, one does not describe individual vehicles. Rather, one describes for each road section the aggregated variables. That is, one can specify the density k, i.e. how close in space vehicles are together. Furthermore, one can specify the flow q i.e. the number of vehicles passing a reference point per unit of time. Finally, one can describe the average speed u of the vehicles on a road section. Other words for flow are throughput, volume, or intensity; we will strictly adhere to the term flow to indicate this concept.

All of the mentioned macroscopic variables have their microscopic counterpart. This is summarized in Table 1. The density is calculated as one divided by the average spacing and is calculated over a certain road stretch. For instance, if vehicles have a spacing of 100 meters, there are 1/100 vehicles per meter or 1000/100=10 veh/km. The flow is the number of vehicles that pass a point per unit of time. It can be directly calculated from the headways by dividing one over the average headway. For instance, if all vehicles have a headway of 4 seconds, there are 1/4 vehicles per second.

That means there are 3600(s/h)/4(s/veh) = 900 veh/h. In table 1.1 units are provided and in the conversion from one quantity to the other, one needs to pay attention. Note that provided units are not obligatory:

One can present individual speed in km/h, or density in veh/hm. However, always pay attention to the units before converting or calculating.

Table 1. Overview of the microscopic and macroscopic variables and their relationship; the pointy brackets indicate the mean.

Microscopic	symbol	unit	Macroscopic	symbol	unit	relation
Headway	h	s	Flow	q	vtg/h	$q = \frac{3600}{\langle h \rangle}$
Spacing	s	m	Density	k	vtg/km	$k = \frac{1000}{\langle s \rangle}$
Speed	v	m/s	Average speed	u	km/u	$u = 3.6 \langle v \rangle$

Relation to the microscopic level

The average speed is calculated as an average of the speeds of vehicles at a certain road stretch. This speed differs from the average speed obtained by the average speed of all vehicles passing a certain point. The next section explains the different measuring principles. The full explanation of the differences between the two speeds and how one can approximate the (space) average speed by speeds of vehicles passing a certain location.

Another concept for a traffic flow, in particular in relation to a detector, is the occupancy o. This indicates which fraction of a time a detector embedded in the roadway is occupied, i.e. whether there is a vehicle on top of the detector. Suppose a detector has a length L_{det} and a vehicle a length of L_i. occupancy is defined as the time the detector is occupied, $\tau_{occupied}$ divided by all time, i.e. the time it is occupied and time is not occupied $\tau_{not\ occupied}$.

$$o = \frac{\tau_{occupied}}{\tau_{occupied} + \tau_{not\ occupied}} \qquad 1$$

The occupation time can be derived from the distances and the speed. The distance the vehicle has to cover from the moment it starts occupying the detector up to the time it leaves the detector is its own length plus the length of the detector. Hence, the occupancy time is:

$$\tau_{occupied} = \frac{L_i + L_{det}}{v} \qquad 2$$

Once the first vehicle drives off the detector, the distance for the following vehicle to reach the detector is the gap (i.e., the spacing minus the length of the vehicle) between the vehicles minus the length of the detector. The amount of time this takes is:

$$\tau_{not\ occupied} = \frac{s - L_i - L_{det}}{v} \qquad 3$$

Substituting the expressions for the occupancy time and the non-occupancy time into equation 1 and rearranging the terms, we get:

$$o = \frac{L_i + L_{det}}{s} \qquad 4$$

On practice, the detector length is known for a certain road configuration (usually, there are country specific standards). So assuming a vehicle length, one can calculate the spacing, and hence the density, from the occupancy.

Other levels

Apart from the macroscopic and microscopic traffic descriptions, there are three other levels to describe traffic. They are less common and are therefore not discussed in detail. The levels mentioned here are mainly used in computer simulation models.

Mesoscopic

The term mesoscopic is used for any description of traffic flow that is in between macroscopic and microscopic. It can also be a term for simulation models which calculate some elements macroscopically and some microscopically. For instance, dynasmart Dynasmart (2003), uses such a mesoscopic description.

Submicroscopic

In a submicroscopic description, the total system state is determined by the sub-levels of a vehicle and/or driver. Processes that influence the speed of a vehicle, like for instance mechanically throttle position and engine response, or psychologically speed perception, are explicitly modeled. This allows to explicitly model the (change in) reaction on inputs. For instance, what influence would cars with a stronger engine have on the traffic flow.

Network level

A relatively new way of describing the traffic state is the network level. This has recently gained attention after the publication by Geroliminis and Daganzo (2008). Instead of describing a part of a road as the smallest element, one can take an area (e.g. a city center) and consider this as one unit.

Measuring principles

Whereas the previous sections described which variables are used to describe traffic flow, this section will introduce three principles of measuring traffic flow. These principles are local, instantaneous, and Spatio-temporal.

Local

With local measurements, one observes traffic at one location. This can be for instance a position on the roadway. To measure motorway traffic, often inductive loops are used. These are coils embedded in the pavement in which an electrical current produces a (vertical) magnetic field. If a car enters or leaves this magnetic field, this can be measured in the current of the coil. Thus, one knows how long a loop is occupied. In the US, usually, single loops are used, giving the occupancy of the loop. Using equation 4, this can be translated into density. The detectors also measure the flow. This determination of density builds upon the assumption of the vehicle length being known. One can also measure the length of a vehicle for passing vehicles, using dual loop detectors. These are inductive loops that are placed at a known short distance (order of 1 m) from each other. If one measures carefully the time between the moments the vehicle starts occupying the first loop and the moment it starts occupying the second loop, one can measure its speed. If its speed is known, as well as the time it occupies one loop, the length of the vehicle can also be determined.

Instantaneous

Contrary to local measurements, there are instantaneous measurements. These are measurements that are taken at one moment in time, most likely over a certain road stretch. An example of such a measurement is a real photograph. In such a measurement, one can clearly distinguish spatial characteristics, such as for instance density. However, measuring the temporal component (flow) is not possible.

Spatio-temporal measurements

Apart from local or instantaneous measurements, one can use measurements that stretch over a period of time and a stretch of road. For instance, the trajectories in figure 1 are an example thereof. This section will introduce Edie’s definitions of flow, density, and speed for an area in space and time.

A combination of instantaneous measurements and local measurements can be found in remote sensing observations. These are observations that stretch in both space and time. For instance, the trajectories presented in figure 1 can be observed using a camera mounted on a high point or a helicopter. One can see a road stretch, and observe it for a period of time.

Measuring average speed by definition requires an observation that stretches over time and space. At one location, one cannot determine speed, nor at one moment. One needs at least two locations close by (several meters) or two-time instances close by. Ignoring these short distances one can calculate a local mean speed based on the speeds of the vehicles passing by the location. Ignoring the short times, one can calculate the time mean speed from the speed of the vehicles on currently the road. At this moment, we suffice by mentioning these average speeds are different.

Figure 4 shows the same trajectories as figure1, but in figure 4 an area is selected. Trajectories within this area in space and time are colored red. Note that a selected area is not necessarily square. It is even possible to have a convex area, or boundaries moving backward and forwards in time. The definitions as introduced here will hold for all types of areas, regardless of their shape in space-time.

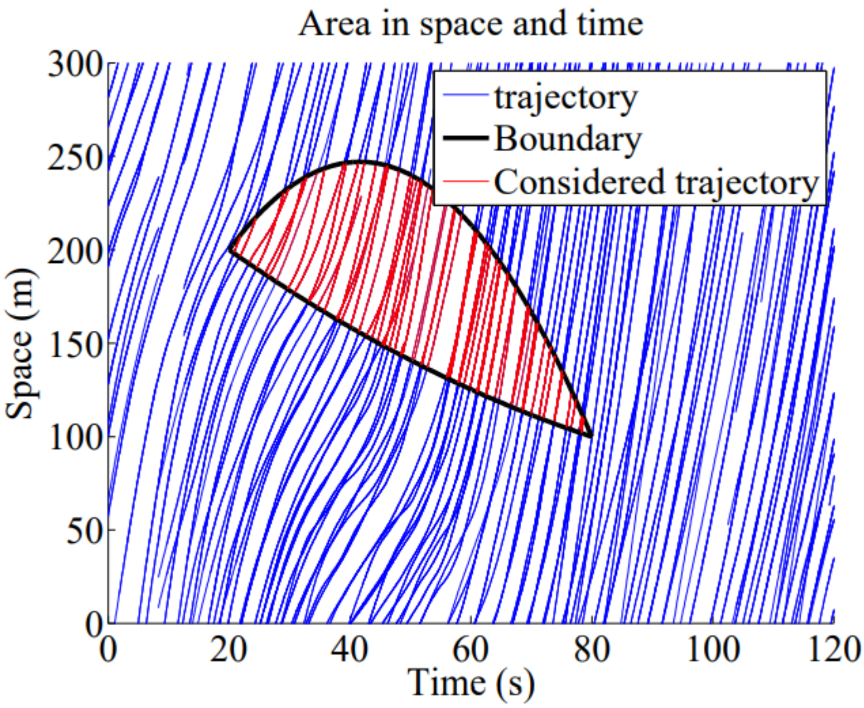


Figure 4: Vehicle trajectories and the selection of an area in space and time.

Let us consider the area X. We indicate its size by W_X, which is expressed in km-h, or any other unit of space times time. For all vehicles, we consider the distance they drive in area X, which we call $d_{X,i}$. Adding these for all vehicles i gives the total distance covered in area X, indicated by TD:

$$TD = \sum_{all\ vehicle\ i} d_{X.i} \tag{5}$$

For a rectangular area in space and time, the distance covered might be the distance from the upstream end to the downstream end, but the trajectory can also begin and/or end at the side of the area, at a certain time. In that case, the distance is less than the full distance.

Similarly, we can define the time a vehicle spends in area X, $t_{X,i}$, which we can sum for all vehicles i to get the total time spent in area X, indicated by TT.

$$TT = \sum_{all\ vehicle\ i} t_{X.i} \tag{6}$$

Obviously, both quantities grow in principle with the area size. Therefore, the traffic flow is best characterized by the quantities TD/WX and TT /WX. This gives the flow and the density respectively:

$$q = \frac{TD}{W_x} \qquad 7$$

$$k = \frac{TT}{W_x} \qquad 8$$

Intuitively, the relationship is best understood reasoning from the known relations of density and flow. Starting with a situation of 1000 veh/h at a cross-section, and an area of 1 h and 2 km. In 1 hour, 1000 vehicles pass by, which all travel 2 kilometers in the area (There are the vehicles that cannot cover the 2 km because the time runs out, but there are just as many which are in the section when the time window starts). So the total distance is the flow times the size of the area: TD = q.WX. This can be simply rewritten to equation 8.

A similar relation is constructed for the density, considering again the rectangular area of 1 hour times 2 kilometers. Starting with a density of 10 veh/km, there are 20 vehicles in the area, which we all follow for one hour. The total time spent is hence 10*2*1, or TT = k.WX. This can be rewritten to equation 8. The average speed is defined as the total distance divided by the total time, so:

$$u = \frac{TD}{TT} \qquad 9$$

The average travel time over a distance l can be found as the average of the time a vehicle travels over a distance l. In an equation, we find:

$$\langle tt \rangle = \langle \frac{1}{v} \rangle = l \, \langle \frac{1}{v} \rangle \qquad 10$$

In this equation, tt indicates the travel time and the pointy brackets indicate the mean. This can be measured for all vehicles passing a road stretch, for instance at a local detector. Note that the mean travel time is not equal to the distance divided by the mean speed:

$$\langle tt \rangle = l \, \langle \frac{1}{v} \rangle \neq l \, \frac{1}{\langle v \rangle} \qquad 11$$

In fact, it can be proven that in case speeds of vehicles are not the same, the average travel time is underestimated if the mean speed is used.

$$\langle tt \rangle = l \, \langle \frac{1}{v} \rangle \leq l \, \frac{1}{\langle v \rangle} \qquad 12$$

The harmonically averaged speed (i.e., 1 divide by the average of 1 divided by the speed) does provide a good basis for the travel time estimation. In an equation, we best first define the pace, p_i:

$$p_i = \frac{1}{v_i} \qquad 13$$

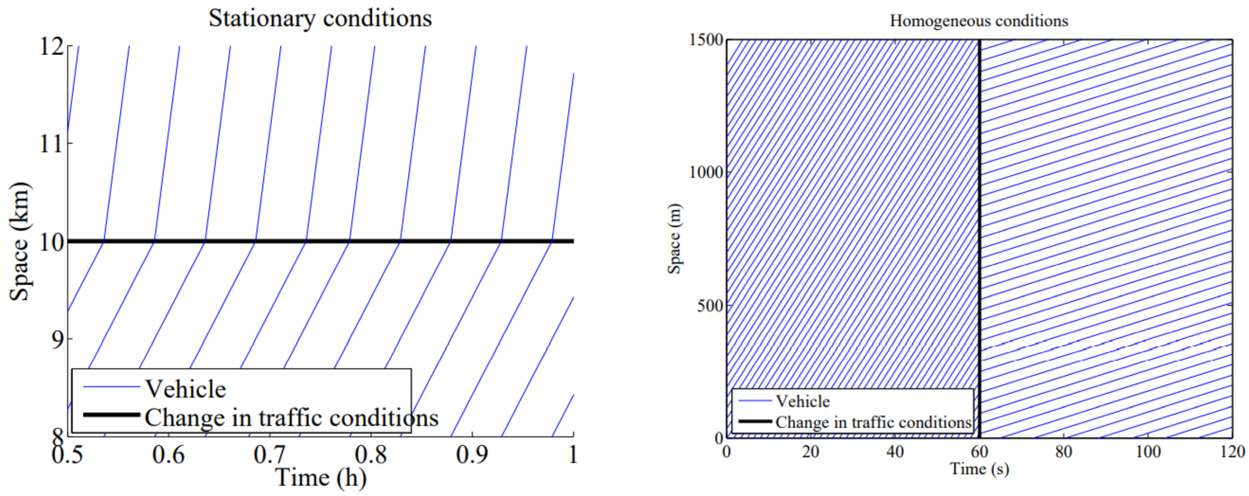
The harmonically averaged speed now is:

$$\langle v \rangle_{harmonically} = \frac{1}{\langle p \rangle} = \frac{1}{\langle \frac{1}{v_i} \rangle} \qquad 14$$

The same quantity is required to find the space mean speed. In short, differences can be several tens of percent.

Stationarity and homogeneity

Traffic characteristics can vary over time and/or over space. There are dedicated names for traffic if the state does not change. Traffic is called stationary if the traffic flow does not change over time (but it can change over space). An example can be for instance two different road sections with different characteristics. An example is given in figure 5(a), where there first is a low speed, then the speed of the vehicles is high. Traffic is called homogeneous if the traffic flow does not change over space (but it can change over time). An example is given in figure 5(b), where at time 60 the speed decreases at the whole road section. This is much less common than stationary conditions. For these conditions to occur, externally the traffic regulations have to change. For instance, the speed limits might change at a certain moment in time (lower speeds at night).



(a) Stationary conditions (b) Homogeneous conditions

Figure 4: Stationarity and homogeneity.

Exercises

Q1/ what is higher, the space mean speed of the time mean speed? Why?

Answer:

Time mean speed, since the faster cars are weighted higher

Q2/ suppose there are two classes of vehicles, fast vehicles driving 120 km/h and slow vehicles moving at 90 km/h. At the entrance to the road (x=0), an equal number of each type of vehicles is measured under a total flow of 1000 veh/h (so 500 veh/h each). Calculate the density on the road. Base your answer on the density of the slow vehicles and the fast vehicles.

Answer:

The partial densities can be calculated by k=q/v per class,

Leading to k1=500/120=4.2 veh/km

And k2 =500/90=5.6 veh/km.

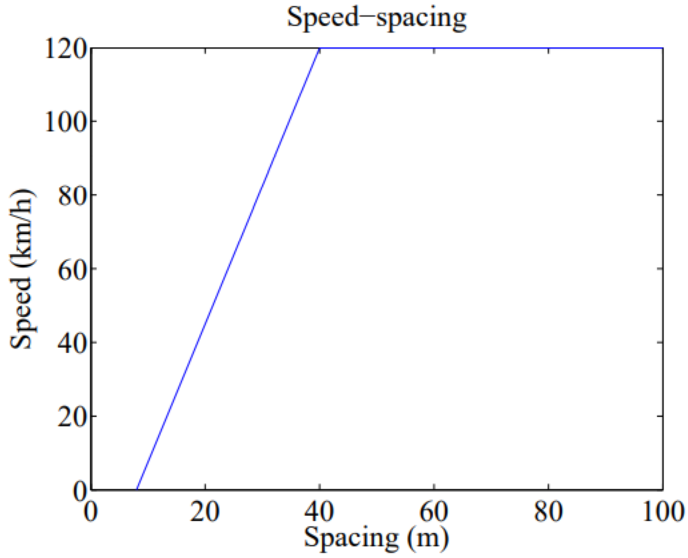
The total density is the sum of these two, 9.7 veh/km

Q3/ One can argue that there are three levels at which traffic can be described: microscopic, macroscopic, and network level. On the microscopic level, traffic behavior can be indicated by a spacing-speed diagram.

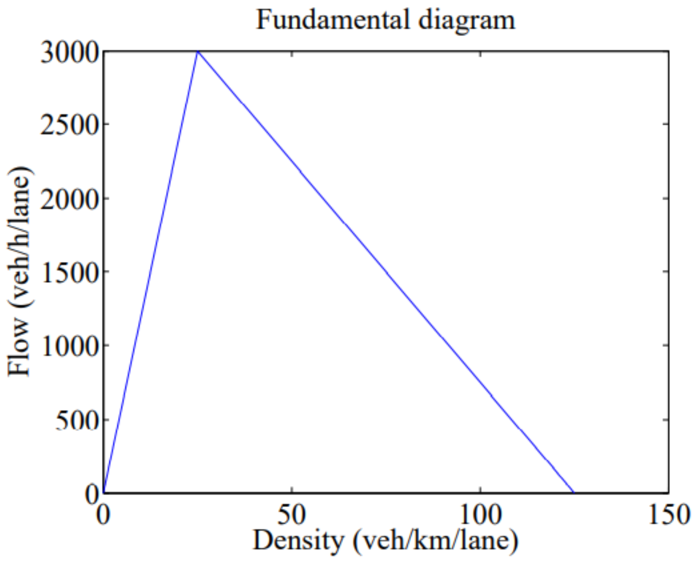
- Sketch this diagram
- Sketch the matching diagram in the flow-density plane; indicate how the values for this graph can be obtained from the previous graph
- Give is the name for the flow aggregated on the network level
- Give the name for the density aggregated on the network level
- Sketch the matching Network Fundamental Diagram for the network level in the same graph as the fundamental diagram of question b, and briefly comment on the difference.

Answer:

- Speed increases with increasing spacing No values for smaller spacing than a critical value, and for higher spacing the speed is constant.

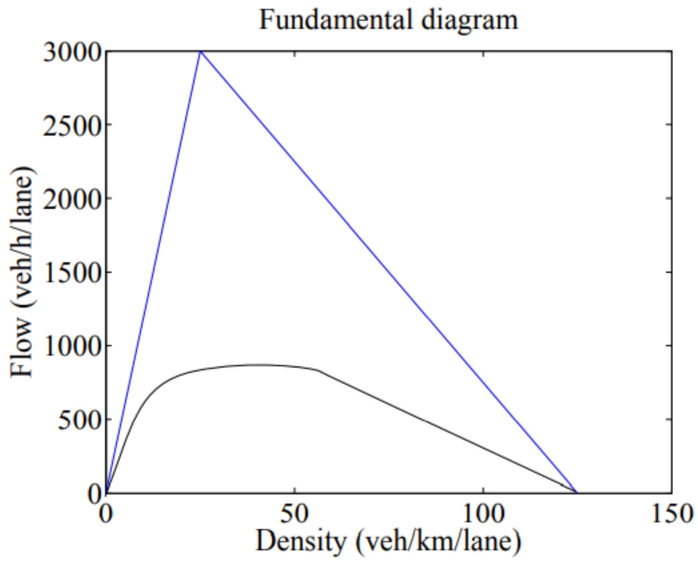


- Free flow speed is slope, critical density is 1/critical spacing, jam density = 1/min spacing.



A piecewise linear speed-spacing diagram leads to a triangular fundamental diagram.

- Production
- Accumulation
- It is lower and it has a flattened top. Plotting below: The right part is mainly theoretical since one cannot reach that state.


Fundamental diagram
3000
2500
2000
1500
1000
500
0
0
50
100
150
Flow (veh/h/lane)
Density (veh/km/lane)