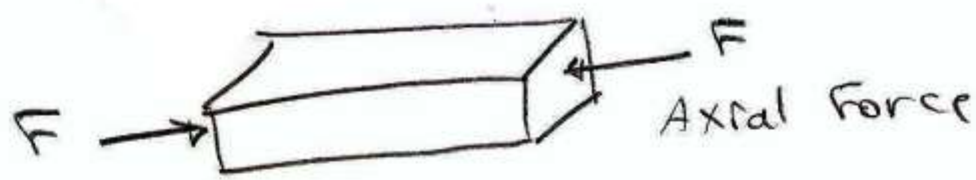
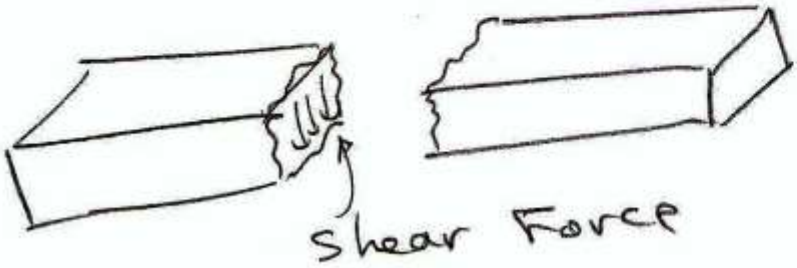
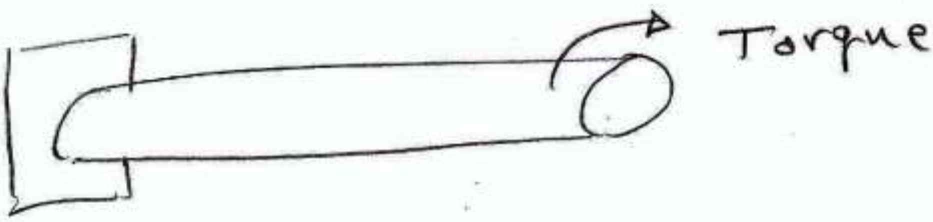
① Bending stresses in Beams

تتعرض الاعضاء الانشائية لقوى او عزوم خارجية مسببة اجهادات على الاعضاء الانشائية. ويمكن ان تكون احداها او عدة منها وهي:-

1- Axial Force
2- Shear Force
3- Bending moment
4- Torsion







Bending

(2) Bending Stresses

الكلمة Bent بالانكليزية هي نفس Flex وتعني انحناء او انثناء .

وقد تتعرض الـ Beam الى عدة قوى في نفس الوقت مثل Axial ، shear ، Torsion وتسبب اجهادات تسمى اجهادات مركبة Compound وهذا ليس موضوع دراستنا حالياً .

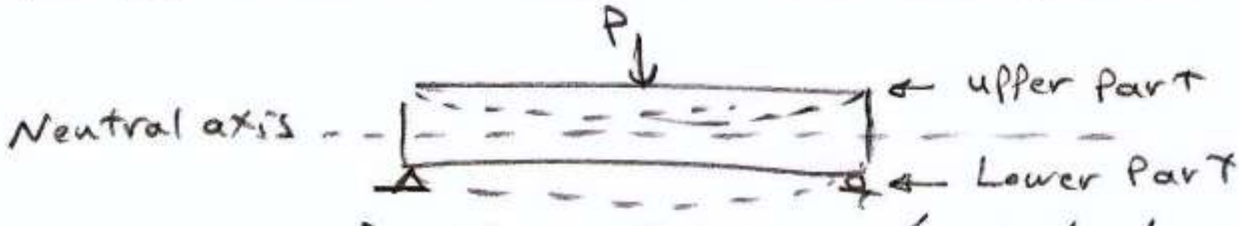
ويطلق على العزم المسلط على الـ Beam الذي يسبب Bending moment يسمى Pure Bending او Flexure .

وهنا يتوضح الفرق بين الكلمتين Bent و Flex .

خلاصة :- اذا لم يسلط على الـ Beam قوى Axial او Torsion او shear نطلق على العزم Flexural Bending .

وفي هذه الحالة يجب ان نلتزم بقانون هوك وشروطه Hook's Law

1- plane section remain plane after bending
2- The upper part subjected to Compression
3- The Lower part subjected To Tension
4- The Neutral axis is un stressed.



او ممكن يكون بالعكس اذا P من الاسفل

(3) Bending stresses

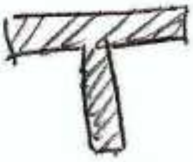
قوانين :-

$$f = \frac{M * C}{I}$$

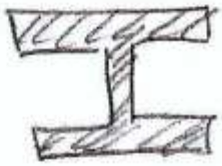
ضغط

- f : flexural stress وحدته N/mm^2 (MPa) او lb/in^2 (psi)
- M : قيمة العزم وحدته N.mm او lb.in تؤخذ من رسم الـ Bending diagram وهي قيمة العزم في اي موقع على طول الـ span وتكون اعلى قيمة اذا طلب Max f
- C : وهي المسافة على محور الـ Y فوق سنتر المقطع الى اي نقطة مطلوبة ، واذا كان المطلوب Max f تكون $C = \frac{h}{2}$
- I : عزم القصور الذاتي Moment of Inertia وحدته mm^4 او in^4 ويؤخذ من الـ cross section للـ Beam مثلاً المستطيل h (b) فان $I = \frac{bh^3}{12}$

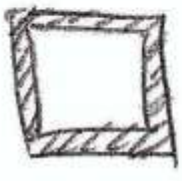
ملاحظة : المقطع العرضي (cross section) لكل عتبة تختلف عن الاخرى فهناك مقطع دائري او مقطع مستطيل او حرف T او مقطع



I



او مقطع مجوف



وهناك مقطع متناظر ومقطع غير متناظر لذلك فان I تختلف من مقطع الى آخر ويجب ان نجيد ايجاد قيمتها

(4) Bending Stresses

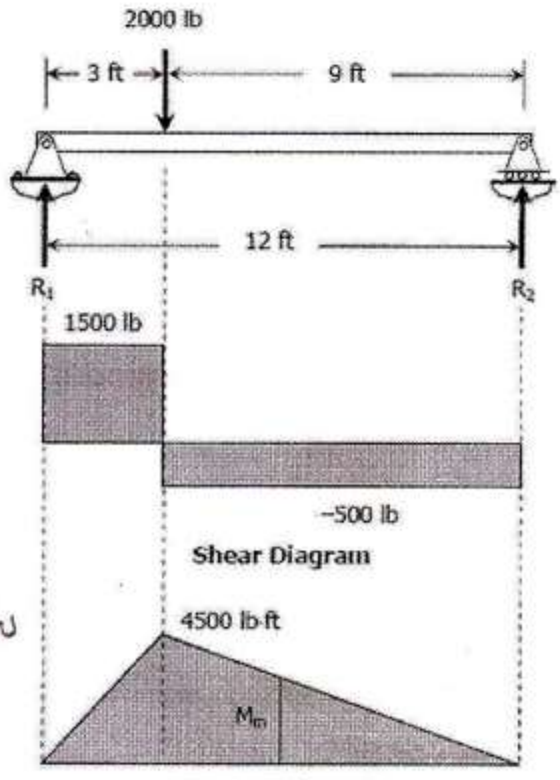
Problem

A simply supported beam, 2 in wide by 4 in high and 12 ft long is subjected to a concentrated load of 2000 lb at a point 3 ft from one of the supports. Determine the maximum fiber stress and the stress in a fiber located 0.5 in from the top of the beam at midspan.

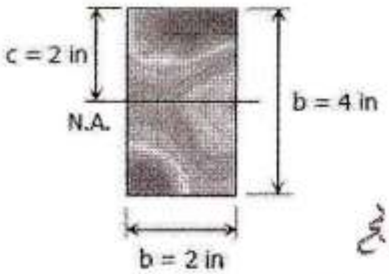
Solution

$\Sigma M_{R2} = 0$
$12R_1 = 9(2000)$
$R_1 = 1500 lb$

$\Sigma M_{R1} = 0$
$12R_2 = 3(2000)$
$R_2 = 500 lb$



Maximum fiber stress:



لتحويل فوت الى إنج

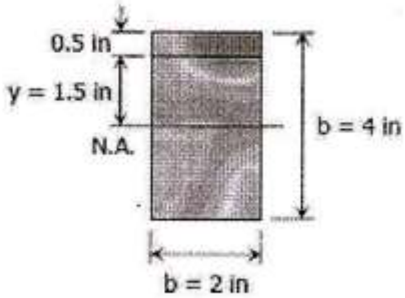
$(f_b)_{max} = Mc/I = 4500(12)(2)/2(4^3)/12$
$(f_b)_{max} = 10{,}125$ psi *answer*

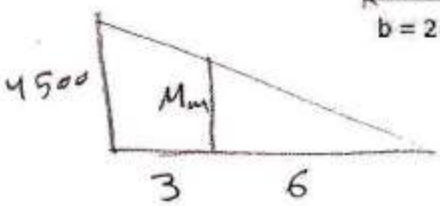
Stress in a fiber located 0.5 in from the top of the beam at midspan:

$M_m/6 = 4500/9$
$M_m = 3000 lb\cdot ft$

تحويل ft الى inch

$f_b = My/I$
$f_b = 3000(12)(1.5)/2(4^3)/12$
$f_b = 5{,}062.5$ psi *answer*



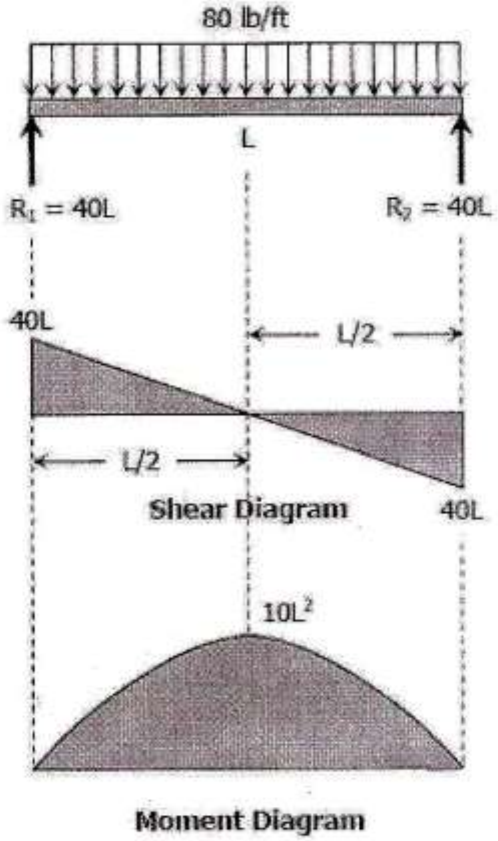


$$\frac{4500}{(3+9)} = \frac{M_m}{6} \Rightarrow M_m = 3000 \text{ lb.ft}$$

(5) Bending Stresses

Problem

A simply supported rectangular beam, 2 in wide by 4 in deep, carries a uniformly distributed load of 80 lb/ft over its entire length. What is the maximum length of the beam if the flexural stress is limited to 3000 psi?



By symmetry:

$$R_1 = R_2 = \tfrac{1}{2}(80L)$$

$$R_1 = R_2 = 40L$$

$$(f_b)_{max} = \frac{Mc}{I}$$

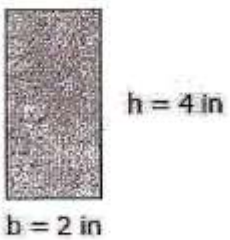
Where

$$(f_b)_{max} = 3000\ \text{psi}$$

$$M = 10L^2\ \text{lb}\cdot\text{ft}$$

$$c = h/2 = 2\ \text{in}$$

$$I = \frac{bh^3}{12} = \frac{2(4^3)}{12} = \tfrac{32}{3}\ \text{in}^4$$



$$3000 = \frac{10L^2(12)(2)}{32/3} \implies L = 133.33\ \text{in}$$

$L = 11.55\ \text{ft}$ *answer*

(6) Bending Stresses

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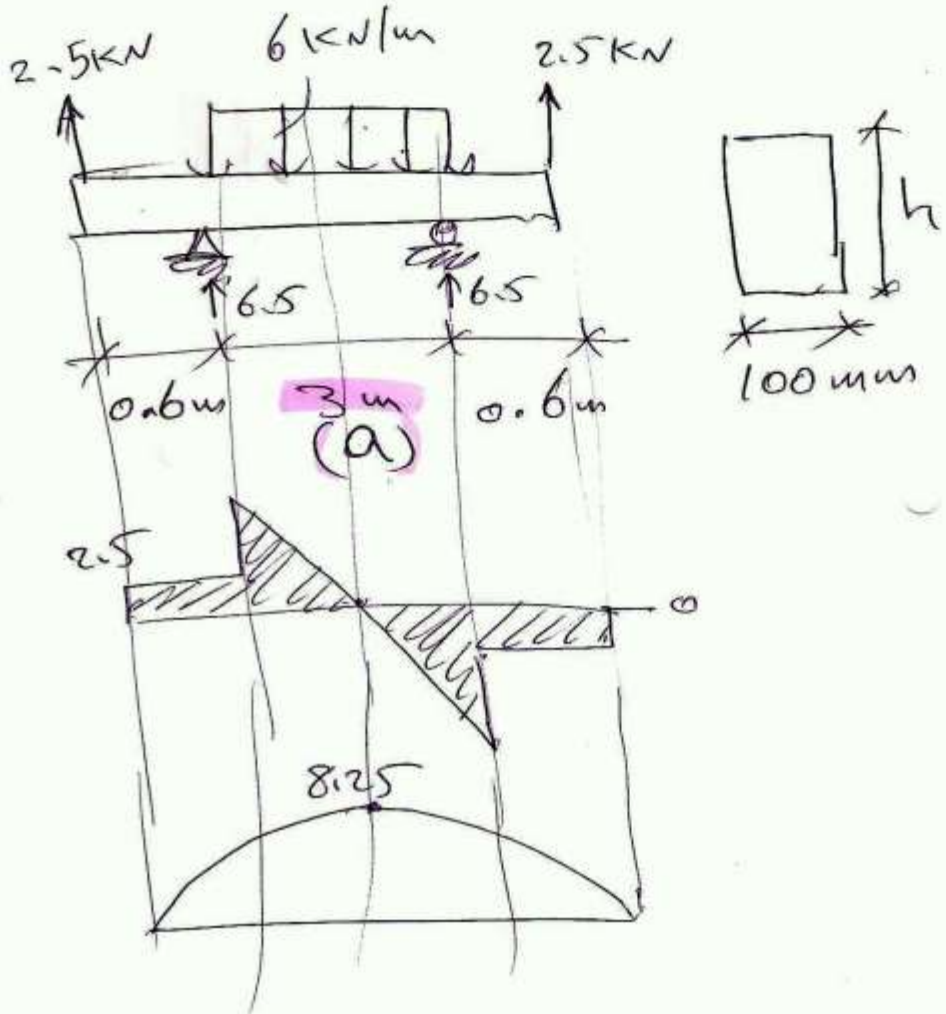
Allowable normal stress is 12 MPa. Find h ?



$$I = \frac{bh^3}{12}$$

$$I = \frac{0.1\,h^3}{12}$$

$$I = \frac{h^3}{120}$$

$$\sigma = \frac{Mc}{I} \leftarrow \frac{h}{2}$$

$$12 * 10^6 = \frac{8.25 * \frac{h}{2}}{h^3/120}$$

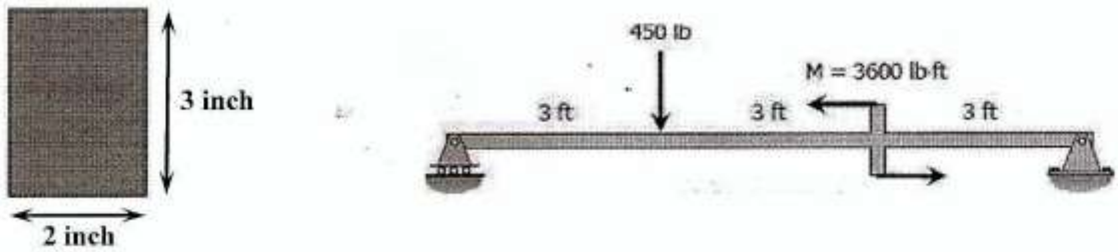
$$h = 203 \text{ mm}$$

(7) Bending Stresses

Q4/ A rectangular steel beam, 2 in wide by 3 in deep, is loaded as shown in the figure.

1-Draw Shear and Bending Diagram.

2- Determine the magnitude and the location of the maximum flexural stress.



Answer:

$\Sigma M_{R_2} = 0$

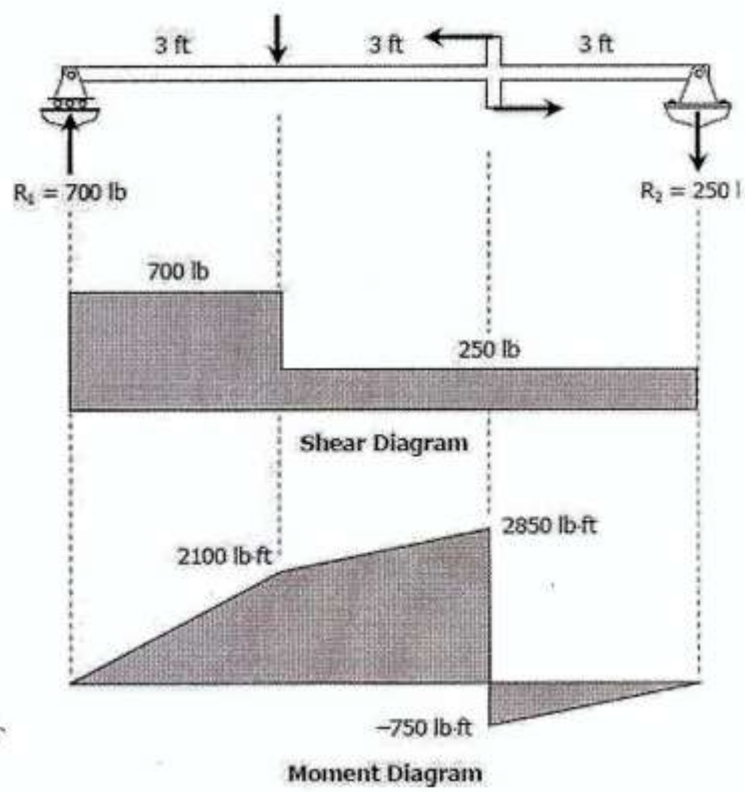
$9R_1 = 6(450) + 3600$

$R_1 = 700\ lb$

$\Sigma M_{R_1} = 0$

$9R_2 + 3(450) = 3600$

$R_2 = 250\ lb$



Max flexural stress $= \frac{Mc}{I}$

$I = \frac{bh^3}{12} = \frac{2(3)^3}{12} = 4.5\ in^4$

$c = \frac{h}{2} = \frac{3}{2} = 1.5\ inch$

Max Moment $= 2850\ lb.ft$

$f = \frac{2850(12)(1.5)}{4.5}$

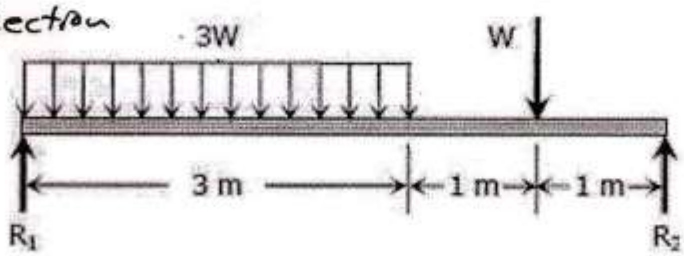
$f = 11,400\ psi$ @ 3 ft from right support

(8) Bending Stresses

Q7/ A beam with an S380 × 74 section carries a total uniformly distributed load of 3W and a concentrated load W, as shown in Figure. Determine W if the flexural stress is limited to 120 MPa. ~~And the~~ $\frac{I}{c}$ =1060x10³ mm³.

where $\frac{I}{c} = S$ = Elastic section Modulus

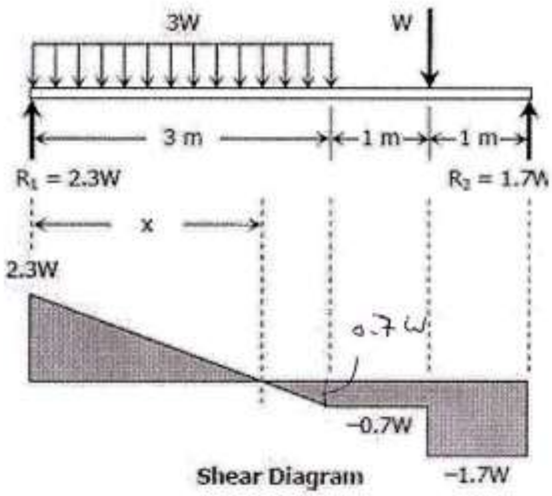
$\sigma_{max} = \frac{M}{S}$



Solution: $R_1 = 2.3W$

$R_2 = 1.7W$

$\frac{2.3W}{x} = \frac{0.7W}{3-x}$ ∴ $x = 2.3m$



M_{max} @ shear = zero

$= \frac{1}{2} * x * (2.3W)$

$= \frac{1}{2} * 2.3 * (2.3W) = 2.645W$

$I/c = 1060*10^3 \ mm^3$ (given)

$f_{max} = \frac{Mc}{I} = 2.645W * 1000 * \frac{1}{1060*10^3}$ &

$\therefore 120 = \frac{2.645W * 1000}{1060*10^3}$

$\therefore W = 48090.74 \ N$

(9) Bending Stresses

Problem

A 50-mm diameter bar is used as a simply supported beam 3 m long. Determine the largest uniformly distributed load that can be applied over the right two-thirds of the beam if the flexural stress is limited to 50 MPa.

$\Sigma M_{R1} = 0$

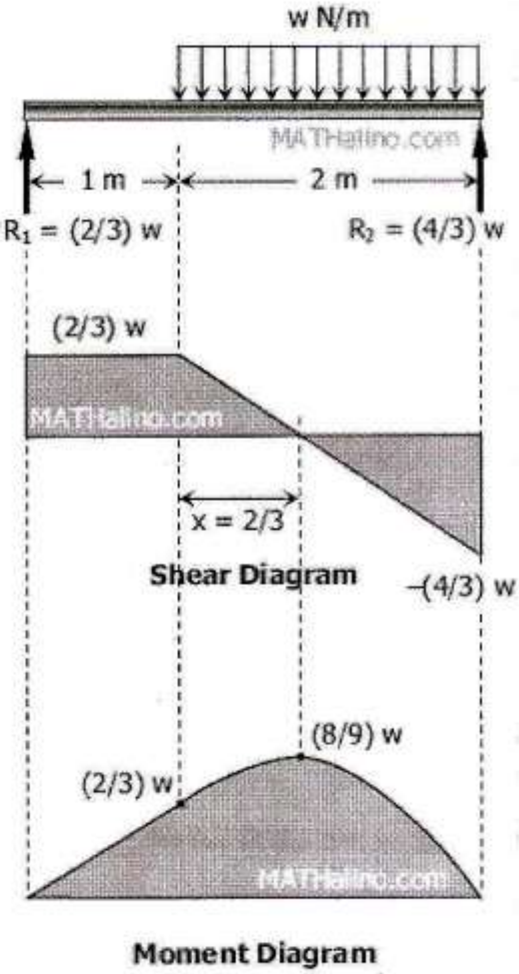
$3R_2 = 2w(2)$

$R_2 = \frac{4}{3}w$

$\Sigma M_{R2} = 0$

$3R_1 = 2w(1)$

$R_1 = \frac{2}{3}w$



$$(f_b)_{max} = \frac{Mc}{I}$$

Where

$(f_b)_{max} = 50\ \text{MPa}$

$M = \frac{8}{9}\ \text{N}\cdot\text{m}$

$c = 25\ \text{mm}$

$I = \frac{\pi r^4}{4} = \frac{\pi(25^4)}{4}$

$I = 97656.25\pi\ \text{mm}^4$

$$50 = \frac{\frac{8}{9}w(1000)(25)}{97656.25\pi}$$

$w = 690.29\ \text{N/m}$

لإيجاد قيمة $x = \frac{2}{3}$ كما يلي:

$$\frac{\frac{4}{3}w}{2-x} = \frac{\frac{2}{3}w}{x}$$

$\therefore x = \frac{2}{3}$

كيفية ايجاد قيمة العزم $\frac{8}{9}w$ كما يلي:

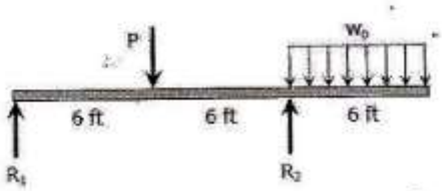
$= \frac{1}{2} \times$ مساحة المثلث

$= \frac{1}{2} \times \frac{4w}{3} \times \frac{4}{3} = \frac{8}{9}w$

(10) Bending Stresses

Problem 526

A wood beam 6 in wide by 12 in deep is loaded as shown in Fig. P-526. If the maximum flexural stress is 1200 psi, find the maximum values of w_o and P which can be applied simultaneously?



$\Sigma M_{R2} = 0$

$12R_1 + 3(6w_o) = 6P$

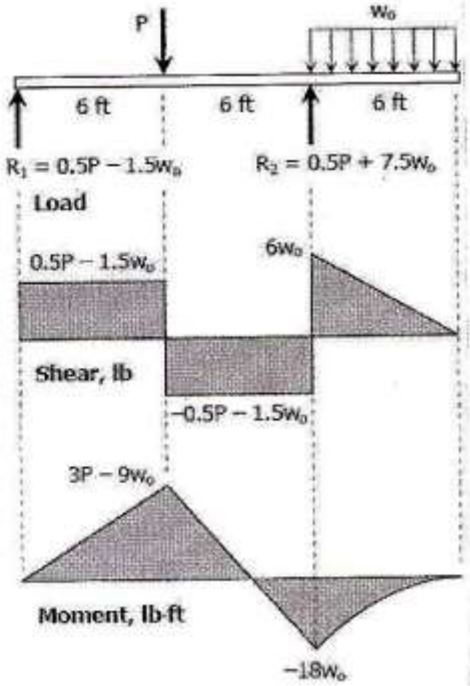
$R_1 = 0.5P - 1.5w_o$

$\Sigma M_{R1} = 0$

$12R_2 = 6P + 15(6w_o)$

$R_2 = 0.5P + 7.5w_o$

$(f_b)_{max} = \dfrac{Mc}{I}$



Where:

$f_b = 1200\text{ psi}$

$c = \frac{1}{2}h = \frac{1}{2}(12) = 6\text{ in}$

$I = \dfrac{bh^3}{12} = \dfrac{6(12^3)}{12} = 864\text{ in}^4$

For moment at R_2:

$1200 = \dfrac{18w_o(6)(12)}{864}$

$w_o = 800\text{ lb/ft}$ *answer*

For moment under P:

$1200 = \dfrac{(3P - 9w_o)(6)(12)}{864}$

$14\,400 = 3P - 9w_o$

$14\,400 = 3P - 9(800)$

$P = 7200\text{ lb}$ *answer*

(11) Bending stresses

Problem 522

A box beam is composed of four planks, each 2 inches by 8 inches, securely spiked together to form the section shown in Fig. P-522. Show that $I_{NA} = 981.3\ in^4$. If w_o = 300 lb/ft, find P to cause a maximum flexural stress of 1400 psi.

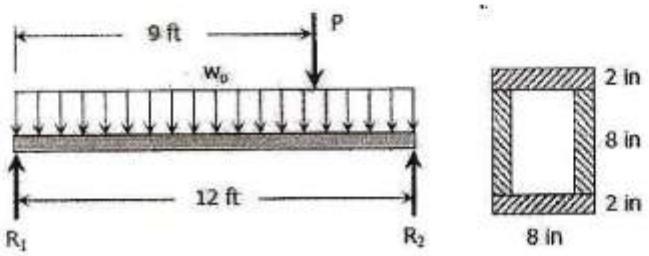
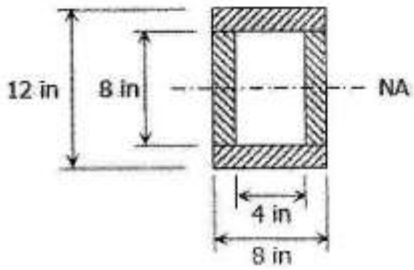


Figure P-522 and P-523



$$I_{NA} = \frac{8(12^3)}{12} - \frac{4(8^3)}{12}$$

$$I_{NA} = 981.33\ \text{in}^4$$

$$\Sigma M_{R1} = 0$$

$$12R_2 = 300(12)(6) + 9P$$

$$R_2 = 1800 + 0.75P$$

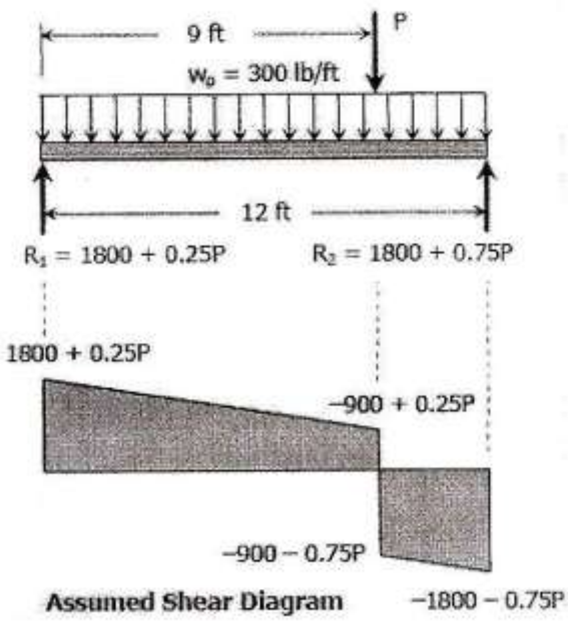
$$M = \tfrac{1}{2}\,[\,(1800 + 0.25P) + (-900 + 0.25P)\,]\,(9)$$

$$M = 4050 + 2.25P\ \text{lb}\cdot\text{ft}$$

$$(f_b)_{max} = \frac{Mc}{I}$$

$$1400 = \frac{(4050 + 2.25P)(6)(12)}{981.33}$$

$$P = 6680.63\ \text{lb}$$



Assumed Shear Diagram

Check if the shear at P is positive as assumed

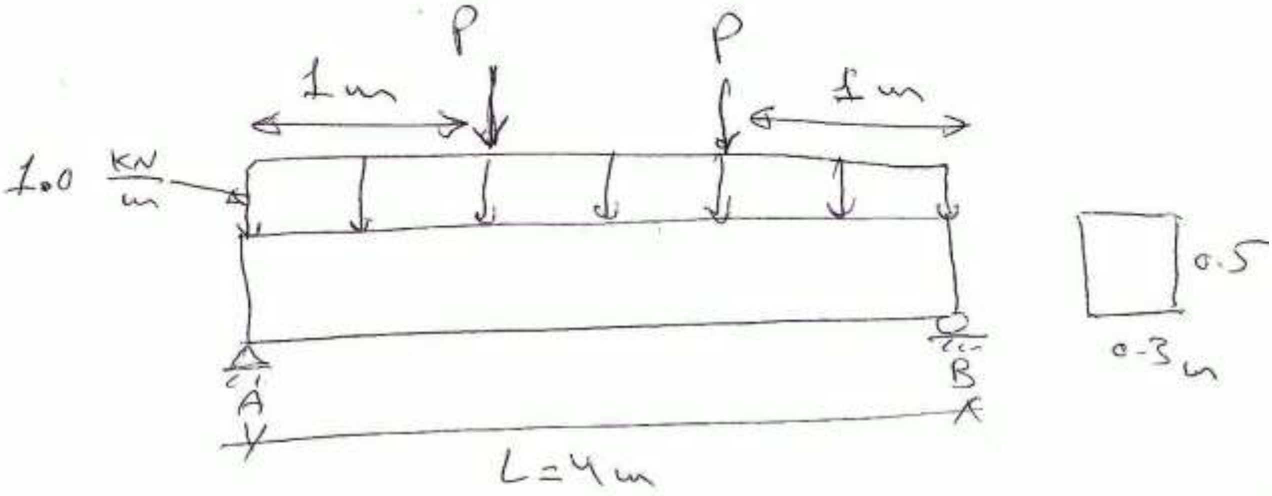
$$-900 + 0.25P = -900 + 0.25(6680.63)$$

$$-900 + 0.25P = 770.16\ \text{lb} \qquad \textit{(okay!)}$$

Thus, P = 6680.63 lb. *answer*

(12) A simply supported wooden beam 0.3 m by 0.5 m weighing 1 KN/m supports two concentrated loads of 10 KN each as shown below. Determine the maximum bending stress developed in the beam?

23/2/2015



Ans: if P = 10 KN then

Then $M_{max} = 12$ KN.m

and $I = 0.003125\ m^4$

$\sigma_{max} = 960\ KN/m^2$

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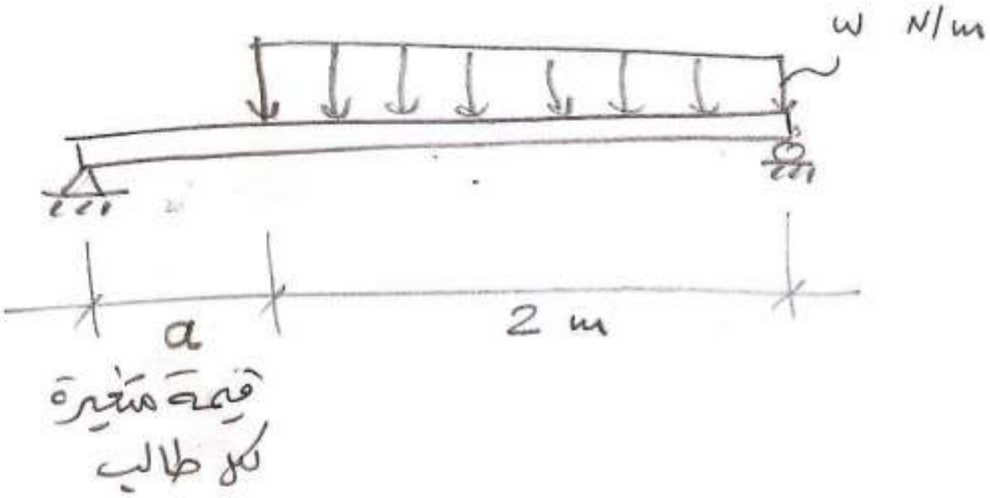

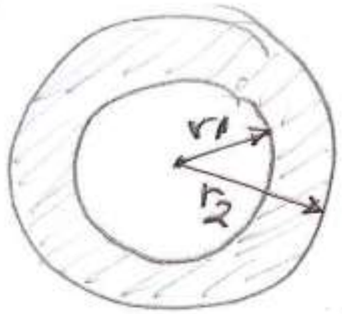

Beam Cross Section

flexural stress = 100 Mpa

r_1 = 50 mm

r_2 = 100 mm

a = Variable value for each student

Required :-

1- Find Reactions.
2- Draw shear and Bending diagram
3- Find the largest W in N/m unit