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Al-Mustansiriyah University

Faculty of Civil Engineering

Water Resources Engineering Department

The Second Course

SOIL MECHANICS

CHAPTER SIX

2020-2021

Third

Stage Students

Undergraduate students (3th stage students)
Faculty of Engineering
Mustansiriyah University
Water Resources Engineering Department

CHAPTER SIX
STRESSES WITHEN THE SOIL

Stresses Within Soil

- Soil consists of: solids, water and/or air
- The engineer needs to know the distribution of stresses at any point in the soil mass to analyze problems such as:

- Settlement of soils
- Bearing capacity of foundations
- Stability of slopes
- Lateral pressure on retaining walls

Components of Stresses in Soil

1. Existing overburden pressure due to soil self weight)

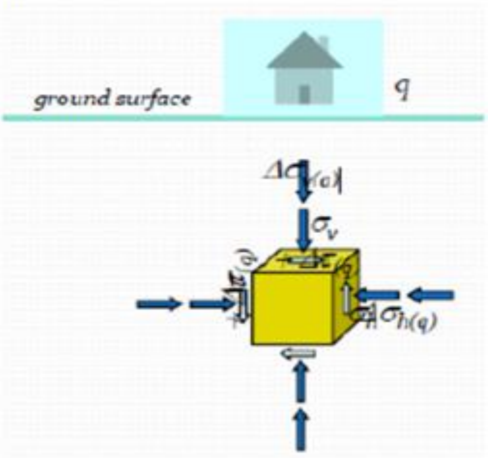
σ_v = vertical normal overburden pressure

σ_h = horizontal normal overburden pressure = $k\sigma_v$

since, $\sigma_v \neq \sigma_h$

τ = shear stresses accompanying σ_v and σ_h

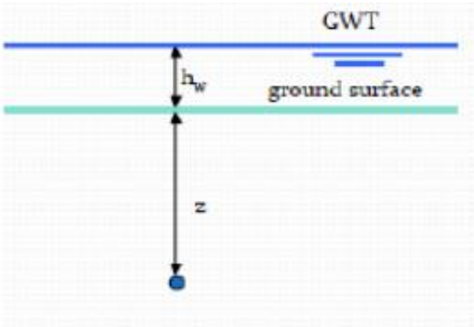
2. Added loads and/or excavation



Stresses Due To Overburden Pressure

- For a saturated soil deposit (γ) with groundwater table above
- The total vertical stress (σ) at depth (z) equals the weight of solids and water per unit area above that depth:

$$\sigma = \gamma_{sat} z + \gamma_w h_w$$



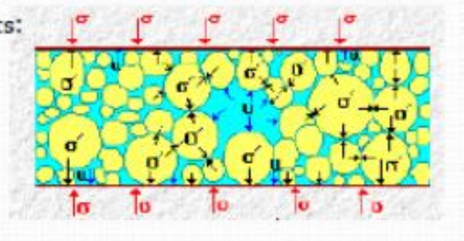
- The total vertical stress (σ) is divided into 2 parts:

1. Portion carried by water in voids = pore-water pressure (u)

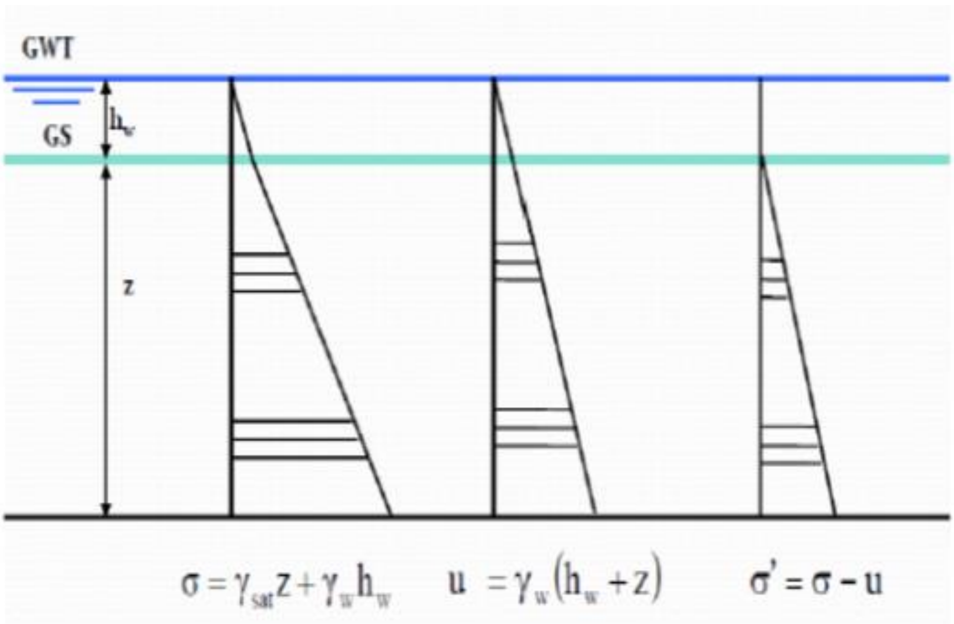
2- Portion carried by solids at their point of contact = effective stress (σ')

The total vertical stress (σ) can be expressed as:

$$\sigma = u + \sigma'$$



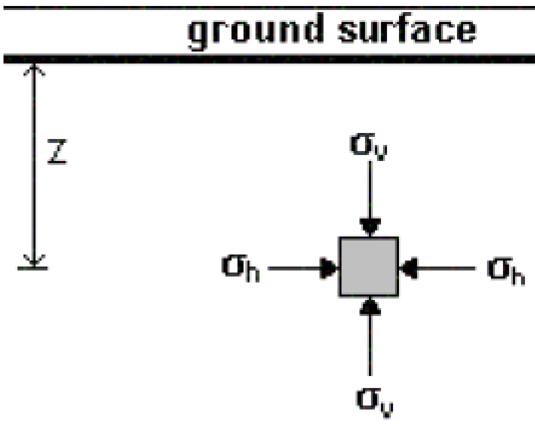
Overburden Stress Distribution with Depth



Total Stress

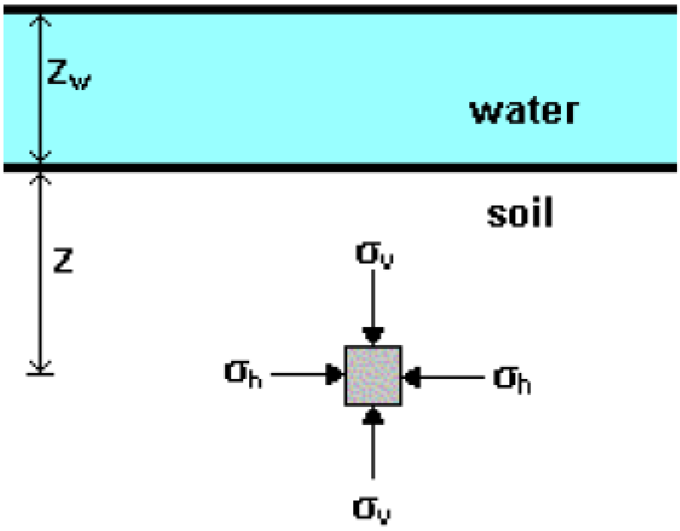
When a load is applied to soil, it is carried by the solid grains and the water in the pores. The total vertical stress acting at a point below the ground surface is due to the weight of everything that lies above, including soil, water, and surface loading. Total stress thus increases with depth and with unit weight.

Vertical total stress at depth z, sv = g.Z



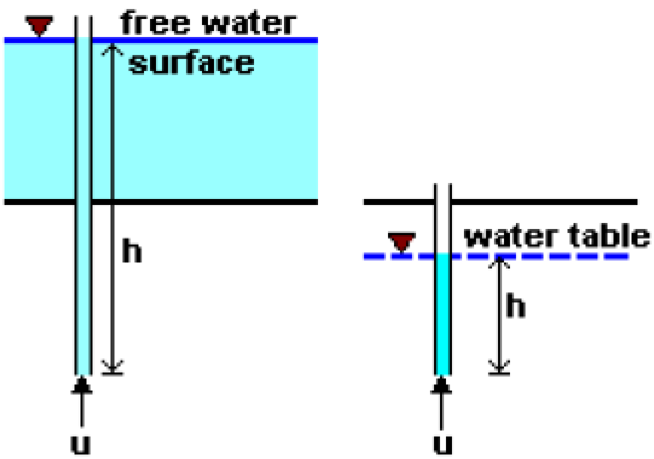
Below a water body, the total stress is the sum of the weight of the soil up to the surface and the weight of water above this.

sv = g.Z + gw.Zw



Pore Water Pressure The pressure of water in the pores of the soil is called pore water pressure (u). The magnitude of pore water pressure depends on:

- **The depth below the water table.**
- **The conditions of seepage flow**

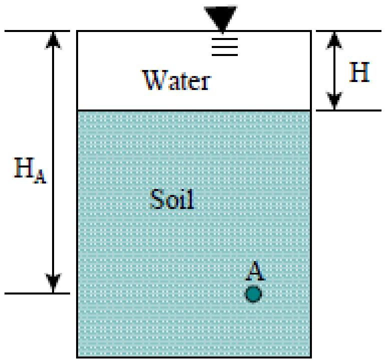


Under hydrostatic conditions, no water flow takes place, and the pore pressure at a given point is given by: u = gw.h

where h = depth below water table or overlying water surface

The natural level of ground water is called the water table or the phreatic surface. Under conditions of no seepage flow, the water table is horizontal. The magnitude of the pore water pressure at the water table is **zero**. Below the water table, pore water pressures are **positive**.

The total stress at A is calculated from:



The weight of the water above A

$\sigma = H\gamma w + (H_A - H)\ \gamma_{sat}$

σ= Total Stress at A

γw = Unit Weight of Water

γsat = Saturated Unit Weight

H_A = Height of A to Top of water

H = Height of water

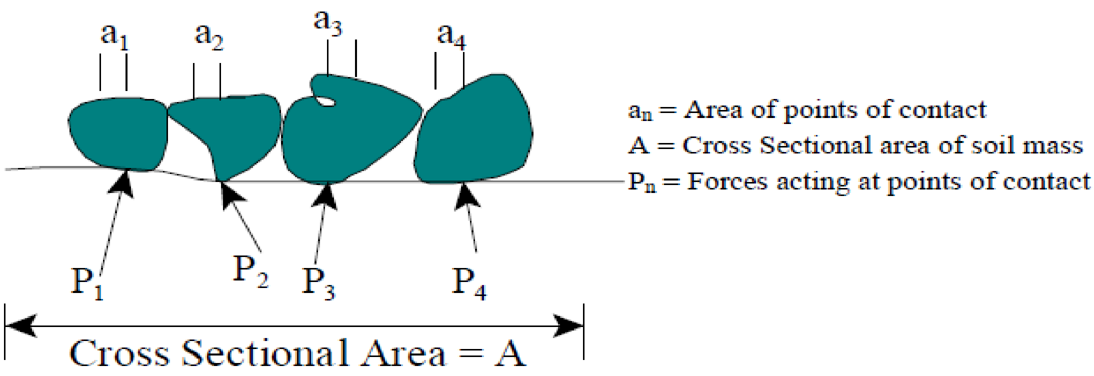
- σ is the stress applied to the soil by its own weight
- As you go deeper in the soil mass, the stress increases

• Like in a swimming pool, as you go deeper, the stress of the weight of the water increases

• The soil carries the stress in 2 ways:

• A portion is carried by the water (acts equally in all directions)

• A portion is carried by the soil solids at their point of contact.



The sum of the vert. components of the forces at their points of contact per unit of X sectional area is the effective stress.

• The sum of vertical components of forces over the area is the effective stress F'

σ/= (P1v+P2v+P3v .....+Pnv) / A

If:

as = a1 + a2 + a3 +...an

• **Effective Stress Concept**

• Soil is a multi- phase system

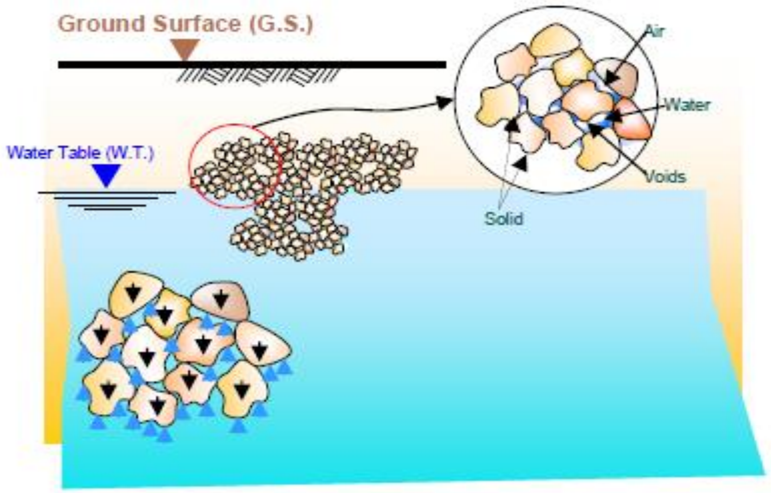
• To perform any kind of analysis - we must understand stress distribution

• The concept of effective stress:

• The soil is "loaded" (footing for example)

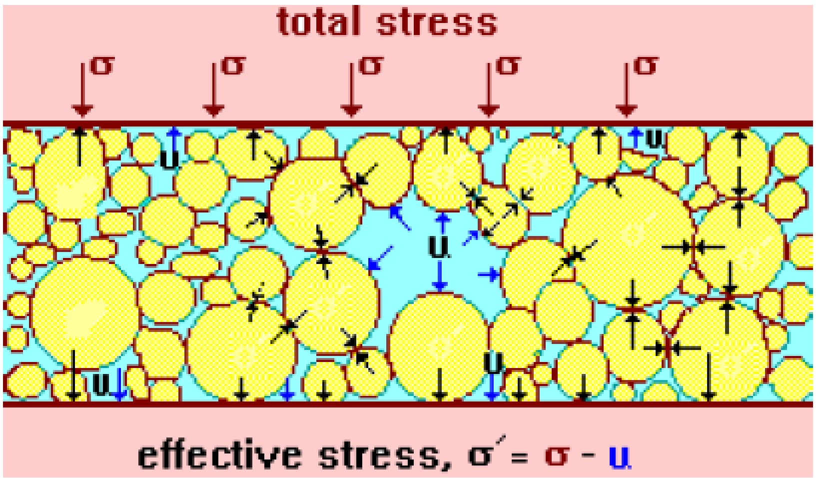
• The resulting stress is transmitted to the soil mass

• The soil mass supports those stresses at the point to point contacts of the individual soil grains



Effective Stress Concept

In a saturated soil system, as the voids are completely filled with water, the pore water pressure acts equally in all directions. The effective stress is not the exact contact stress between particles but the distribution of load carried by the soil particles over the area considered. It cannot be measured and can only be computed.



If the total stress is increased due to additional load applied to the soil, the pore water pressure initially increases to counteract the additional stress. This increase in pressure within the pores might cause water to drain out of the soil mass, and the load is transferred to the solid grains. This will lead to the increase of effective stress.

$$Total\ Stress\ at\ Point\ O = \sigma = h_1\gamma_d + h_2\gamma_{sat}$$

where:

h_1 = thickness of the dry soil layer

h_2 = thickness of the saturated soil layer

γ_d = Dry Unit Weight

γ_{sat} = Saturated Unit Weight

$$Total\ Stress\ at\ Point\ O = \sigma = \bar{\sigma} + u$$

where:

$\bar{\sigma}$ = Effective Stress

u = Pore Water Pressure $= h_2\gamma_w$

$$Effective\ Stress\ at\ Point\ O = \bar{\sigma} = \sigma - u$$

$$= h_1\gamma_d + h_2(\gamma_{sat} - \gamma_w) = h_1\gamma_d + h_2\gamma'$$

where:

γ' = Submerged Unit Weight

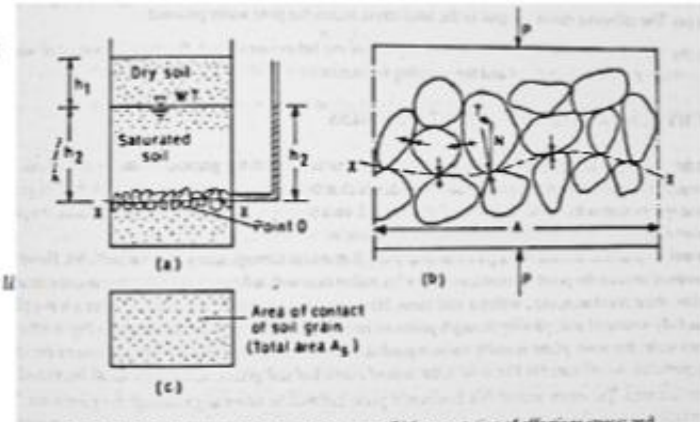
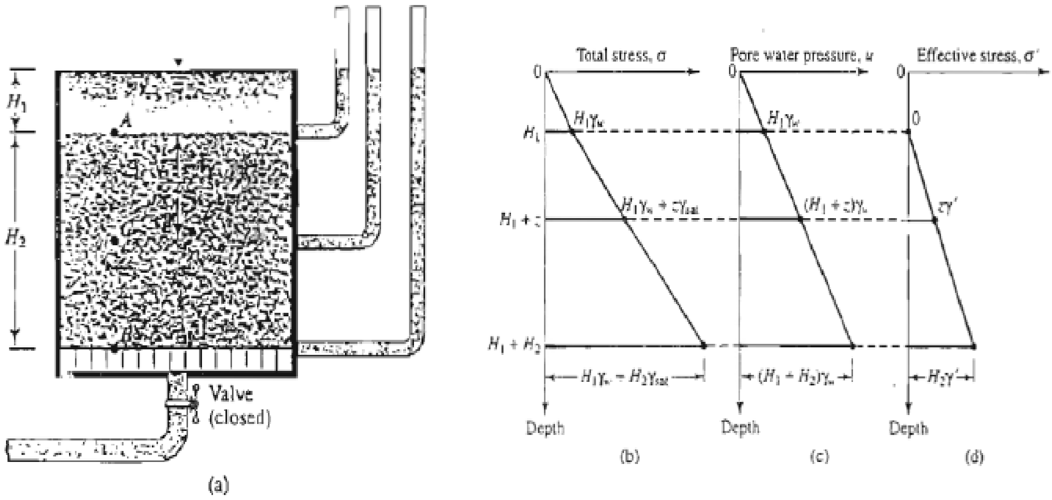


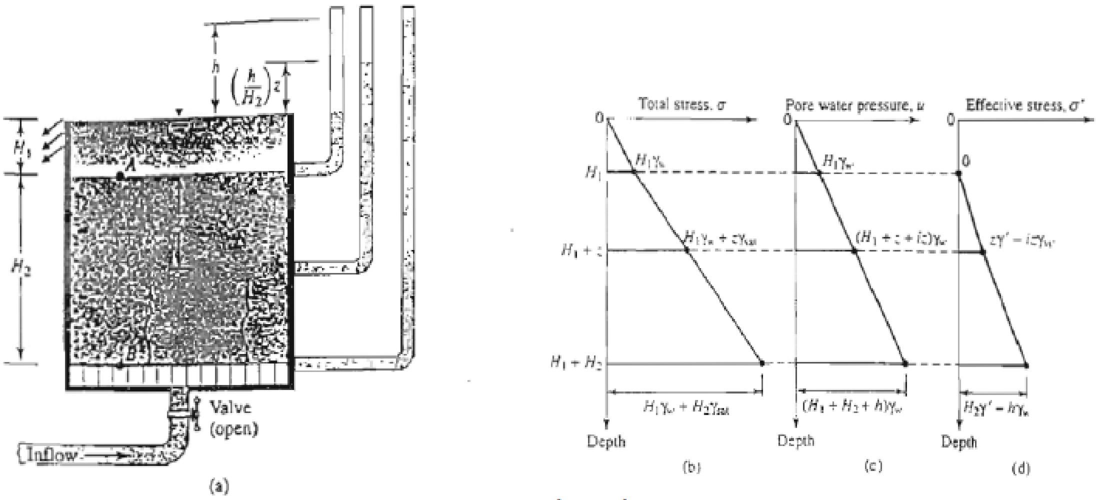
Fig. 6.1 (a) Soil mass and pore water pressure; (b) interpretation of effectives stress; and (c) sectional plan of the contact area

Effective Stress in Saturated Soil with no Seepage



Effective Stress in Saturated Soil with Seepage

Upward flow

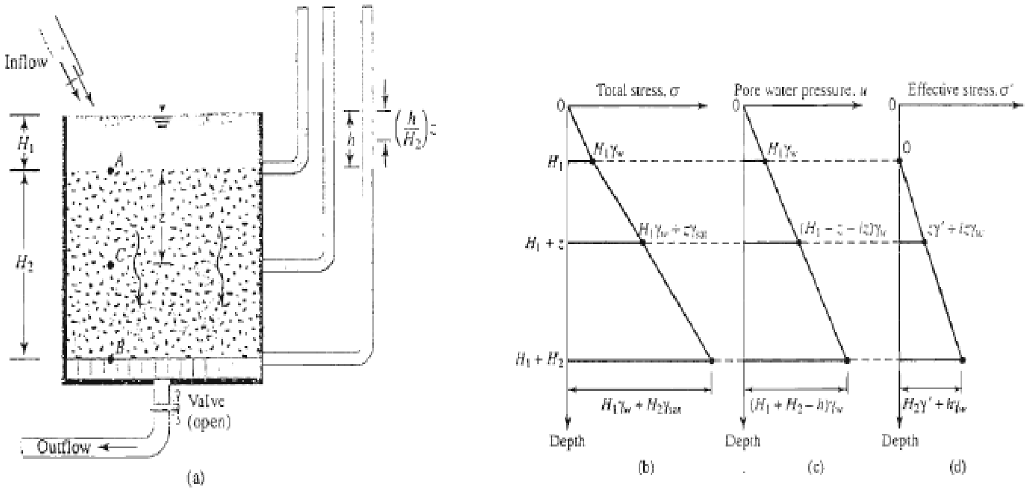


And limiting conditions may occur when $\sigma'_c = z\gamma' - iz\gamma_w = 0$ which lead to

i_{cr} = *critical hydraulic gradient*

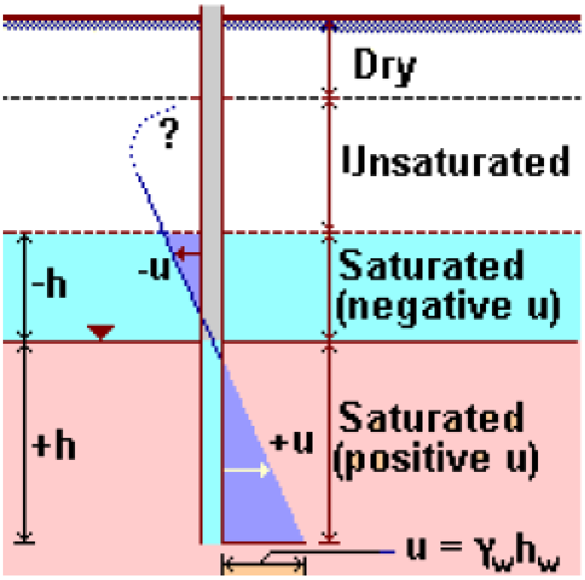
$$i_{cr} = \frac{\gamma'}{\gamma_w}$$ *for most soils 0.9-1.1 ith average value of 1*

Downward flow

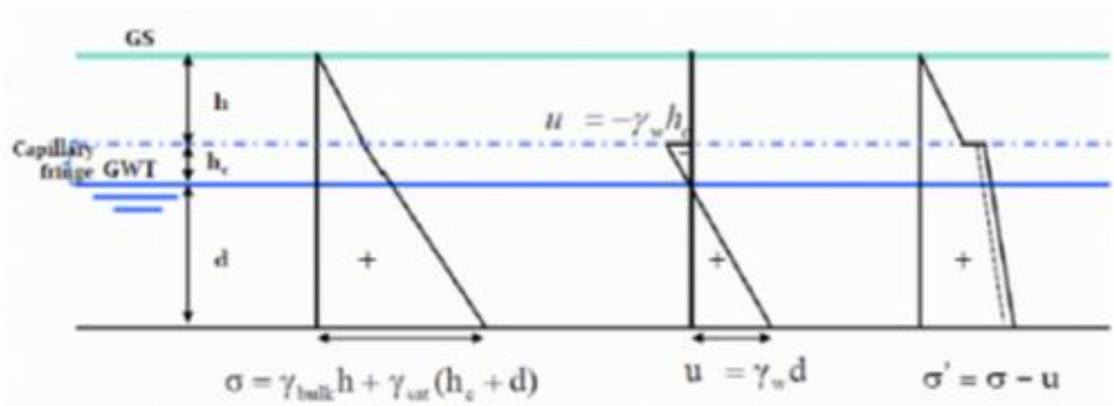


Effective Stress in Unsaturated

Zone Above the water table, when the soil is saturated, pore pressure will be negative (less than atmospheric). The height above the water table to which the soil is saturated is called the capillary rise, and this depends on the grain size and the size of pores. In coarse soils, the capillary rise is very small.



- The capillary phenomenon and its effects in soil
  - The capillary phenomenon is the presence of water above the ground water table held by surface tension forces.
  - Since capillary water has a negative pore water pressure (held in tension), capillary phenomenon causes an increase in effective stress.
  - The thickness of the capillary zone depends on the size of the soil pores which is related to permeability.



Examples:

Example: For the subsoil conditions shown in fig. below draw the total, effective stress diagrams upto a depth of 8 m. Neglect capillary flow.

Unit Weight of the partially saturated sand above the water table

$$\gamma_{t(sand)} = \frac{Gs + Se}{1+e}\gamma_w = \frac{2.65+0.4\times0.6}{1+0.6}\times10 = 18\,kN/m^3 (\gamma_w \approx 10kN/m^3)$$

Unit Weight of saturated sand

$$\gamma_{sat(sand)} = \frac{Gs+(1)e}{1+e}\gamma_w = \frac{2.65+1\times0.6}{1+0.6}\times10 = 20.3\,kN/m^3 (\gamma_w \approx 10kN/m^3)$$

Unit Weight of saturated clay

$$\gamma_{sat(clay)} = \frac{Gs(1+w)}{1+wGs}\gamma_w = \frac{2.70(1+0.45)}{1+0.45\times2.70}\times10 = 17.67\,kN/m^3$$

$(e = wG \text{ when } S = 1.\ \gamma_w \approx 10kN/m^3)$

At elevation $-3m$:

$\sigma = 3\times18 = 54\,kN/m^2,\ u = 0, \sigma' = \sigma - u = 54\,kN/m^2$

At elevation $-5m$:

$\sigma = 3\times18 + 2\times20.3 = 94.6\,kN/m^2$

$u = 2\times10 = 20\,kN/m^2$

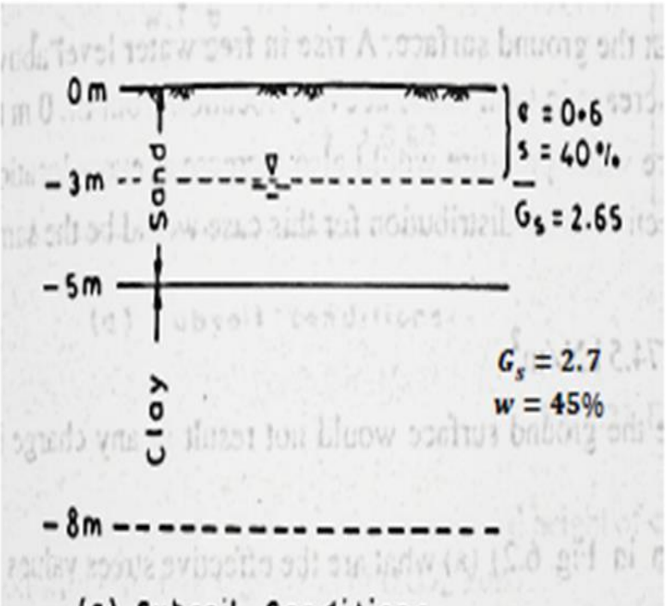
$\sigma' = \sigma - u = 74.6\,kN/m^2$

At elevation $-8m$:

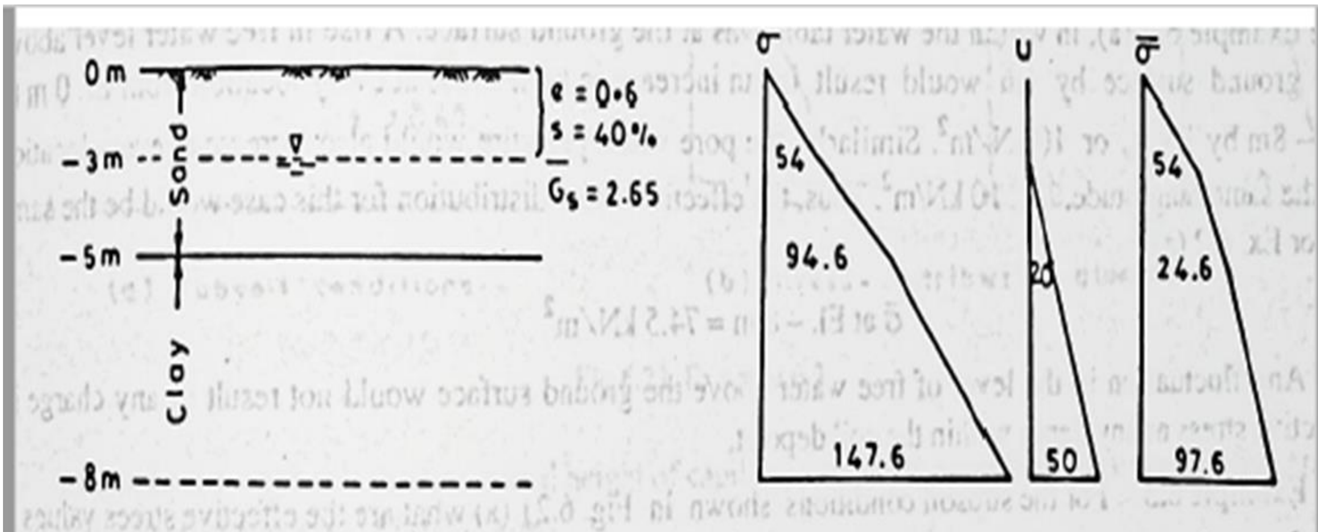
$\sigma = 94.6 + 3\times17.67 = 147.6\,kN/m^2$

$u = 5\times10 = 50\,kN/m^2$

$\sigma' = \sigma - u = 97.6\,kN/m^2$



(a) Subsoil Conditions



(a) Subsoil Conditions (b) Stress-distribution diagrams

For previous example, if the water table rises upto the ground surface, what is the change in effective stress at elevation -8m?

At elevation $-8m$:

$\sigma = 5 \times 20.3 + 3 \times 17.67 = 154.4 kN/m^2$

$u = 8 \times 10 = 80 kN/m^2$

$\sigma' = \sigma - u = 154.4 - 80 = 74.5 kN/m^2$

The effective stress has decreased by $97.6 - 74.5 = 23.1 kN/m^2$.

What is the change in effective stress at elevation -8m, if in previous example, the water table is lowered by 2 m?

At elevation $-8m$:

$\sigma = 5\times18+3\times17.67 = 147.6\,kN/m^2$

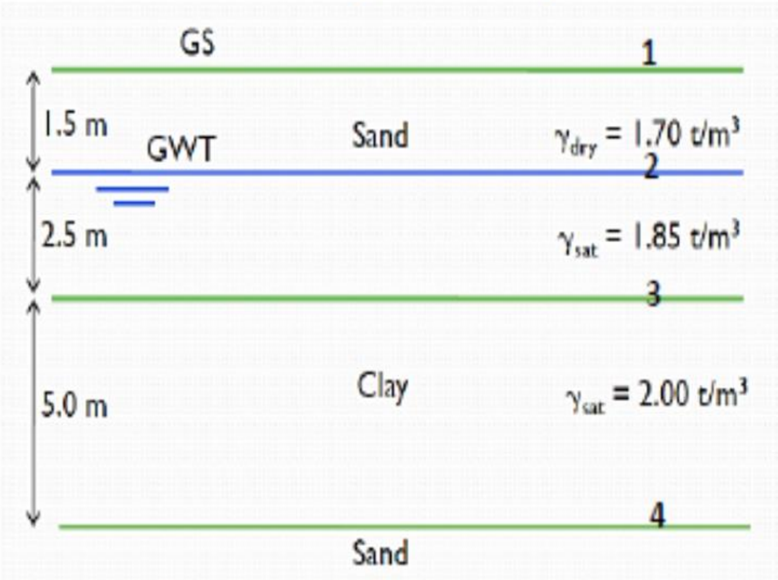
$u = 3\times10 = 30\,kN/m^2$ (since *the Water Table is now at elevation* $-5m$.)

$\sigma' = 147.6-30 = 117.6\,kN/m^2$

The effective stress has increased by $117.6-97.6 = 20\,kN/m^2$.

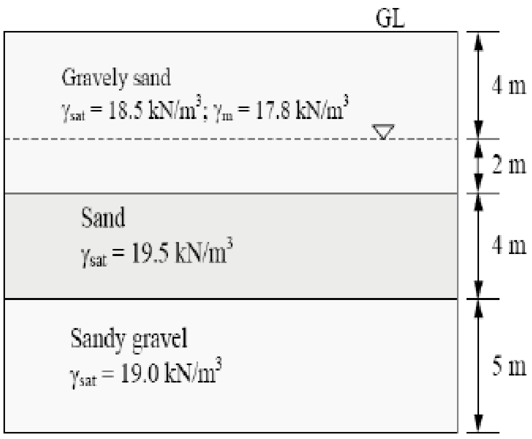
- In general, it can be understood that a rise in position of water table results in a decrease in effective stress while a lowering of the water table brings about an increase in effective stress.
- This effect has an important fact on bearing capacity and settlement of foundations.

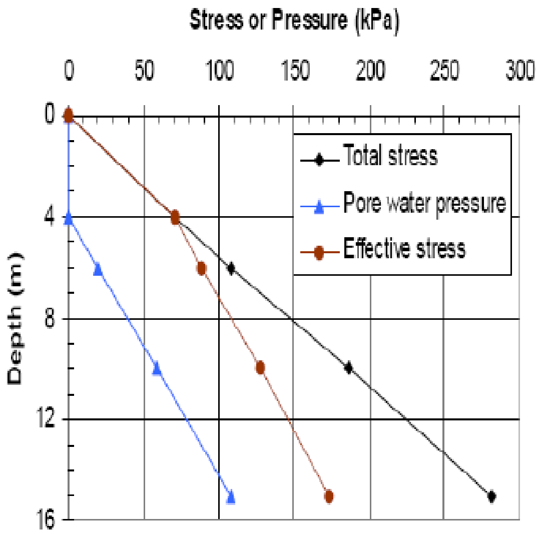
Example: For the soil formation shown plot the variation of the vertical shown, total and effective stresses as well as the pore water pressure with depth.



Point	σ_v (t/m²)	u (t/m²)	σ'_v (t/m²)
1	0	0	0
2	1.7x1.5=2.55	0	2.55-0=2.55
3	2.55+1.85x2.5=7.175	1x2.5=2.5	7.175-2.5=4.675
4	7.175+2.0x5.0 =17.175	1x7.5=7.5	17.175-7.5=9.675

EXAMPLE1. Plot the variation of total and effective vertical stresses, and pore water pressure with depth for the soil profile shown below in Fig.





Solution:

Within a soil layer, the unit weight is constant, and therefore the stresses vary linearly. Therefore, it is adequate if we compute the values at the layer interfaces and water table location, and join them by straight lines.

At the ground level,

$\sigma_v = 0$; $\sigma_v' = 0$; and u=0

At 4 m depth,

$\sigma_v = (4)(17.8) = 71.2$ kPa; u = 0

$\therefore \sigma_v' = 71.2$ kPa

At 6 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) = 108.2$ kPa

u = (2)(9.81) = 19.6 kPa

$\therefore \sigma_v' = 108.2 - 19.6 = 88.6$ kPa

At 10 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) = 186.2$ kPa

u = (6)(9.81) = 58.9 kPa

$\therefore \sigma_v' = 186.2 - 58.9 = 127.3$ kPa

At 15 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) + (5)(19.0) = 281.2$ kPa

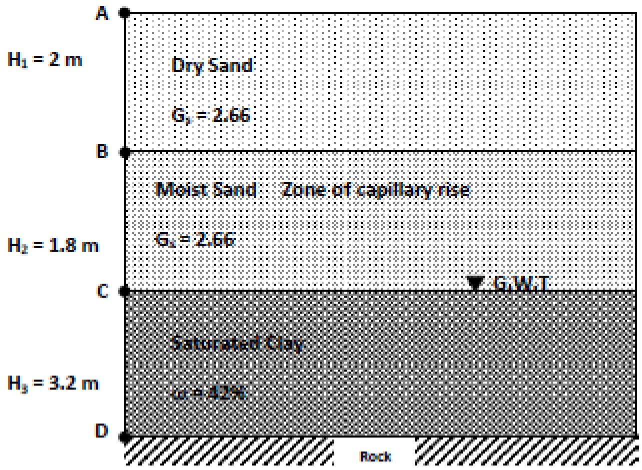
u = (11)(9.81) = 107.9 kPa

$\therefore \sigma_v' = 281.2 - 107.9 = 173.3$ kPa

The values of σ_v, u and σ'_v computed above are summarized in Table 1.

depth (m)	σ_v (kPa)	u (kPa)	σ_v' (kPa)
0	0	0	0
4	71.2	0	71.2
6	108.2	19.6	88.6
10	186.2	58.9	127.3
15	281.2	107.9	173.3

EXAMPLE2. Plot the variation of total and effective vertical stresses, and pore water pressure with depth for the soil profile shown below in Fig.



Dry sand $\gamma_d = \frac{G_s}{1+e}\gamma_w = \frac{2.66}{1+0.55}9.81 = 16.84$ kN/m^3

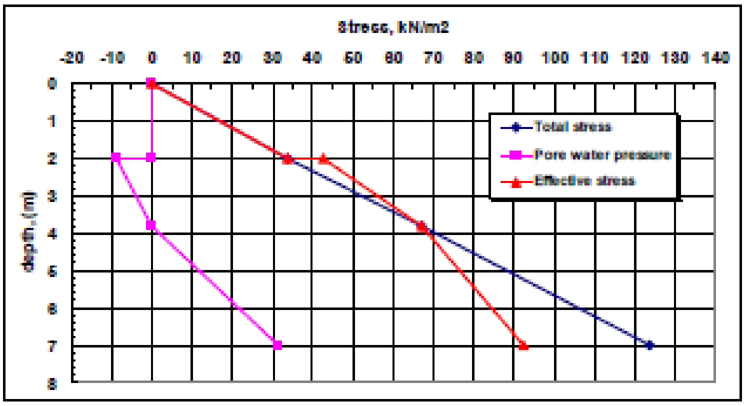
Moist sand $\gamma_t = \frac{G_s + Se}{1+e}\gamma_w = \frac{2.66 + 0.5 * 0.55}{1+0.55}9.81 = 18.58$ kN/m^3

Saturated Clay $e = \frac{G_s\omega}{S} = \frac{2.71 * 0.42}{1} = 1.138$

$\gamma_{sat} = \frac{G_s + e}{1+e}\gamma_w = \frac{2.66 + 1.138}{1+1.138}9.81 = 17.66$ kN/m^3

Point	σ_v kN/m^2	u kN/m^2	σ'_v kN/m^2
A	0	0	0
B	2*16.84=33.68	0	33.68
		$- S \gamma_w H_2 = - 0.5*9.81*1.8 = -8.83$	33.68-(-8.83) = 42.51
C	2*16.84+1.8*18.58 = 67.117	0	67.117
D	2*16.84+1.8*18.58+3.2*17.66 =123.68	3.2*9.81=31.39	123.68-31.39 = 92.24

The plot is shown below in Fig.

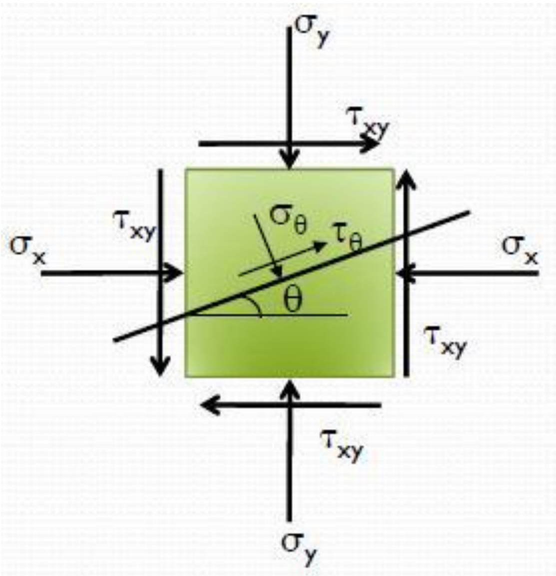


Variation of σ_v, u and σ'_v with depth

Normal and Shear Stresses in a Soil Mass

Considering a certain point inside the soil mass and knowing the normal (σ) and shear stresses (τ) acting on two planes at this point:

- What is the maximum and minimum normal stresses (magnitude and direction)?
- What is the maximum shear stresses (magnitude and direction)?
- What is the normal (σ) and shear (τ) stresses acting on any plane?

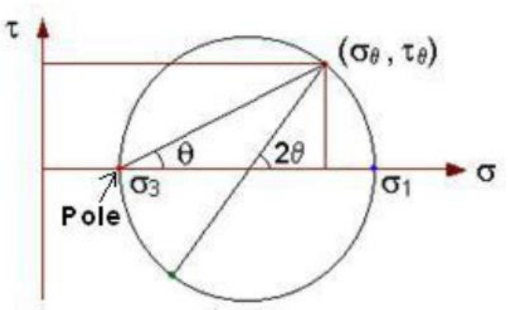


Mohr Circle of Stresses

In soil testing, cylindrical samples are commonly used in which radial and axial stresses act on principal planes. The vertical plane is usually the minor principal plane whereas the horizontal plane is the major principal plane. The radialstress (sr) is the minor principal stress (s3), and the axial stress (sa) is the major principal stress**(s1).**

To visualize the normal and shear stresses acting on any plane within the soil sample, a graphical representation of stresses called the Mohr circle is obtained by plotting the principal stresses. The sign convention in the construction is to consider compressive stresses as positive and angles measured counter-clockwise also positive.

Draw a line inclined at angle with the horizontal through the pole of the Mohr circle so as to intersect the circle. The coordinates of the point of intersection are the normal and shear stresses acting on the plane, which is inclined at angle within the soil sample.



Normal stress $\sigma_\theta = \frac{(\sigma_1 + \sigma_3)}{2} + \frac{(\sigma_1 - \sigma_3)}{2}\cos 2\theta$

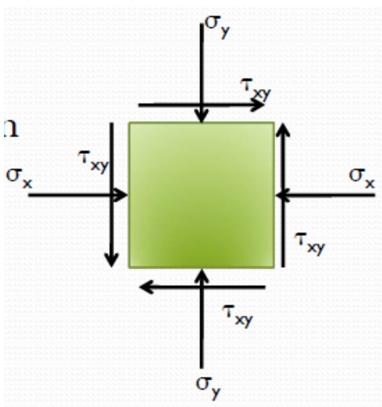
Shear stress $\tau_\theta = \frac{(\sigma_1 - \sigma_3)}{2}\sin 2\theta$

The plane inclined at an angle of 45° to the horizontal has acting on it the maximum shear stress equal to $\frac{\sigma_1 - \sigma_3}{2}$, and the normal stress on this plane is equal to $\frac{\sigma_1 + \sigma_3}{2}$.

Mohr Circle Presentation

It is a graphical method to present the state of stress along any plane passing through any point within the soil mass.

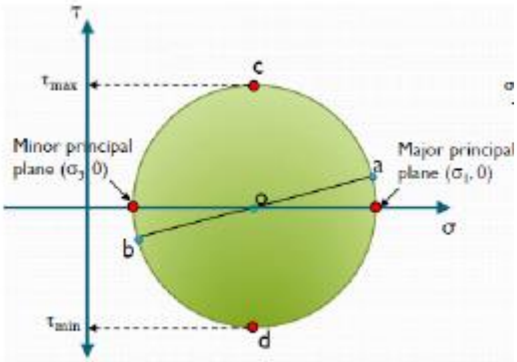
- Need to define σ and τ sign conventions: For σ:
- Compression (+ve sign)
- Tension (-ve sign) For τ:
- Rotation anticlockwise (+ve sign)



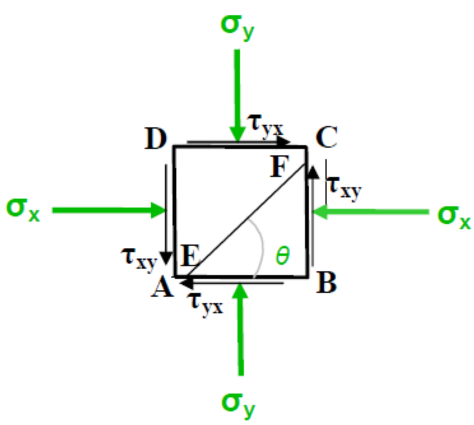
- **Maximum and minimum normal stresses:**
- **Major principal plane (σ1,0)**
- **Minor principal plane (σ3,0)**
- **Maximum and minimum shear stresses:**
- **Plane c ((1 + σ3)/2, τmax)**
- **Plane d ((σ1 + σ3)/2, τmin)**

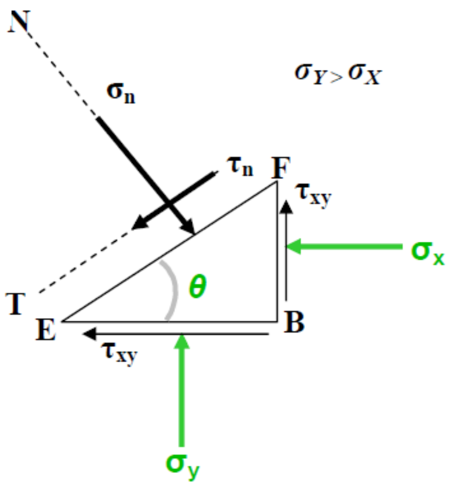
$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$



Normal and Shear Stresses on a Plane





From geometry for the free body diagram EBF

$$\overline{EB} = \overline{EF}\cos\theta$$

$$\overline{FB} = \overline{EF}\sin\theta$$

Summing forces in N and T direction, we have

$$\sigma_n(\overline{EF}) = \sigma_x(\overline{EF})\sin^2\theta + \sigma_y(\overline{EF})\cos^2\theta + 2\tau_{xy}(\overline{EF})\sin\theta\cos\theta$$

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \text{.......(1)}$$

Again

$$\tau_n(\overline{EF}) = -\sigma_x(\overline{EF})\sin\theta\cos\theta + \sigma_y(\overline{EF})\sin\theta\cos\theta - \tau_{xy}(\overline{EF})\cos^2\theta + \tau_{xy}(\overline{EF})\sin^2\theta$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \qquad \text{......(2)}$$

If $\tau_n = 0$ then

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_y - \sigma_x} \qquad \text{.......(3)}$$

This eq. gives 2 values of θ that are 90° apart, this means that there are 2 planes that are right angles to each other on which shear stress = 0, such planes are called *principle planes* and the normal stress that act on the principle planes are to as *principle stresses*.

To find the principle stress substitute eq.3 into eq.1, we get

$$\sigma_n = \sigma_1 = \frac{\sigma_y + \sigma_x}{2} + \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau^2_{xy}} \qquad \textit{major principle stress}$$

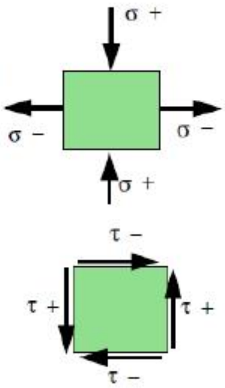
$$\sigma_n = \sigma_3 = \frac{\sigma_y + \sigma_x}{2} - \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau^2_{xy}} \qquad \textit{minor principle stress}$$

These stresses on any plane can be found using *Mohr's circle*

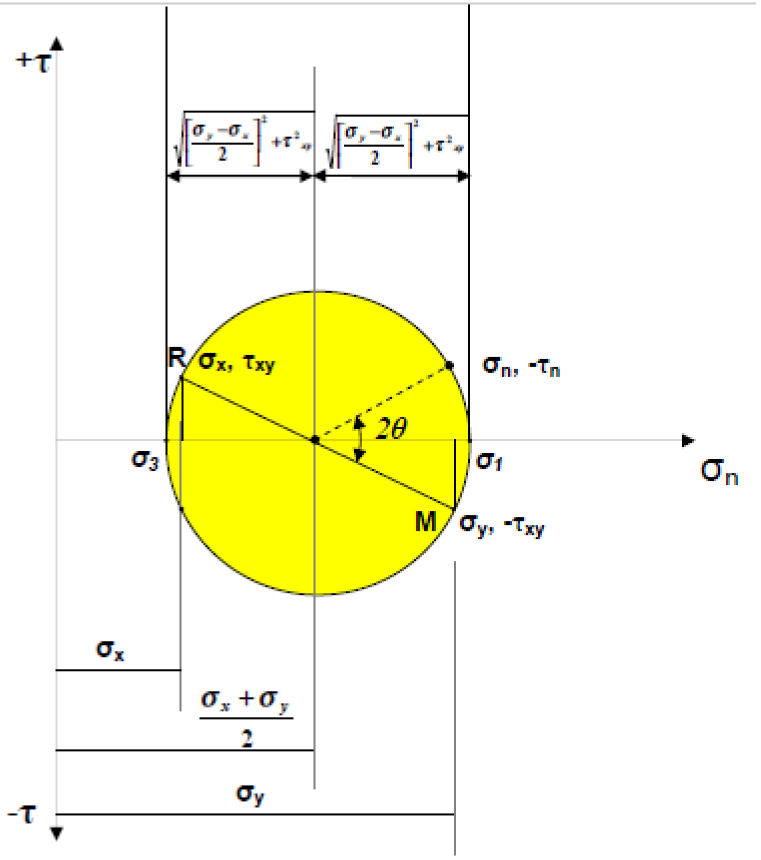
- ***Mohr's circle***

Mohr's Circle Sign Conventions:

- Compressive normal stresses are positive
- Shear stresses are positive, if when they act on two opposing faces, they tend to produce a counterclockwise rotation.



Refer to the element shown in Fig. above



Pole Method

a) Draw the circle.

b) To locate the pole P:

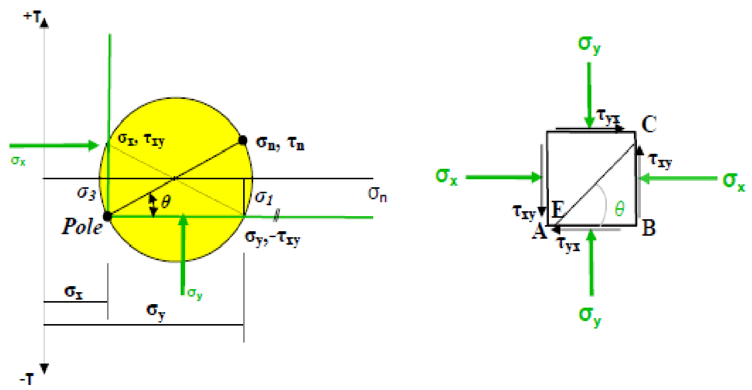
1) Through the point representing the stresses on the first reference plane (x−plane), draw the orientation of the first reference plane (x−plane is vertical).

2) The point where this line intersects the Mohr's Circle is the pole P.

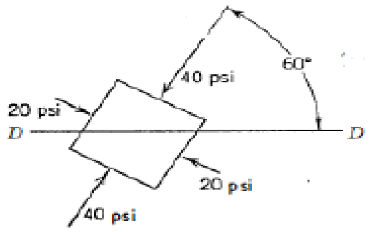
c) To find the stresses on a plane of any orientation:

1) Draw a line through the pole P parallel to the plane;

2) The point where this line intersects the Mohr's circle gives the stresses (σn, τn) on the plane of interest.

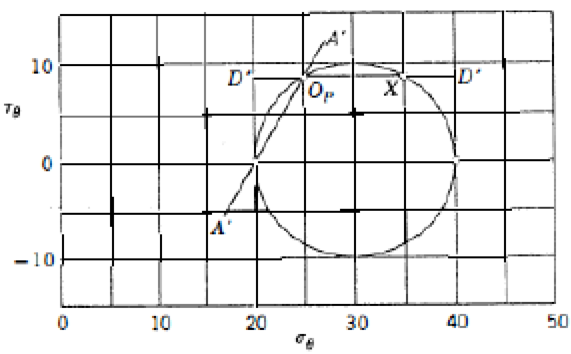


Example 2 : Find the stresses on Horizontal plane D-D?

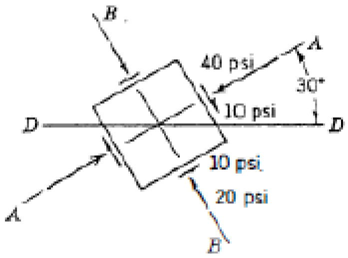


1-Draw point 1(40,0), point (20,0)

2- From point (20,0) draw a line through point (20,0) ,parallel to the plane that the force (20,0) acts .
3- The point of intersection with Mohr –circle represent the OP.
4- From the op draw a horizontal line (line parallel to horizontal plane) the point of intersection with Mohr circle represent the stresses on horizontal plane. (35, 8.7).



Example 3 :

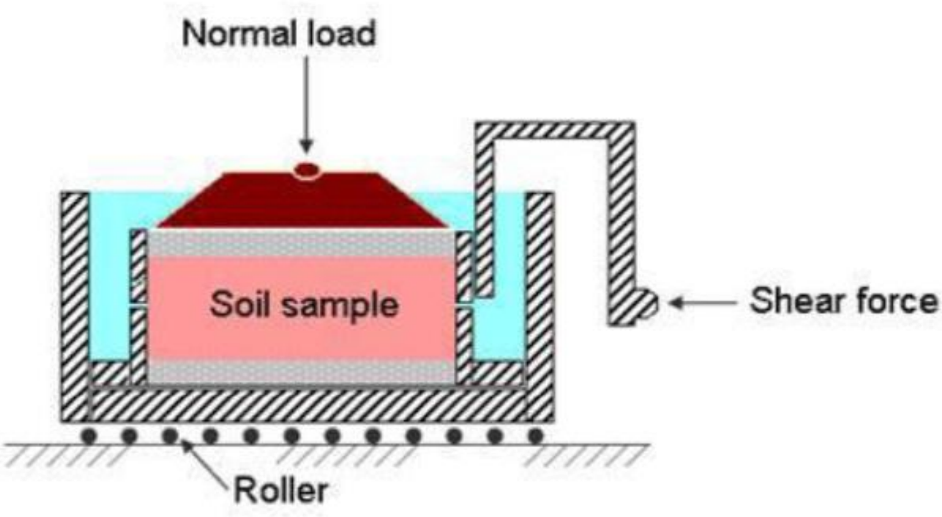


Find the Magnitude and direction of priciple stresses.

Methods of Shear Strength Determination

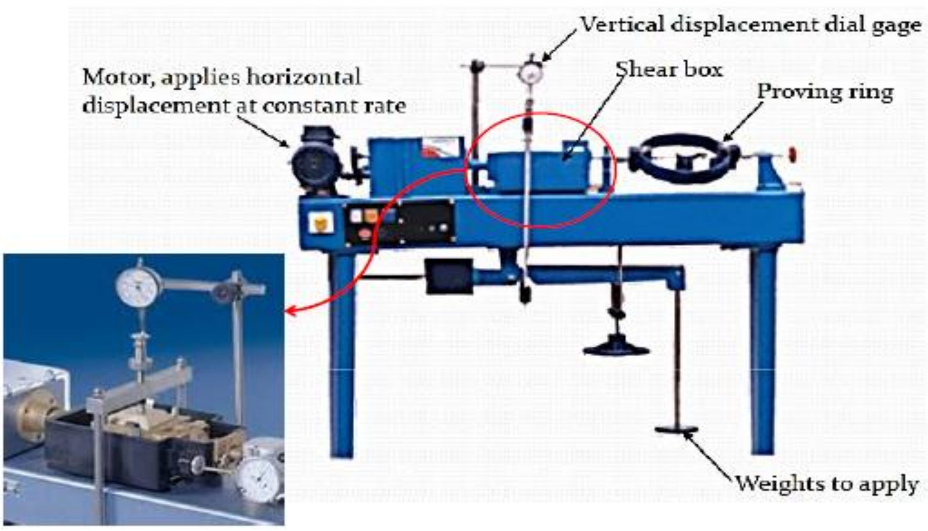
Direct Shear Test

The test is carried out on a soil sample confined in a metal box of square cross-section which is split horizontally at mid-height. A small clearance is maintained between the two halves of the box. The soil is sheared along a predetermined plane by moving the top half of the box relative to the bottom half. The box is usually square in plan of size 60 mm x 60 mm. A typical shear box is shown.



If the soil sample is fully or partially saturated, perforated metal plates and porous stones are placed below and above the sample to allow free drainage. If the sample is dry, solid metal plates are used. A load normal to the plane of shearing can be applied to the soil sample through the lid of the box.
Tests on sands and gravels can be performed quickly, and are usually performed dry as it is found that water doesnot sig nificantly affect the drained strength. For clays, the rate of shearing must be chosen to prevent excess pore pressures building up.

As a vertical normal load is applied to the sample, shear stress is gradually applied horizontally, by causing the two halves of the box to move relative to each other. The shear load is measured together with the corresponding shear displacement. The change of thickness of the sample is also measured.
A number of samples of the soil are tested each under different vertical loads and the value of shear stress at failure is plotted against the normal stress for each test. Provided there is no excess pore water pressure in the soil, the total and effective stresses will be identical. From the stresses at failure, the failure envelope can be obtained.

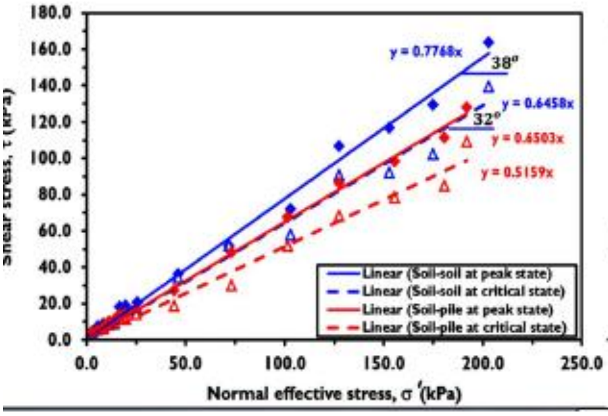


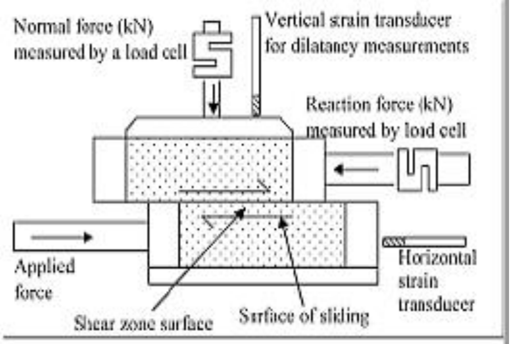
For a given test, the normal stress can be calculated as

$$\sigma = \text{Normal stress} = \frac{\text{Normal force}}{\text{Cross-sectional area of the specimen}}$$

The resisting shear stress for any shear displacement can be calculated as

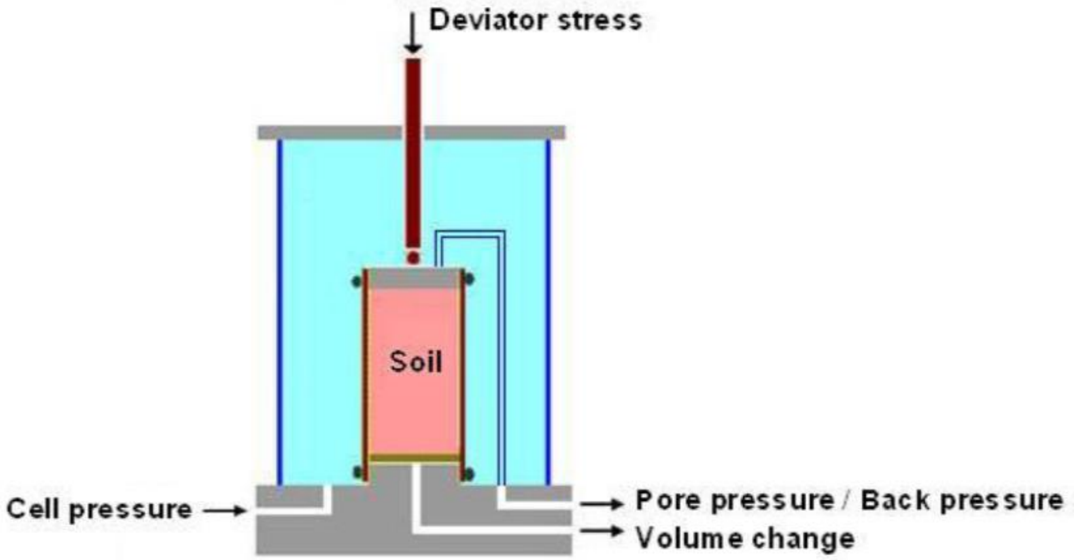
$$\tau = \text{Shear stress} = \frac{\text{Resisting shear force}}{\text{Cross-sectional area of the specimen}}$$





TriaxialTest

The triaxial test is carried out in a cell on a cylindrical soil sample having a length to diameter ratio of 2. The usual sizes are 76 mm x 38 mm and 100 mm x 50 mm. Three principal stresses are applied to the soil sample, out of which two are applied water pressure inside the confining cell and are equal. The third principal stress is applied by a loading ram through the top of the cell and is different to the other two principal stresses.A typical triaxial cell is shown.



The soil sample is placed inside a rubber sheath which is sealed to a top cap and bottom pedestal by rubber Orings. For tests with pore pressure measurement, porous discs are placed at the bottom, and sometimes at the top of the specimen. Filter paper drains may be provided around the outside of the specimen in order to speed up the consolidation process. Pore pressure generated inside the specimen during testing can be measured by means of pressure transducers. The triaxial compression test consists of two stages:

First stage: In this, a soil sample is set in the triaxial cell and confining pressure is then applied. Second stage: In this, additional axial stress (also called deviator stress) is applied which induces shear stresses in the sample. The axial stress is continuously increased until the sample fails.

During both the stages, the applied stresses, axial strain, and pore water pressure or change in sample volume can be measured.

TestTypes

There are several test variations, and those used mostly in practice are:

UU (unconsolidated undrained) test: In this, cell pressure is applied without allowing drainage. Then keeping cell pressure constant, deviator stress is increased to failure without drainage.

CU (consolidated undrained) test: In this, drainage is allowed during cell pressure application. Then without allowing further drainage, deviator stress is increased keeping cell pressure constant.

CD (consolidated drained) test: This is similar to CU test except that as deviator stress is increased, drainage is permitted. The rate of loading must be slow enough to ensure no excess pore water pressure develops.

In the UU test, if pore water pressure is measured, the test is designated by . In the CU test, if pore water pressure is measured in the second stage, the test is symbolized as .

During the shearing process, the soil sample experiences axial strain, and either volume change or development of pore water pressure occurs. The magnitude of shear stress acting on different planes in the soil sample is different. When at some strain the sample fails, this limiting shear stress on the failure plane is called the shear strength.

The triaxial test has many advantages over the direct shear test:

• The soil samples are subjected to uniform stresses and strains.
• Different combinations of confining and axial stresses can be applied.
• Drained and undrained tests can be carried out.
• Pore water pressures can be measured in undrained tests.
• The complete stress-strain behaviour can be determined.