Beam Reactions

Loads on Members

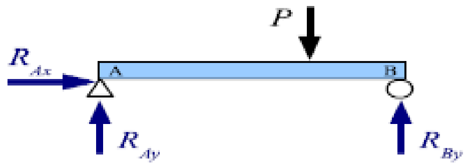
Assume ½ inch diameter steel rod and pull it lengthwise with a load P, the rod will develop a tensile stress $\sigma = \frac{P}{A}$ where A is the cross-section area of the rod.

Case 1: loading the rod in tension parallel to its axis makes the rod a tensile member.

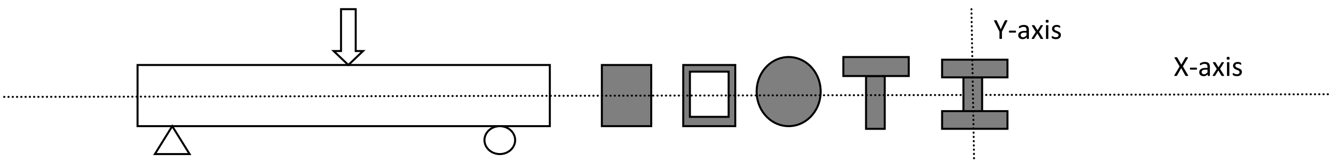
Case 2: loading the rod in compression parallel to its axis makes the rod a compressive member.

Case 3: twist the rod with torque T, then we call it a torsion shaft.

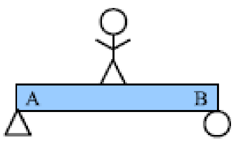
Case 4: if the loading is perpendicular (transverse) to its axis, so that the rod bends, then the rod called a beam.

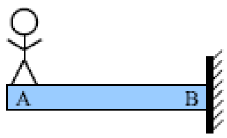


Beam: a member resist forces applied to its axis, set on a support.



Cross Section Beams

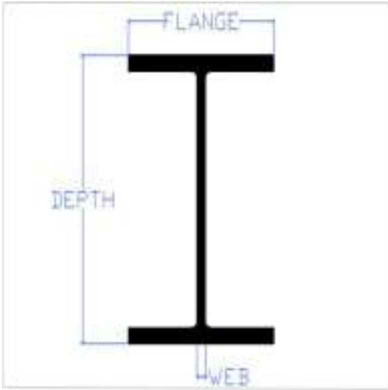


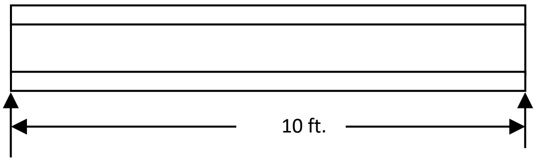


Self Weight

Example 1: For the U.S. Customary (متعارف عليه) W-beam W24x162 (I), given the length of the beam is 10 feet long. What is the total weight W of the beam?

help: for the US system designation (تسمية) system W24x162, has two numbers: the 1st is the nominal depth 24 inch, and the 2nd is the weight per unit length ω=162 Ib./ft..





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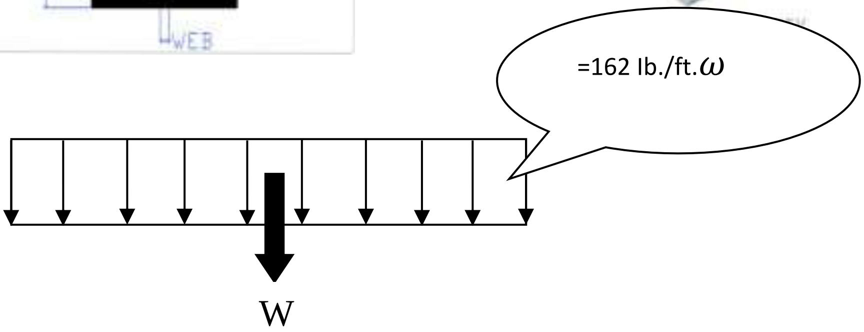


Solution: Total weight W= $\omega \times l = 162 \frac{Ib.}{ft.} \times 10\, ft = 1.620\, Ib.$

Example 2: For the SI (metric), a W-beam W250x115 (I), given a length 4m. What is the total weight W of the beam?

Help:

- For the SI (metric) beams are designated by mass, not weight: a W250x115 wide flange beam has a nominal depth of 250mm and mass per unit length of 115 kg/m.
- The SI unit of force and weight is the Newton (N) defined as 1N=1$\frac{\text{kg.m}}{\text{s}^2}$.
- The acceleration of gravity is 9.81 m/s^2.

Solution: $weight\ per\ unit\ length\ (\omega) = 115\,\frac{\text{kg}}{\text{m}} \times 9.81\,\frac{\text{m}}{\text{s}^2} = 1.13\ \text{kn/m}$

$$\therefore \text{W} = \omega \times l = 1.13\frac{\text{kn}}{\text{m}} \times 4\text{m} = 4.51\ \text{kn}$$

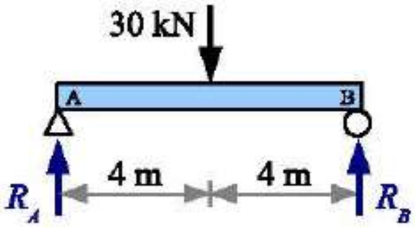
Help: You can also calculate the weight per unit length if you know the cross-sectional area and the specific weight of the material. Specific weight is weight divided by volume:

$$\gamma = \frac{W}{V} = \frac{W}{A \times l} \Rightarrow \gamma \times A = \frac{W}{l} = weight\ per\ unit\ length = \omega$$

Steel density=7850 kg/m^3=490 $Ib./ft^3$.

Reactions for Simply-Supported Simple Beams

Example1: Calculate the reaction forces R_A and R_B for a beam with a 30 kN load at the midspan.
Solution: Divide the total load by 2 to obtain the reaction forces



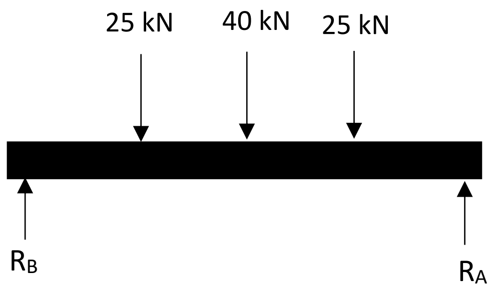
$$R_A = R_B = \frac{P}{2} = \frac{30\,kN}{2} = 15\,kN$$

اذا كان الحمل (Load) في منتصف العارضة او الجسر (Beam) فلا داعي ان تضيع وقتك بالحسابات، مباشرة اقسم الحمل على ٢.

وهناك حالات أخرى يكون فيها الحمل متناظر (Symmetry) وابعاد الحمل عن الـ (Support) أيضا Symmetry. الحل يكون بجمع كل الاحمال المتناظرة البعد ثم القسمة على ٢.

25+40+25=90 kN

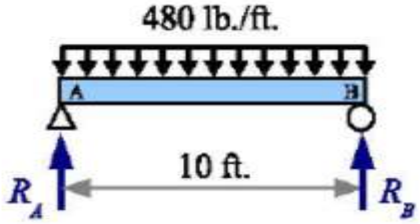
$$RA = RB = \frac{90}{2} = 45\ kN$$



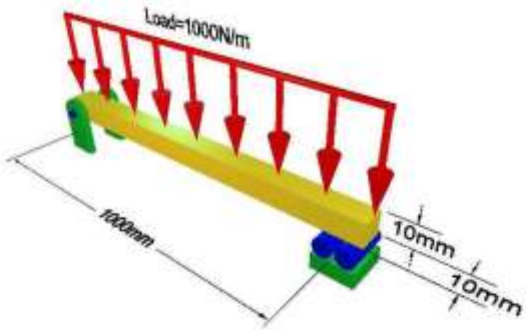
Example 2: Calculate the reaction forces R_A and R_B for a 10-ft. beam with a 480 lb./ft. uniformly distributed load. Report the answer in kips.

Answer:

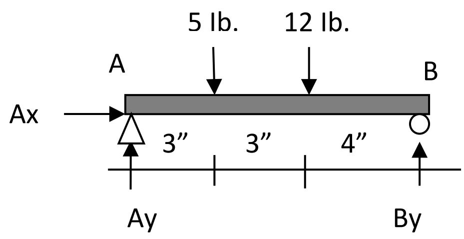
$$R_A = R_B = \frac{W}{2} = \frac{wL}{2} = 2.4\,kips$$



في حالة الحمل المنتشر، نضرب مقدار الحمل المنتشر في طول الـ Span للحمل المنتشر فيكون لدينا حمل مركز في الوسط، ثم نقوم بقسمة مقدار الحمل المركز على ٢، هذا في حالة الحمل المنتشر متناظر اما اذا كان غير متناظر فنلجا للحسابات التقليدية باستخدام معادلات القوى.



Example 3: Calculate the reaction forces Ax, Ay, and By?



Sol. $\curvearrowright + \sum M_A = 0 = 5 \times 3 + 12 \times 6 - By \times (3+3+4)$

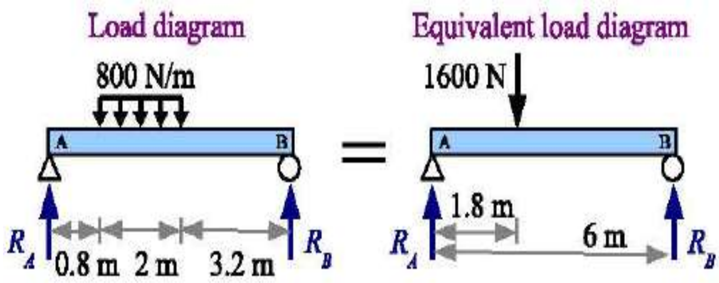
$\therefore By = 8.7\ Ib.$

$\uparrow + \sum Fy = 0 = Ay + By - 5 - 12 \Rightarrow Ay + 8.7 - 17 = 0 \Rightarrow \therefore Ay = 8.3\ Ib.$

$\overset{+}{\rightarrow} \sum Fx = 0 = Ax \quad \Rightarrow \therefore Ax = 0$

تم اعتبار فرضية العزم حول A مع عقارب الساعة.

Example 4: Calculate the reaction forces R_A and R_B for a beam with a uniform distributed load of 800 N/m. Report the result in N.



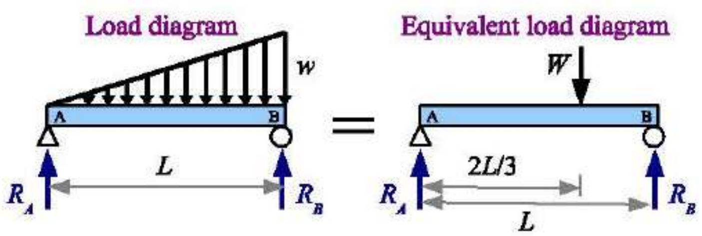
Solution Draw an equivalent load diagram, placing the equivalent load at the centroid of the distributed load. Use the equivalent load diagram to find the reaction forces.

$\curvearrowright + \sum M_A = 0 \Rightarrow \therefore R_B = 480\ N$

$\uparrow + \ \sum F_y = 0 = R_A - 1600\,\text{N} + 480\,\text{N}$. Solve for the reaction force $R_A = 1600\,\text{N} - 480\,\text{N} = 1120\,\text{N}$.

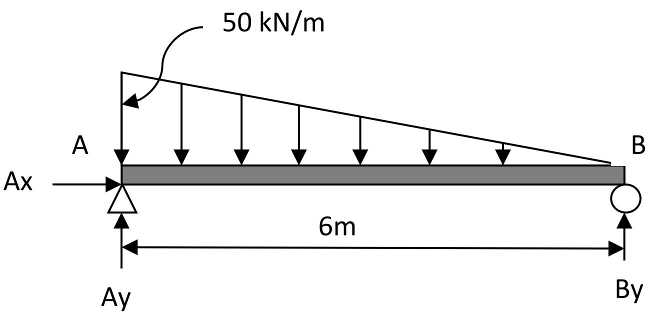
Hint:

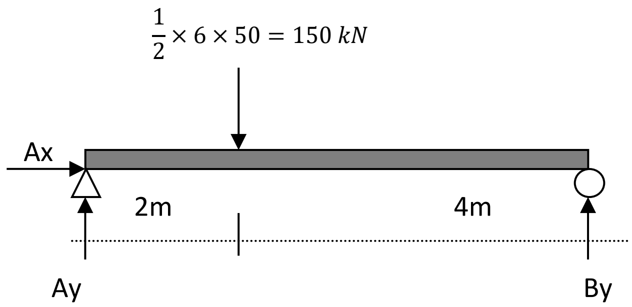
Use the same approach for a nonuniformly distributed load. Again, the location of the equivalent load is at the centroid of the distributed load. The centroid of a triangle is one third of the distance from the wide end of the triangle, so the location of the equivalent load is one third of the distance from the right end of this beam, or two thirds of the distance from the left end.



The load varies from 0 at the left end to w at the right end; therefore, the total load is the average of these loads times the beam length: $W = \left(\frac{0+w}{2}\right) L = \frac{wL}{2}$. **Or we can say the area of the triangle**

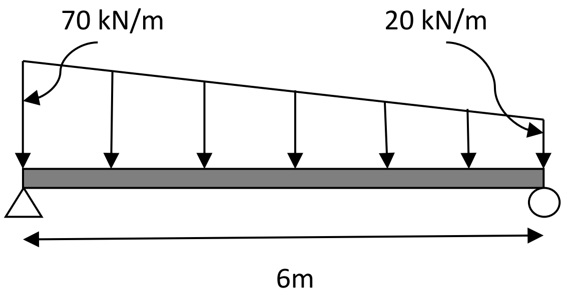
Example 5: calculate the reaction forces?

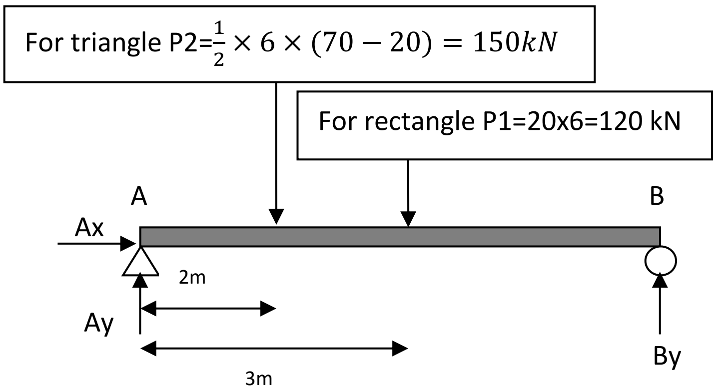




$\curvearrowleft + \sum M_A = 0 \Rightarrow \therefore B_y = 50\ kN$, $\uparrow + \sum F_y = 0$: A_y=100kN, $\xrightarrow{+} \sum Fx = 0 \Rightarrow Ax = 0$

Example 6: calculate the reaction forces?





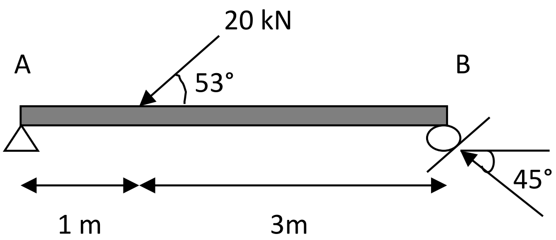
Sol.

$\curvearrowleft + \sum M_A = 0 \Rightarrow$ 150*2+120*3-B_y*6=0 $\Rightarrow \therefore B_y = 110\ kN \uparrow$

$\Rightarrow A_y + B_y - 150 - 120 = 0$ $\Rightarrow \therefore A_y = 160\ kN \uparrow$

$\xrightarrow{+} \sum Fx = 0$ $\Rightarrow A_x = 0$

Example 7: calculate the reaction forces?



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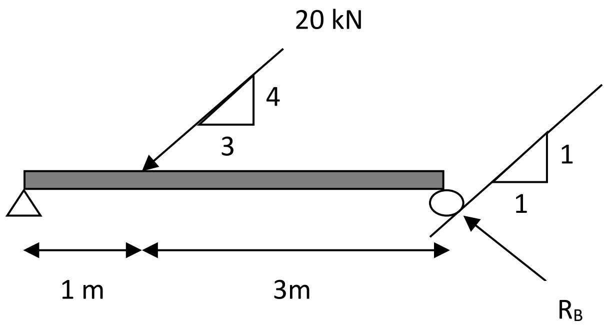
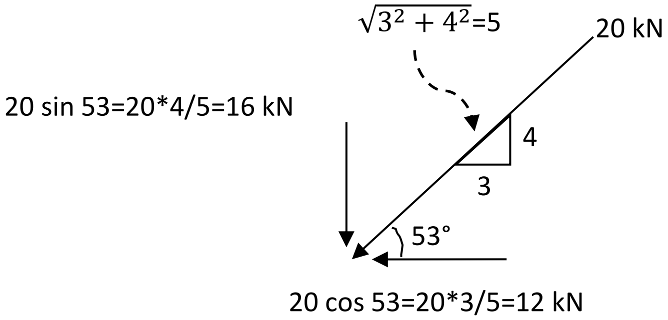
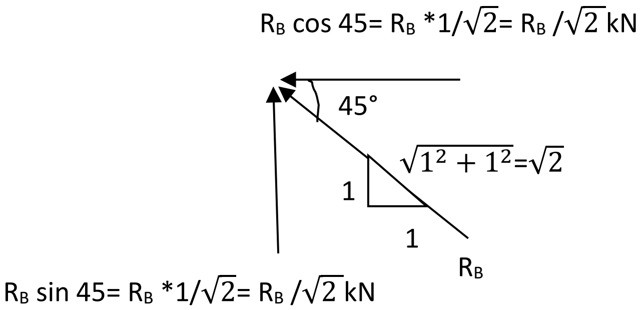
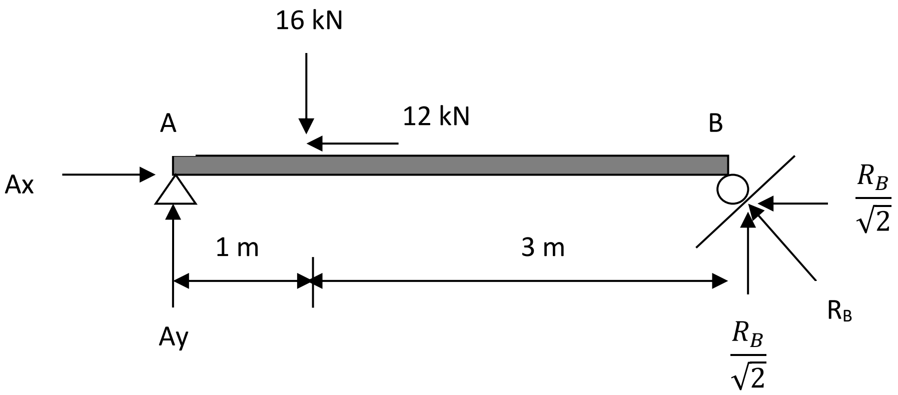








ملاحظة: ان المسند B هو Roller وليس Hinge اي يوجد قوة واحدة فقط عمودية على السطح ولكون السطح مائل تم تحليلها الى مركبتين عمودية وافقية. ولتبسيط الحل ناخذ العزم حول نقطة B.

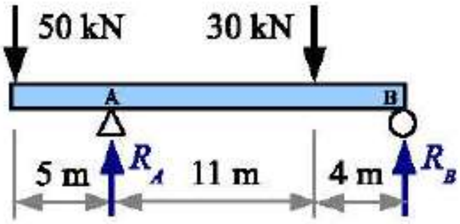
$\circlearrowright + \sum M_B = 0 \Rightarrow Ay \times 4 - 16 \times 3 = 0 \Rightarrow \quad \therefore Ay = 12 \text{ kN} \uparrow$

$\uparrow + \sum Fy = 0 \Rightarrow Ay - 16 + \frac{R_B}{\sqrt{2}} = 0 \Rightarrow R_B = 4\sqrt{2} \text{ kN} \nwarrow$

$\overset{+}{\rightarrow} \sum Fx = 0 \quad Ax - 12 - \frac{R_B}{\sqrt{2}} = 0 \Rightarrow Ax - 12 - \frac{4\sqrt{2}}{\sqrt{2}} = 0 \Rightarrow Ax = 16 \text{ kN} \rightarrow$

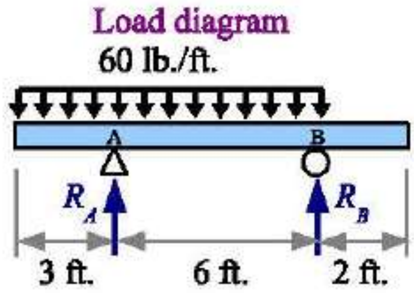
Home Work 1: calculate the reaction forces?

Answer: R_A=74.67 kN, R_B=5.33 kN

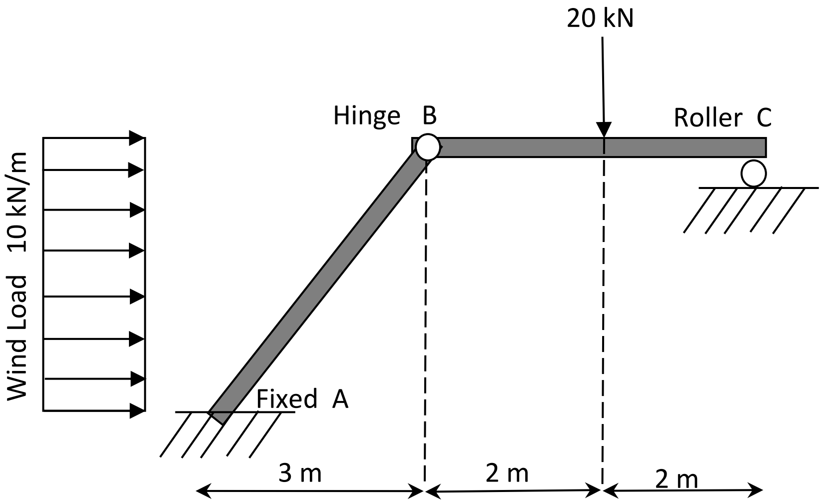


Home Work 2: calculate the reaction forces?

Answer: R_A=405 Ib., R_B=135 Ib.



Home Work 3:
Calculate the reaction forces?
Ans.
C_y=10kN upward
B_y=10kN upward
A_x=40kN left-hand
A_y=10kN upward
M_A=110kN.m Counterclockwise



ملاحظة:

العزم ينتج من قوة في ذراعها وعندما تكون مسافة الذراع =صفر فان مقدار العزم =صفر أيضا بغض النظر عن قيمة القوة او اتجاهها. لذلك عندما نحسب مقدار العزم حول الـ Support فان مقدار الـ Reaction نضربه في الذراع (صفر) فيكون قيمة العزم =صفر سواء كانت الفرضية للعزم مع عقارب الساعة او عكس عقارب الساعة. فلذلك لا تدخل قيمة الـ Reaction في الحساب.

سؤال/ هل يمكن ان نحل السؤال بان نعطي فرضية العزم عكس عقارب الساعة؟ وهل ستكون النتيجة للـ Reaction نفس الناتج؟

الجواب / نعم تعطي نفس الناتج. ولكن لتوحيد الحلول نفترض دائما ان العزم حول الـ Support مع عقارب الساعة. وينطبق ذلك أيضا على فرضية اتجاه القوة R_x للـ Reaction في الـ Hinge باتجاه اليمين دائما, واتجاه القوة R_y للاعلى دائما.