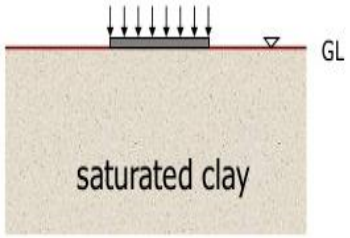
Chapter Three

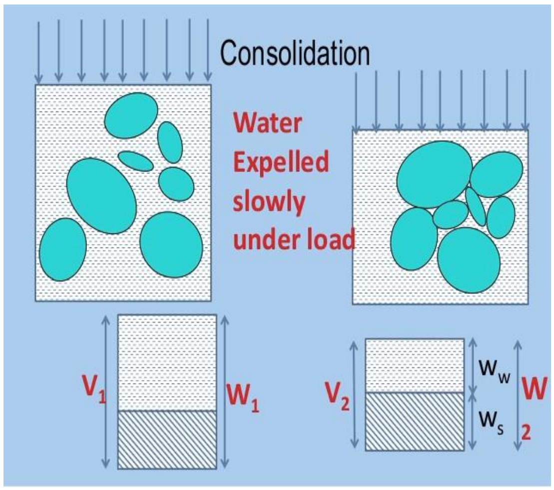
Consolidation

What is consolidation?

* When a saturated soil is loaded externally,

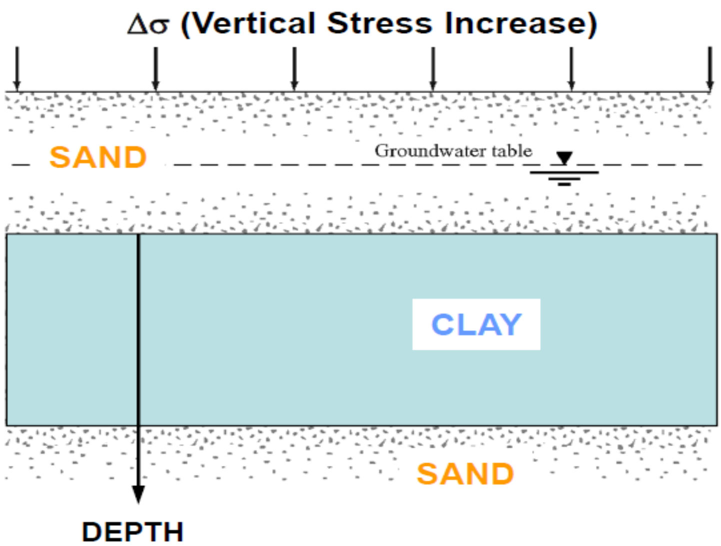


the water is squeezed out of the soil and the soil shrinks, over a long time may be up to several years depending upon the permeability of the soil this whole phenomena is called consolidation.



Fundamentals of Consolidation

Volume change in saturated soils caused by the expulsion of pore water from loading.
Saturated Soils: Δσ causes *u* to increase immediately
Sands: Pore pressure increase dissipates rapidly due to high permeability.
Clays: Pore Pressure dissipates slowly due to low permeability.



Mechanics of Consolidation

- Spring Piston Model
- A cylindrical vessel with compartments marked by pistons separated by springs
- Pistons has perforations to allow water to flow through
- Piezometers inserted at middle of each compartment
- Space between springs filled with water.

- Apply pressure $\boxed{\Delta\sigma}$ on the top most piston.
- Immediately on application of load,
  - Length of springs remain unchanged
  - As a result, the entire $\boxed{\Delta\sigma}$ is borne by water in the vessel. $\boxed{\Delta u}$ is the excess hydrostatic pressure
  - Initial rise in water level $\boxed{h = \frac{\Delta\sigma}{\gamma_w}}$

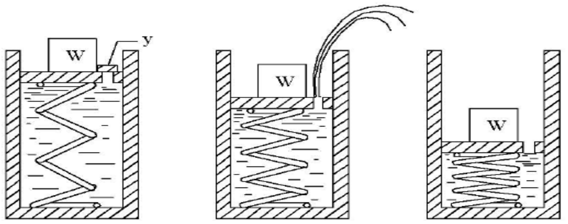
- After time t, flow of water through the perforations has begun at upper compartments. There is a corresponding decrease in volume, the upper springs have compressed a little.
- They carry portion of applied loads and a drop in pressure occurs. The isochrone for $t=t_1$ represents the pore distribution at upper and lower compartments.
- The decrease in soil volume by the squeezing out of pore water on account of gradual dissipation of excess hydrostatic pressure induced by an imposed total stress is called consolidation.

Terzaghi's 1-D Theory of Consolidation

- A Theory to predict the pore pressures at any elapsed time and at any location is required to predict the time rate of consolidation of a consolidating layer.
- Assumptions:

1. Compression and flow is 1-D
2. Darcy's Law is valid
3. Soil is homogeneous and completely saturated
4. Soil grains and water are both incompressible
5. Strains are small. Applied del sigma produce virtually no changes in thickness,
6. k, av are constant.
7. No secondary compression.

THE SPRING ANALOGY



(a) Initial Loading

Water takes load

Soil (i.e. spring) has no load

(b) Dissipation of Excess Water Pressure

Water dissipating

Soil starts to

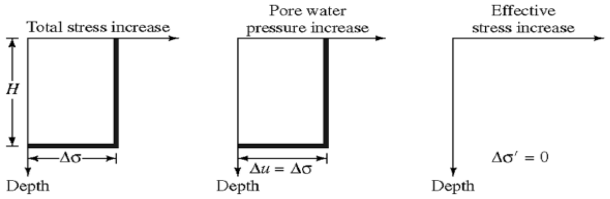
(c) Final Loading

Water dissipated

Soil has load

Slide 5 of 74

At Time of Initial Loading (t = 0)

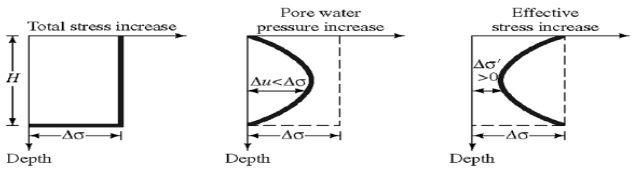


Variation in Total, Pore water, and Effective Stresses in Clay Layer
Figure 7.1b. Das FGE (2005)

Pore water takes initial change in vertical loading ($\Delta\sigma = \Delta u$) since water is incompressible

Soil skeleton does not see initial loading

Between time t = 0 to t = ∞

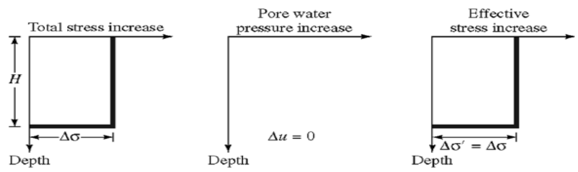


Variation in Total, Pore water, and Effective Stresses in Clay Layer
Figure 7.1c. Das FGE (2005)

Pore water increase due to initial loading dissipates

Soil skeleton takes loading as pore pressure decreases

At time t = ∞



Variation in Total, Pore water, and Effective Stresses in Clay Layer
Figure 7.1e. Das FGE (2005)

Pore water increase due to initial loading completely dissipated ($\Delta u = 0$)

Soil skeleton has taken loading. Effective stress increase now equals vertical stress increase ($\Delta\sigma = \Delta'\sigma$)

The excess hydrostatic pressure can be given as:

$$\Delta u = \frac{P}{A}$$

$$P = P_s + P_w$$

where P_s = load carried by the spring and P_w = load carried by the water.

when the valve is closed after the placement of the load P:

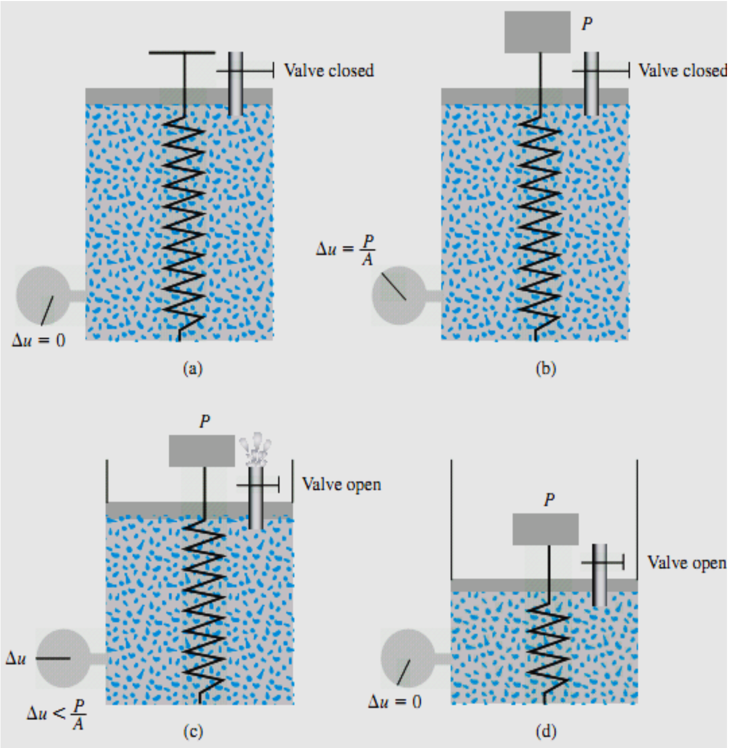
$$P_s = 0 \quad \text{and} \quad P_w = P$$

if the valve is opened:

$$P_s > 0 \quad \text{and} \quad P_w < P$$

After some time, the excess hydrostatic pressure will become zero and the system will reach a state of equilibrium:

$$P_s = P \quad \text{and} \quad P_w = 0$$



One-Dimensional Consolidation Test

- 1-D consolidation testing procedure was first suggested by Terzaghi.
- The main purpose of consolidation tests is to obtain data which is used in predicting the rate and amount of settlement of structures founded on clay.

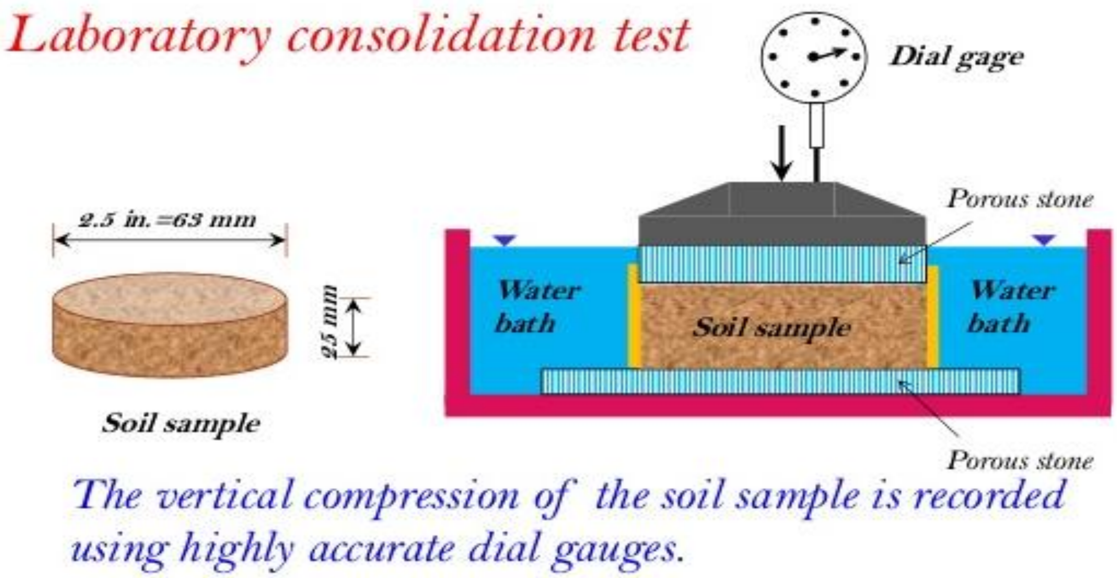
The most important soil properties found by a consolidation test are:

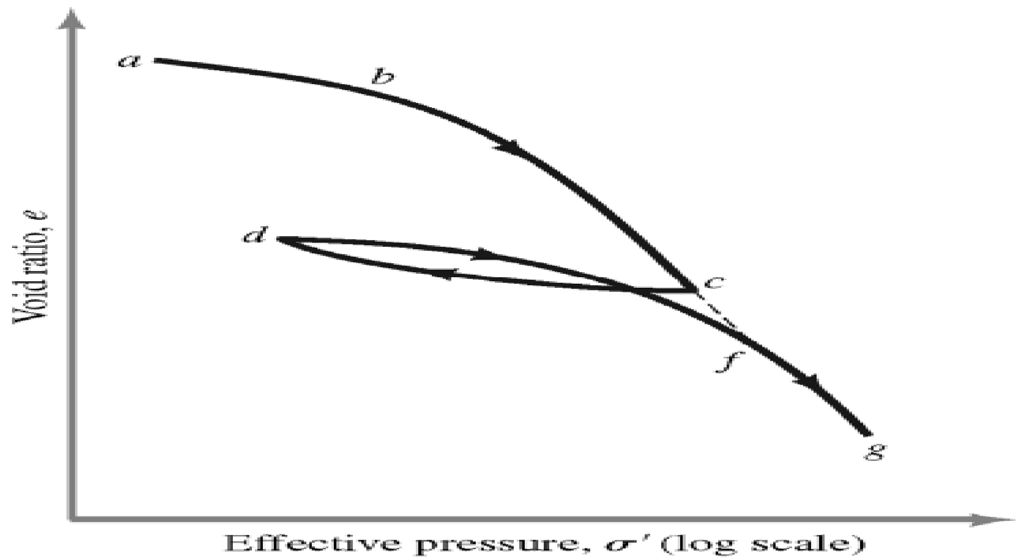
1. The pre-consolidation pressure ($\boldsymbol{\sigma c'}$) is the maximum stress that the soil had subjected to in the past;
2. The compression index (Cc) is the compressibility of a normally consolidated soil;
3. iii. The recompression index (Cr) is the compressibility of an over-consolidated soil;
4. iv. The coefficient of consolidation (Cv) is the rate of compression under a load increment.

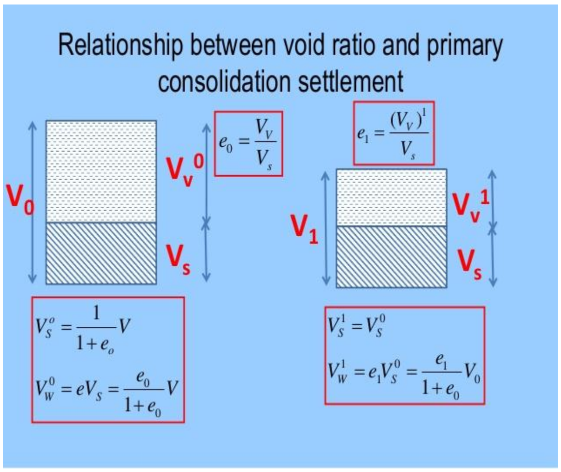
Test Procedure

- The soil specimen is placed inside a metal ring with two porous stones, one at top of the specimen and another at bottom.
- The specimens are usually 75 mm in diameter and 19 mm thick.
- The load on the specimen is applied through a lever arm, and compression is measured by a dial gauge. The specimen is kept under water during the test.

- Each load usually is kept for 24 hours. After that, the load is doubled and the compression measurement is continued.
- At the end of the test, the dry weight of the test specimen is determined.





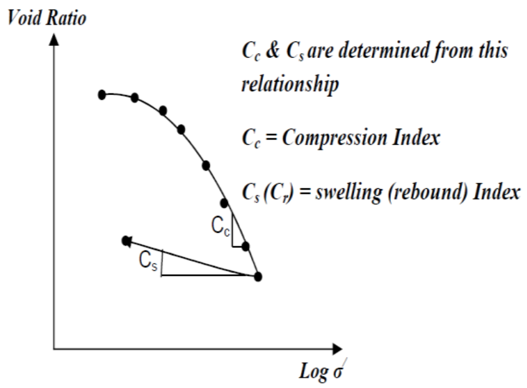


Relationship between void ratio and primary consolidation settlement

$$\begin{aligned} \Delta V &= V_1 - V_0 \\ &= V_V^1 - V_V^0 \\ &= V_W^1 - V_W^0 \\ &= \frac{e_1 - e_0}{1+e_0}V \\ &= \frac{\Delta e}{1+e_0}V \end{aligned}$$

$$\Delta V = \frac{\Delta e}{1+e_0}V$$

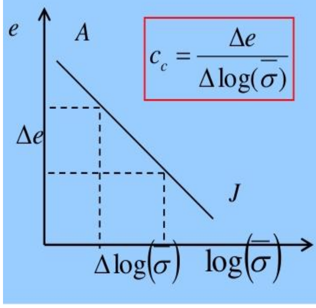
$$\Delta H = \frac{\Delta e}{1+e_0}H$$



Change in void ratio vs. vertical effective stress

Definitions

Compression Index

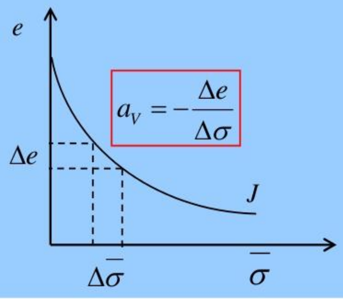


$$\boxed{c_c = \frac{\Delta e}{\Delta \log(\bar{\sigma})}}$$

$$\boxed{c_c = \frac{e_2 - e_1}{\log(\frac{\bar{\sigma}_2}{\bar{\sigma}_1})}}$$

Definitions

Coefficient of Compressibility



$$\boxed{a_V = -\frac{\Delta e}{\Delta \bar{\sigma}}}$$

Definitions

Coefficient of Volume Compressibility

$$\boxed{m_V = -\frac{\Delta V}{V} \Big/ \Delta \bar{\sigma}}$$

$$\boxed{m_V = -\frac{\Delta e}{1 + e_0} \Big/ \Delta \bar{\sigma} = \frac{a_V}{1 + e_0}}$$

Compression Index (C_c) Estimates from Other Laboratory Tests

Soil	C_c Equation	Reference
Undisturbed Clays	$C_c = 0.009(LL - 10)$	Terzaghi & Peck (1967)
Disturbed Clays	$C_c = 0.007(LL - 10)$	
Organic Soils, Peat	$C_c = 0.0115W_n$	EM 1110-1-1904
Clays	$C_c = 1.15(e_o - 0.35)$	
	$C_c = 0.012W_n$	
	$C_c = 0.01(LL - 13)$	
Varved Clays	$C_c = (1 + e_o) - [0.1 + 0.006(W_n - 25)]$	
Uniform Silts	$C_c = 0.20$	

Compression Index (C_c) Estimates from Other Laboratory Tests

Soil	C_c Equation	Reference
Clays	$C_c = 0.141G_s^{1.2}\left(\frac{1+e_o}{G_s}\right)^{2.38}$	Rendon-Herrero (1983)
Clays	$C_c = 0.2343\left[\frac{LL}{100}\right]G_s$	Nagaraj & Murty (1985)

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Clays	$C_c = 0.2343\left[\frac{LL}{100}\right]G_s$	Nagaraj & Murty (1985)

Where:

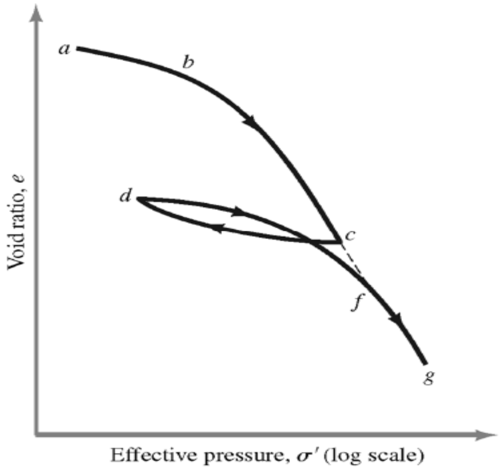
G_s = Specific Gravity of Solids
LL = Liquid Limit (in %)
W_n = Natural Water Content
e_o = Initial Void Ratio

Final e – log σ´ plots consist of results of numerous load & unload increments

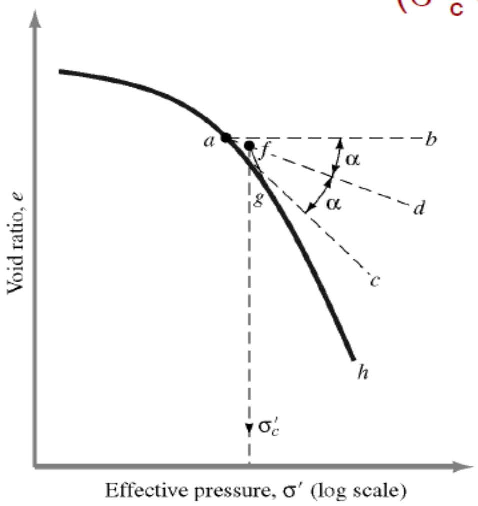
Two Definitions of Clays based on Stress History:

Normally Consolidated (NC):
The present overburden pressure (a.k.a. effective in-situ stress) is the most the soil has ever seen.

Overconsolidated Clay (OC):
The present overburden pressure is less than the soil has experienced in the past. The maximum effective past pressure is called the preconsolidation pressure (σ'_c) or ***Maximum Past Pressure (σ'_{vm})***



Determination of Maximum Past Pressure (σ'_c or σ'_{vm})



Graphical Method (Casagrande, 1936)

1. Visually identify point of minimum radius of curvature on *e*-log σ´ curve (i.e. Point *a*).
2. Draw horizontal line from Point *a* (i.e. Line *ab*).
3. Draw Line *ac* tangent to Point *a*.
4. Draw Line ad bisecting Angle *bac*.
5. Project the straight line portion of *gh* on *e*-log σ´ curve to intersect Line *ad*. This intersection (Point *f*) is the maximum past pressure (a.k.a. preconsolidation pressure).

OverConsolidaiton Ratio (OCR)

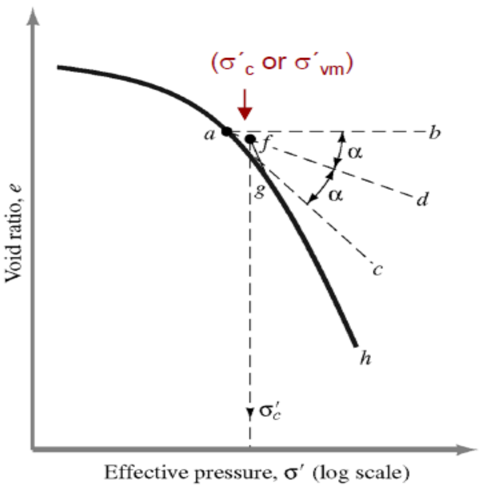


Figure 7.8. Das FGE (2005).

$$OCR = \frac{\sigma'_c}{\sigma'}$$

Where:

σ'_c (a.k.a. σ'_{vm}) = Preconsolidation Pressure (a.k.a Maximum Past Pressure).

σ' = Present Effective Vertical Stress

General Guidelines:
NC Soils: $1 \le OCR \le 2$
OC Soils : $OCR > 2$

Possible Causes of OC Soils:
Preloading (thick sediments, glacial ice); fluctuations of GWT, underdraining, light ice/snow loads, desiccation above GWT, secondary compression.

1D Consolidation Testing

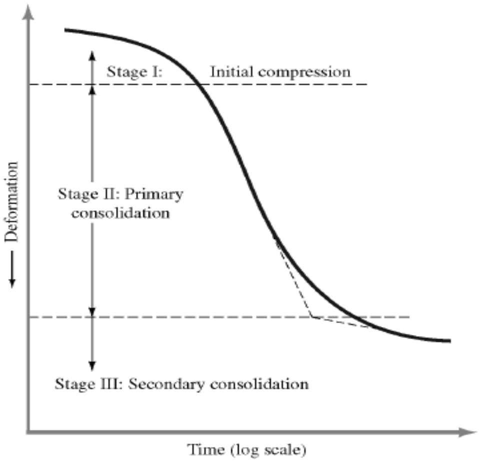
Load Increment Data

THREE STAGES

Stage I: Initial Compression
Primarily caused by preloading.

Stage II: Primary Consolidation
Excess pore water pressure dissipation and corresponding soil volume change.

Stage III: Secondary Consolidation
Occurs after excess pore water pressure dissipation. Due to plastic deformation/ readjustment of soil particles.

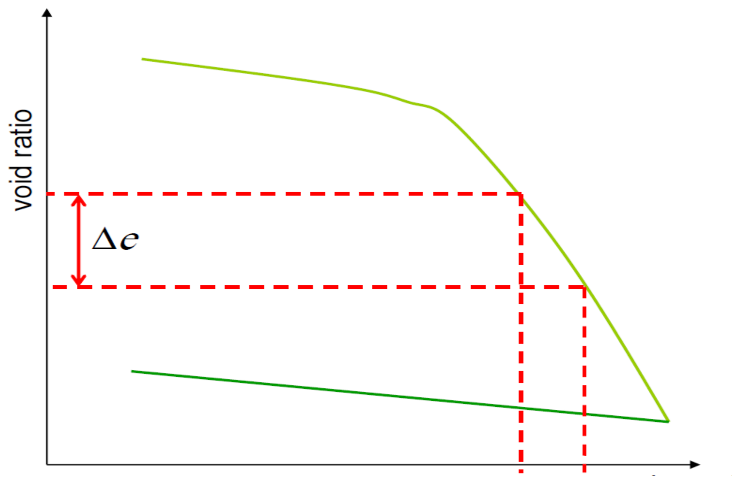


Calculation of Primary Consolidation Settlement

I) Using e - log σ_v

If the e-log σ' curve is given, Δe can simply be picked off the plot for the appropriate range and pressures.

$$S_c = \frac{\Delta e}{1+e_o} H$$



II) Using m_v

$$S_C = m_v . H . \Delta\sigma$$

$$m_v = \frac{\Delta e}{\Delta\sigma(1+e_0)}$$

Disadvantage

m_v is obtained from e vs. $\Delta\sigma$ which is nonlinear and m_v is stress level dependent. This is on contrast to C_c which is constant for a wide range of stress level.

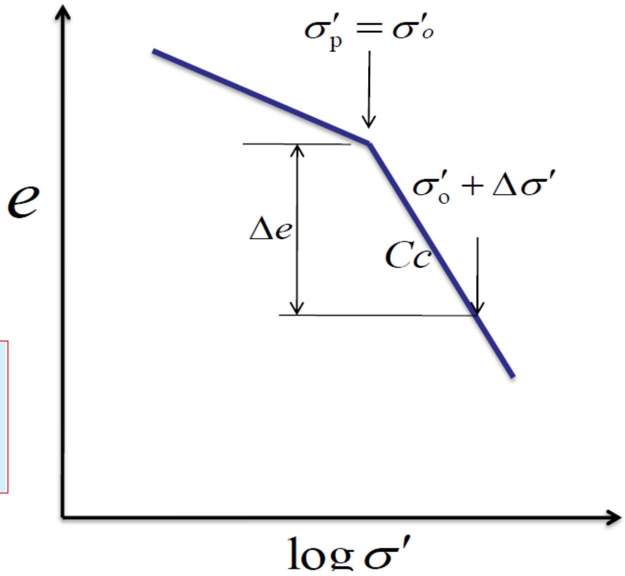
III) Using Compression and Swelling Indices

a) Normally Consolidated Clay ($\sigma'_0 = \sigma'_c$)

$$S_c = \frac{\Delta e}{1+e_o} H$$

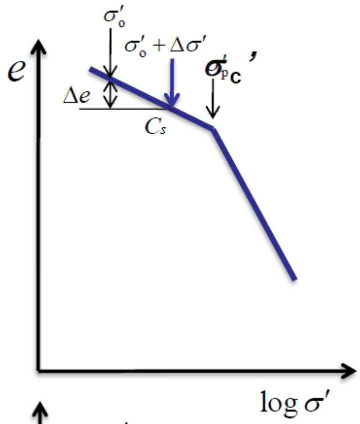
$$\Delta e = C_c \log\left(\frac{\sigma'_o + \Delta\sigma}{\sigma'_o}\right)$$

$$\boxed{S_c = \frac{C_c H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)}$$



b) Overconsolidated Clays

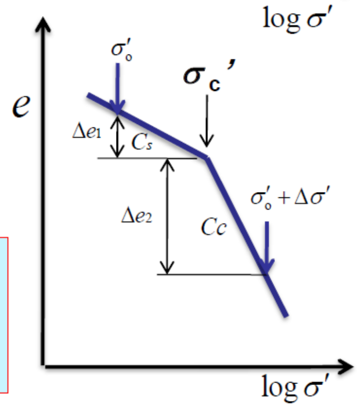
$$S_c = \frac{\Delta e}{1+e_o} H$$



Case I: $\sigma'_0 + \Delta\sigma' \leq \sigma'_c$

$$\Delta e = C_s[\log(\sigma'_o + \Delta\sigma') - \log \sigma'_o]$$

$$\boxed{S_c = \frac{C_s H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)}$$



Case II: $\sigma'_0 + \Delta\sigma' > \sigma'_c$

$$\boxed{S_c = \frac{C_s H}{1 + e_o} \log \frac{\sigma'_c}{\sigma'_o} + \frac{C_c H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right)}$$

Summary of calculation procedure

Calculate σ'_o at the middle of the clay layer

Determine σ'_c from the e-log σ' plot (if not given)

Determine whether the clay is N.C. or O.C.

Calculate $\Delta\sigma$

Use the appropriate equation

- **If N.C.**

$$S_c = \frac{C_c H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

- **If O.C.**

$$S_c = \frac{C_s H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right) \qquad \text{If } \sigma'_o + \Delta\sigma \le \sigma'_c$$

$$S_c = \frac{C_s H}{1 + e_o} \log\frac{\sigma'_c}{\sigma'_o} + \frac{C_c H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right) \qquad \text{If } \sigma'_o + \Delta\sigma > \sigma'_c$$

Secondary Consolidation Settlement

- In some soils (especially recent organic soils) the compression continues under constant loading after all of the excess pore pressure has dissipated, i.e. after primary consolidation has ceased.
- This is called secondary compression or creep, and it is due to plastic adjustment of soil fabrics.
- Secondary compression is different from primary consolidation in that it takes place at a constant effective stress.
- This settlement can be calculated using the secondary compression index, C_α.
- The Log-Time plot (of the consolidation test) can be used to estimate the coefficient of secondary compression C_α as the slope of the straight line portion of e vs. log time curve which occurs after primary consolidation is complete.

- **The magnitude of the secondary consolidation can be calculated as:**

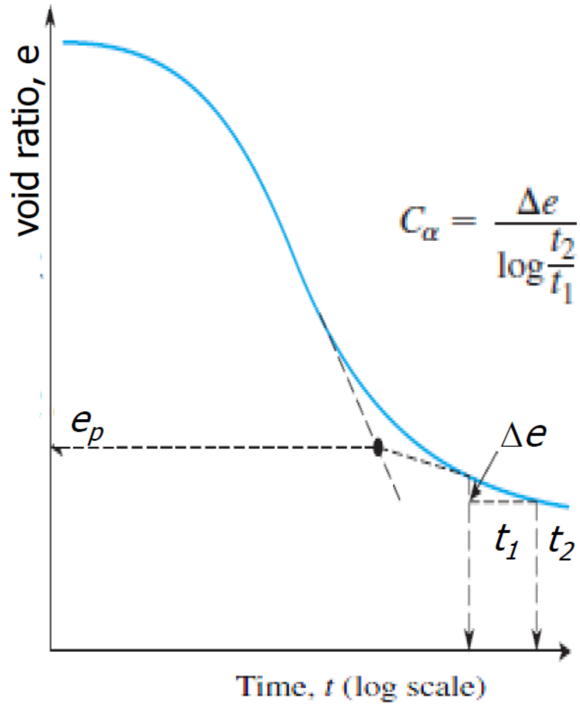
$$\boxed{S_s = \frac{H}{1+e_p}\Delta e}$$

e_p void ratio at the end of primary consolidation,
H thickness of clay layer.

$$\Delta e = C_\propto \log(t_2/t_1)$$

C_α = coefficient of secondary compression

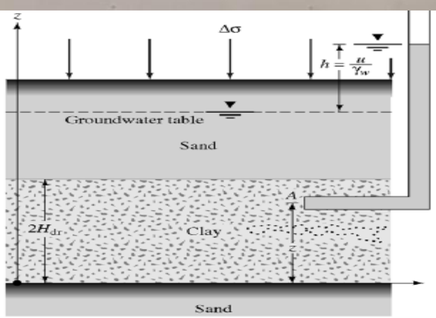
$$\boxed{S_s = \frac{C_\alpha H}{1+e_p}\log\left(\frac{t_2}{t_1}\right)}$$



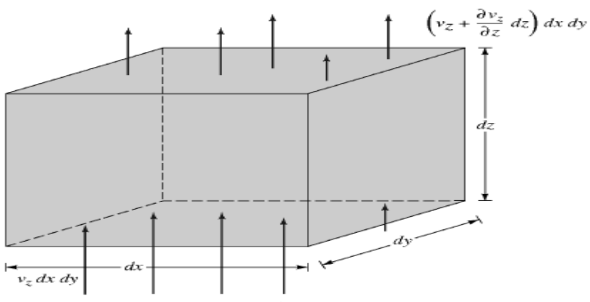
TIME RATE OF CONSOLIDATION

Terzaghi proposed a theory which related the following three quantities:

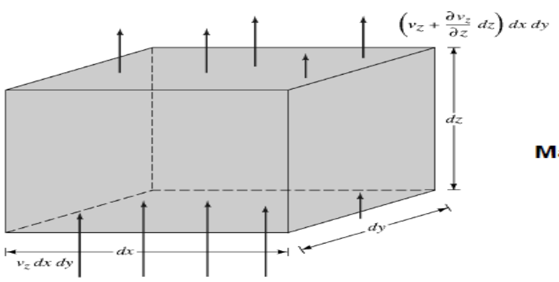
ⓐ The excess pore water pressure (u_e)
ⓑ The depth (z) below the top surface of the clay layer.
ⓒ The time (t) from the instantaneous application of a total stress increment.



Clay Layer Undergoing Consolidation



Flow of Water @ Point A



(Rate of Water Outflow) –

(Rate of Water Inflow) =

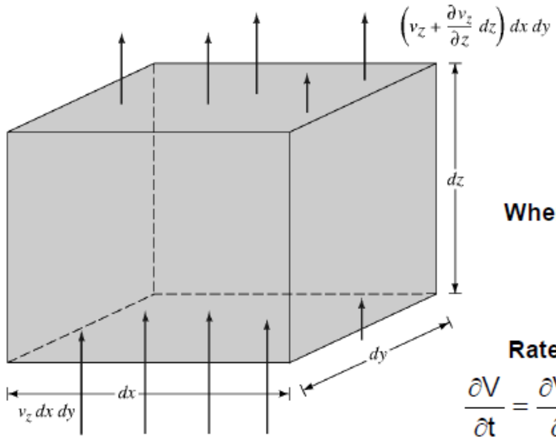
(Rate of Volume Changes)

Mathematical Equation:

$$\left(v_z + \frac{\partial v_z}{\partial z}dz\right)dxdy - v_z dxdy = \frac{\partial V}{dt}$$

or

$$\frac{\partial v_z}{\partial z}dxdydz = \frac{\partial V}{dt}$$



$$\frac{\partial v_z}{\partial z}\,dxdydz = \frac{\partial V}{\partial t}$$

Using Darcy's Law ($v = ki$)

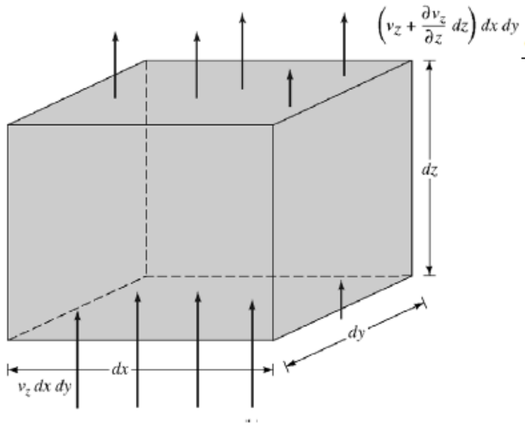
$$v_z = ki = -k\frac{\partial h}{\partial z} = -\frac{k}{\gamma_w}\frac{\partial u}{\partial z}$$

Where u = excess pore pressure. From algebra:

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{dxdydz}\frac{\partial V}{\partial t}$$

Rate of change in V = Rate of Change in V_v

$$\frac{\partial V}{\partial t} = \frac{\partial V_v}{\partial t} = \frac{\partial(V_s + eV_s)}{\partial t} = \frac{\partial V_s}{\partial t} + V_s\frac{\partial e}{\partial t} + e\frac{\partial V_s}{\partial t}$$



Flow of Water @ Point A

Figure 7.17b. Das FGE (2005).

$$\frac{\partial V}{\partial t} = \frac{\partial V_v}{\partial t} = \frac{\partial(V_s + eV_s)}{\partial t} = \frac{\partial V_s}{\partial t} + V_s\frac{\partial e}{\partial t} + e\frac{\partial V_s}{\partial t}$$

Assuming soil solids are incompressible

$$\frac{\partial V_s}{\partial t} = 0$$

and

$$V_s = \frac{V}{1+e_o} = \frac{dxdydz}{1+e_o}$$

e_o = Initial Void Ratio. Substituting:

$$\frac{\partial V}{\partial t} = \frac{dxdydz}{1+e_o}\frac{\partial e}{\partial t}$$

Combining equations:

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{1+e_o}\frac{\partial e}{\partial t}$$

The change in void ratio is caused by the increase in effective stress. Assuming linear relationship between the two:

$$\partial e = a_v \partial(\Delta\sigma') = -a_v \partial u$$

a_v = Coefficient of Compressibility. Can be considered constant over narrow pressure increases. Combining equations:

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = -\frac{a_v}{1+e_o}\frac{\partial u}{\partial t} = -m_v\frac{\partial u}{\partial t}$$

m_v = Coefficient of Volume Compressibility.

$$m_v = \frac{a_v}{1+e_o}$$

Slide 43 of 74

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = -\frac{a_v}{1+e_o}\frac{\partial u}{\partial t} = -m_v\frac{\partial u}{\partial t}$$

a_v = Coefficient of Compressibility.
m_v = Coefficient of Volume Compressibility.

$$m_v = \frac{a_v}{1+e_o}$$

Rearranging Equations:

$$\frac{\partial u}{\partial t} = c_v\frac{\partial^2 u}{\partial z^2}$$

Where c_v = Coefficient of Consolidation.

$$c_v = \frac{k}{(\gamma_w m_v)}$$

Slide 44 of 74

The mathematical solution of Terzaghi's theory present a new characteristic of soil which is called "the coeff. of consolidation" designated by "C_v" which can be defined as:

$$C_v = \frac{k}{m_v * \gamma_w}$$

where,

k, the coeff. of permeability.

mv: the coeff. of volume change. $\frac{a_v}{1+e} = \frac{\Delta e}{\Delta\sigma} * \frac{1}{1+e}$

Cv: the coeff. of consolidation which can be obtained from the results of the oedometer test using two different methods (or techniques) of dealing with data obtained from the test.

Degree of Consolidation $U_z\%$ درجة الانضمام

is the ratio of the amount of the dissipated excess pore water pressure, (any pressure in water above the static pore pressure), to the initial excess pore water pressure, for an element of soil located at depth (z) below the top surface of the clay layer at a specific time.

درجة الانضمام: تمثل النسبة المئوية لتسرب ضغط الماء الزائد في نقطة محددة وزمن معين.

لذلك فإن درجة الانضمام = مقدار ما تسرب من ضغط الماء الزائد / مقدار ما سيتسرب من ضغط الماء الزائد عند انتهاء الانضمام

وتعتمد درجة الانضمام على خواص التربة الطينية وموقع النقطة المطلوبة والزمن المحدد.

The degree of consolidation can be represented by the following terms

① In terms of void ratio:

$$U_z\% = \frac{e_o - e_t}{e_o - e_f} * 100$$

where, e_o, e_f : initial and final void ratios respectively.
e_t : void ratio at time in question.

② In terms of effective stress:

$$U_z\% = \frac{\bar{P}_t - \bar{P}_o}{\bar{P}_f - \bar{P}_o} * 100$$

where: $\bar{P}_o$, $\bar{P}_f$: initial and final overburden effective stress.
$\bar{P}_t$: effective stress at time in question.

③ In terms of excess pore water pressure:

$$U_z\% = \frac{U_i - U_e}{U_i} * 100 \quad \text{or} \quad U_z\% = \left(1 - \frac{U_e}{U_i}\right) * 100$$

where: U_i : initial excess p.w.p. $(= \Delta\sigma_v)$
U_e : excess p.w.p at time in question.

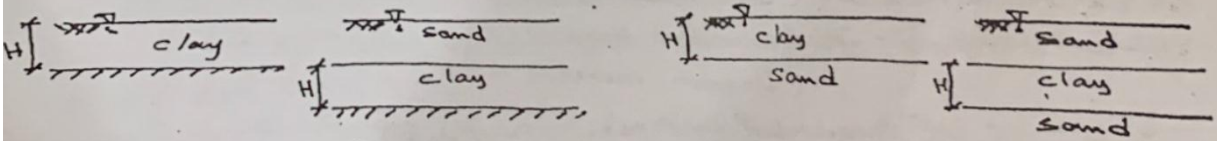
Relation between "C_v" and "U_z %"

"C_v" is related to the degree of consolidation (U_z%) through a dimensionless factor called "Time Factor" designated by "T_v"

$$T_v = \frac{C_v\, t}{d^2}$$

where
- C_v : coeff. of consolidation
- t : time in question.
- T_v : time factor
- d = whole thickness of clay layer if one way drainage.
- or $d = \frac{1}{2}$ thickness of clay layer if two way drainage.

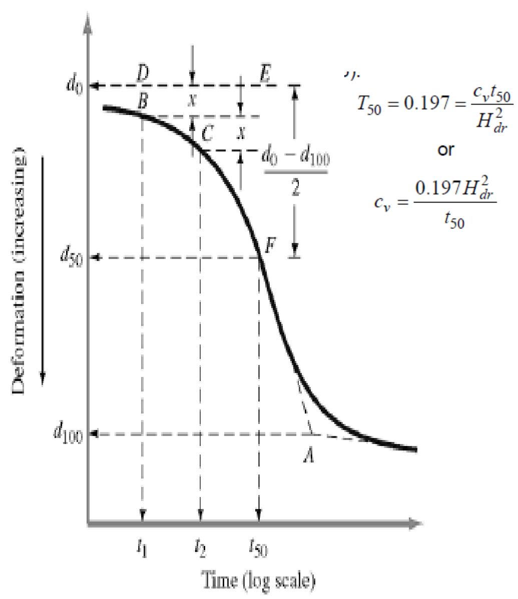
(d) تمثل اطول مسار لقطرة الماء للخروج من الطبقة الطينية.



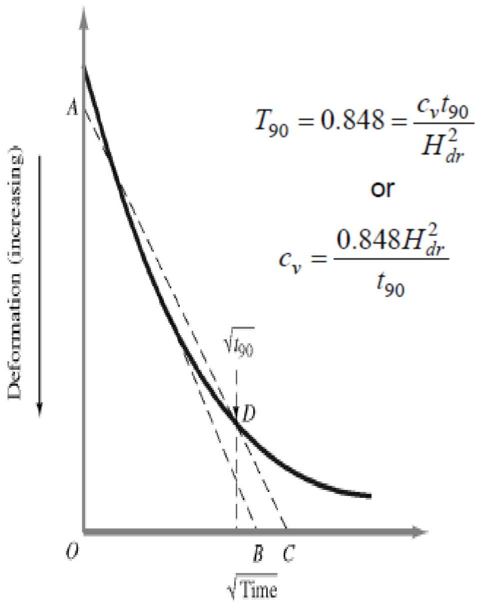
One Way Drainage
$d = H$

Two Way Drainage
$d = \frac{H}{2}$

The time factor "T_v" is related to the degree of consolidation (U_z),



Logarithm of Time Method
(Casagrande and Fadum, 1940)



Square Root of Time Method
(Taylor, 1942)

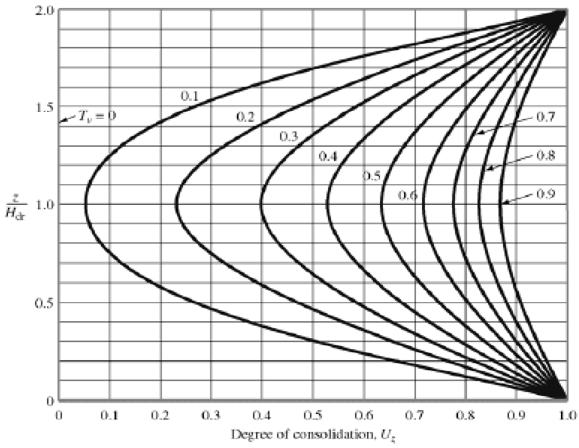
$$T_v = \frac{c_v t}{H^2{}_{dr}} = \textbf{TIME FACTOR}$$

$$\text{For } U = 0\% \text{ to } 60\%, T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$$

$$\text{For } U > 60\%, T_v = 1.781 - 0.933\log(100 - U\%)$$

Because consolidation progress by dissipation of excess pore pressure, the degree of consolidation (U_z) at a distance z at any time t is:

$$U_z = \frac{u_o - u_z}{u_o} = 1 - \frac{u_z}{u_o}$$



Settlement after a Specific Time : "Sct"

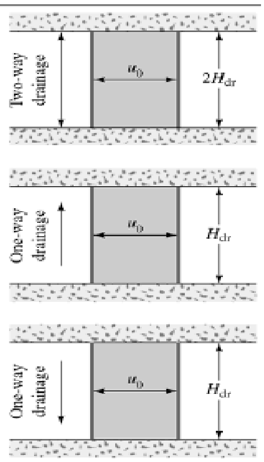
$$S_{ct} = U_{av.} * S_{cf}$$

* من خلال احدى قوانين (Scf) السابقة يتم حساب مقدار الهطول النهائي (Scf)
* من الزمن المحدد وقيمة (Cv) نجد قيمة (Tv) من قانون ($C_v = \frac{T_v d^2}{t}$).
* من العلاقة بين (Tv) و (Uav) في شكل (27.3) أو المخططات السابقة. نجد قيمة (Uav).
* نجد قيمة Sct من القانون اعلاه.

Time Rate of Consolidation

Variation of T_v with U
Table 7.1 Das PGE (2006).

U (%)	T_v	U (%)	T_v	U (%)	T_v
0	0	34	0.0907	68	0.377
1	0.00008	35	0.0962	69	0.390
2	0.0003	36	0.102	70	0.403
3	0.00071	37	0.107	71	0.417
4	0.00126	38	0.113	72	0.431
5	0.00196	39	0.119	73	0.446
6	0.00283	40	0.126	74	0.461
7	0.00385	41	0.132	75	0.477
8	0.00502	42	0.138	76	0.493
9	0.00636	43	0.145	77	0.511
10	0.00785	44	0.152	78	0.529
11	0.0095	45	0.159	79	0.547
12	0.0113	46	0.166	80	0.567
13	0.0133	47	0.173	81	0.588
14	0.0154	48	0.181	82	0.610
15	0.0177	49	0.188	83	0.633
16	0.0201	50	0.197	84	0.658
17	0.0227	51	0.204	85	0.684
18	0.0254	52	0.212	86	0.712
19	0.0283	53	0.221	87	0.742
20	0.0314	54	0.230	88	0.774
21	0.0346	55	0.239	89	0.809
22	0.0380	56	0.248	90	0.848
23	0.0415	57	0.257	91	0.891
24	0.0452	58	0.267	92	0.938
25	0.0491	59	0.276	93	0.993
26	0.0531	60	0.286	94	1.055
27	0.0572	61	0.297	95	1.129
28	0.0615	62	0.307	96	1.219
29	0.0660	63	0.318	97	1.336
30	0.0707	64	0.329	98	1.500
31	0.0754	65	0.304	99	1.781
32	0.0803	66	0.352	100	∞
33	0.0855	67	0.364		



Different types of drainage with u_0 constant