Semiconductor Optoelectronics

Lecture 5: The pn Junction Diode
Dark Current Analysis

Asst. Prof. Dr. Ghusoon Mohsin Ali

4th year Electronics & Communication

Department of Electrical Engineering

College of Engineering

Mustansiriyah University

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Donald A. Neamen

Chapter 8: The pn Junction Diode

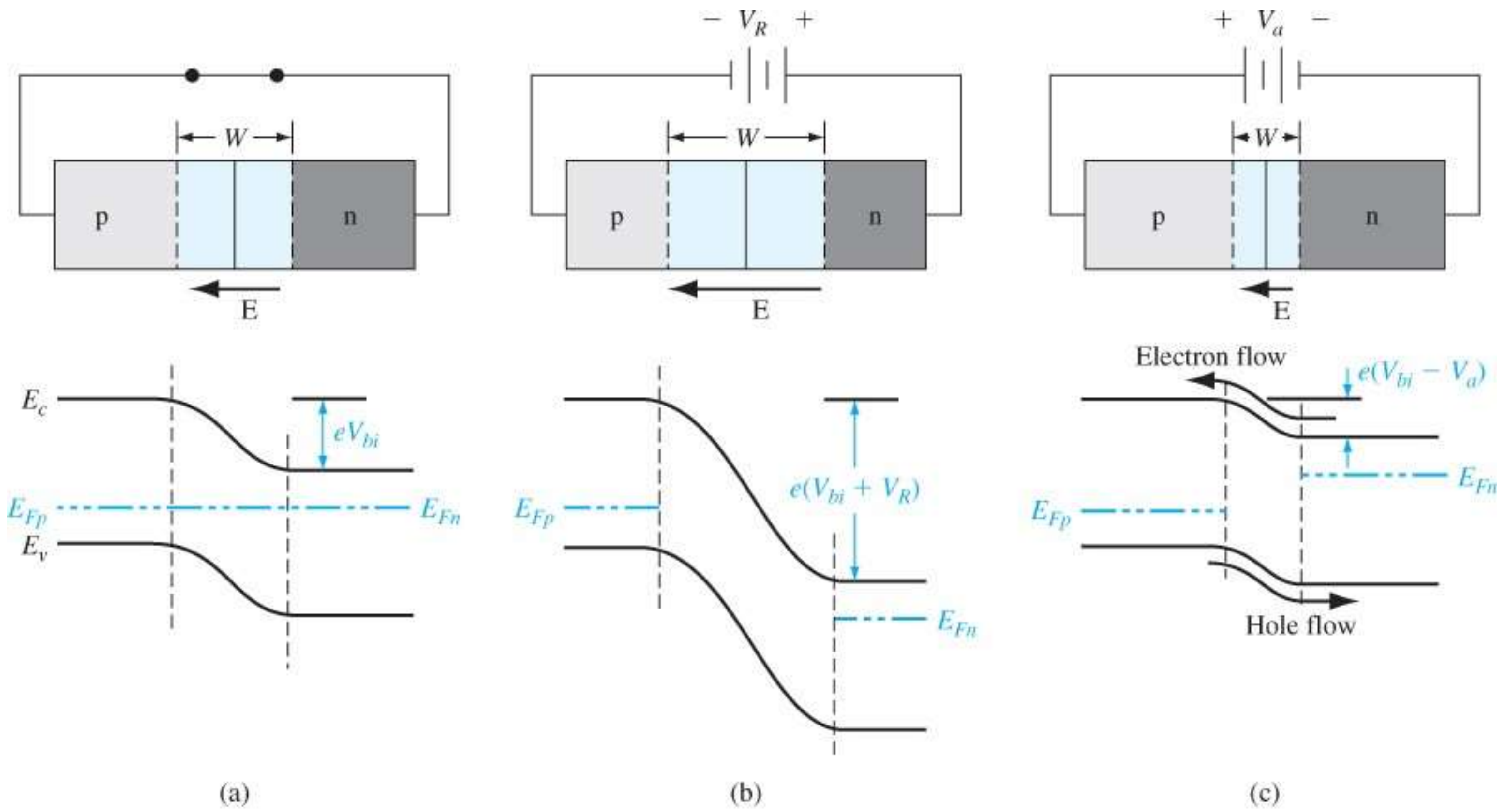

Figure 8.1 | A pn junction and its associated energy-band diagram for (a) zero bias, (b) reverse bias, and (c) forward bias.

In Figure 8.1c, the total potential barrier is reduced. There will be a diffusion of holes from the p region across the space charge region where they will flow into the n region. Similarly, there will be a diffusion of electrons from the n region across the space charge region where they will flow into the p region.

The ideal current–voltage relationship of a pn junction is derived on the basis of four assumptions.

1.The abrupt depletion layer approximation applies. The space charge regions have abrupt boundaries, and the semiconductor is neutral outside of the depletion region.

2. The Maxwell–Boltzmann approximation applies to carrier statistics.

3. The concepts of low injection and complete ionization apply.

4a. The total current is a constant throughout the entire pn structure.

4b. The individual electron and hole currents are continuous functions through the pn structure.

4c. The individual electron and hole currents are constant throughout the depletion region.

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

The built-in potential barrier prevents this large density of electrons from flowing into the p region.

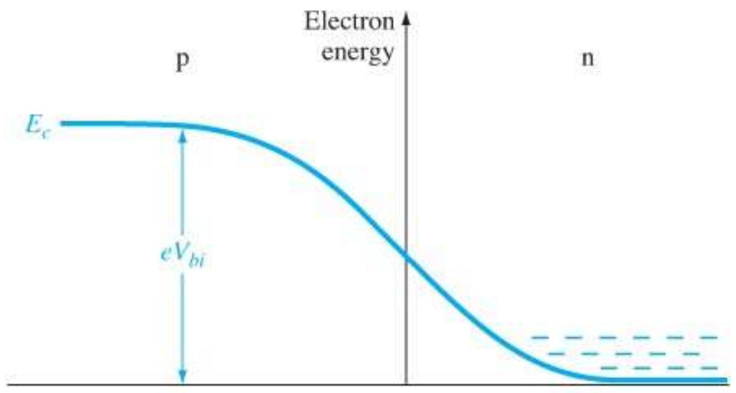

Figure 8.2 | Conduction-band energy through a pn junction.

Table 8.1 | Commonly used terms and notation for this chapter

Term	Meaning
N_a	Acceptor concentration in the p region of the pn junction
N_d	Donor concentration in the n region of the pn junction
$n_{n0} = N_d$	Thermal-equilibrium majority carrier electron concentration in the n region
$p_{p0} = N_a$	Thermal-equilibrium majority carrier hole concentration in the p region
$n_{p0} = n_i^2/N_a$	Thermal-equilibrium minority carrier electron concentration in the p region
$p_{n0} = n_i^2/N_d$	Thermal-equilibrium minority carrier hole concentration in the n region
n_p	Total minority carrier electron concentration in the p region
p_n	Total minority carrier hole concentration in the n region
$n_p(-x_p)$	Minority carrier electron concentration in the p region at the space charge edge
$p_n(x_n)$	Minority carrier hole concentration in the n region at the space charge edge
$\delta n_p = n_p - n_{p0}$	Excess minority carrier electron concentration in the p region
$\delta p_n = p_n - p_{n0}$	Excess minority carrier hole concentration in the n region

$$\frac{n_i^2}{N_a N_d} = \exp\left(\frac{-eV_{bi}}{kT}\right) \quad (8.1)$$

$$n_{p0} = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right) \quad (8.4)$$

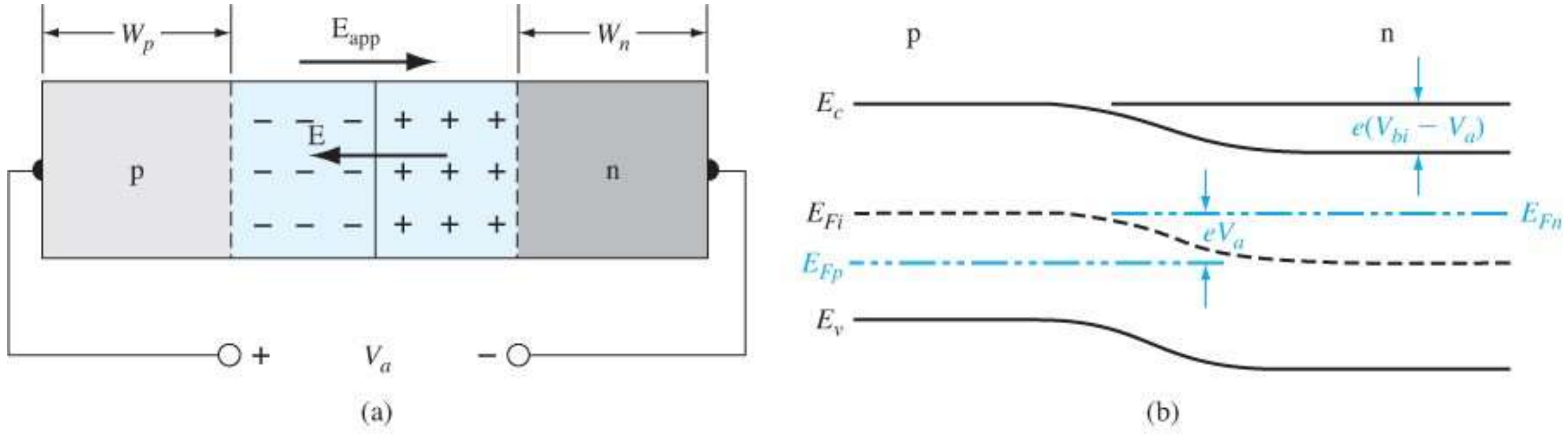

Figure 8.3 | (a) A pn junction with an applied forward-bias voltage showing the directions of the electric field induced by V_a and the space charge electric field. (b) Energy-band diagram of the forward-biased pn junction.

The electric field Eapp induced by the applied voltage is in the opposite direction to the thermal-equilibrium space charge electric field, so the net electric field in the space charge region is reduced below the equilibrium value. The electric field force that prevented majority carriers from crossing the space charge region is **reduced**; majority carrier electrons from the n side are now **injected** across the depletion region into the p material, and majority carrier holes from the p side are **injected** across the depletion region into the n material.

$$n_{p0} = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right) \tag{8.4}$$

The potential barrier V_{bi} in Equation (8.4) can be replaced by $(V_{bi} - V_a)$ when the junction is forward biased. Equation (8.4) becomes

$$n_p = n_{n0} \exp\left(\frac{-e(V_{bi} - V_a)}{kT}\right) = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right) \exp\left(\frac{+eV_a}{kT}\right) \tag{8.5}$$

$$n_p = n_{p0} \exp\left(\frac{eV_a}{kT}\right) \tag{8.6}$$

$$p_n = p_{n0} \exp\left(\frac{eV_a}{kT}\right) \tag{8.7}$$

where p_n is the concentration of minority carrier holes at the edge of the space charge region in the n region. Figure 8.4 shows these results. By applying a forward-bias

Comment of Ex 8.1

The minority carrier concentrations can increase by many orders of magnitude when a relatively small forward-bias voltage is applied.

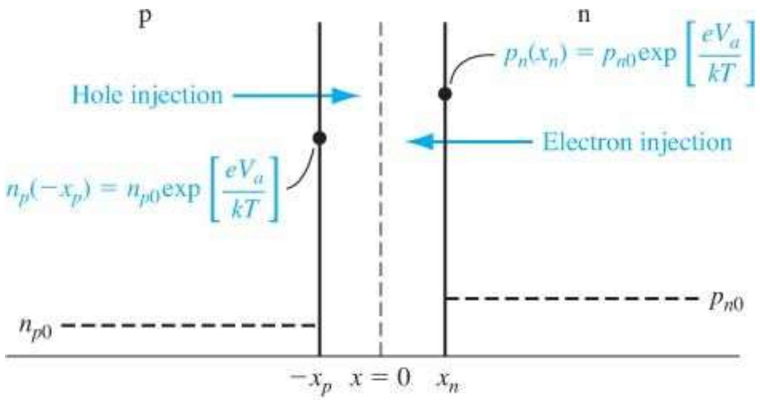

Figure 8.4 | Excess minority carrier concentrations at the space charge edges generated by the forward-bias voltage.

Example 1

A silicon pn junction has impurity doping concentration of $N_d = 2 \times 10^{15}$ cm^{-3} and $N_a = 8 \times 10^{15}$ cm^{-3}. Determine the minority carrier concentration at the edges of the space charge region for

a) $V_a = 0.45$V
b) $V_a = 0.55$V
c) $V_a = -0.55$V

$$n_{po} = \frac{n_i^2}{N_a} = \frac{\left(1.5\times10^{10}\right)^2}{8\times10^{15}} = 2.8125\times10^{4}\,\text{cm}^{-3}$$

$$n_p\left(-x_p\right) = n_{po}\exp\left(\frac{V_a}{V_t}\right)$$

$$n_p\left(-x_p\right) = \left(2.8125\times10^{4}\right)\exp\left(\frac{0.45}{0.0259}\right) = 9.88\times10^{11}\,\text{cm}^{-3}$$

$$p_{no} = \frac{n_i^2}{N_d} = \frac{\left(1.5\times10^{10}\right)^2}{2\times10^{15}} = 1.125\times10^{5}\,\text{cm}^{-3}$$

$$p_n\left(x_n\right) = p_{no}\exp\left(\frac{V_a}{V_t}\right)$$

$$p_n\left(x_n\right) = \left(1.125\times10^{5}\right)\exp\left(\frac{0.45}{0.0259}\right) = 3.95\times10^{12}\,\text{cm}^{-3}$$

Example 1

A silicon pn junction has impurity doping concentration of $N_d = 2 \times 10^{15}$ cm^{-3} and $N_a = 8 \times 10^{15}$ cm^{-3}. Determine the minority carrier concentration at the edges of the space charge region for

b) $V_a = 0.55\text{V}$

$$p_n(x_n) = (1.125\times10^5)\exp\left(\frac{0.55}{0.0259}\right) = 1.88\times10^{14}\,\text{cm}^{-3}$$

$$n_p(-x_p) = (2.8125\times10^4)\exp\left(\frac{0.55}{0.0259}\right) = 4.69\times10^{13}\,\text{cm}^{-3}$$

Example 1

A silicon pn junction has impurity doping concentration of $N_d = 2 \times 10^{15}$ cm^{-3} and $N_a = 8 \times 10^{15}$ cm^{-3}. Determine the minority carrier concentration at the edges of the space charge region for

c) $V_a = -0.55$V

$$p_n(x_n) = \left(1.125\times10^5\right)\exp\left(\frac{-0.55}{0.0259}\right) \cong 0\text{cm}^{-3}$$

$$n_p(-x_p) = \left(2.8125\times10^4\right)\exp\left(\frac{-0.55}{0.0259}\right) \cong 0\text{ cm}^{-3}$$

If a reverse-biased voltage greater than a few tenths of a volt is applied to the pn junction, then we see from Equations (8.6) and (8.7) that **the minority carrier concentrations at the space charge edge are essentially zero.**

8.1.4 Minority Carrier Distribution

$$D_p \frac{\partial^2(\delta p_n)}{\partial x^2} - \mu_p \mathrm{E} \frac{\partial(\delta p_n)}{\partial x} + g' - \frac{\delta p_n}{\tau_{p0}} = \frac{\partial(\delta p_n)}{\partial t} \qquad (8.8)$$

where $\delta p_n = p_n - p_{n0}$ is the excess minority carrier hole concentration and is the difference between the total and thermal equilibrium minority carrier concentrations.

p and n regions. In the n region for $x > x_n$, we have that $\mathrm{E} = 0$ and $g' = 0$. If we also assume steady state so $\partial(\delta p_n)/\partial t = 0$, then Equation (8.8) reduces to

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0 \qquad (x > x_n) \qquad (8.9)$$

where $L_p^2 = D_p \tau_{p0}$. For the same set of conditions, the excess minority carrier electron concentration in the p region is determined from

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = 0 \qquad (x < x_p) \qquad (8.10)$$

where $L_n^2 = D_n \tau_{n0}$.

The boundary conditions for the total minority carrier concentrations are

$$p_n(x_n) = p_{n0} \exp\left(\frac{eV_a}{kT}\right) \tag{8.11a}$$

$$n_p(-x_p) = n_{p0} \exp\left(\frac{eV_a}{kT}\right) \tag{8.11b}$$

$$p_n(x \to +\infty) = p_{n0} \tag{8.11c}$$

$$n_p(x \to -\infty) = n_{p0} \tag{8.11d}$$

The general solution to Equation (8.9) is

$$\delta p_n(x) = p_n(x) - p_{n0} = Ae^{x/L_p} + Be^{-x/L_p} \qquad (x \geq x_n) \tag{8.12}$$

and the general solution to Equation (8.10) is

$$\delta n_p(x) = n_p(x) - n_{p0} = Ce^{x/L_n} + De^{-x/L_n} \qquad (x \leq -x_p) \tag{8.13}$$

concentrations are then found to be, for $(x \geq x_n)$,

$$\boxed{\delta p_n(x) = p_n(x) - p_{n0} = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_n - x}{L_p}\right)} \tag{8.14}$$

and, for $(x \leq -x_p)$,

$$\boxed{\delta n_p(x) = n_p(x) - n_{p0} = n_{p0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_p + x}{L_n}\right)} \tag{8.15}$$

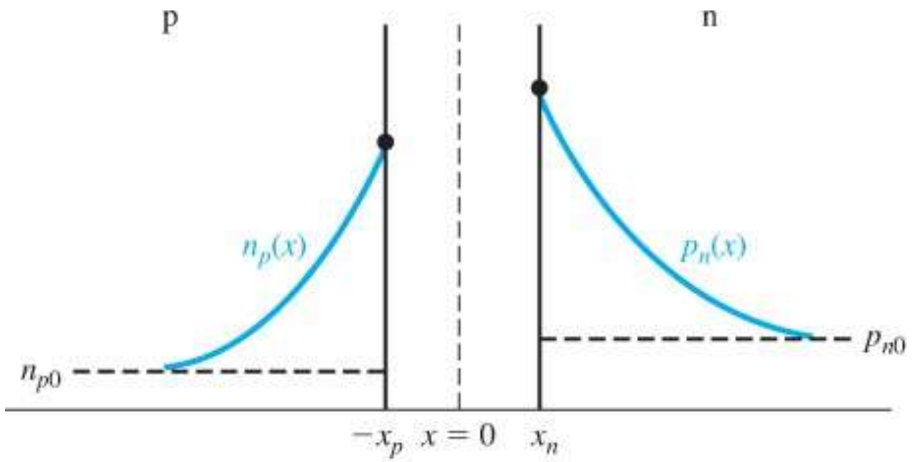

Figure 8.5 | Steady-state minority carrier concentrations in a pn junction under forward bias.

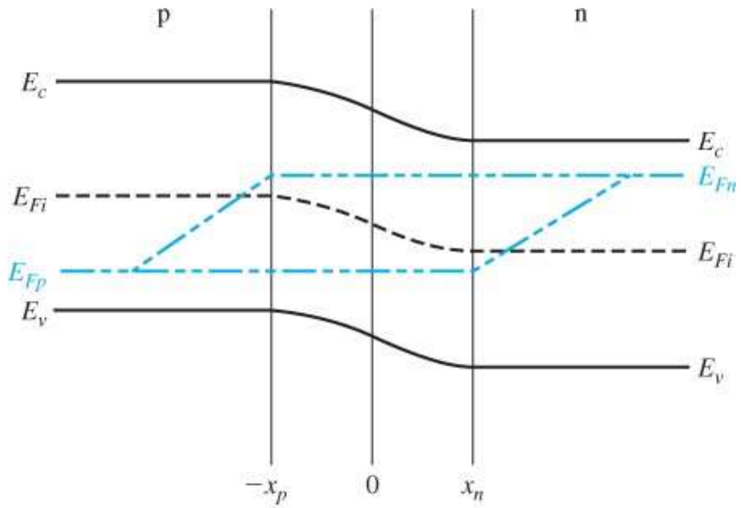

Figure 8.6 | Quasi-Fermi levels through a forward-biased pn junction.

levels to these regions. We had defined quasi-Fermi levels in terms of carrier concentrations as

$$p = p_o + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right) \tag{8.16}$$

$$n = n_o + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right) \tag{8.17}$$

The quasi-Fermi levels are linear functions of distance in the neutral p and n regions close to the space charge edge in the p region, EFn - EFi > 0 which means that δn > ni. Further from the space charge edge, EFn - EFi < 0 which means that δn < ni and the excess electron concentration is approaching zero

At the space charge edge at $x = x_n$, we can write, for low injection

$$n_o p_n(x_n) = n_o p_{no} \exp\left(\frac{V_a}{V_t}\right) = n_i^2 \exp\left(\frac{V_a}{V_t}\right) \tag{8.18}$$

Combining Equations (8.16) and (8.17), we can write

$$np = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right) \tag{8.19}$$

Comparing Equations (8.18) and (8.19), the difference in quasi-Fermi levels is related to the applied bias V_a and represents the deviation from thermal equilibrium.

8.1.5 Ideal pn Junction Current

the total pn junction current will be the minority carrier hole diffusion current at x = xn plus the minority carrier electron diffusion current at x = -xp.

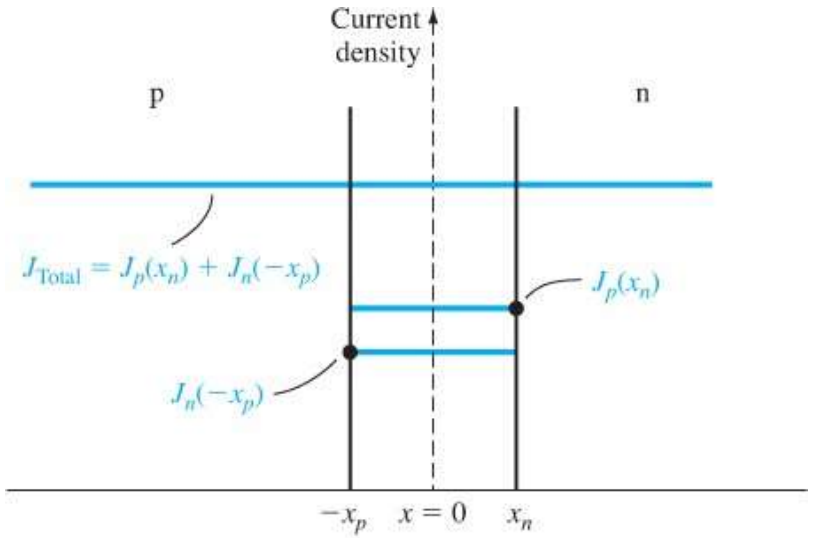

Figure 8.7 | Electron and hole current densities through the space charge region of a pn junction.

We can calculate the minority carrier hole diffusion current density at $x = x_n$ from the relation

$$J_p(x_n) = -eD_p \frac{dp_n(x)}{dx}\bigg|_{x=x_n} \tag{8.20}$$

$$J_p(x_n) = -eD_p \frac{d(\delta p_n(x))}{dx}\bigg|_{x=x_n} \tag{8.21}$$

Taking the derivative of Equation (8.14) and substituting into Equation (8.21), we obtain

$$\boxed{J_p(x_n) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]} \tag{8.22}$$

$$J_n(-x_p) = eD_n \frac{d(\delta n_p(x))}{dx}\bigg|_{x=-x_p} \tag{8.23}$$

ve obtain

$$\boxed{J_n(-x_p) = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]} \tag{8.24}$$

The total current density in the pn junction is then

$$J = J_p(x_n) + J_n(-x_p) = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right]\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \quad (8.25)$$

Equation (8.25) is the ideal current–voltage relationship of a pn junction.

We may define a parameter J_s as

$$J_s = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right] \quad (8.26)$$

$$J = J_s\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \quad (8.27)$$

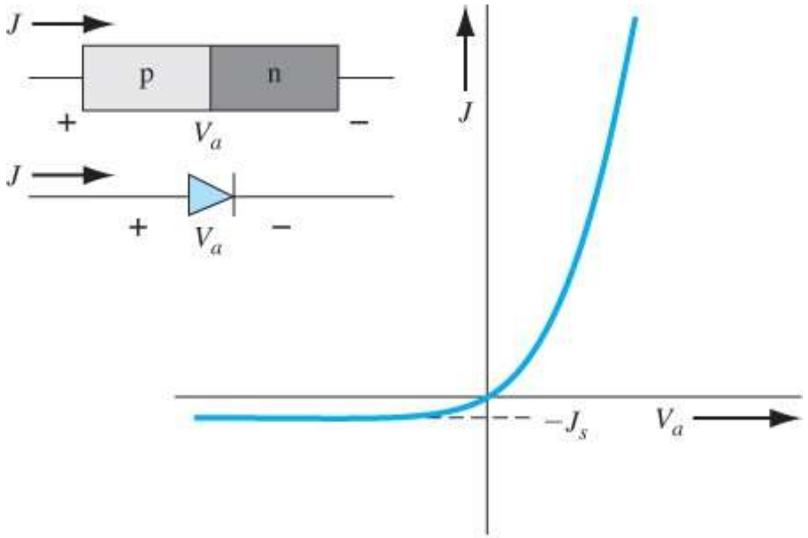

Figure 8.8 | Ideal *I–V* characteristic of a pn junction diode.

If the voltage Va becomes negative (reverse bias) by a few kT eV, then the reverse-biased current density becomes independent of the reverse-biased voltage.
The parameter Js is then referred to as **the reverse-saturation** current density.

Example 2

Consider a GaAs pn junction diode at T=300K. The parameters of the device are $N_d=2\times10^{16}$ cm^{-3}, $N_a=8\times10^{15}$ cm^{-3}, $D_n=210$ cm^2/s, $D_p=8$ cm^2/s, $\tau_{no}=10^{-7}$s and $\tau_{po}=5\times10^{-8}$ s. Determine the ideal reverse-saturation current density.

$$J_s = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{po}}}\right]$$

$$=\left(1.6\times10^{-19}\right)\left(1.8\times10^{6}\right)^2\times\left[\frac{1}{8\times10^{15}}\sqrt{\frac{210}{10^{-7}}}+\frac{1}{2\times10^{16}}\sqrt{\frac{8}{5\times10^{-8}}}\right]$$

$$J_s = 3.30\times10^{-18}\ \text{A/cm}^2$$

Example 3

Consider a GaAs pn junction with doping concentration $N_a = 5 \times 10^{16}$ cm^{-3}, $N_d = 10^{16}$ cm^{-3}. The junction cross-sectional area is $A = 10^{-3}$ and the applied forward bias voltage is $V_a = 1.10$V. Calculate the

a) Minority electron diffusion current at the edge of the space charge region

$$J_n(-x_p) = \frac{eD_n n_{po}}{L_n} \exp\left(\frac{V_a}{V_t}\right) = \frac{en_i^2}{N_a}\sqrt{\frac{D_n}{\tau_{no}}} \cdot \exp\left(\frac{V_a}{V_t}\right)$$

$$= \frac{(1.6\times10^{-19})(1.8\times10^{6})^2}{5\times10^{16}}\sqrt{\frac{205}{5\times10^{-8}}} \times \exp\left(\frac{1.10}{0.0259}\right)$$

$$J_n(-x_p) = 1.849 \text{ A/cm}^2$$

$$I_n = AJ_n(-x_p) = (10^{-3})(1.849)\,\text{A}$$

$$I_n = 1.85\,\text{mA}$$

Example 3

Consider a GaAs pn junction with doping concentration N_a=5 ×10^{16} cm^{-3}, N_d=10^{16} cm^{-3}. The junction cross-sectional area is A=10^{-3} and the applied forward bias voltage is V_a=1.10V. Calculate the

b) Minority hole diffusion current at the end of the space charge region

$$J_p\left(x_n\right)=\frac{eD_p p_{no}}{L_p}\exp\left(\frac{V_a}{V_t}\right)=\frac{en_i^2}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\cdot\exp\left(\frac{V_a}{V_t}\right)$$

$$=\frac{\left(1.6\times10^{-19}\right)\left(1.8\times10^{6}\right)^2}{10^{16}}\sqrt{\frac{9.80}{10^{-8}}}\times\exp\left(\frac{1.10}{0.0259}\right)$$

$$J_p\left(x_n\right)=4.521$$

$$I_p=AJ_p\left(x_n\right)=\left(10^{-3}\right)\left(4.521\right)\text{A}$$

$$I_p=4.52\text{ mA}$$

Example 3

Consider a GaAs pn junction with doping concentration N_a=5 $\times 10^{16}$ cm^{-3}, N_d=10^{16} cm^{-3}. The junction cross-sectional area is A=10^{-3} and the applied forward bias voltage is V_a=1.10V. Calculate the

c) Total current in the pn junction diode

$$I_n = 1.85 \text{ mA}$$

$$I_p = 4.52 \text{ mA}$$

$$I = I_n + I_p = 1.85 + 4.52 = 6.37 \text{ mA}$$

Example 4

Calculate the applied reverse-biased voltage at which the ideal reverse current in a pn junction diode at T=300K reaches 90% of its reverse-saturation current value

We have $I = I_S\left[\exp\left(\frac{V}{V_t}\right) - 1\right]$

Rewrite to $\frac{I}{I_S} + 1 = \exp\left(\frac{V}{V_t}\right)$ so that $V = V_t \ln\left(\frac{I}{I_S} + 1\right)$

In reverse bias, is negative, so at $\frac{I}{I_S} = -0.90$

We have $V = (0.0259)\ln(1 - 0.90) = -59.6\text{ mV}$

Example 6

A silicon pn junction with a cross-sectional area of 10^{-4} cm^2 has the following properties at T=300K

n region	p region
$N_d = 10^{17}$ cm^{-3}	$N_{da} = 5\times 10^{15}$ cm^{-3}
$\tau_{p0} = 10^{-7}$ s	$\tau_{n0} = 10^{-6}$ s
$\mu_n = 850$ $cm^2/V\text{-}s$	$\mu_n = 1250$ $cm^2/V\text{-}s$
$\mu_p = 320$ $cm^2/V\text{-}s$	$\mu_p = 420$ $cm^2/V\text{-}s$

a- Calculate the values of the Fermi level with respect to the intrinsic level on each side of the junction.

b- Calculate the reverse-saturation current I_s and determine the forward-bias current I at a forward-bias voltage of 0.5V.

n side

$$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right) = (0.0259)\ln\left(\frac{10^{17}}{1.5\times10^{10}}\right)$$

$$E_F - E_{Fi} = 0.407 \text{ eV}$$

p side

$$E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{5\times10^{15}}{1.5\times10^{10}}\right)$$

$$E_{Fi} - E_F = 0.329 \text{ eV}$$

Example 6

Calculate the reverse-saturation current I_s and determine the forward-bias current I at a forward-bias voltage of 0.5V.

We have

$$D_n = (1250)(0.0259) = 32.4\ \text{cm}^2/\text{s}$$

$$D_p = (320)(0.0259) = 8.29\ \text{cm}^2/\text{s}$$

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

$$= (1.6\times10^{-19})(1.5\times10^{10})^2 \times \left[\frac{1}{5\times10^{15}}\sqrt{\frac{32.4}{10^{-6}}} + \frac{1}{10^{17}}\sqrt{\frac{8.29}{10^{-7}}}\right]$$

$$J_S = 4.426\times10^{-11}$$

$$I_S = AJ_S = \left(10^{-4}\right)\left(4.426\times10^{-11}\right) = 4.426\times10^{-15}\ \text{A}$$

$$I = I_S \exp\left(\frac{V_D}{V_t}\right) = \left(4.426\times10^{-15}\right)\exp\left(\frac{0.5}{0.0259}\right)$$

$$I = 1.07\times10^{-6}\ \text{A} = 1.07\,\mu\text{A}$$