Miscellaneous worked examples

Ex. 1 A flat plate solar collector has the specifications listed below. Find the maximum plate temperature and the working fluid temperature at 0.8 of the collector length, the useful heat gain and the collector efficiency.

Collector length = 1 m, Collector width = 1 m, Number of risers = 8, Plate thickness = 2 mm, Riser outside diameter = 1.2 cm, Riser inside diameter = 1 cm, Bond thickness = 2 mm, Bond width = 4 mm, Metal thermal conductivity = 380 W/(m °C), Overall heat loss coefficient = 5 W/(m² °C), Convection heat transfer coefficient inside risers = 200 W/(m² °C), Irradiance = 1000 W/m², Ambient temperature = 10 °C, Transmittance−absorptance product = 0.85 , Collector inlet temperature = 10 °C, Total mass flow rate = 0.02 kg/s, working fluid specific heat = 4180 J/(kg °C).

Sol.

$$R_b = \frac{t_b}{w_b k_b} = \frac{0.002}{0.004 * 380} = 0.0013\ (m\ °C)/W$$

$$R_f = \frac{1}{\pi D_i h_f} = \frac{1}{\pi * 0.01 * 200} = 0.1592\ (m\ °C)/W$$

$$m = \sqrt{\frac{U_L}{k\delta_p}} = \sqrt{\frac{5}{380 * 0.002}} = 2.5649\ 1/m$$

$$W=1/8=0.125\ m$$

$$F_f = \frac{\tanh\left[\frac{m(W-D)}{2}\right]}{\frac{m(W-D)}{2}} = \frac{\tanh\left[\frac{2.5649 * (0.125-0.012)}{2}\right]}{\frac{2.5649 * (0.125-0.012)}{2}} = 0.9931$$

$$F = \frac{\frac{1}{U_L}}{W\left[\frac{1}{U_L[D+F_f(W-D)]} + R_b + R_f\right]}$$

$$= \frac{\frac{1}{5}}{0.125 * \left(\frac{1}{5 * [0.012 + 0.9931 * (0.125-0.012)]} + 0.0013 + 0.1592\right)}$$

$$= \frac{\frac{1}{5}}{0.125 * (1.61 + 0.0013 + 0.1592)} = 0.9037$$

$$\frac{I_T\tau\alpha - U_L(T_f - T_a)}{I_T\tau\alpha - U_L(T_{fi} - T_a)} = e^{\frac{-nW\,U_L F}{\dot{m}\,c_{pf}}y}$$

$$\frac{1000 * 0.85 - 5 * (T_f - 10)}{1000 * 0.85 - 5 * (10 - 10)} = e^{\frac{-8*0.125*5*0.9037}{0.02*4180}*0.8} \qquad T_f = 17.194\ ^\circ C = T_b$$

$$\frac{T - T_a - \frac{I_T\tau\alpha}{U_L}}{T_b - T_a - \frac{I_T\tau\alpha}{U_L}} = \frac{\cosh(mx)}{\cosh\left(\frac{m(W-D)}{2}\right)}$$

Maximum plate temperature at length y=0.8 m occurs at midpoint between two risers where x=0

$$\frac{T_{p\_max} - 10 - \frac{1000 * 0.85}{5}}{17.194 - 10 - \frac{1000 * 0.85}{5}} = \frac{\cosh(2.5649 * 0)}{\cosh\left(\frac{2.5649 * (0.125 - 0.012)}{2}\right)}$$

$T_{p\_max} = 18.8872\ ^oC$

$$F_R = \frac{\dot{m}c_{pf}}{A_r U_L}\left(1 - e^{\frac{-A_r U_L F}{\dot{m}\,c_{pf}}}\right) = \frac{0.02 * 4180}{1 * 5} * \left(1 - e^{\frac{-1*5*0.9037}{0.02*4180}}\right) = 0.8797$$

$$Q_u = A_c F_R[\tau\alpha I_T - U_L(T_{fi} - T_a)] = 1 * 0.8797 * \left(850 - 5 * (10 - 10)\right)$$

$$Q_u = 747.7450\ W$$

$$\eta_c = \frac{Q_u}{A_c I_T} = \frac{747.745}{1000} = 0.747 = 74.7\%$$

Ex. 2 A concentrating solar collector operates at irradiance of 750 W/m^2 and ambient temperature of 15 °C. Water enters the collector at 25 °C with a mass flow rate of 0.005 kg/s. The collector length and width are 2 m and 1 m respectively and the internal and external absorber diameters are 0.02 m and 0.022 m respectively. The overall heat loss coefficient and the convection heat transfer coefficient inside the absorber are 8 $W/(m^2\ °C)$ and 500 $W/(m^2\ °C)$ respectively. The reflectivity of the reflector is 0.9 and the transmittance–absorptance product of the absorber is 0.8. Determine the exit temperature of the water and the collector efficiency. Take water specific heat as 4180 J/(kg °C). The thermal conductivity of the absorber is 380 W/(m °C).

Sol.

$$A_a = L_c * W_c = 2 * 1 = 2\ m^2$$

$$A_r = \pi\ D_o\ L_c = 3.14159*0.022*2 = 0.1382\ m^2$$

$$C = A_a / A_r = 2/0.1382 = 14.4718$$

$$R_f = \frac{1}{h_{fi}\pi D_i L} + \frac{ln\left(\frac{D_o}{D_i}\right)}{2\pi k L} = R_f = \frac{1}{500 * \pi * 0.02 * 2} + \frac{ln\left(\frac{0.022}{0.02}\right)}{2 * \pi * 380 * 2} = 0.0159\ °C/W$$

$$F = \frac{\frac{1}{A_r U_L}}{\frac{1}{A_r U_L} + R_f} = \frac{\frac{1}{0.1382 * 8}}{\frac{1}{0.1382 * 8} + 0.0159} = 0.9827$$

$$F_R = \frac{\dot{m}c_{pf}}{A_r U_L}\left(1 - e^{\frac{-A_r U_L F}{\dot{m}c_{pf}}}\right) = \frac{0.005 * 4180}{0.1382 * 8} * \left(1 - e^{\frac{-0.1382*8*0.9827}{0.005*4180}}\right) = 0.9576$$

$$Q_u = A_a F_R\left[\rho\tau\alpha I_T - \frac{U_L}{C} * (T_{fi} - T_a)\right]$$

$$= 2 * 0.9576 * \left[0.9 * 0.8 * 750 - \frac{8}{14.4718} * (25 - 15)\right] = 1023.6\ W$$

$$Q_u = \dot{m}c_{pf}(T_{fo} - T_{fi})$$

$$1023.6 = 0.005 * 4180 * (T_{fo} - 25)$$

$$T_{fo} = 74\ °C$$

$$\eta_c = \frac{Q_u}{A_a I_T} = \frac{1023.6}{2 * 750} = 0.6824 = 68.24\%$$