محاضرة / 1

التاريخ / 2019/3/17

الكورس ~~الاول~~ الثاني

السعر/ ٥,٠٠

Engineering Mechanics

الميكانيك الهندسي

لطلبة الدراسات الاولية

المرحلة الاولى

قسم الهندسة الموارد المائية

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Chapter - 6 -

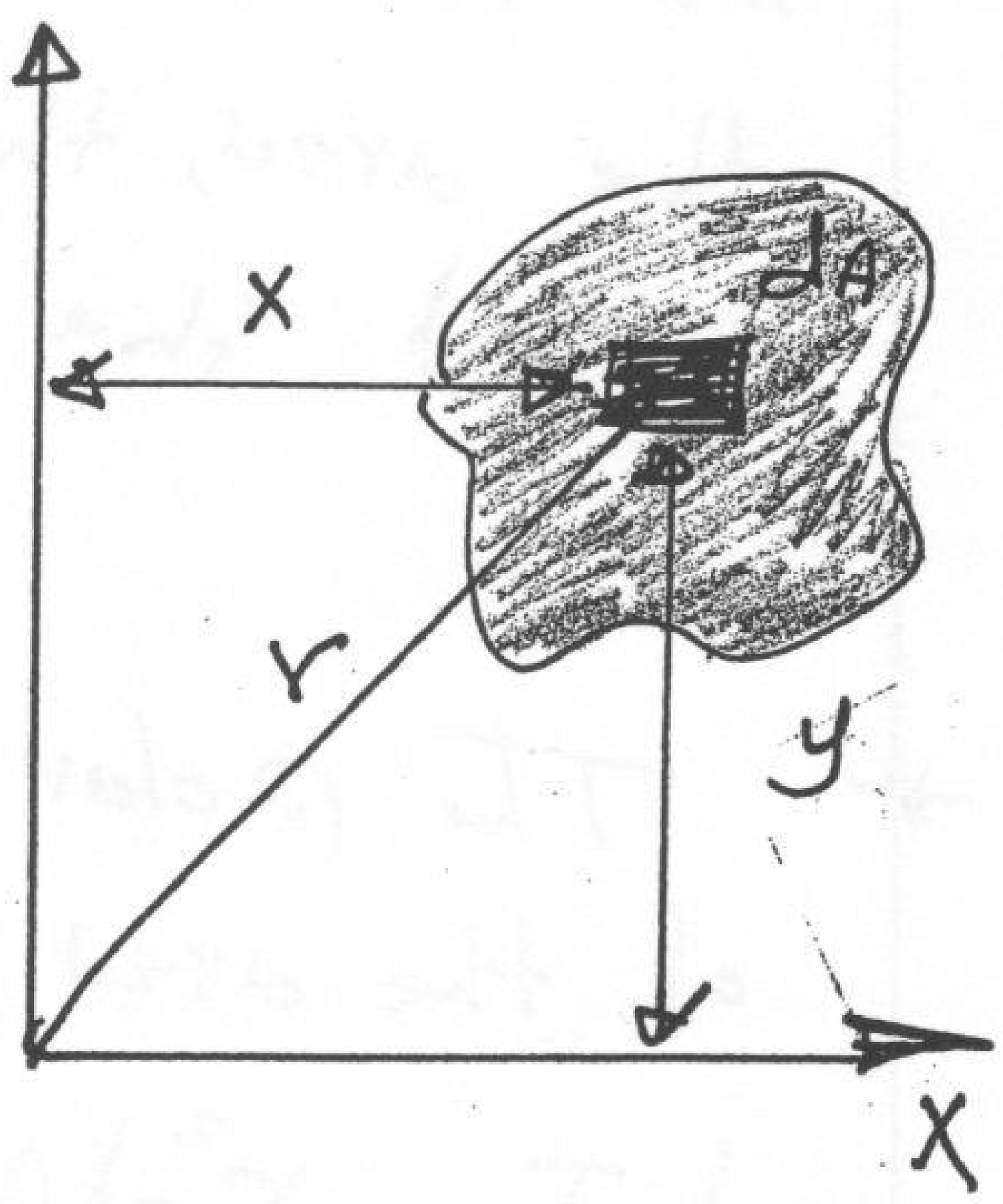
Seconed moment or moment of Inertia.

- The mom. of inertia is a mathematical expression, and it is difficult to obtain aphysical concept. of the quantity.

- In determining stresses and deflections in beams an expression similar to the moment of inertia is used and it is called the second moment of an area.

The second moment of an area.

- The second mom. or mon. of inertia of an element of area, such as dA, with respect to any axis is defined as the product of the area of the element and the square of the distance from the axis to the element.



- The second mom. of the element dA with respect to y- axis is $dI_y = x^2\,dA$

م.د.هشام عبد الحسين

∴ The second mom. of the total area is

$$I_y = \int x^2 \cdot dA$$

- Similarly, the second mom. of the element dA

$$dI_x = y^2\, dA$$

and $I_x = \int y^2 dA$

** If the moment axis is in the plane of the area, the second mom. of the area is called the rectangular moment of inertia.

** If the moment axis is ⊥ to the plane of the area, the second mom. of the area is called the polar moment of Inertia

* The polar mom. of inertia of the element of the area dA is

$$dJ_o = r^2 dA = (x^2+y^2)\, dA$$

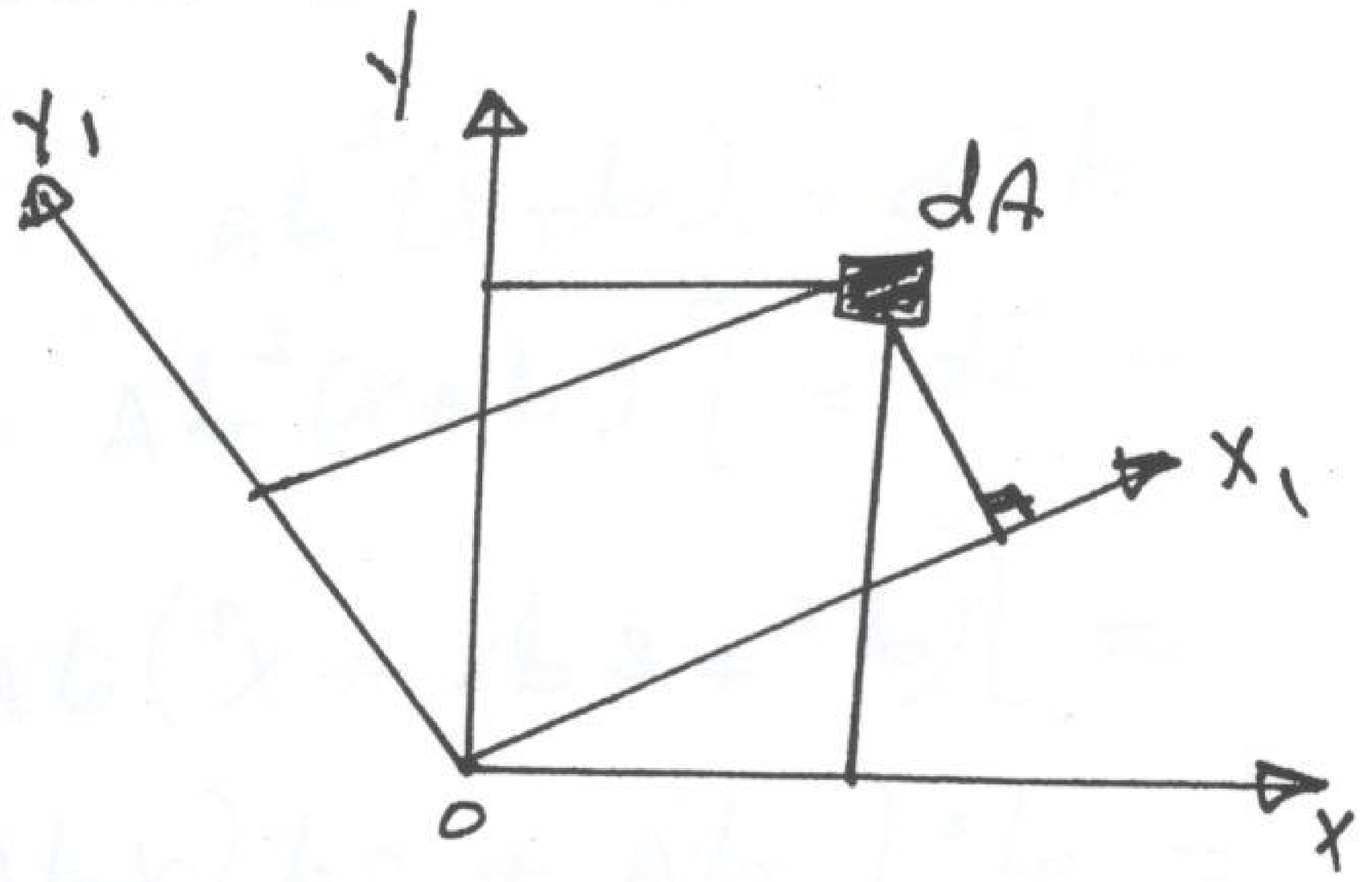
$$\therefore J_o = \int (x^2+y^2)\, dA = \int x^2 dA + \int y^2 dA$$

$$\boxed{\therefore \quad J_o = I_y + I_x}$$

Note:- The polar mom. of inertia of an area is equal to the sum of rectangular moment of inertia with respect to any two ⊥ axes. intersecting the polar axis

$$J_o = I_y + I_x$$

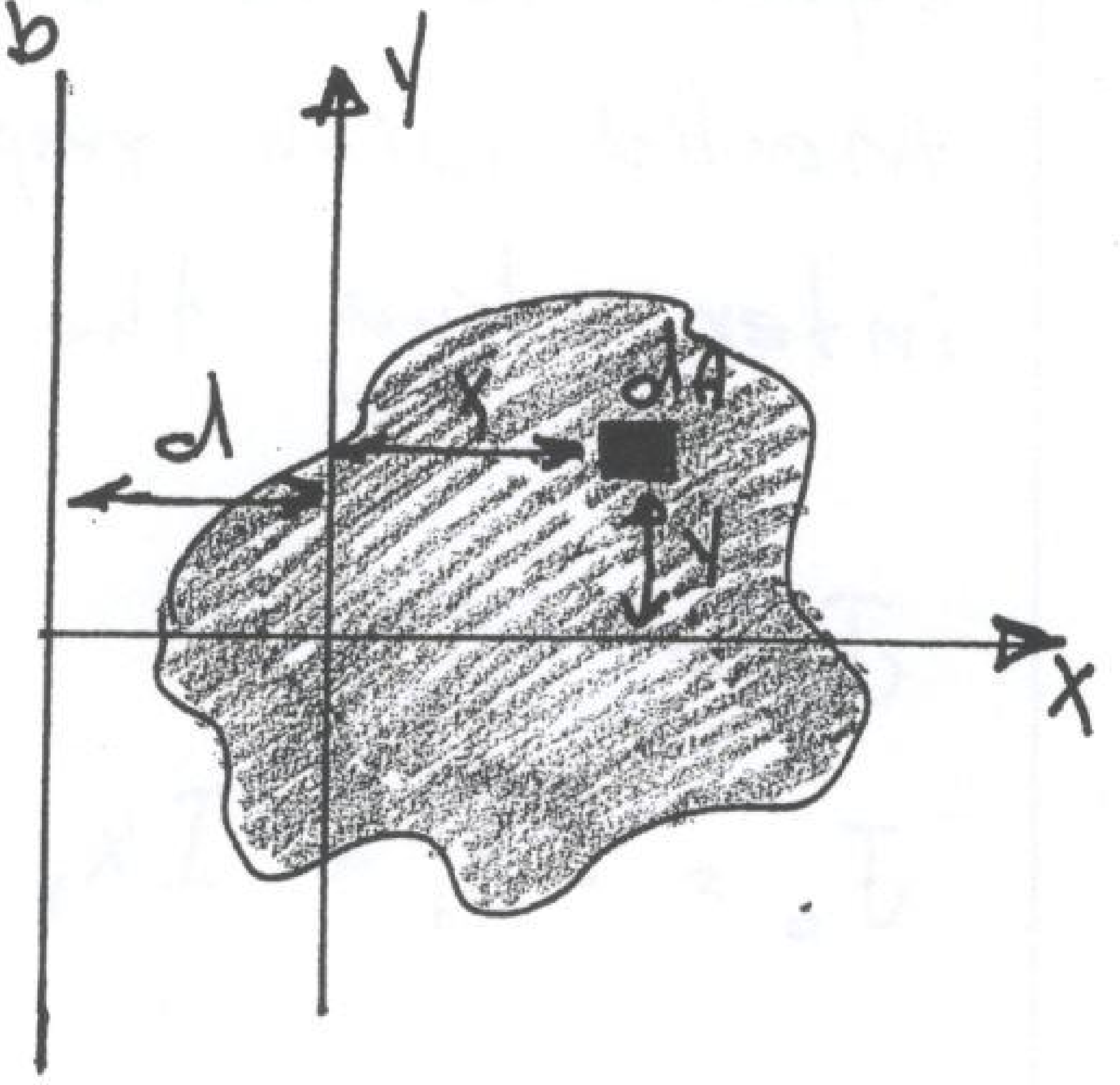
$$J_o = I_{y_1} + I_{x_1}$$



The moment of inertia of an area is always positive and has dimensions of (length) (m^4, mm^4, cm^4)

The parallel_ Axis Theorem for Area

The mom. of inertia of the element of area (dA), with respect to the b-axis is.



$$dI_b = (d+x)^2 dA$$

$$\therefore I_b = \int (d+x)^2 dA$$

$$= \int (d^2 + 2dx + x^2) dA$$

$$= d^2 \int dA + 2d \int x dA + \int x^2 dA$$

$$I_b = Ad^2 + 2d \int x dA + I_y$$

If the y-axis passes through the centroid of the area, then the first mom. of the area ($\int x dA$) is zero and.

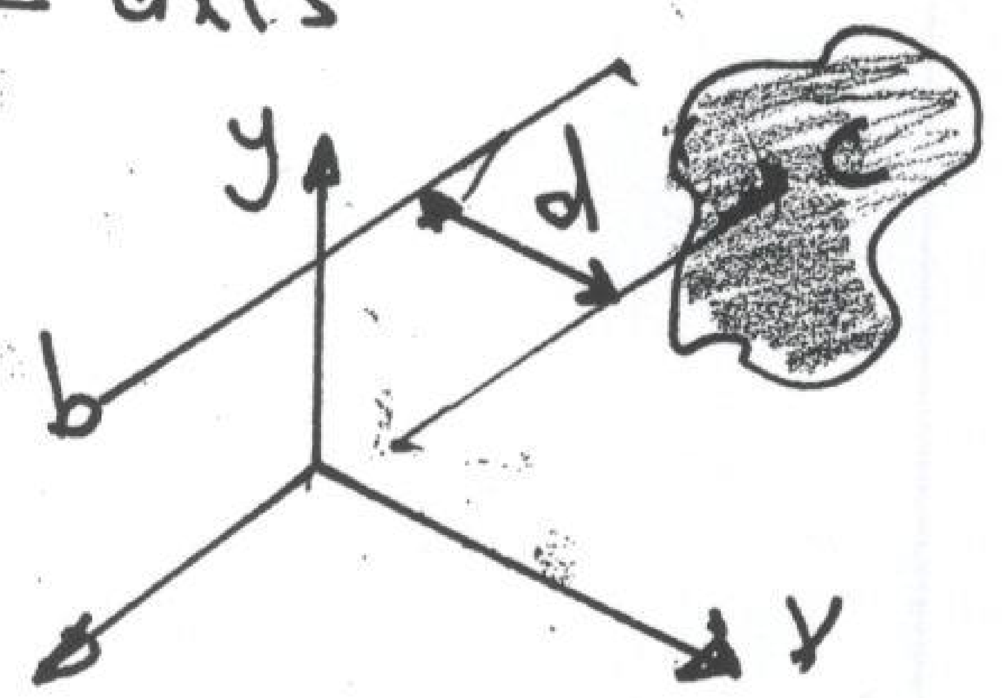
$$\boxed{I_b = Ad^2 + I_c}$$

I_c: The second mom. of the area with respect to an-axis through the centroid and // to the b-axis

Similarly

$$J_b = Ad^2 + J_c$$

$\therefore$ the parallel-axis theorm can be stated as follows.



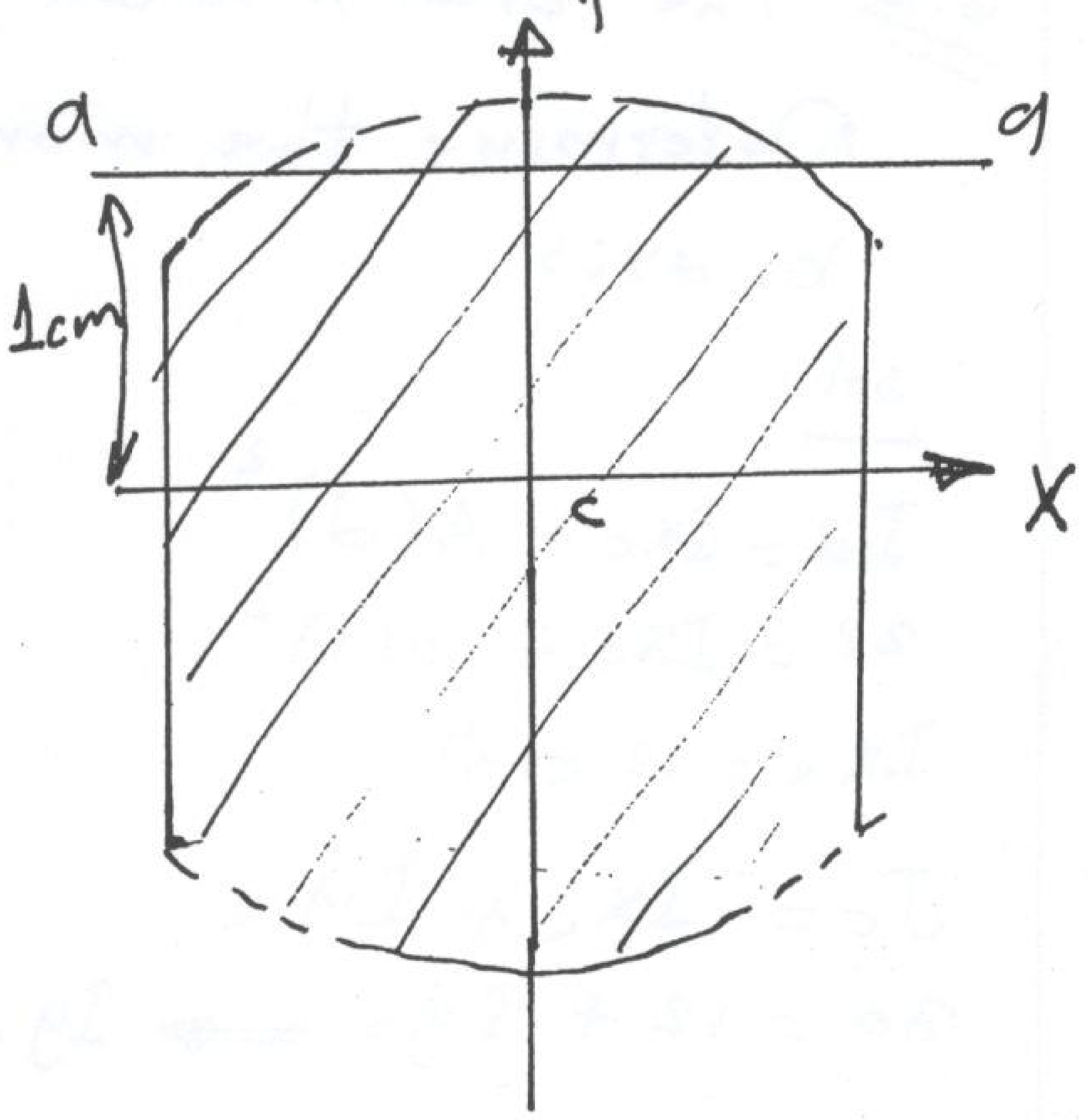
The moment of Inertia

The mom. of inertia of an area with respect to any axis is equal to the mom. of inertia with respect to a // axis through the controid of the area plus the product of the area and the square of the distance between the two axes.

Ex:- the 6 cm^2 shaded area (shown in fig) has a polar mom. of inertia with respect to an axis through the controid c of 6.5 cm^4

The mom. of inertia of the area with respect to y-axis is 4.5 cm^4. Determine the mom. of inertia with respect to the a-axis

solution



$$J_c = I_{y_c} + I_{x_c}$$

$$6.5 = 4.5 + I_{x_c}$$

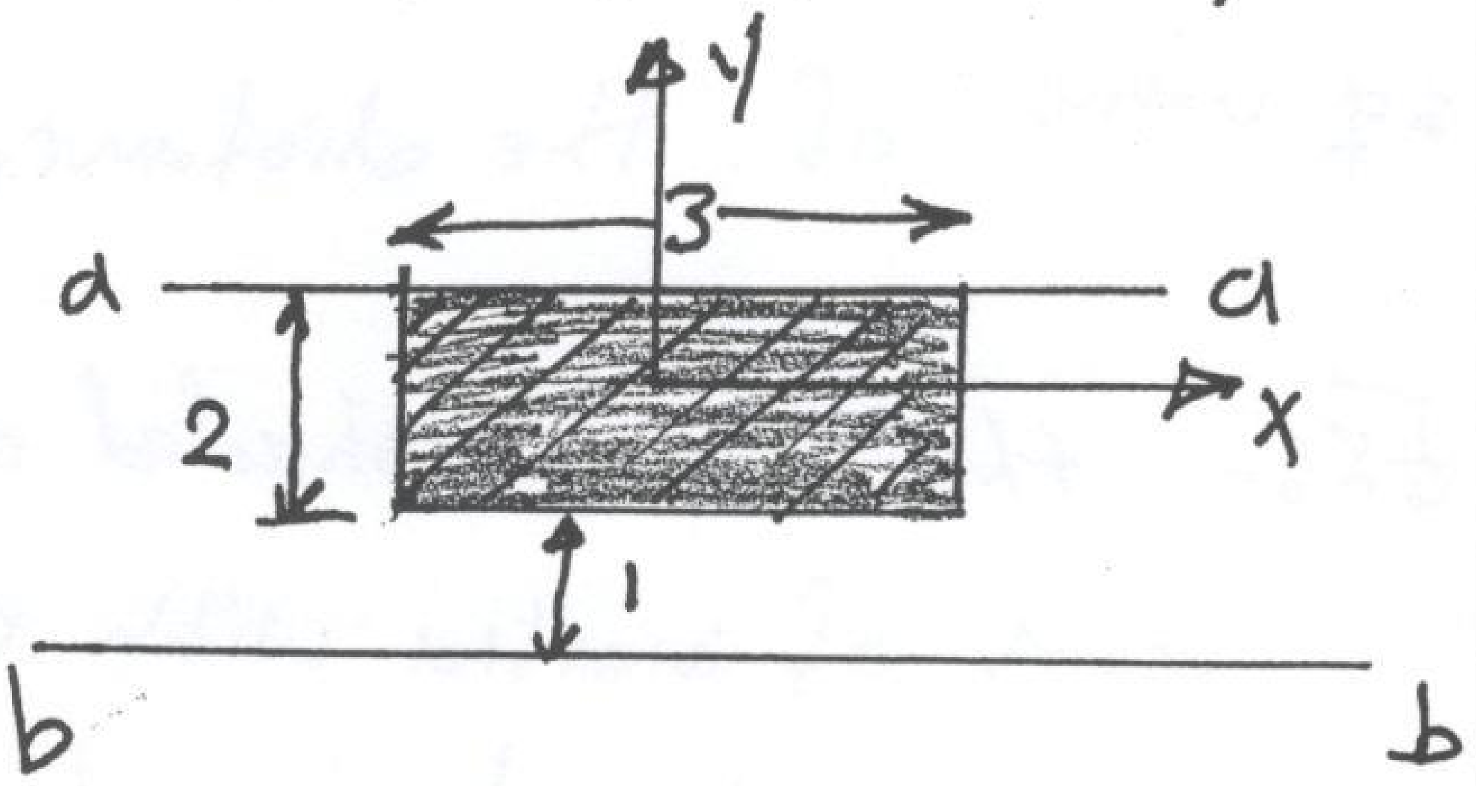
$$\therefore \; I_{x_c} = 2 \, cm^4$$

$$I_a = I_{x_c} + A \cdot d^2$$

$$= 2 + 6(1)^2$$

$$I_a = 8 \, cm^4$$

6.1 $I_a = 8\,cm^4$ for the shaded area. Determine moment of inertia of the area with respect to b-axis.



solution

$I_a = I_{xc} + A(d)^2$

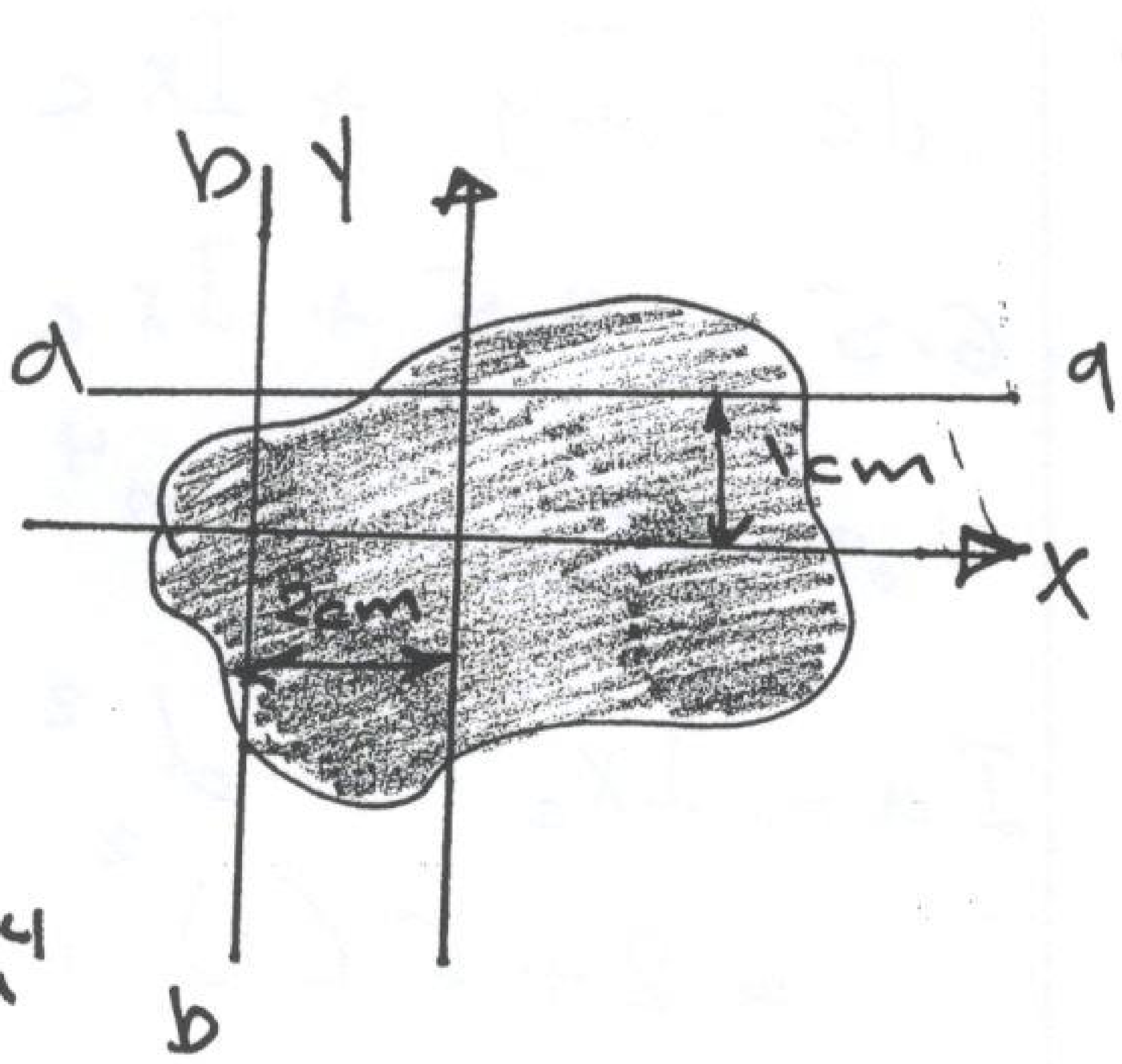
$8 = I_{xc} + 6$

$\therefore I_{xc} = 2\,cm^4$

$I_b = I_{xc} + A(d)^2$

$I_b = 2 + 6(2)^2 = 26$

6.2 The area is $10\,cm^2$, $J_c = 30\,cm^4$, $I_a = 22\,cm^4$. Determine the mom. of inertia with respect to b-axis



sol.

$I_a = I_{xc} + A(d)^2$

$22 = I_{xc} + 10(1)^2$

$I_{xc} = 12\,cm^4$

$J_c = I_{xc} + I_{yc}$

$30 = 12 + I_{yc} \Longrightarrow I_{yc} = 18\,cm^4$

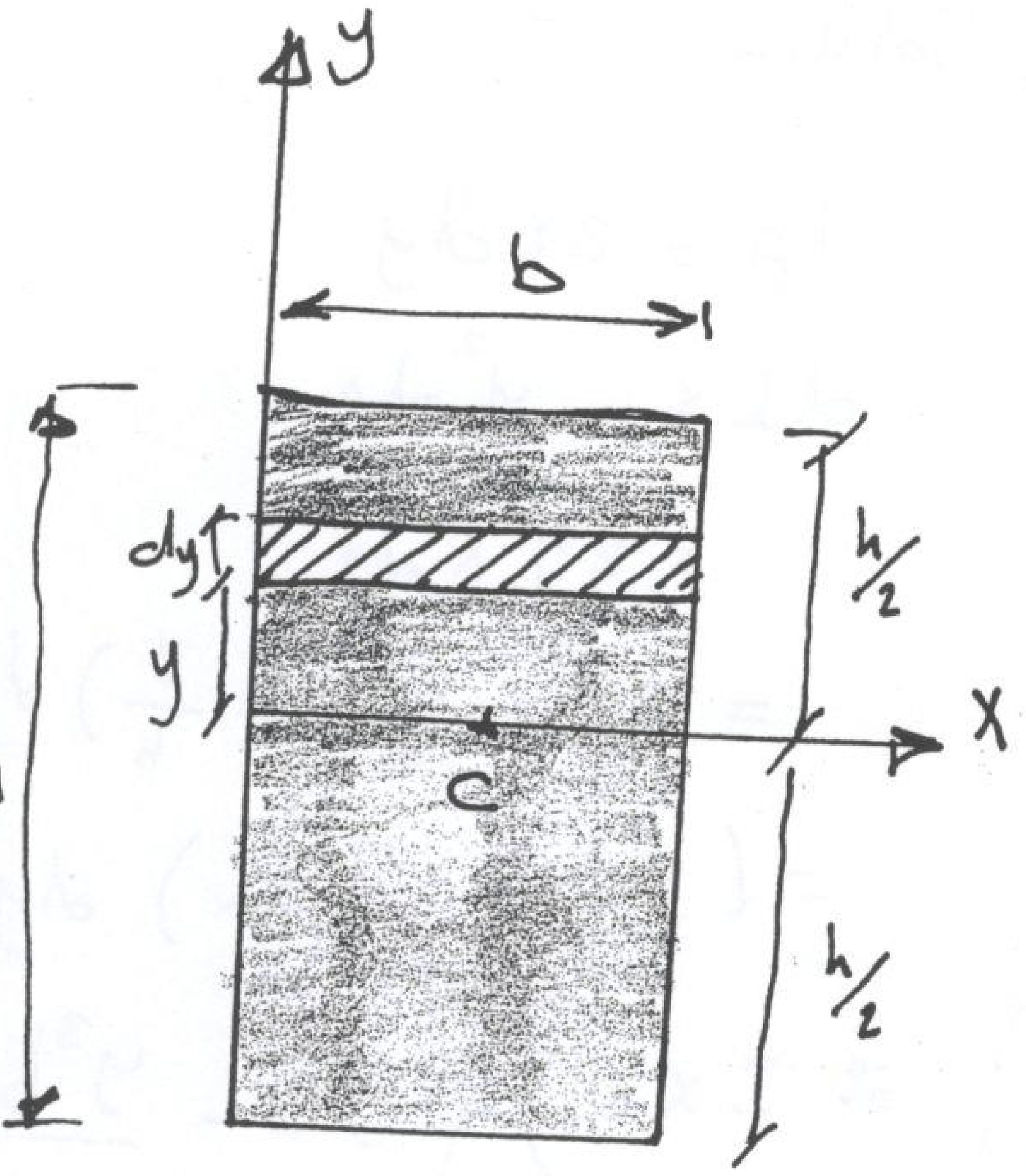
$\therefore I_b = I_{yc} + A(d)^2$

$= 18 + 10(2)^2$

$= 58\,cm^4$

Second Moment of Area
By Integration

Ex: Determine the second moment of a rectangular area with respect to an axis through the centroid parallel to the base of the rectangle.



$dA = b\,dy$

$dIx_c = y^2\,dA$

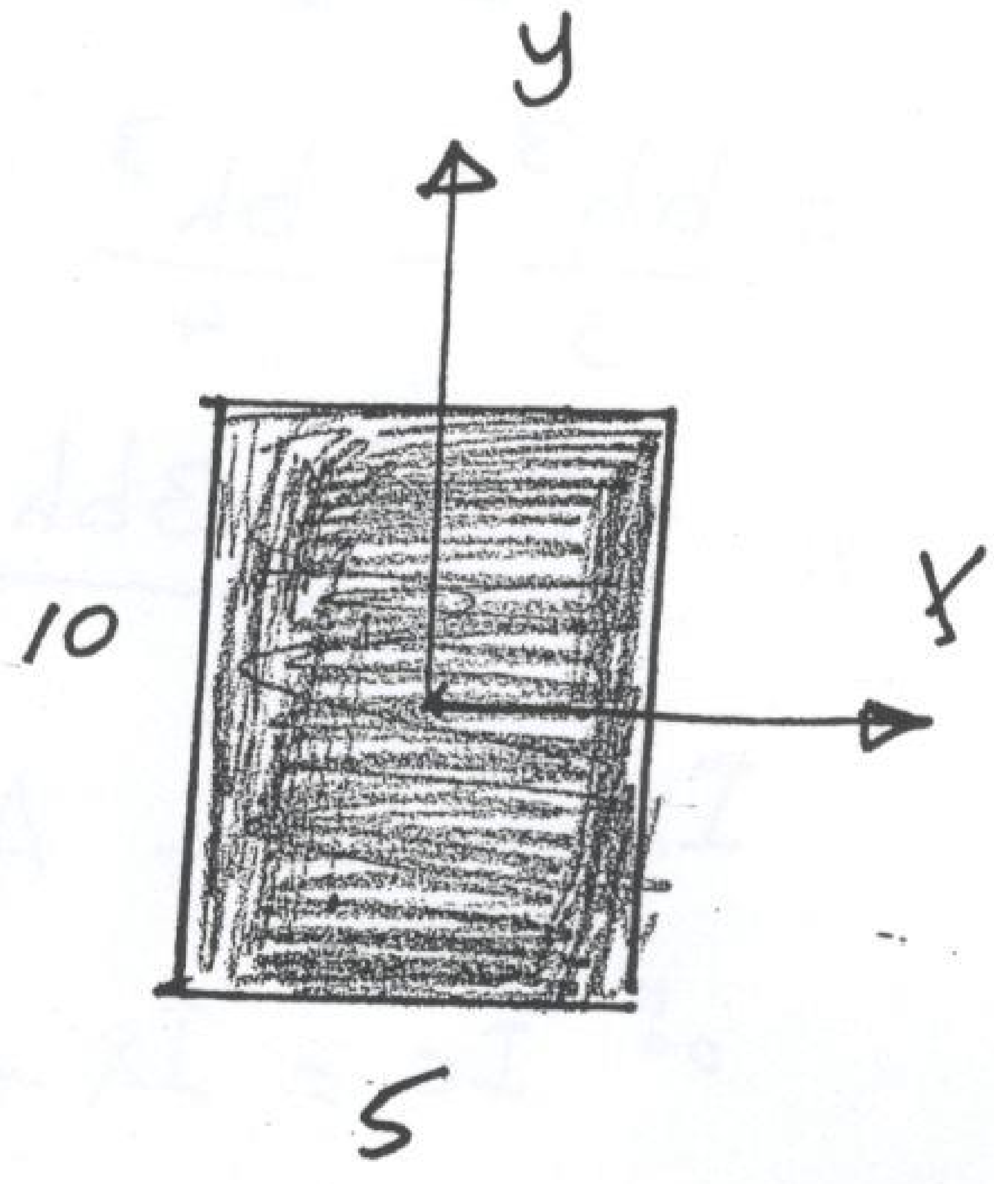
$= y^2 \cdot b\,dy$

and

$$Ix_c = b\int_{-h/2}^{h/2} y^2\,dy = \left.\frac{by^3}{3}\right]_{-h/2}^{h/2}$$

$$Ix_c = \frac{b}{3}\left[\frac{h^3}{8} - \frac{(-h^3)}{8}\right] = \frac{bh^3}{12}$$

$$\boxed{I_c = \frac{bh^3}{12}}$$

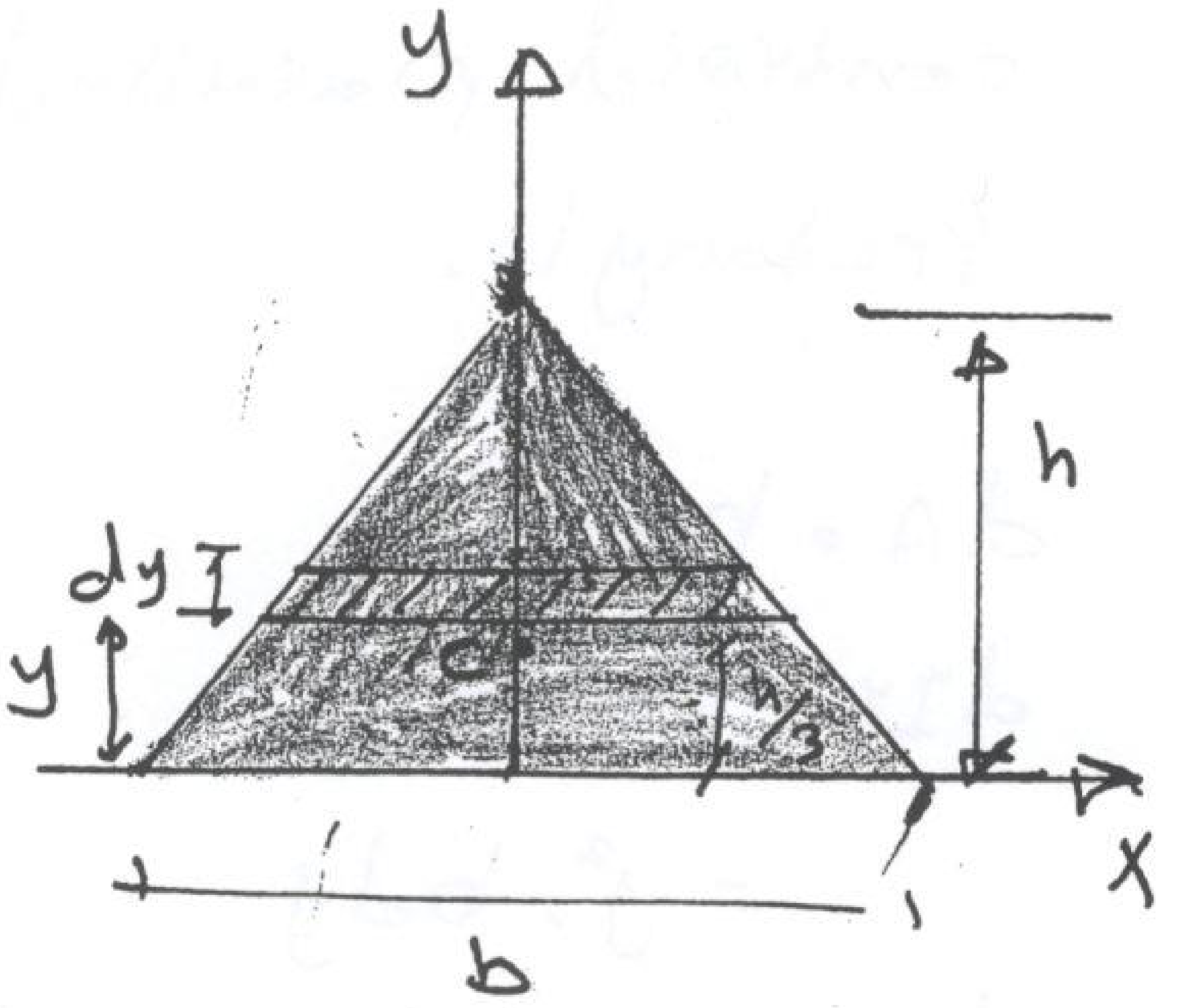


$$Ix_c = \frac{5(10)^3}{12}$$

$$Iy_c = \frac{10(5)^3}{12}$$

Moment of Inertia

6.4// Determine the moment of inertia of any triangular area of altitude h and base b with respect to an axis through the centroid parallel to the base.



Solu:-

$$dA = 2x\,dy$$

$$dI_x = y^2\,dA$$

$$= 2y^2 x\,dy$$

$$= 2y^2 \cdot \frac{b}{2}\left(1 - \frac{y}{h}\right) dy$$

$$= \left(y^2 b - \frac{y^3 b}{h}\right) dy$$

$$\therefore I_x = \int_0^h \left(y^2 b - \frac{y^3 b}{h}\right) dy$$

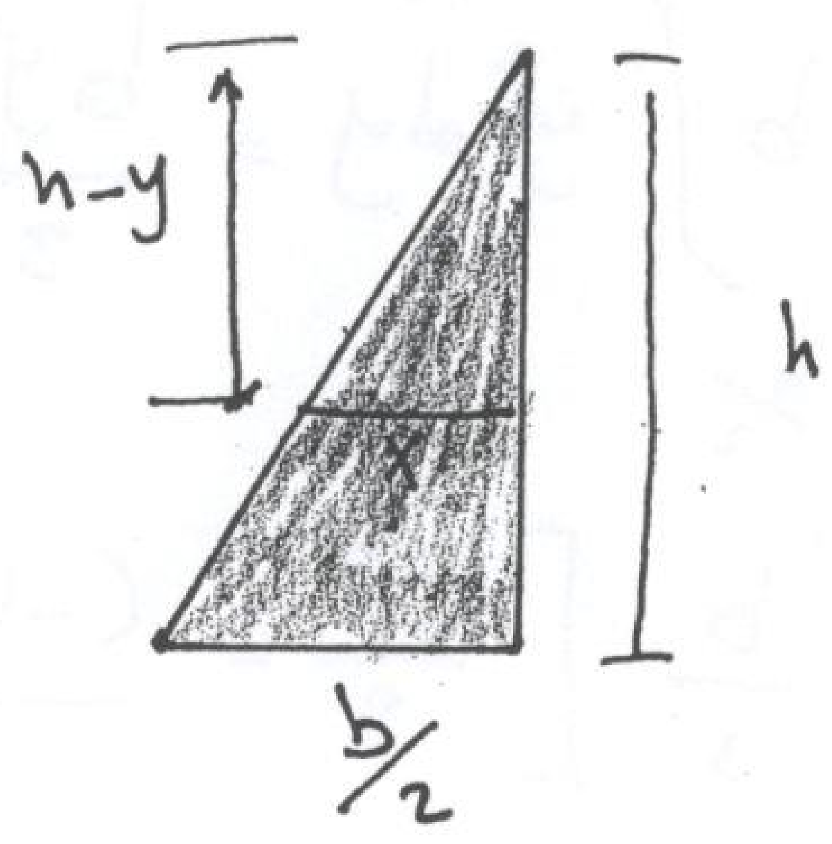
$$I_x = \left[\frac{y^3}{3} b - \frac{y^4 b}{4h}\right]_0^h$$

$$= \frac{bh^3}{3} - \frac{bh^3}{4}$$

$$= \frac{4bh^3 - 3bh^3}{12} = \frac{bh^3}{12}$$

$$I_x = I_c + A\left(\frac{h}{3}\right)^2$$

$$\therefore I_c = I_x - A\left(\frac{h}{3}\right)^2$$



$$\frac{h}{b/2} = \frac{h-y}{x}$$

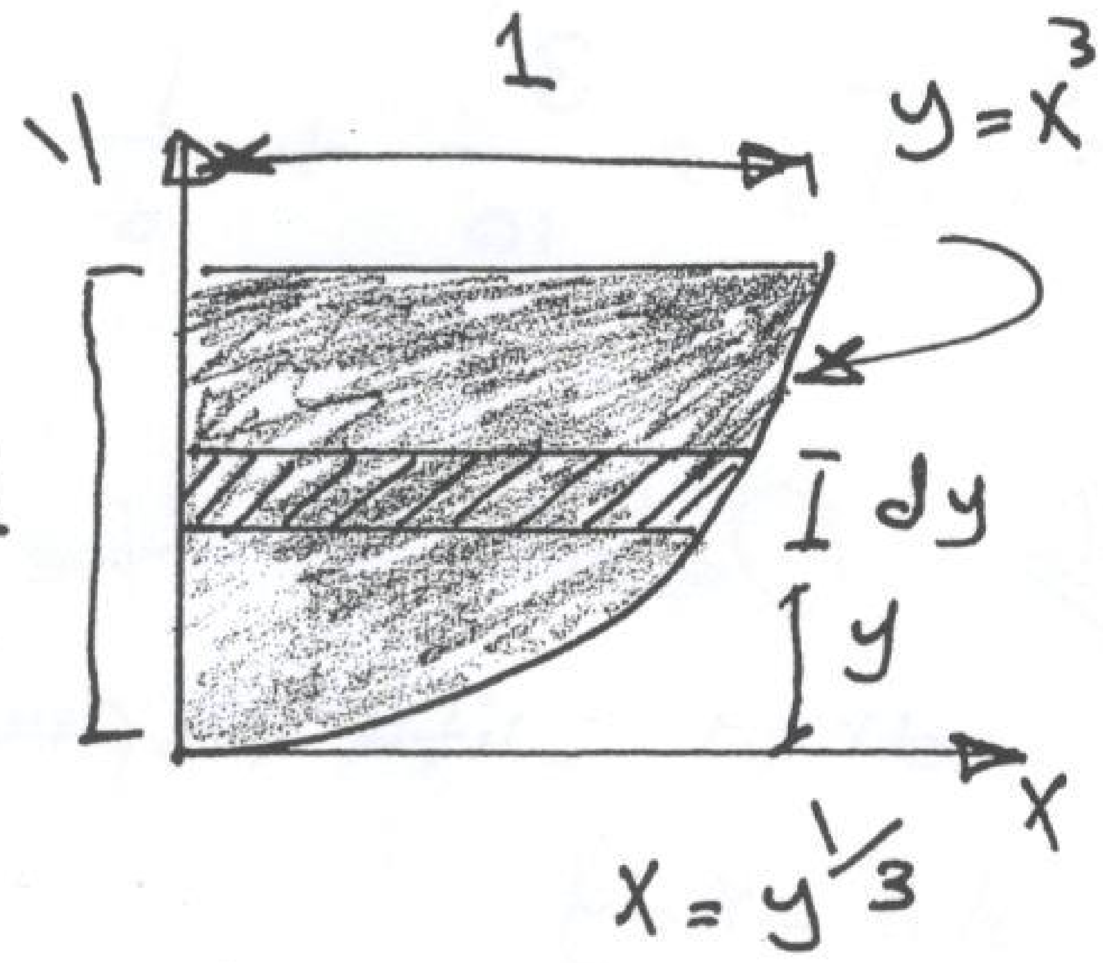
$$\therefore x = \frac{b/2\,(h-y)}{h}$$

$$x = \frac{b}{2}\left(1 - \frac{y}{h}\right)$$

$$= \frac{bh^3}{12} - \frac{bh}{2} \cdot \frac{h^2}{9}$$

$$I_c = \frac{3bh^3 - 2bh^3}{36} = \frac{bh^3}{36}$$

6.6 Determine the polar moment of inertia of the shaded area with respect to the axis through the origin.



Solution:

$J_o = I_x + I_y$

$dA = x\,dy$

$dI_x = dA\,d^2 + I_c$

$\therefore dI_x = dA\,y^2 + \cancel{\frac{x\,dy^3}{12}} \to 0$

$x y^2\,dy = y^{1/3} y^2\,dy$

$= y^{7/3}\,dy$

$$\therefore I_x = \int_0^1 y^{7/3}\,dy = \frac{y^{10/3}}{10/3}\Bigg]_0^1 = \frac{3}{10}$$

$dI_y = dA\,d^2 + I_c$

$= dA\left(\frac{x}{2}\right)^2 + \frac{x^3}{12}\,dy$

$= x\left(\frac{x^2}{4}\right)dy + \frac{x^3}{12}\,dy$

$$\therefore\ dI_y = \left(\frac{x^3}{4} + \frac{x^3}{12}\right) dy$$

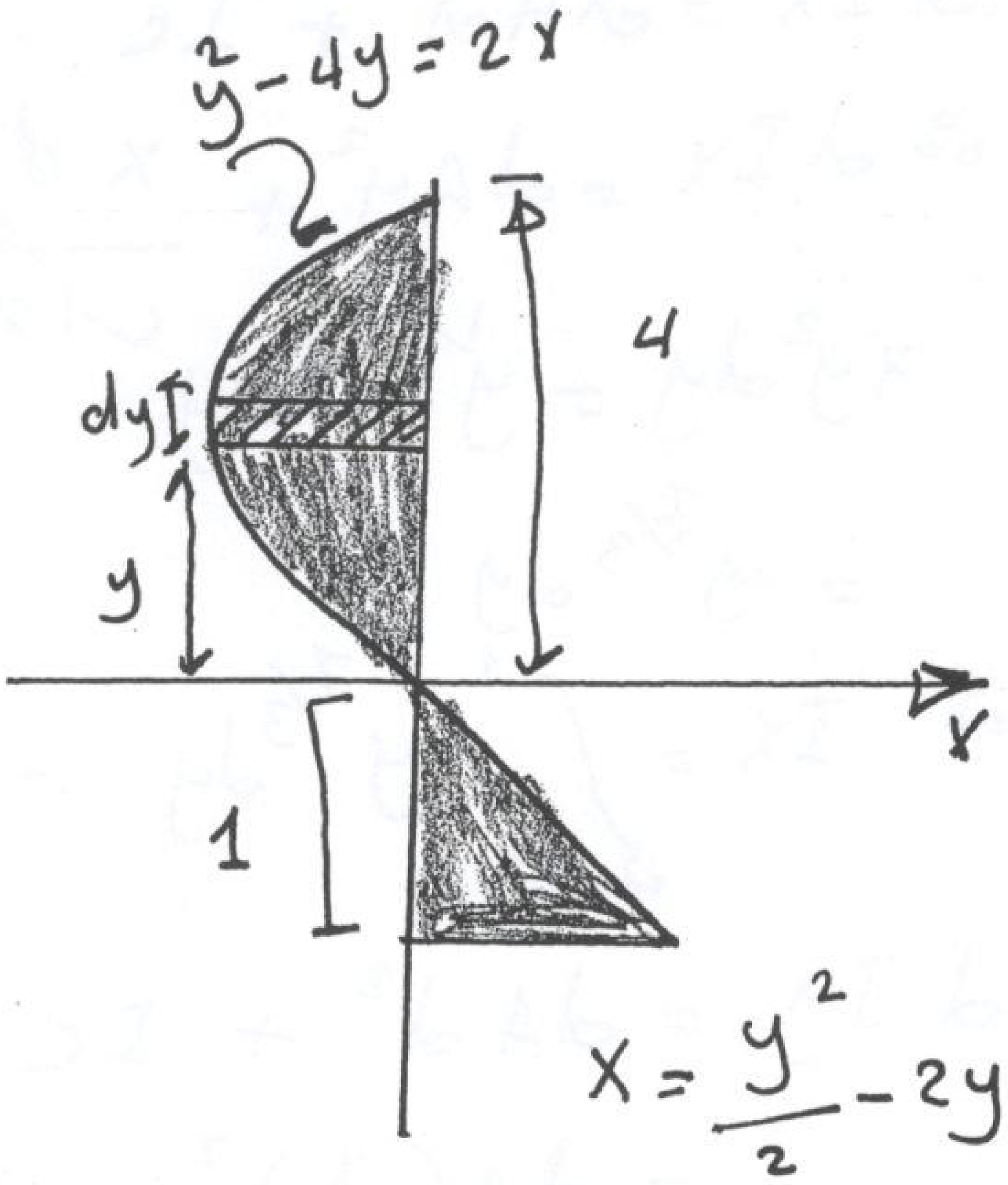
$$= \left(\frac{y}{4} + \frac{y}{12}\right) dy$$

$$I_y = \int_0^1 \left(\frac{y}{4} + \frac{y}{12}\right) dy$$

$$= \frac{y^2}{8} + \frac{y^2}{24}\Big]_0^1 = \frac{1}{8} + \frac{1}{24} = \frac{1}{6}$$

$$\therefore\ J_o = \frac{3}{10} + \frac{1}{6} = 0.4666 \text{ cm}^4$$

6.11 // Determine the second mom. of the shaded area with respect to the x-axis



$$dA = x\,dy$$

$$dI_x = y^2\,dA$$

$$= y^2 x\,dy$$

$$= y^2\left(\frac{y^2}{2} - 2y\right) dy$$

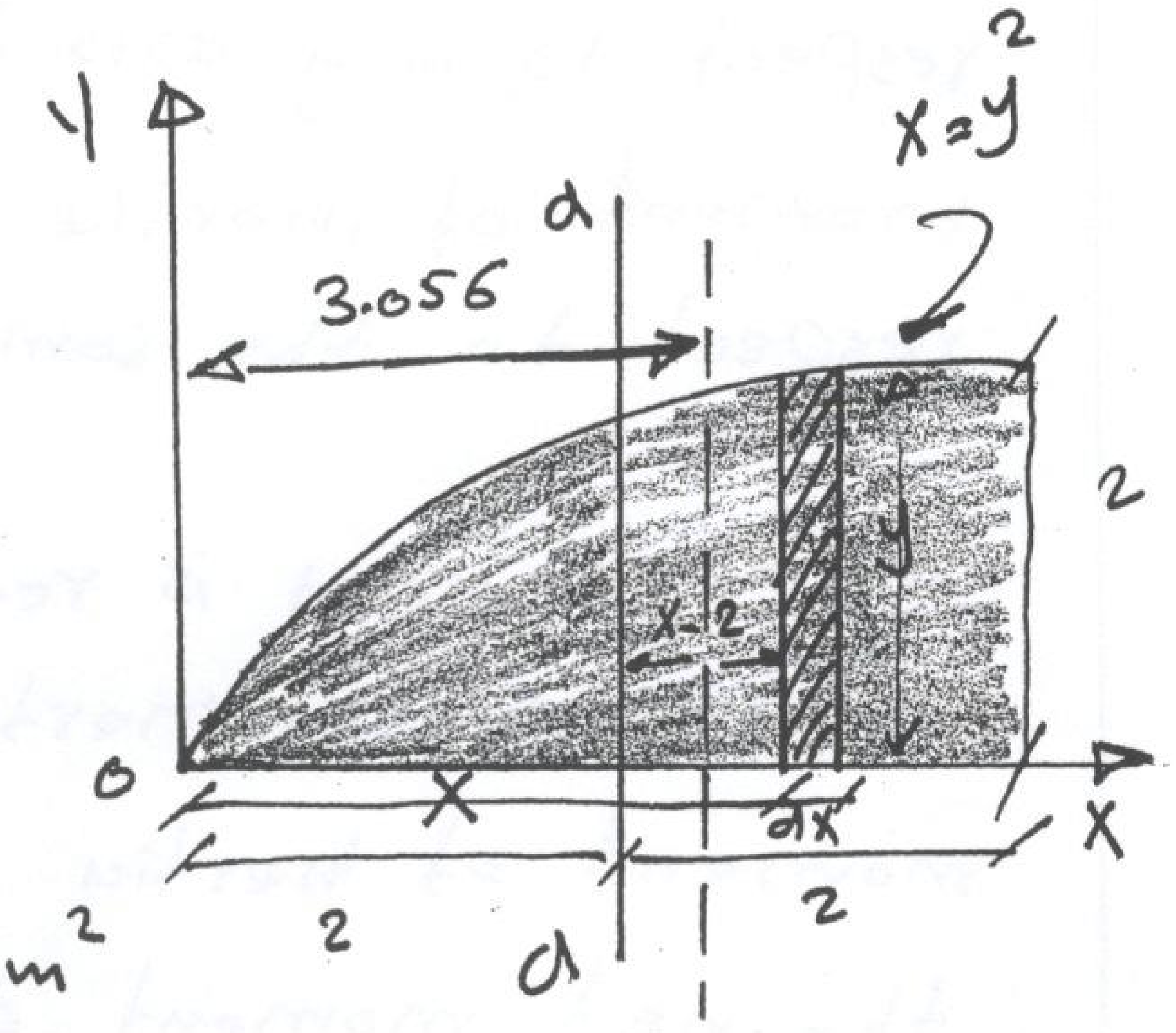
$$= \left(\frac{y^4}{2} - 2y^3\right) dy$$

$$\therefore\ I_x = \int_{-1}^{4} \left(\frac{y^4}{2} - 2y^3\right) dy$$

$$= \left[\frac{y^5}{10} - \frac{2y^4}{4}\right]_{-1}^{4}$$

$$I_x = \left|-25.6\right| + 0.6 \Longrightarrow I_x = 26.2 \text{ cm}^4$$

6.14 Determine the mom. of inertia of the shaded area with respect to the a-axis.



Solu:.

$dA = y\,dx$

$= x^{1/2}\,dx$

$\therefore A = \int_2^4 x^{1/2}\,dx$

$= \left[\dfrac{x^{3/2}}{3/2}\right]_2^4 = 3.4477\ cm^2$

$\bar{x} = \dfrac{\int x\,dA}{\int dA}$

$\int x\,dA = \int_2^4 x \cdot x^{1/2}\,dx = \left[\dfrac{x^{5/2}}{5/2}\right]_2^4 = 10.537\ cm^3$

$\therefore \bar{x} = \dfrac{10.537}{3.4477} = 3.056\ cm$

$I_y = I_c + Ad^2$

$dI_y = x^2\,dA \Longrightarrow x^2 x^{1/2}\,dx = x^{5/2}\,dx$

$\therefore I_y = \int_2^4 x^{5/2}\,dx = \left[\dfrac{x^{7/2}}{7/2}\right]_2^4 = 33.338$

$I_c = I_y - Ad^2$

$= 33.338 - 3.4477\,(3.056)^2$

$= 1.139\ cm^4$

$I_a = I_c + A(d)^2 \Longrightarrow 1.139 + 3.4477\,(1.056)^2$

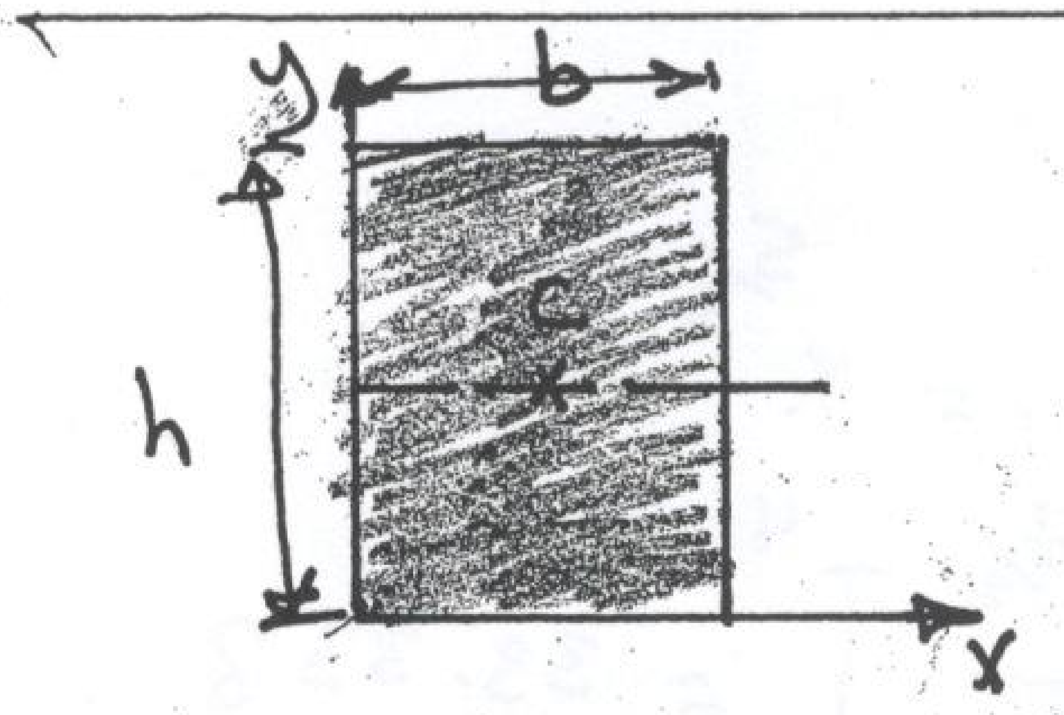
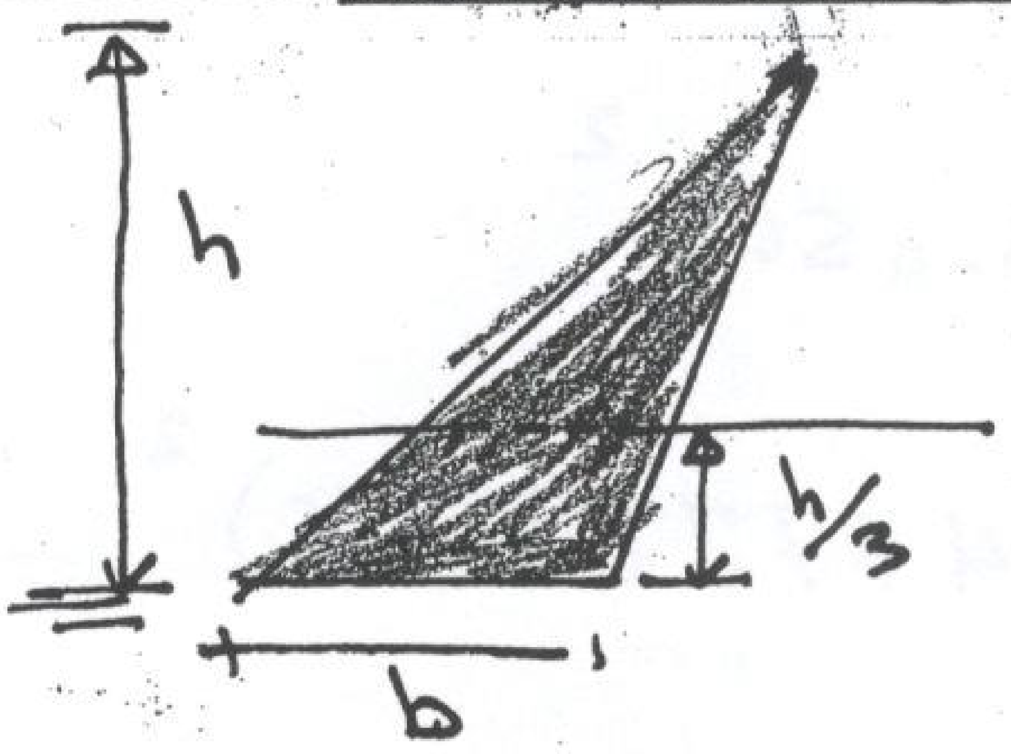
$I_a = 4.98\ cm^4$

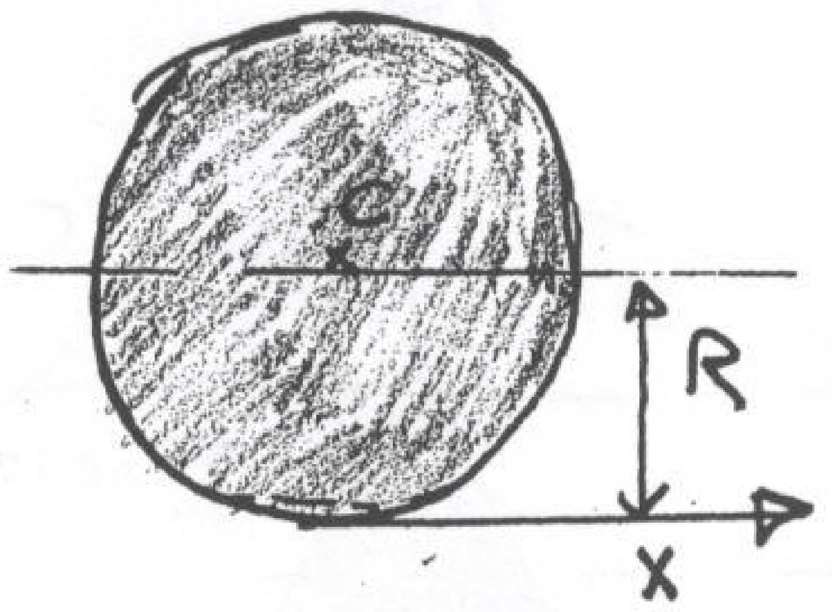
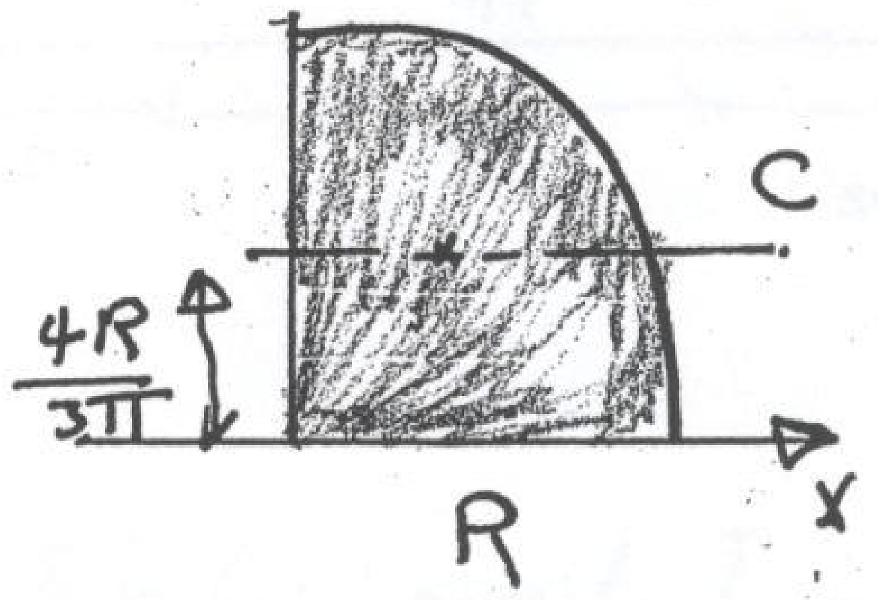
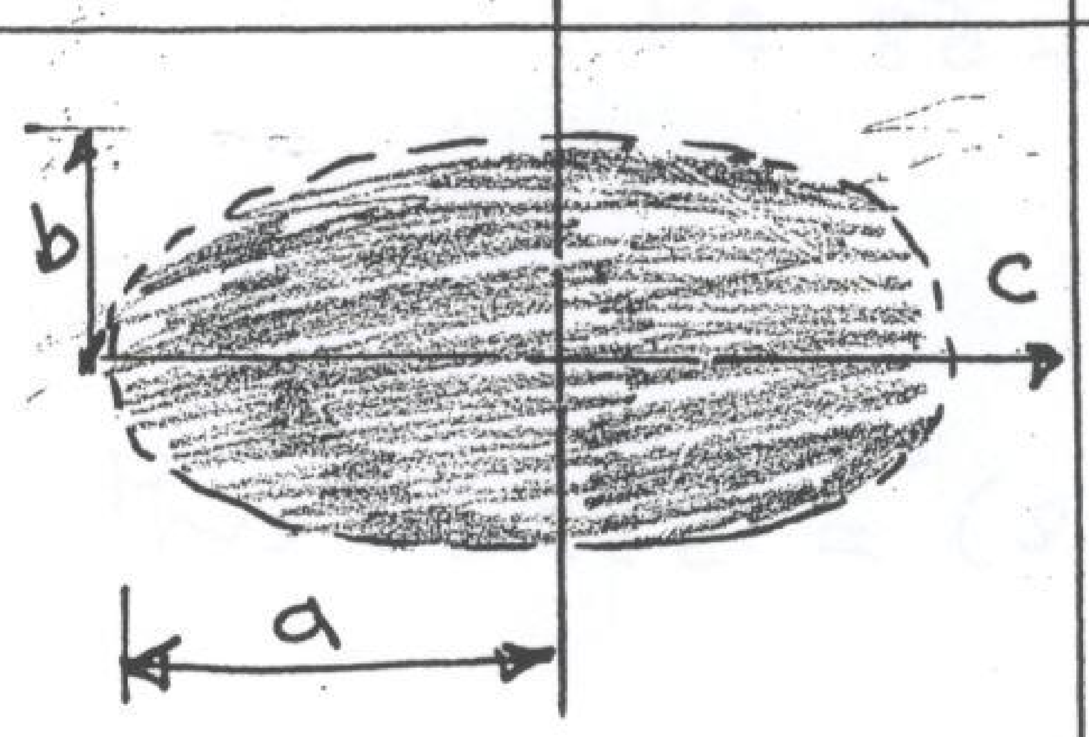
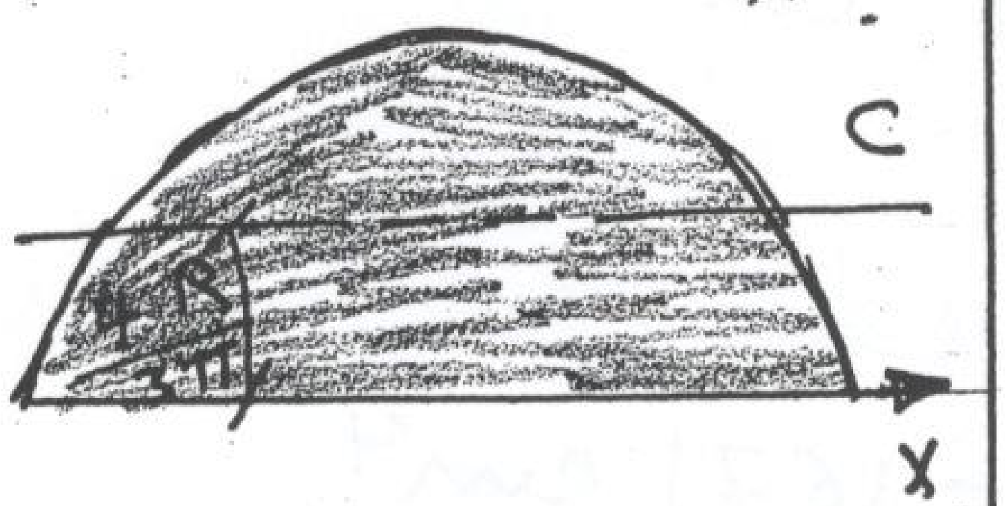
Moment of Inertia of Composite Area.

The mom. of inertia of a composite area with respect to any axis is equal to the sum of the moment of inertia of its component areas with respect to the same axis.

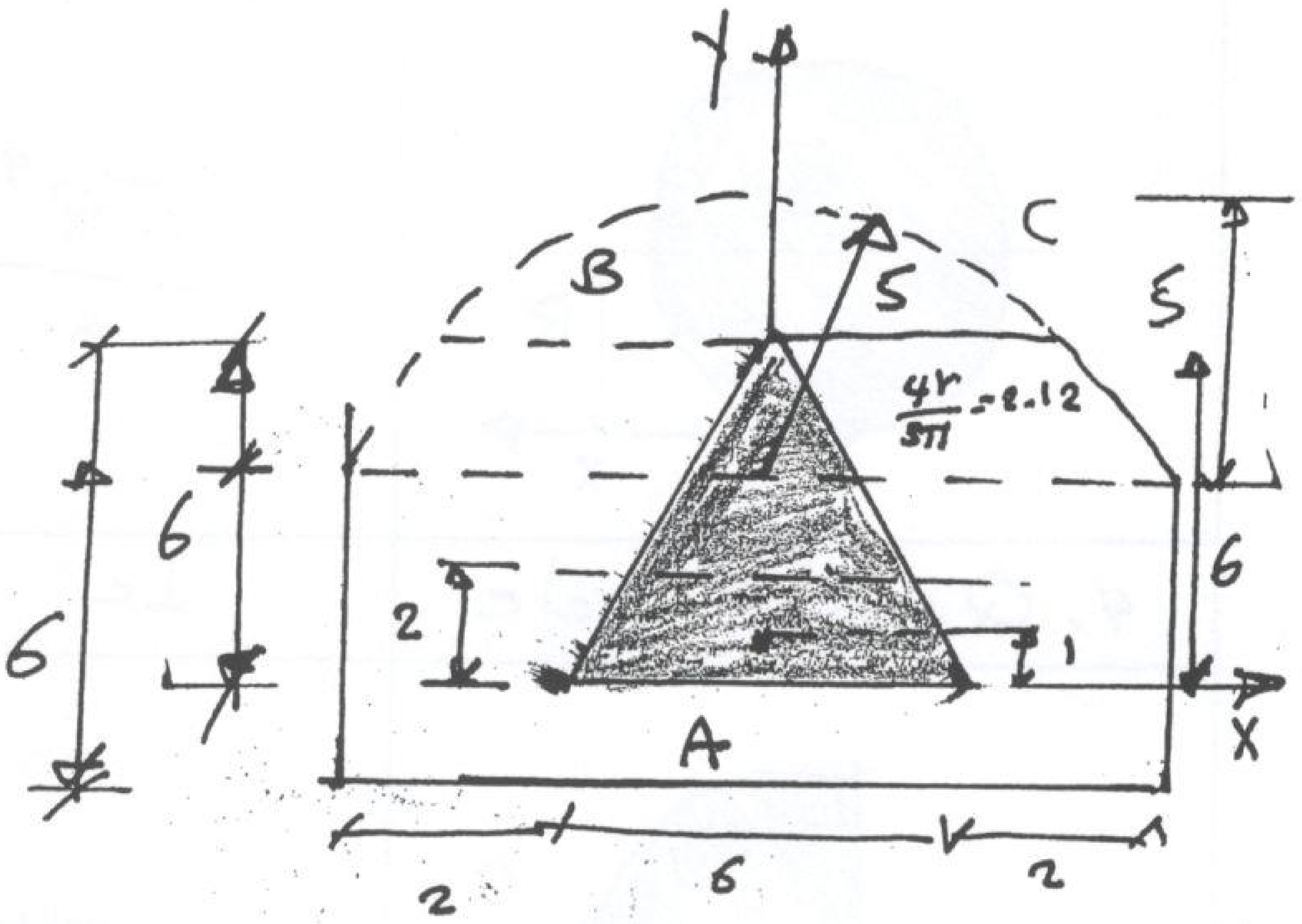
When an area is removed from the larger area its moment of inertia is subtracted from the moment of inertia of the larger area to obtain the net moment of inertia.

Moment of Inertia of Common Geometric Areas.

1- Rectangle	I_c	I_x
	$\frac{bh^3}{12}$	$\frac{bh^3}{3}$
any Triangle		
	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

3. Circle	I_c	I_x
	$\frac{\pi R^4}{4}$	$\frac{5\pi R^4}{4}$
4. Quarter circle	I_c	I_x
	$0.0549R^4$	$\frac{\pi R^4}{16}$
5. Ellipse	I_c	I_x
	$\frac{\pi}{4} ab^3$	
	$0.1094R^4$	$\frac{\pi R^4}{8}$

Ex:- Determine the mom. of inertia of the shaded area with respect to the x-axis.



Solution

For area A:

$$I_x = I_c + Ad^2$$

$$= \frac{bh^3}{12} + Ad^2$$

$$= \frac{10(6)^3}{12} + 10(6)(1)^2$$

$$= 240 \text{ cm}^4$$

For area B:-

$$I_{dia} = I_c + Ad^2 \implies I_c = I_{diam} - Ad^2$$

$$= \frac{\pi R^4}{8} - \frac{\pi(5)^2}{2}(2.12)^2 = 68.94$$

$$I_x = I_c + A(d)^2$$

$$= 68.94 + \frac{\pi(5)^2}{2}(6.12)^2 = 1539.77 \text{ cm}^4$$

For area C:-

$$I_x = \frac{bh^3}{36} + Ad^2 \qquad \text{or } I_x = \frac{bh^3}{12}$$

$$= \frac{6(6)^3}{36} + 18 * 2^2 = 108 \text{ cm}^4 \qquad = 108$$

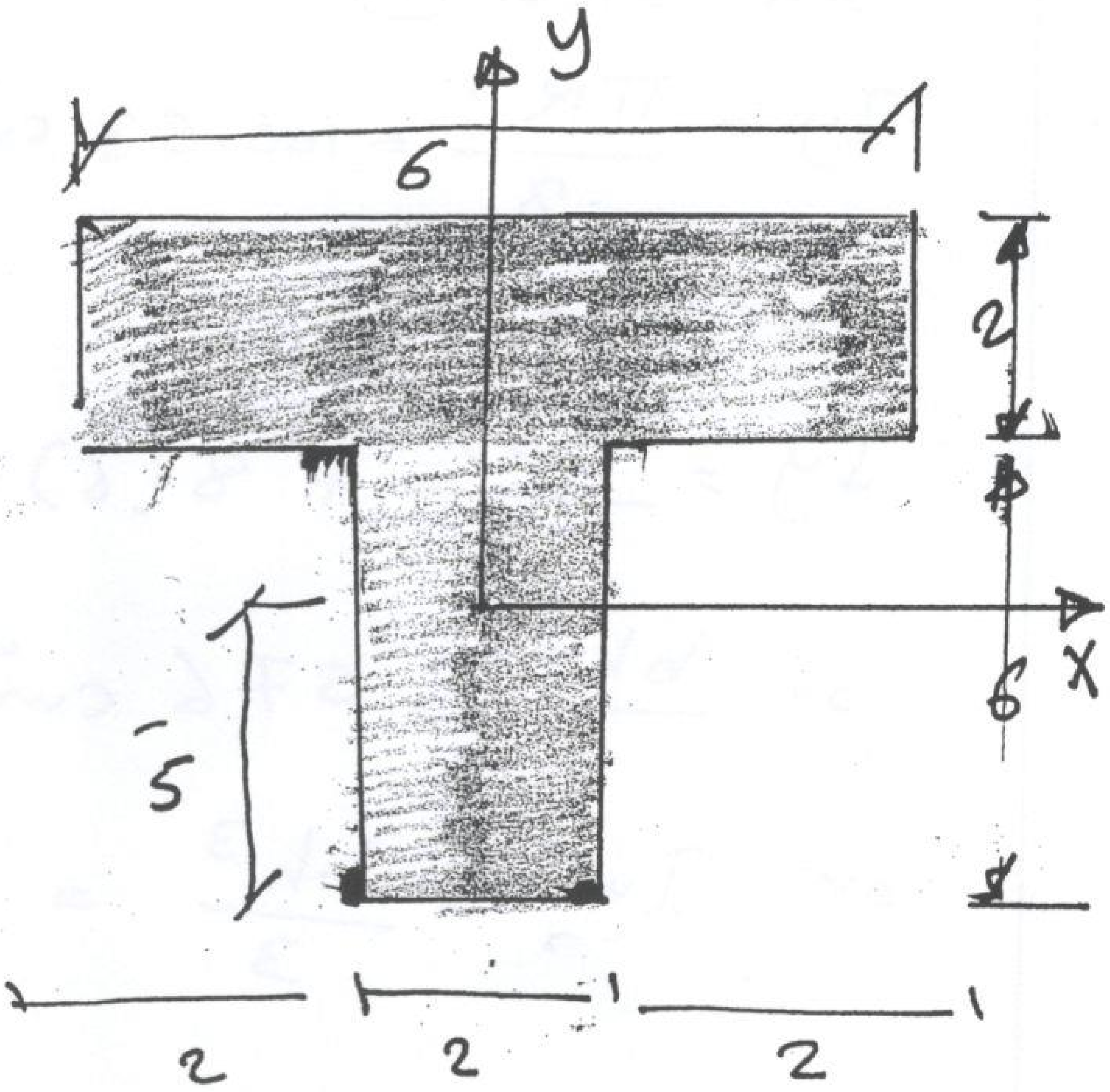
For composit area

$$I_x = 240 + 1539 - 108 = 1671 \text{ cm}^4$$

6.28 Determine the moments of inertia of the T section with respect to horizontal and vertical axes through it centroid.

$\bar{x} = 0$

$\bar{y} = \frac{M_x}{A} = \frac{6*2*7+6*2*3}{12+12}$



$I_{cx} = I_{cx_1} + I_{cx_2}$

$I_{cx} = \left[\frac{6(2)^3}{12} + 12(2)^2\right]$

$+ \left[\frac{2(6)}{12} + 12(2)^2\right]$

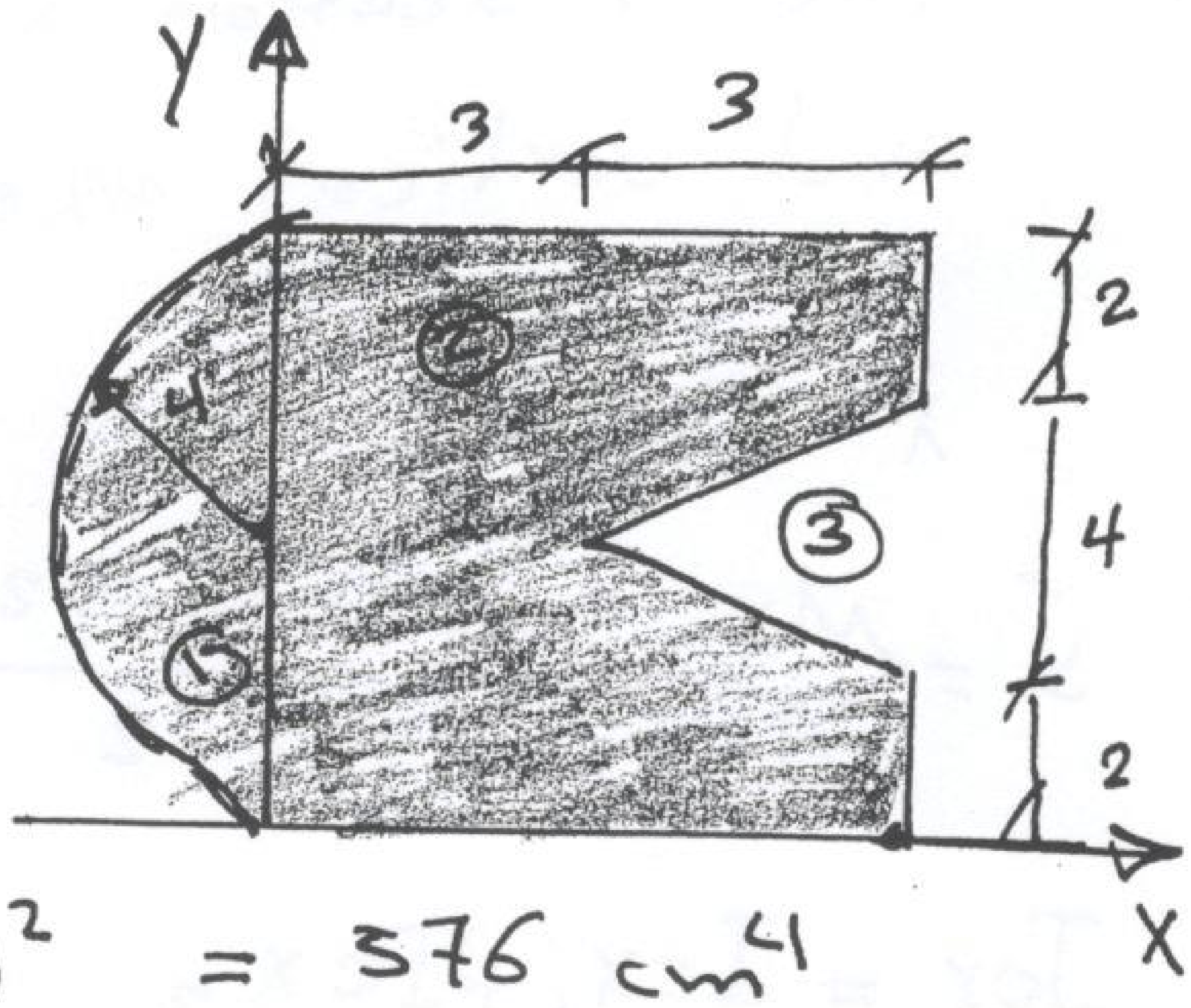
$= 52 + 84 = 136\ cm^4$

$I_{cy} = (I_x)_1 + (I_{yc})_2$

$= \frac{2(6)^3}{12} + \frac{6(2)^3}{12}$

$= 36 + 4 = 40\ cm^4$

6.36 Determine the moment of inertia of the shaded area with respect to the y-axis



Solution

For area 1

$$Iy = \frac{\pi R^4}{8} = 100.53 \text{ cm}^4$$

For area 2

$$Iy = \frac{8(6)^3}{12} + 8(6)(3)^2 = 576 \text{ cm}^4$$

$$\text{or } \frac{bh^3}{3} = 576 \text{ cm}^4$$

$$\text{or } Iy_x = \frac{bh^3}{3} = \frac{8(6)^3}{3} = 576 \text{ cm}^4$$

For area 3

$$Iy = \frac{4(3)^3}{36} + \frac{4 * 3}{2}(5)^2$$

$$= 153 \text{ cm}^4$$

∴ For total area :-

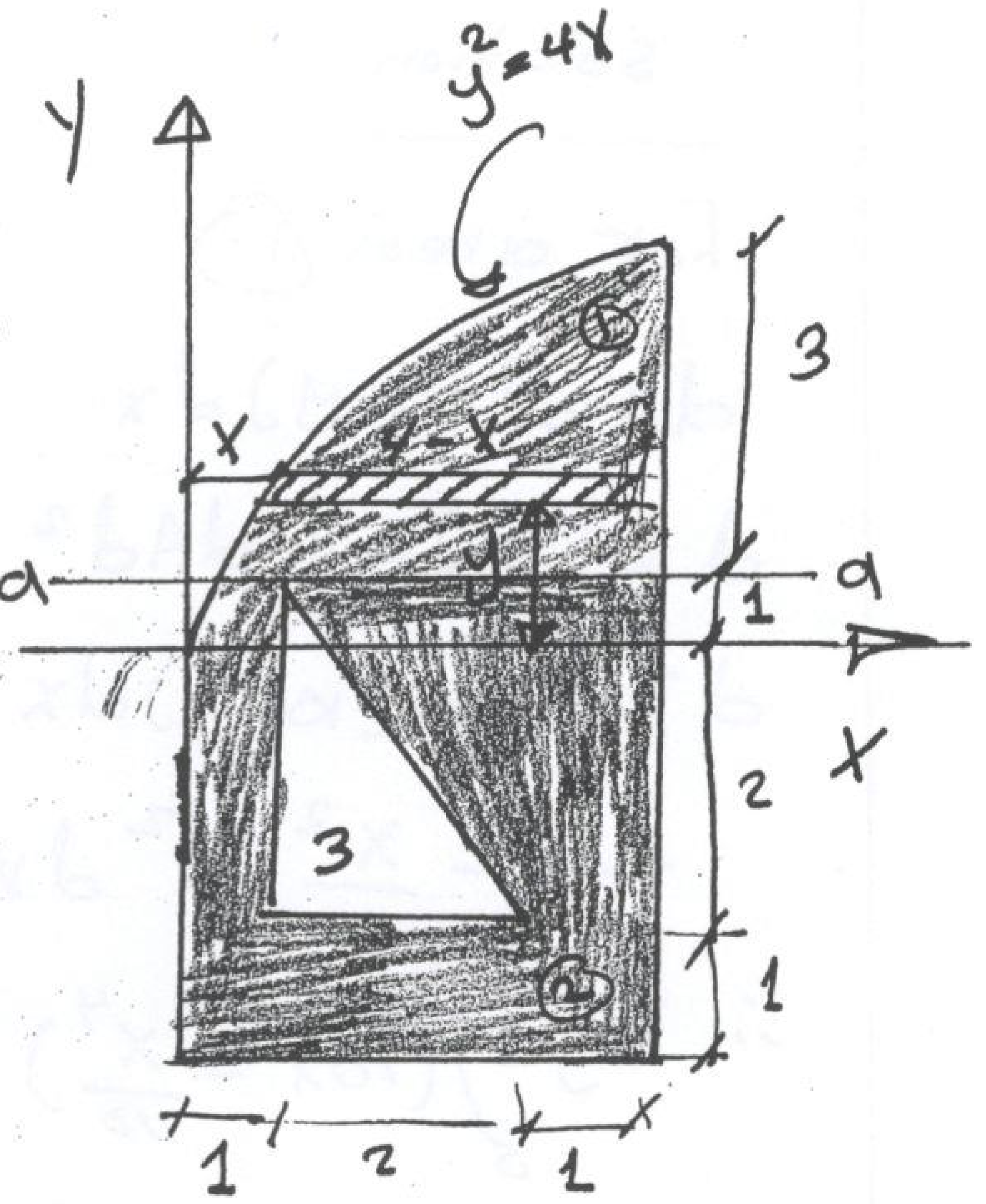
$$Iy = 100.53 + 576 - 153$$

$$= 523.53 \text{ cm}^4$$

6.44

Determin the moment of inertia of the area with respect to the a-axis

Solution



area ①

$$dI_a = dA\, d^2 + I_c \qquad \frac{(4-x)\,y^3}{2}$$

$$= (4-x)dy + (y-1)^2 + 0$$

$$dI_a = \left(4 - \frac{y^2}{4}\right)(y-1)^2 dy$$

$$\therefore I_a = \int_0^4 \left(4 - \frac{y^2}{4}\right)(y^2 - 2y + 1)\, dy$$

$= 12.8 \text{ cm}^4$

area ②:

$$I_a = I_c + A d^2$$

$$= \frac{4(3)^2}{12} + 3(4)(2.5)^2 = 84 \text{ cm}^4$$

area ③ : $I_a = I_c + A d^2$

$$= \frac{2(3)^3}{36} + \frac{2 * 3}{2}(2)^2$$

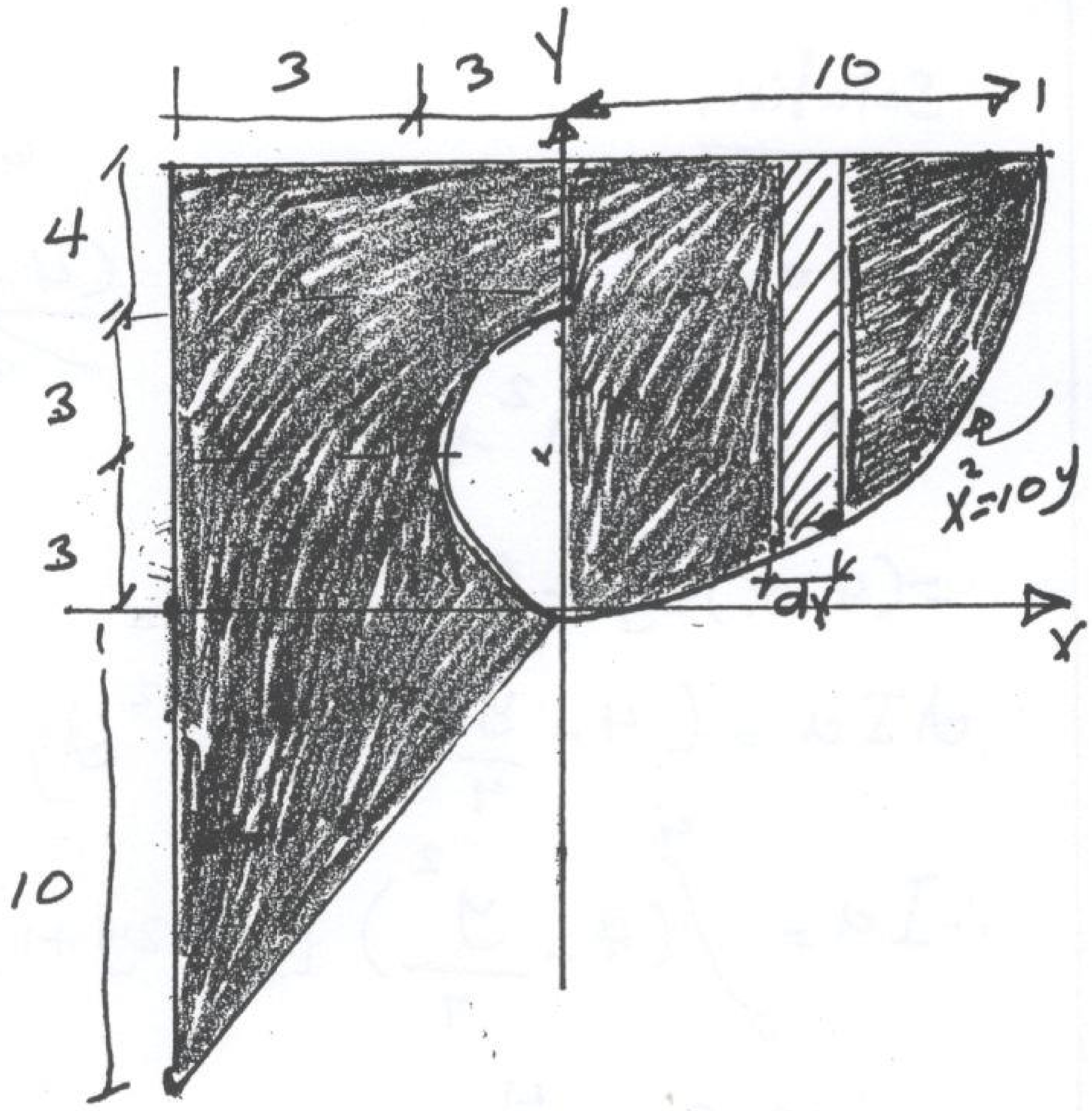
$= 13.5 \text{ cm}^4$

For total area

$I_a = 12.8 + 84 - 13.5$

$= 83.3 \text{ cm}^4$

6.52 Determin the moment of inertia of the shadad area with respect to the y-axis



Solution

For area ①

$dA = (10-y)\,dx$

$dI_y = I_c + dA\,d^2$

$dI_y = 0 + (10-y)\,dx\,x^2$

$= \left(10 - \frac{x^2}{10}\right)x^2\,dy$

$\therefore I_y = \int_0^{10}\left(10x^2 - \frac{x^4}{10}\right)dy$

$I_y = 1333\ cm^4$

For area ②

$I_y = \frac{bh^3}{3} \Longrightarrow \frac{10(6)^3}{3} = 720\ cm^4$

For area ③

$I_y = \frac{10(6)^3}{36} + \frac{10 \times 6}{2}(4)^2 = 540\ cm^4$

for area ④

$I_y = \frac{\pi R^4}{8} = \frac{\pi(3)^2}{8} = 31.808\ cm^4$

Iy. Total are

$I_y = 1333 + 720 + 540 - 31.8$

$= 2561.2\ cm^4$