The term *composite* is used for a material composed of two different materials bonded together with one serving as the matrix surrounding fibres or particles of the other. This section is a consideration of the various forms composites can take and the resulting properties.

There are many examples of composite materials encountered in everyday products. Composites can be classified into four categories:

- ## Fibre reinforced

  The fibres may be continuous throughout the matrix or short fibres, and aligned in all the same direction or randomly arranged. For example, there are composites involving glass or carbon fibres in polymers, ceramic fibres in metals and metal fibres in ceramics. Many plastics are glass fibre reinforced, the result being a much stronger and stiffer material than given by the plastic alone. Polymer composites are used for such applications as car instrument panels, domestic shower units and crash helmets. Vehicle tyres are rubber reinforced with woven cords. A common example of a metal-reinforced ceramic composite is reinforced concrete. The composite material enables loads to be carried that otherwise could not have been carried by the concrete alone. Ceramic-reinforced metal composites are used for rocket nozzles, wire-drawing dies, cutting tools and other applications where hardness and performance at high temperatures might be required.

- ## Particle reinforced

  Cermets, composites involving ceramic particles in a metal matrix, are widely used for the tips of cutting tools. Ceramic particles are hard but brittle and lack toughness; the metal is soft and ductile. Embedding the ceramic particles in the metal gives a material that is strong, hard and tough. Glass spheres are widely used with polymers to give a composite which is stronger and stiffer than the polymer alone.

- ## Dispersion strengthened

  The strength of a metal can be increased by small particles dispersed throughout it. Thus solution treatment followed by precipitation hardening for an aluminium–copper alloy can lead to a fine dispersion of an aluminium–copper compound throughout the alloy. The result is a higher tensile strength because the movement of dislocations through the alloy is hindered. Another way of introducing a dispersion of small particles throughout a metal is sintering.

- ## Laminates

  Laminates are composites in which materials are sandwiched together. Plywood is an example where thin sheets of wood are bonded together to give a stronger laminated structure.

4.3.1 Fibre-reinforced materials

The main functions of the fibres in a composite are to carry most of the load applied to the composite and provide stiffness. For this reason, fibre materials have high tensile strength and a high elastic modulus. Ceramics are frequently used for the fibres in composites. Ceramics have high values of tensile strength and tensile modulus, the useful asset of low density, but are brittle. In the presence of quite small surface flaws can markedly reduce the tensile strength. By incorporating such fibres in a ductile matrix it is possible to form a composite which makes use of the high strength–high elastic modulus properties of the fibres and the protective properties of the matrix material to give a composite with properties considerably better than with just the matrix material alone or the properties of damaged fibre material.

The properties required of a suitable matrix material are that:

- It adheres to the fibre surfaces so that forces applied to the composite are transmitted to the fibres so that they can assume the primary responsibility for the strength of the composite.

- It protects the fibre surfaces from damage.

- It keeps the fibres apart to hinder crack propagation.

The fibres used may be continuous, i.e. in lengths running the full length of the composite, or discontinuous, i.e. in short lengths. They may be aligned so that they are all lying in the same direction or randomly orientated (Figure 4.37). Aligning them all in the same direction gives a directionality to the properties of the composite. Table 4.14 gives the properties of some commonly used fibre reinforcing materials.
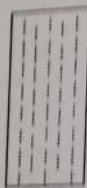
(a) Continuous, aligned

(b) Discontinuous, aligned

(c) Discontinuous, random

Figure 4.37 Forms of fibre reinforcements

Table 4.14 Properties of fibres

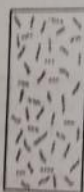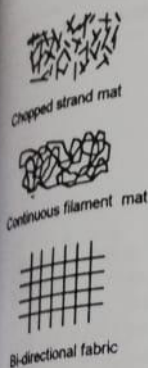| Fibre	Density Mg/m³	Tensile modulus GPa	Tensile strength MPa
Alumina	3.2	170	2100
Silicon carbide (Nicalon)	2.6	250	2200
Boron	2.65	420	3500
Carbon	1.8	250	2700
E-Glass	2.5	70	2200*
Polyethylene (Spectra 1000)	0.97	172	2964
Polyamide (Aramid) (Kevlar 49)	1.45	125	3000

Note: *3500 MPa freshly drawn.

Glass fibres are widely used for reinforcing polymers. The fibres may be long lengths running through the length of the composite or discontinuous short lengths randomly orientated within the composite. Another form of composite uses glass fibre mats or cloth in the plastic. These may be in the form of randomly

orientated fibre bundles which may be chopped or continuous and are loosely held together with a binder or a woven fabric (Figure 4.38). The effect of the fibres is to increase both the tensile strength and tensile modulus, the amount of change depending on both the form the fibres take and the amount. The continuous fibres give the highest tensile strength and tensile modulus composite but with a high directionality of properties. The strength along the direction of the fibres could be as high as 800 MPa while that at right angles to the fibre direction may be as low as 30 MPa, i.e. just about the strength of the plastic alone. Randomly orientated short fibres do not lead to this directionality of properties but do not give such high strengths and tensile modulus. Table 4.15 gives examples of the strength and modulus values obtained with glass fibre-reinforced polymer.

Table 4.15 *Properties of reinforced polyester*

Composite	% by weight of glass fibre	Tensile modulus GPa	Tensile strength MPa
Polyester alone		2 to 4	20 to 70
With short random fibres	10 to 45	5 to 14	40 to 180
With plain weave cloth	45 to 65	10 to 20	250 to 350
With long fibres	50 to 80	20 to 50	400 to 1200

Fibre-reinforced metals are a later development than fibre-reinforced polymers, having only been developed since the 1970s when the production of fibres of boron and silicon carbide became feasible. Because the coefficient of thermal expansion of ceramics is less than that of metals, e.g. that for silicon carbide being about one-fifth that of aluminium, such composites lead to a reduction in the coefficient. Likewise the thermal and electrical conductivities of ceramics is less than that of metals and so the composites have lower thermal and electrical conductivities. For example, the thermal conductivity of aluminium is 201 W m^{-1} K^{-1} and an aluminium–silicon carbide composite with 15% of silicon carbide has a thermal conductivity of 140 W m^{-1} K^{-1}. Significant increases in the tensile modulus of elasticity can be obtained by including ceramic fibres in metals, the improvement being particularly significant for those metals such as aluminium and magnesium that have low modulus values. For example, the modulus of elasticity for aluminium is about 70 GPa, while that of an aluminium–silicon carbide composite with 50% continuous silicon carbide fibres is about 200 GPa. Significant increases in the tensile strength of metals can be obtained by the inclusion of ceramic fibres. Table 4.16 gives some examples of fibre-reinforced metals and the tensile strengths achieved.

Chopped strand mat

Continuous filament mat

Bi-directional fabric

Figure 4.38 *Examples of mats and fabrics*

Key points

In designing fibre-based composites, the designer has to consider where the fibres should be placed and their directions. Does the product require high stiffness and high strength in some particular direction or directions? If so, continuous aligned fibres in such directions needs to be considered.

Table 4.16 *Properties of reinforced metals*

Composite	Tensile strength MPa
Aluminium + 50% silica fibres	900
Aluminium + 50% boron fibres	1100
Nickel + 8% boron fibres	2700
Nickel + 40% tungsten fibres	1100
Copper + 50% tungsten fibres	1200
Copper + 80% tungsten fibres	1800

Ceramic matrix composites, such as silicon carbide fibre-reinforced glass ceramics and silicon carbide whisker-reinforced alumina, enable the elastic modulus, strength and toughness of the ceramic matrix material to be improved. For example, a $Li_2O–Al_2O_3–MgO–SiO_2$ glass ceramic with a tensile modulus of 86 GPa has as a composite with 50% silicon carbide fibre reinforcement a modulus of about 130 GPa, its toughness improved from 1.5 MN $m^{-3/2}$ to 17 MN $m^{-3/2}$ and its strength improved from 180 MPa to 620 MPa. Silicon carbide-reinforced alumina uses whiskers of silicon carbide, these being single crystals rather than the polycrystalline form of fibres. With 25% silicon carbide whiskers, the tensile modulus is improved from 340 GPa to 390 GPa, the strength from 300 MPa to 900 MPa and the fracture toughness from 4.5 MN $m^{-3/2}$ to 8.0 MN $m^{-3/2}$.

Continuous fibre composites

Composites made with continuous aligned fibres all aligned in the same direction have properties which depend on the direction in which the material is stretched, the modulus of elasticity and strength in the direction which the fibres are aligned being markedly different from its modulus and strength in the direction at right angles to the fibres. The term *isotropic material* is used for one which has the same properties in all directions, the term *anisotropic material* for one which has properties which depend on the direction. A composite with the continuous aligned fibres is an anisotropic material.

Figure 4.39 Continuous aligned fibres parallel to load direction

Maths in action

The law of mixtures

The mass m_c of a composite is made up of the masses of the matrix m_m and the fibres m_f, i.e.

$$m_c = m_m + m_f \qquad [7]$$

Since mass is volume v times density ρ, then equation [7] can be written as:

$$v_c\rho_c = v_m\rho_m + v_f\rho_f$$

and so:

$$\rho_c = \frac{V_m}{V_c}\rho_m + \frac{V_f}{V_c}\rho_f$$

v_m/v_c is the volume fraction V_m that is matrix and v_f/v_c is the volume fraction V_f that is fibre. Thus:

$$\rho_c = V_m\rho_m + V_f\rho_f$$

[8]

Note that since $v_m = v_c - v_f$ we must have $V_m = 1 - V_f$. Equation [8] can be termed a *law of mixtures*.

Elastic modulus

Now consider the law of mixtures for the elastic modulus of a composite having aligned continuous fibres with the direction of the fibres parallel to the loading direction (Figure 4.39). The load F_c on the composite is the sum of the loads on the matrix F_m and on the fibres F_f:

$$F_c = F_m + F_f$$

[9]

But stress σ is force/area and so for the composite $\sigma_c = F_c/a_c$, for the matrix $\sigma_m = F_m/a_m$ and for the fibres $\sigma_f = F_f/a_f$ where a_c is the cross-sectional area of the composite, a_m the cross-sectional area of that part of the composite that is matrix and A_f that part that is fibre. But $\sigma = E\varepsilon$, where E is the modulus of elasticity and ε the strain, thus $F_c = \sigma_c a_c = E_c\varepsilon a_c$, $F_m = \sigma_m a_m = E_m\varepsilon a_m$ and $F_f = \sigma_f a_f = E_f\varepsilon a_f$. The strain is the same for the matrix and the fibres will be the same as that for the composite if we assume that the fibres are firmly bonded to the matrix. Equation [9] can thus be written as:

$$E_c\varepsilon a_c = E_m\varepsilon a_m + E_f\varepsilon a_f$$

and thus, if we write A_m as the area fraction a_m/a_c and A_f as a_f/a_c:

$$E_c = A_mE_m + A_fE_f$$

[10]

Since $a_m = a_c - a_f$, then $A_m = 1 - A_c$. Equation [10] is the law of mixtures for the tensile modulus when we have continuous fibres aligned parallel to the loading direction. The area fractions are equivalent to the volume fractions and thus equation [10] becomes:

$$E_c = V_mE_m + V_fE_f$$

[11]

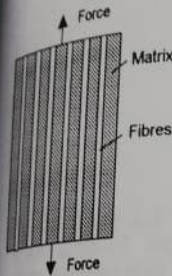Now consider the elastic modulus when the fibres are at right angles to the direction of loading (Figure 4.40).

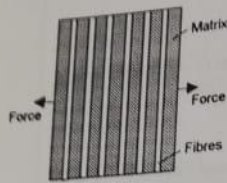**Figure 4.39** Continuous aligned fibres parallel to load direction

Force

Matrix

Fibres

Force

Figure 4.40 Continuous aligned fibres at right angles to load direction

The total extension δ_c of the composite in the direction of the load will be the sum of the extensions of the matrix δ_m and the fibres δ_f:

$$\delta_c = \delta_m + \delta_f \qquad [12]$$

The stress on the matrix will be the same as on the fibres and on the composite as a whole. The strain $\varepsilon = \delta/l$, where l is the original length and thus $\delta = \varepsilon l = (\sigma/E)l$, where σ is the stress. Thus equation [12] becomes:

$$\frac{\sigma}{E_c}l_c = \frac{\sigma}{E_m}l_m + \frac{\sigma}{E_f}l_f$$

and so:

$$\frac{1}{E_c} = \frac{L_m}{E_m} + \frac{L_f}{E_f} \qquad [13]$$

where L_m is the matrix length fraction l_m/l_c and L_f is the fibre length fraction l_f/l_c. The length fractions are equivalent to the volume fractions and thus equation [13] becomes:

$$\frac{1}{E_c} = \frac{V_m}{E_m} + \frac{V_f}{E_f} \qquad [14]$$

Equation [11] will give a different elastic modulus value to that given by equation [14]. For an epoxy-glass composite with the volume fraction of fibres as 65%, equation [11] gives for the modulus of elasticity when the load is at 0° to the fibres (modulus for epoxy 3.5 GPa, modulus for the glass fibres 70 GPa):

$$E_c = V_m E_m + V_f E_f = 0.35 \times 3.5 + 0.65 \times 70 = 47 \text{ GPa}$$

and equation [14] gives

$$\frac{1}{E_c} = \frac{V_m}{E_m} + \frac{V_f}{E_f} = \frac{0.35}{3.5} + \frac{0.65}{70} = 0.11$$

and hence $E_c = 9$ GPa when the load is at 90° to the fibres. Figure 4.41 shows, for a glass fibre epoxy composite, how the modulus of elasticity varies with the angle between the load direction and the direction of the fibres.

Figure 4.41 Modulus variation with angle

Strength

Now consider the strength of fibre-reinforced composites. This is more difficult to predict than the stiffness because there are a number of ways the composite can fail: fibre failure, matrix failure or fibre–matrix interface failure.

Figure 4.42 Strength with angle

The matrix has a lower strength than the fibres. Thus if we have a very small volume fraction then when the matrix fails, the load has to be carried by the fibres but as there are not many fibres to carry the load complete failure follows. Thus when we have the load applied in a direction parallel to the fibres, equation [9], $F_c = F_m + F_f$ gives:

$$\sigma_c a_c = \sigma_m a_m + \sigma_f a_f$$

where σ_c is the composite failure stress, σ_m the matrix failure stress and σ_f is the fibre stress when the matrix fails. Thus:

$$\sigma_c = \sigma_m A_m + \sigma_f A_f = \sigma_m V_m + \sigma_f V_f \qquad [15]$$

and because there are so few fibres we can generally neglect their contribution and write:

$$\sigma_c = \sigma_m V_m \qquad [16]$$

At large fibre volume fractions, the fibres will take most of the load and after matrix failure will still be able to carry a load and so determine the strength of the composite. Then we must have:

$$\sigma_c = \sigma_f V_f \qquad [17]$$

where σ_f is the fibre failure stress.

As with the modulus of elasticity, the strength depends on the orientation of the fibres relative to the load direction. Figure 4.42 shows the effect of fibre orientation on the strength of a typical continuous fibre composite, glass fibre-reinforced epoxy.
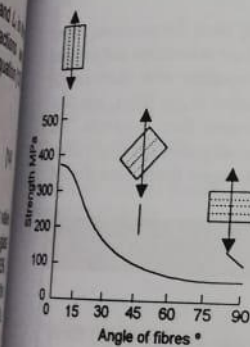

Figure 4.42 Strength variatiom with angle

Example

A composite has continuous aligned glass fibres with a tensile modulus of 76 GPa and strength of 1500 MPa in a matrix of polyester with a tensile modulus of 3 GPa and strength of 50 MPa. Determine the tensile modulus and strength of the composite when the load is applied in a direction parallel to the fibres and the volume fraction is 60%.

Using equation [11]: $E_c = V_m E_m + V_f E_f = 0.4 \times 3 + 0.6 \times 76 = 46.8$ GPa. If we assume that the fibres will continue to take the load after matrix failure, then equation [17] gives for the composite strength: $\sigma_c = \sigma_f V_f = 1500 \times 0.6 = 900$ MPa.

Example

A column of reinforced concrete has steel reinforcing rods running through its entire length and parallel to the column axis. If the concrete has a modulus of elasticity of 15 GPa and a tensile strength of 2.8 MPa and the steel rods a modulus of elasticity of 210 GPa and a tensile strength of 400 MPa, determine the modulus of elasticity and tensile strength of the composite if the steel rods occupy 10% of the cross-sectional area of the column.

Using equation [11]: $E_c = V_m E_m + V_f E_f = 0.9 \times 15 + 0.1 \times 210 = 34.5$ GPa. Using equation [15]: $\sigma_c = \sigma_m V_m + \sigma_f V_f = 0.9 \times 2.8 + 0.1 \times 400 = 42.5$ MPa.

Discontinuous fibres

The properties of composites containing short discontinuous fibres depends on the lengths of the fibres, their orientation and volume fraction. The tensile modulus of composites with short length fibres all aligned is, in a direction parallel to the fibres, less than that of the comparable aligned continuous fibre composite. The tensile modulus of composites with non-aligned short length fibres is less than that for a composite with the same fibres when aligned.

Maths in action

Stress transfer

To consider the effects of the length of short length fibres we need to consider the stress transfer that takes place between the matrix and fibres. When a load is applied to a composite it is applied to the matrix and transferred to the fibres by some combination of shear and tensile or compressive stresses acting across the interface. The nature of the bond between the matrix material and the fibres thus plays a critical role in determining the properties of the composite

For a composite under tension, we can think of a discontinuous fibre in the matrix as being stretched as a result of interfacial shear stresses acting on the surfaces of the fibre (Figure 4.43). These shear stresses will be a maximum at the ends of the fibre where it stretches the most and fall as we consider points on the fibre nearer to the middle. The tensile stress in the fibre will be zero at the ends and build up as the shear stress decreases. Provided the fibre is long enough we can consider the shear stresses decreasing to zero and the tensile stress rising to a maximum value in the central region of the fibre.
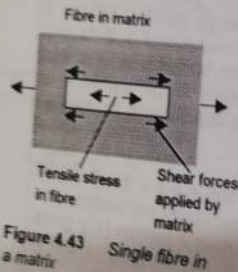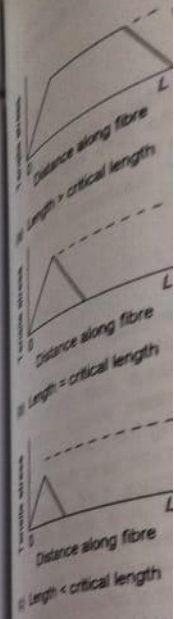
Fibre in matrix

Tensile stress in fibre   Shear forces applied by matrix

Figure 4.43   Single fibre in a matrix

(a) Length > critical length

(b) Length = critical length
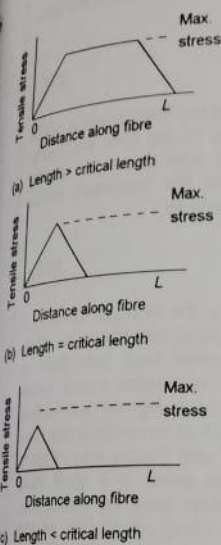
(c) Length < critical length

Figure 4.44   Stress transfer for different length fibres
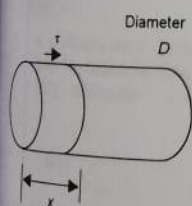
Figure 4.45   Shear force at interface

Table 4.17 Typical values of L_c /D ratios

Fibre	Matrix	L_c/D ratio
Glass	Polyester	40
Glass	Polypropylene	140
Carbon	Epoxy	35
Boron	Epoxy	35
Silicon carbide	Alumina	10
Boron	Aluminium	20

If the fibre is not long enough, the shear stress will not fall to zero and the tensile stress will not reach the maximum possible value. In order to achieve this maximum level of tensile stress, the fibre length must be at least equal to a *critical length* L_c. Figure 4.44 shows the tensile stress patterns we might expect for different fibre lengths.

Consider the interfacial shear stress acting on a single fibre in a matrix (Figure 4.45). If τ is the average interfacial shear stress then the shear force acting on a section of the fibre, length x and of uniform cross-sectional diameter D, is:

shear force = shear stress × area = $\tau \pi D x$

This shear force results in a longitudinal stress in the fibre of σ_f. Thus:

$$\sigma_f \tfrac{1}{4}\pi D^2 = \tau \pi D x$$

and so:

$$\sigma_f = \frac{4\tau x}{D}$$

The stress increases from zero at the end of a fibre, i.e. when $x = 0$, to its maximum possible value when $x = \frac{1}{2}L$. Hence, the maximum value of the tensile stress is:

$$\text{maximum } \sigma_f = \frac{2\tau L_c}{D} \qquad [18]$$

The critical length to diameter ratio L_c/D must not be less than $\sigma_f/2\tau$ if the composite is to realise the potential of the fibre. If, for a glass fibre–polyester composite the maximum value of stress the fibres can withstand is 1500 MPa and the shear strength is 25 MPa, then L_c/D is 30. For fibres of diameter 5 μm then L_c is 0.15 mm. If the fibres used are of greater diameter then the critical length is increased. Table 4.17 shows some typical values of the L_c/D ratio.

If the fibre length L is longer than the critical length and we assume a linear variation of stress with distance, as in Figure 4.44(a), then the average tensile stress in the fibre is given the area under the stress–fibre length graph divided by the fibre length:

$$\text{average stress} = \left[(L - L_c) + \tfrac{1}{2}L_c \right] \frac{\text{max. } \sigma_f}{L}$$

$$= \left[1 - \frac{L_c}{2L} \right] \text{max. } \sigma_f \qquad [19]$$

$$= \left[1 - \frac{L_c}{2L} \right] \frac{2\tau L_c}{D} \qquad [20]$$

When the fibre length is equal to the critical length, as in Figure 4.44(b), then the average tensile stress in the fibre is half the maximum stress:

$$\text{average stress} = \frac{\tau L_c}{D}$$
[21]

When the fibre length is less than the critical length, as in Figure 4.44(c), the tensile stress in the fibre will never reach the maximum value. The area under the tensile stress–fibre length graph is half the maximum stress value attained, this being $2\tau L/D$ and so:

$$\text{average stress} = \frac{\tau L}{D}$$
[22]

As the above equations indicate, the average stress in a short fibre will be less than the maximum stress the fibre can withstand. With a continuous fibre the fibre is assumed to be entirely at the maximum stress value (Figure 4.46) when it fails, this not being the case with a short fibre because of the drop off of stress near the fibre ends. Compare Figure 4.46(a) with Figure 4.44. Figure 4.46(b) shows how the average stress in a short fibre compares with the maximum stress value for a fibre, i.e. the value at which continuous fibres would fail, as the length of fibre, in terms of multiples of the critical length, is increased. The average stress value can be used with the rule of mixtures, equation [15], to give a value for the failure stress of the composite.
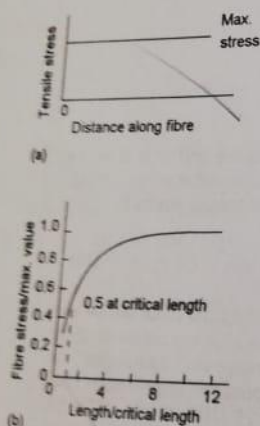
Tensile modulus

A modified rule of mixtures can be used, incorporating a length efficiency parameter η_L to take account of the fibres not being continuous. Thus, equation [11] becomes:

$$E_c = V_m E_m + \eta_L V_f E_f$$
[23]

For carbon fibres of length 0.1 mm and diameter 8 μm in epoxy the efficiency parameter is 0.20. With these fibres of length 1.0 mm, the efficiency parameter is 0.89 and as the length increases so the parameter approaches 1.

For non-aligned short length fibres an orientation efficiency parameter η_0 can be used to take account of this, equation [18] then becoming:

$$E_c = V_m E_m + \eta_0 \eta_L V_f E_f$$
[24]

For completely three-dimensionally random fibres, the orientation efficiency parameter has the value 0.2.

Figure 4.46 (a) Stress transfer for continuous fibres, (b) average stress in short fibres

Example

A glass fibre–polyester composite contains 60% by volume of fibres, the fibres being of length 3 mm. If the failure stress for the fibres is 1500 MPa, the shear strength 25 MPa and the matrix has a tensile strength of 50 MPa, determine the strength of the composite.

The critical length is, as derived above, 0.15 mm and so the fibres have a length which is greater than the critical length. Using equation [19], the maximum value of the average stress acting on the fibres is:

$$\text{max. average stress} = \left[1 - \frac{L_c}{2L}\right]\text{max.}\,\sigma_f$$

$$= \left[1 - \frac{0.15}{6}\right]1500 = 1462.5 \text{ MPa}$$

Hence, using equation [15], the strength of the composite is:

$$\text{strength} = \sigma_m V_m + \sigma_f V_f = 50 \times 0.4 + 1462.5 \times 0.6$$

$$= 897.5 \text{ MPa}$$

This compares with a strength of 920 MPa that occurs with the continuous fibres and 475 MPa if the fibres had been equal to the critical length.

4.3.2 Particle-reinforced materials

Particle-reinforced materials have particles with diameters of the order of 1 μm or more dispersed throughout the matrix, the particles often accounting for a quarter to half, or more, of the volume of the composite.

Cermets

Cermets, or cemented carbides, are examples of particle-reinforced composites in which hard ceramic particles are in a metal matrix. The ceramics used have high strengths, high values of tensile modulus and high hardness, but are brittle. By comparison, the metals are weaker and less stiff, but ductile. By incorporating ceramic particles, often about 80% by volume, in a metal matrix, a composite can be produced which is strong, hard and tough and can be used as a tool material.

For example, tungsten carbide is a very hard (about 2000 HV) ceramic with a high tensile modulus but also very brittle. Tools made from this material would thus be extremely brittle. A cermet involving tungsten carbide in a metal matrix, cobalt, can be made by mixing tungsten carbide powder with cobalt powder and heating the compacted powders to a temperature above the melting

Key point

Cermets are an example of a particle-reinforced composite, having hard ceramic particles in a metal matrix.

point of the cobalt. The liquid cobalt then melts and flows round each tungsten carbide particle. After solidification, the cobalt acts as a binder for the tungsten carbide. The composite has a better toughness than the tungsten carbide alone, since crack propagation through the material is hindered. When in use, the tungsten carbide particles in the surface of the material provide the tool with its cutting ability. As the tungsten carbide particles at the cutting surface become blunted, they either fracture or pull out of the cobalt matrix and expose fresh tungsten carbide particles which can continue to provide cutting ability. For a fine cutting tool, the amount of cobalt in the composite is low and the tungsten carbide particles fine so that the tungsten carbide particles pull out easily and the tool remains sharp. For a rough cutting tool, the amount of cobalt is increased to improve toughness and coarser tungsten carbide particles are used. Table 4.18 shows the composition and application of some cobalt–tungsten carbide composites used as tool materials.

Particle-reinforced polymers

Many polymeric materials incorporate fillers, these being particulate. Examples of such fillers are glass beads, silica flour and rubber particles. Such fillers can be regarded as discontinuous fibres which have lengths comparable with their diameter. Their effect on the tensile strength and modulus of elasticity thus tends to be smaller.

The toughness of some polymers is increased by incorporating tiny rubber particles in the polymer matrix. Polystyrene is toughened in this way by polybutadiene particles to give a product referred to as *high impact polystyrene* (HIPS). The rubber particles increase the toughness from about 1 to 1.7 MN m$^{-3/2}$. They block the transmission of cracks and, since they deform readily, absorb energy. The rubber has a lower tensile modulus and tensile strength than that of the matrix material and the net result is a lowering of the tensile modulus and tensile strength, but much greater elongations before breaking and a tougher material.

Table 4.18 Cobalt–tungsten carbide materials

Cobalt %	Tungsten carbide %	Tungsten carbide grain size	Typical applications
3	97		
6	94	Medium	Machining of cast iron, non-ferrous metals and non-metallic materials
6	94	Fine	Machining of non-ferrous and high-temperature alloys
		Medium	General purpose machining for metals other than steels, small and
6	94		medium size compacting dies and nozzles
10	90	Coarse	Machining of cast iron, non-ferrous materials, compacting dies
10	90	Fine	Machining steel, milling, form tools
16	84	Coarse	Percussive drilling bits
16	84	Fine	Mining and metal forming tools
25	75	Coarse	Mining and metal forming tools, medium and large size dies
		Medium	Heavy impact metal forming tools, e.g. heading dies, cold extrusion dies

Carbon black, which consists of very fine particles of carbon, is widely used as a filler with vulcanised rubber. The carbon black enhances the strength, stiffness, hardness, wear resistance and heat resistance of the rubber.

Foams

Foams are a particulate composite in which the component bound by the matrix is not a solid but bubbles of a gas. Such foams are used as cushioning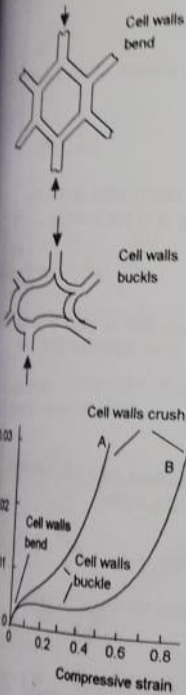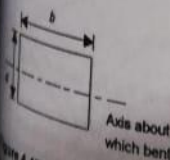 in furniture, energy-absorbent packaging and padding, for thermal insulation, for buoyancy, for structural panels and as the filling in sandwich panels (see later in this chapter). The parameters determining the characteristics of foams are the ratio of bulk density of the foam to that of the unfoamed matrix material, and the cellular structure of the foam. The foam can be open cell, closed cell or a mixture of the two. With a closed-cell structure, the gas bubbles in the foam are discrete and not interconnected, whereas in an open-cell structure the bubbles have coalesced and are interconnected. Structural foams and sandwich foams have solid skins covering the foamed core.

Figure 4.47 shows typical forms of compressive stress–strain graphs for polymeric foams. Over the initial straight-line part of the graph, the cell walls just bend under the action of the applied stress. The next stage is when the walls elastically buckle, often giving a plateau of deformation at almost constant stress. The deformation is still elastic and so recoverable. Finally the cell walls suffer irrecoverable buckling collapse.

With a material used for, say, a cushion, the foam is required to give continually increasing resistance to increasing load and so a plateau on the stress–strain graph is not required. Thus a stress–strain graph of the form shown as A is required. Foams used for packaging need to absorb the energy involved when packages are dropped and so a plateau is highly desirable, like that indicated for B in the figure, since it indicates a high energy absorption. A low-density packaging form with a density ratio, bulk foam to unformed plastic, of 0.01 might be used for packaging small delicate instruments; heavier density foams being used for packaging heavier components.

Foamed plastics have a lower modulus of elasticity, and so are more flexible, than when unfoamed. However, in bending this reduction in modulus can be more than offset by the ability to increase the second moment of area I. The second moment of area of a rectangular cross-section (Figure 4.48) of breadth b and depth d is $bd^3/12$. Thus because of the lower density of the foamed plastic, we can have a much larger cross-section for the same mass and thus a much larger second moment of area. The important parameter in determining the stiffness of a beam subject to bending is the product EI. For example, for a cantilever of length L subject to a force F at its free end, the deflection y of that free end is $FL^3/3EI$. Thus we can minimise the deflection per unit force, i.e. y/F, and so have a stiffer cantilever, if there is a large value of EI. With the foamed plastic, the reduction in E can be more than offset by the increase in I.

Cell walls bend

Cell walls buckls

Cell walls crush

Cell walls bend

Cell walls buckle

Compressive strain

Figure 4.47 Stress–strain graphs for foamed polymers

Axis about which bent

Figure 4.48 Cross-section of beam

Maths in action

For structural and sandwich foams, to a reasonable approximation, the elastic modulus E_f of the foam is related to the modulus E_s of the solid matrix material by:

$$E_f = V_s E_s \qquad [25]$$

where V_s is the fraction of the bulk volume of the foam that is solid matrix and is given by:

$$V_s = \frac{v_s}{v_s + v_g} \qquad [26]$$

where v_s is the volume of the foam that is solid and v_g the volume that is gas. The bulk density ρ_f of the foam is

$$\rho_f = \frac{m_s + m_g}{v_s + v_g}$$

with m_s being the mass of the foam that is solid and m_g the mass of the gas in the foam. Thus equation [26] can be written as:

$$V_s = \frac{v_s \rho_f}{m_s + m_g}$$

Since m_g is very small in comparison with m_s, we can neglect the m_g term. Since $\rho_s = m_s/v_s$ then:

$$V_s \approx \frac{\rho_f}{\rho_s}$$

and so equation [25] can be written as:

$$E_f \approx \frac{\rho_f}{\rho_s} E_s \qquad [27]$$

For foams with uniform densities but without the structural skin, a better relationship has been found experimentally to be:

$$E_f = \left(\frac{\rho_f}{\rho_s}\right)^n E_s \qquad [28]$$

where n has the approximate value of 1.5 for such foams in tension and 2 in compression.

Thus, for example, foamed polystyrene with a volume fraction of polymer of 0.5 would have a tensile modulus of about $0.5^{1.5} = 0.35$ times that of the unfoamed polystyrene. The compression modulus would be about $0.5^2 = 0.25$ times that of the unfoamed polystyrene.
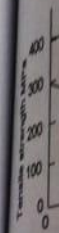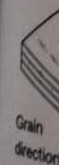
Expanded polystyrene used for thermal insulation and packaging has a volume fraction of about 0.05 and thus a tensile modulus of about $0.05^{1.5} = 0.011$ of that of the unfoamed polystyrene and a compression modulus of $0.05^2 = 0.0025$ that of the unfoamed polystyrene.

4.3.3 Dispersion strengthened metals

One way of introducing a dispersion of small particles throughout a metal uses sintering. This process involves compacting a powdered metal powder in a die and then heating it to a temperature high enough to knit together the particles in the powder. If this is done with aluminium the result is a fine dispersion of aluminium oxide, about 10%, throughout an aluminium matrix. The aluminium oxide occurs because aluminium in the presence of oxygen is coated with aluminium oxide and when the aluminium powder is compacted, much of the surface oxide film becomes separated from the aluminium and becomes dispersed through the metal. The aluminium oxide powder, a ceramic, dispersed throughout the aluminium matrix gives a stronger material than that which would have been given by the aluminium alone. At room temperature, the tensile strength of the sintered aluminium powder (SAP) is about 400 MPa, compared with that of about 90 MPa for the aluminium. The sintered aluminium has an advantage over precipitation-hardened aluminium alloys in that it retains its strength better at high temperatures (Figure 4.49). This is because at the higher temperatures, the precipitate particles in precipitation hardened alloys tend to coalesce or go into solution in the metal.
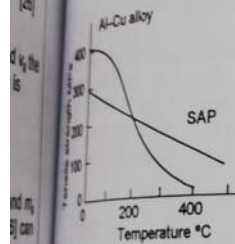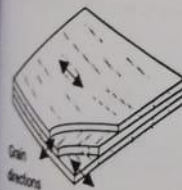
4.3.4 Laminates

Plywood is an example of a laminated material. It is made by gluing together thin sheets of wood with their grain directions at right angles to each other (Figure 4.50). The grain directions are the directions of the cellulose fibres in wood, a natural composite, and thus the resulting structure, the plywood, has fibres in mutually perpendicular directions. Thus, whereas the thin sheet had properties that were directional, the resulting laminate has no such directionality.

It is not only wood that is laminated, metals are too. The *cladding* (Figure 4.51) of an aluminium–copper alloy with aluminium to give a material with a better corrosion resistance is another example. Galvanised steel can be considered a further example, the layer of zinc on the steel giving better corrosion resistance. Steel for use in food containers is often plated with tin to improve its corrosion resistance.
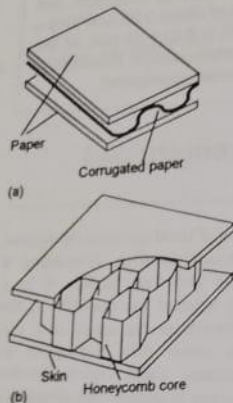
Figure 4.49 The effect of temperature on strength

Figure 4.50 Plywood, 3-ply

Figure 4.51 Clad alloy

Paper   Corrugated paper

(a)

Skin   Honeycomb core

(b)

Figure 4.52   (a) Corrugated cardboard, (b) honeycomb structure
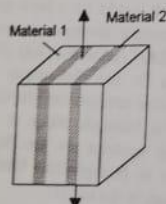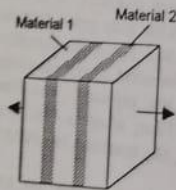

Figure 4.53   Laminate

Figure 4.54   Laminate

Corrugated cardboard is another form of laminated structure (Figure 4.52(a)), consisting of paper corrugations sandwiched between layers of paper. The resulting structure is much stiffer, in the direction parallel to the corrugations, than the paper alone. A structure with a different sandwiched core has an aluminium, polymer or paper *honeycomb* structure (Figure 4.52(b)) sandwiched between thin sheets of metal or polymer. Such a structure has good stiffness and is very light and is often used for structural panels. Another form of sandwich panel has a core of low density, e.g. a foamed polymer, sandwiched between two stronger and stiffer panels.

Maths in action

Consider the elastic modulus for a laminate made up of alternate layers of materials with different stiffnesses and loaded in the way shown in Figure 4.53. If we assume that the bonding between the layers is such that each of the layers is equally strained then the problem of determining the modulus of elasticity of the laminate is just the same as that carried out earlier in this chapter for continuous fibres in a matrix. We can apply the law of mixtures [11] to give:

$$E_c = V_1 E_1 + V_2 E_2 \qquad [29]$$

where E_c is the elastic modulus of the composite, E_1 that of material 1 and E_2 that of material 2. V_1 is the volume ratio of material 1 and V_2 the volume ratio of material 2.

If the material is loaded as shown in Figure 4.54 then we assume that each of the layers is equally stressed and thus, as in equation [14]:

$$\frac{1}{E_c} = \frac{V_1}{E_1} + \frac{V_2}{E_2} \qquad [20]$$

Example

A sheet of plywood consists of three equally thick sheets, the upper and lower sheets having their grain in the same direction and the middle sheet with its grain at right angles. The wood has a tensile modulus for forces in the direction parallel to the grain of 10 GPa and in the transverse direction 0.4 GPa. Determine the tensile modulus of the laminate when loaded in a direction parallel to the grain direction of the outer sheets.

The situation is similar to that shown in Figure 4.53 and thus, using equation [29]: $E_c = V_1 E_1 + V_2 E_2 = (2/3) \times 10 + (1/3) \times 0.4 = 6.8$ GPa.

Figure 4.55   Second m

of area

Example

How thick would a rectangular strip of steel, modulus of elasticity 210 GPa, have to be to have the same bending stiffness as a laminate consisting of two 1.5 mm thick surfaces with an elastic modulus of elasticity 7 GPa and a core of foamed polymer of thickness 6 mm and a density which is 50% of the unfoamed polymer, it having a modulus of elasticity of 7 GPa.

For equal bending stiffness we must have the product EI the same for the laminate as for the steel. For the laminate, the core will have a tensile modulus given by equation [28] as:

$$E_f = \left(\frac{\rho_f}{\rho_s} \right)^{1.5} E_s = 0.5^{1.5} \times 7 = 2.47 \text{ GPa}$$

and a compression modulus of:

$$E_f = \left(\frac{\rho_f}{\rho_s} \right)^{2} E_s = 0.5^{2} \times 7 = 1.75 \text{ GPa}$$

One surface of a bent beam is in tension and one in compression, thus we will use the average modulus of 2.11 GPa for our comparison.

The bending stiffness of the laminate is the sum of the bending stiffness of the core and the surface sheets about the central axis. A rectangular section has a second moment of area about a central axis of $bd^3/12$, where b is the breadth and d the depth (Figure 4.55). The second moment of area for a rectangular section about a parallel axis a distance y from the axis through the centre of the rectangular section is $I + Ay^2$, where A is the area of the section (theorem of parallel axes). Thus for the laminate we have:

$$(EI)_{\text{laminate}} = 2.11 \times \frac{b \times 6^3}{12} + 7 \times 2 \times b \left[\frac{1.5^3}{12} + 1.5 \times 3.75^2 \right]$$

$$= 360b$$

For the steel to have the same value bending stiffness we must have $(EI)_{\text{steel}} = 360b$. This means $I = 360b/210 = 1.7b$. Hence:
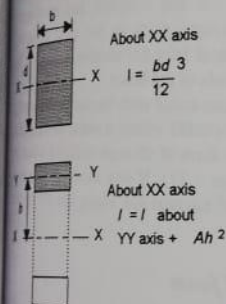
$$\frac{bd^3}{12} = 1.7b$$

and so $d = 2.7$ mm.

Figure 4.55 Second moment of area

About XX axis $I = \dfrac{bd^3}{12}$

About XX axis $I = I$ about YY axis $+ Ah^2$