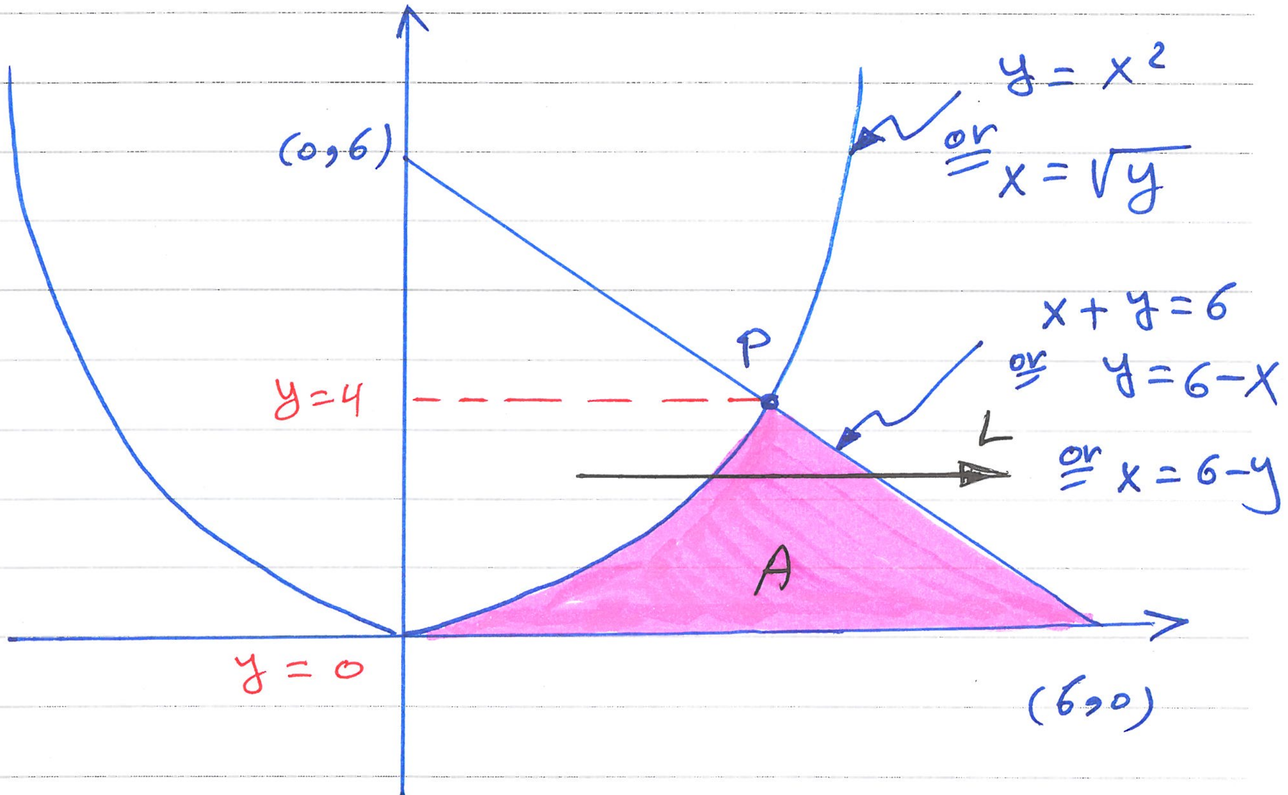
Changing The order in Double Integration:

Ex1: Revers the order of the following Integration:

$$\int_0^4 \int_{\sqrt{y}}^{6-y} f(x,y)\,dx\,dy$$



The intersection Point P

$y = x^2$ and $y = 6 - x$

$x^2 + x - 6 = 0$

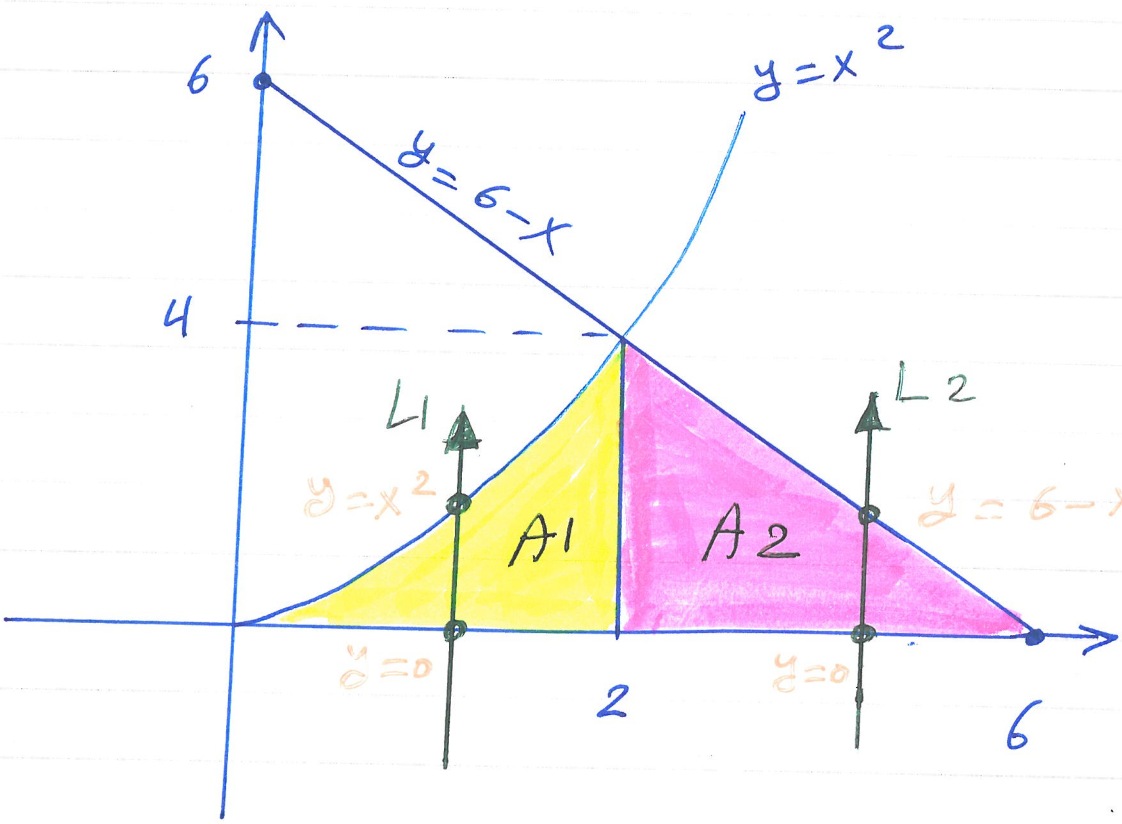
$(x+3)(x-2) = 0$

either $(x+3) = 0 \longrightarrow x = -3$ fail'd

or $(x-2) = 0 \longrightarrow x = 2$

$y = x^2 \therefore y = 4$

at $x = 2 \longrightarrow y = x^2 = 4$

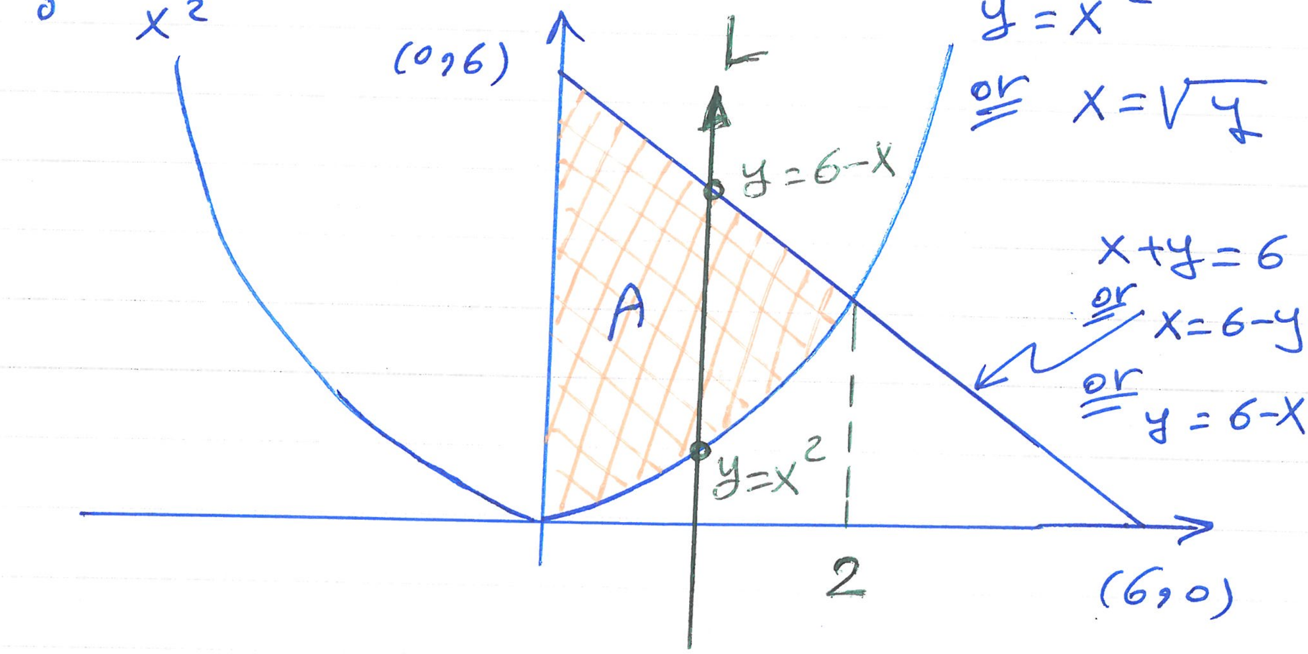


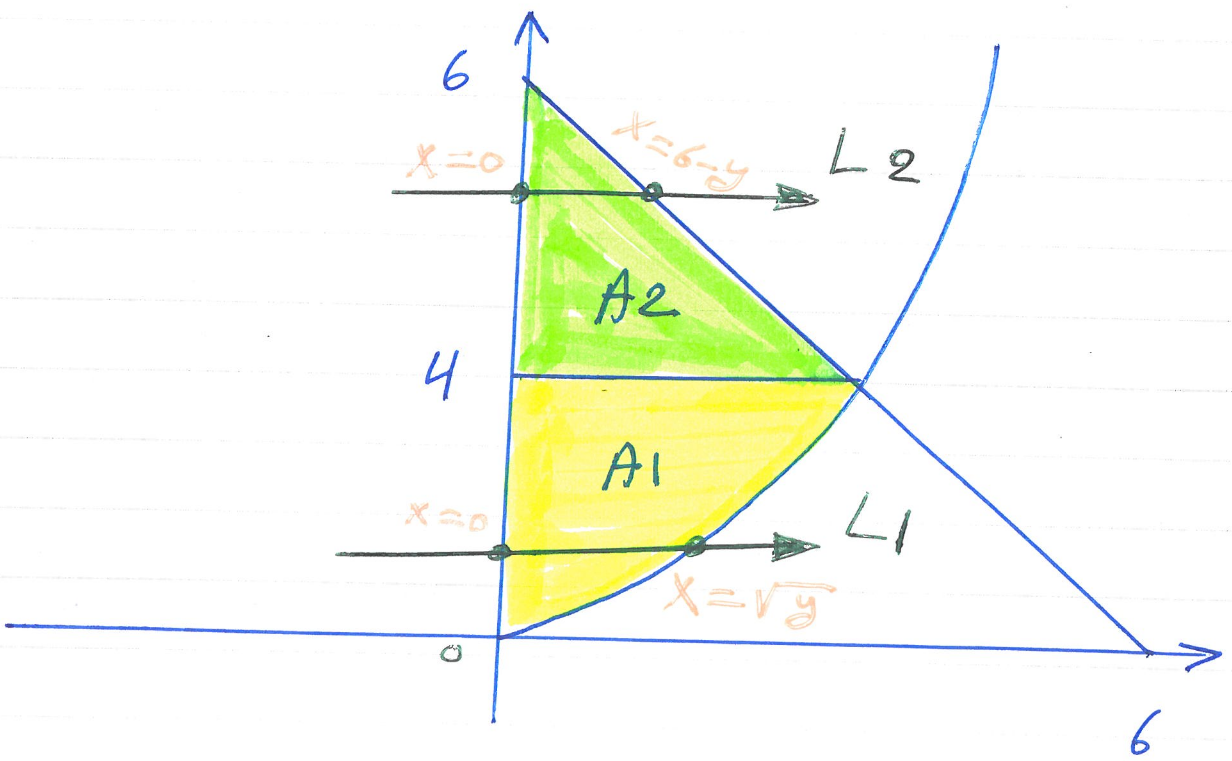
$$\int_{y=0}^{y=4} \int_{x=\sqrt{y}}^{x=6-x} f(x,y)\,dx\,dy$$

$$= \int_{x=0}^{x=2} \int_{y=0}^{y=x^2} f(x,y)\,dy\,dx + \int_{x=2}^{x=6} \int_{y=0}^{y=6-x} f(x,y)\,dy\,dx$$

Ex2: Revers the order of the following integration:

$$\int_0^2 \int_{x^2}^{6-x} f(x,y)\, dy\, dx$$



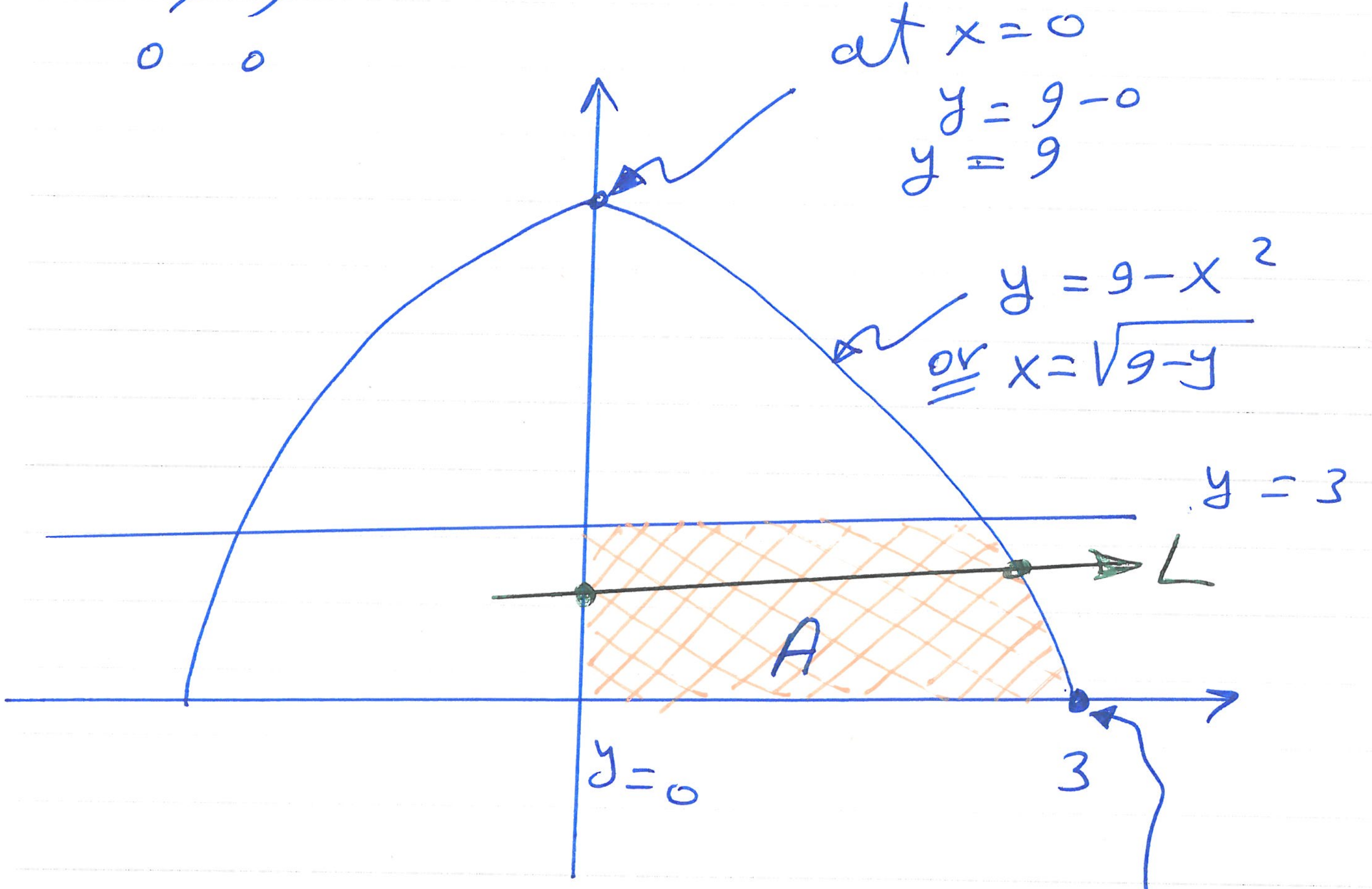


$$\int_{x=0}^{x=2}\int_{y=x^2}^{y=6-x} f(x,y)\,dy\,dx$$

$$= \int_{y=0}^{y=4}\int_{x=0}^{x=\sqrt{y}} f(x,y)\,dx\,dy + \int_{y=4}^{y=6}\int_{x=0}^{x=6-y} f(x,y)\,dx\,dy$$

Ex3: Revers the order of the following integration:

$$\int_0^3 \int_0^{\sqrt{9-y}} f(x,y)\,dx\,dy$$

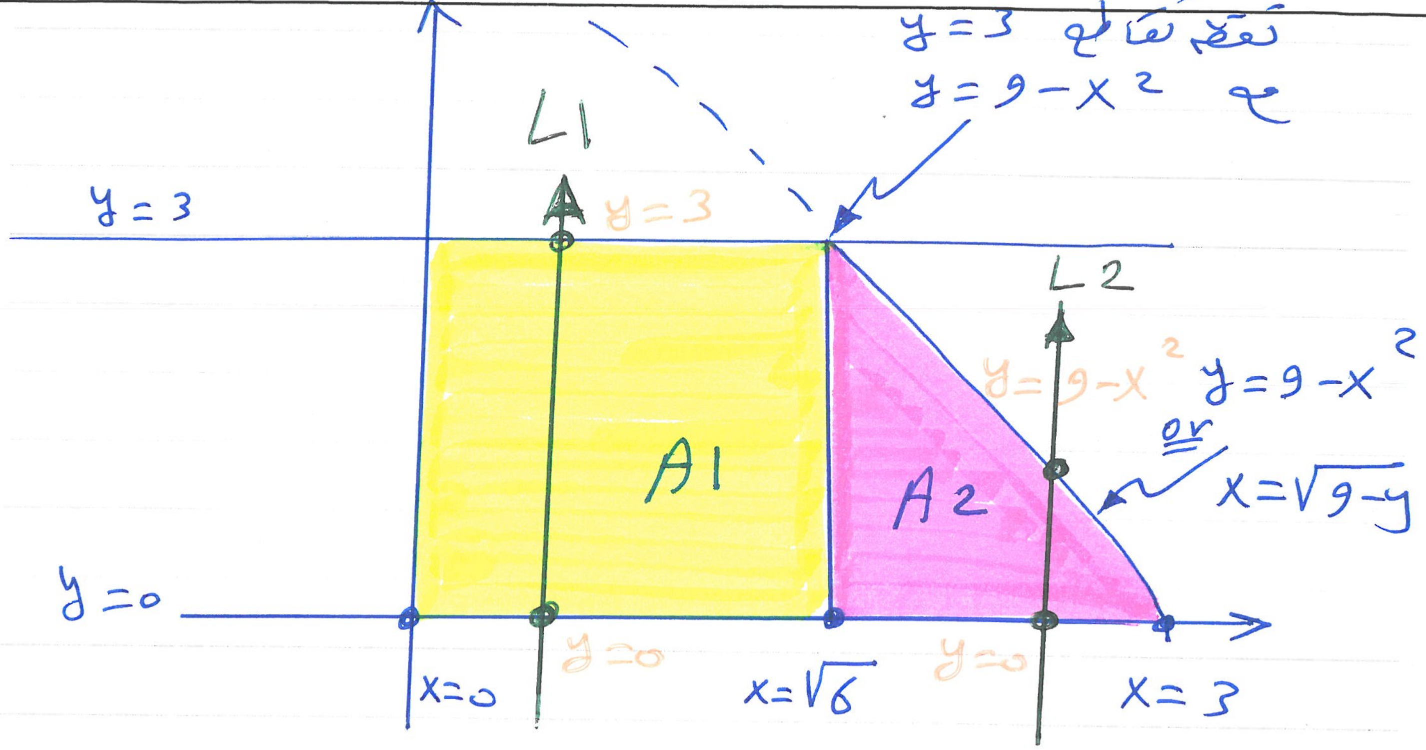


at $y=0$

$x=\sqrt{9-y}$

$=\sqrt{9}$

$=3$



$$y = 3 \ \& \ y = 9 - x^2$$

$$3 = 9 - x^2 \longrightarrow x^2 = 9 - 3 = 6$$

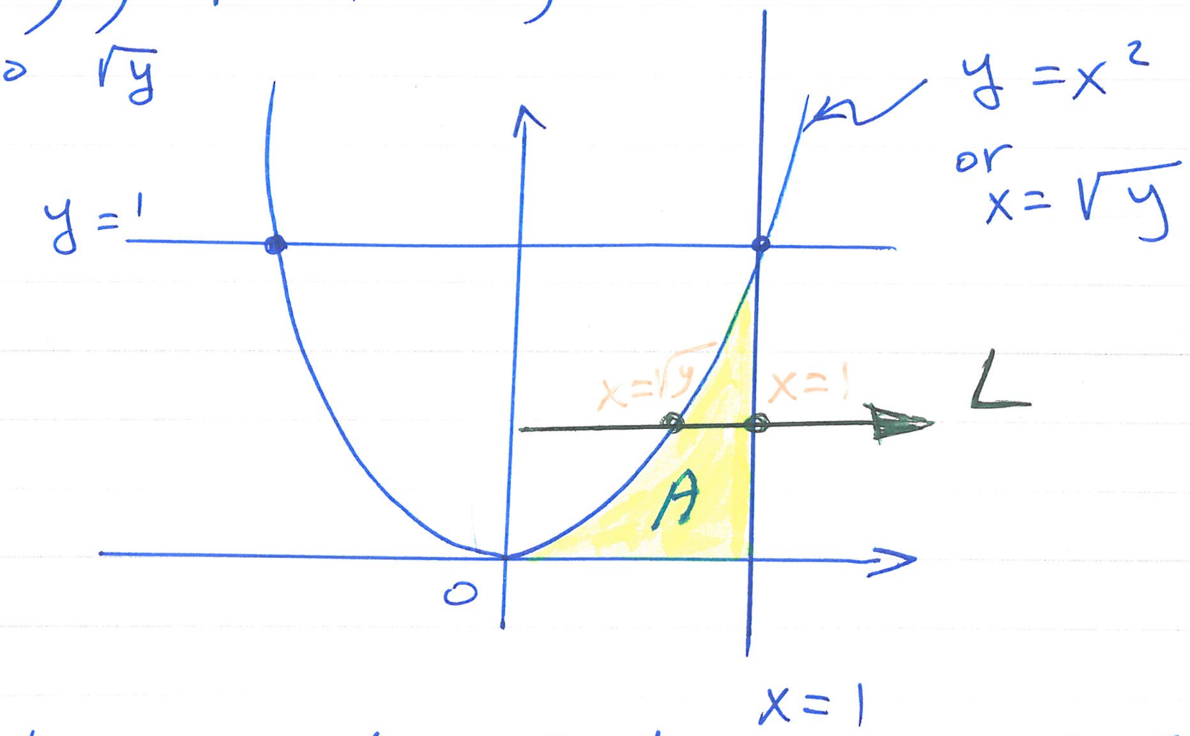
$$x = \sqrt{6}$$

$$\int_{y=0}^{y=3} \int_{x=0}^{x=\sqrt{9-y}} f(x,y)\, dx\, dy$$

$$= \int_{x=0}^{x=\sqrt{6}} \int_{y=0}^{y=3} f(x,y)\, dy\, dx + \int_{x=\sqrt{6}}^{x=3} \int_{y=0}^{y=9-x^2} f(x,y)\, dy\, dx$$

Ex4: Evaluate The integral

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\, dx\, dy$$



The intersection point

$y=1$ & $y=x^2$

$x^2=1 \longrightarrow x=\mp 1$

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$$\int_{y=0}^{y=1} \int_{x=\sqrt{y}}^{x=1} \sqrt{x^3+1}\, dx\, dy = \int_{x=0}^{x=1} \int_{y=0}^{y=x^2} \sqrt{x^3+1}\, dy\, dx$$

$$= \int_0^1 y\sqrt{x^3+1}\,\Big|_{y=0}^{y=x^2}\, dx$$

$$= \int_0^1 \left\{ \left[x^2 \sqrt{x^3+1} \right] - \left[0 \sqrt{x^3+1} \right] \right\} dx$$

$$= \int_0^1 x^2 \sqrt{x^3+1} \, dx$$

$$= \frac{1}{3} \int_0^1 3x^2 \sqrt{x^3+1} \, dx$$

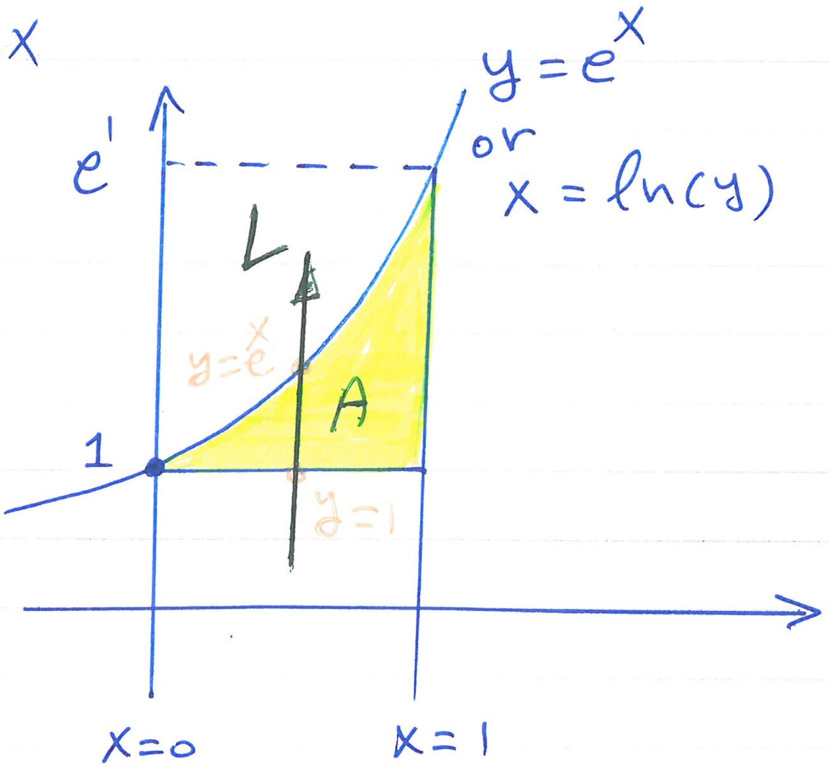
$$= \frac{1}{3} \frac{2}{3} (x^3+1)^{\frac{3}{2}} \Big|_0^1$$

$$= \frac{2}{9} \left[(1^3+1)^{\frac{3}{2}} - (0+1)^{\frac{3}{2}} \right]$$

$$= \frac{4\sqrt{2} - 2}{9}$$

EX5! Revers the order of integration in

$$\int_{0}^{1}\int_{1}^{e^x} dy\,dx$$

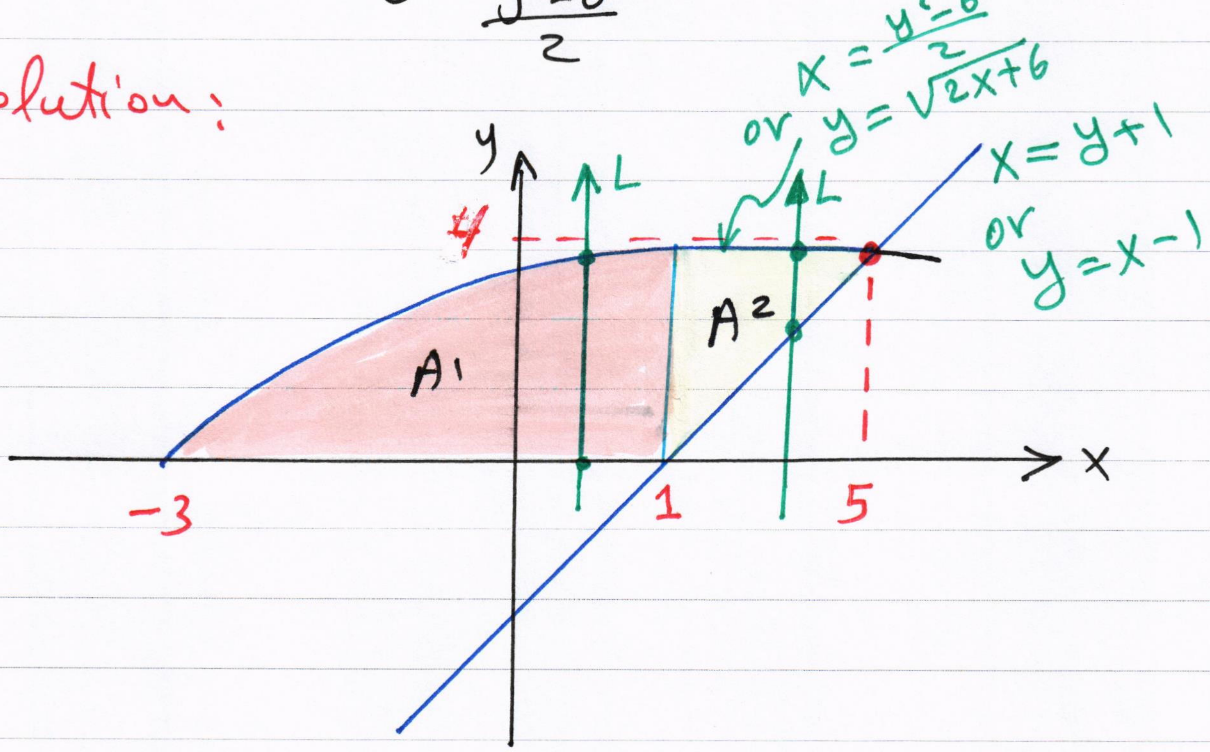


$$\int_{x=0}^{x=1}\int_{y=1}^{y=e^x} dy\,dx = \int_{y=1}^{y=e^1}\int_{x=\ln(y)}^{x=1} dx\,dy$$

Ex 6: Revers the order of the following integration:

$$\int_{0}^{4}\int_{\frac{y^2-6}{2}}^{y+1} f(x,y)\,dx\,dy$$

solution:



To find the intersection points:

$x = \frac{y^2-6}{2}$ and $x = y+1$

$\frac{y^2-6}{2} = y+1$

$y^2 - 6 = 2y + 2$

$y^2 - 2y - 8 = 0$

$$(y-4)(y+2)=0$$

either $y-4=0 \longrightarrow y=4$

$x=y+1=5$

or $y+2=0 \longrightarrow y=-2$, $x=-1$ $\Big\}$ تهمل لانها خارج الرسم

The intersection point is $(5,4)$

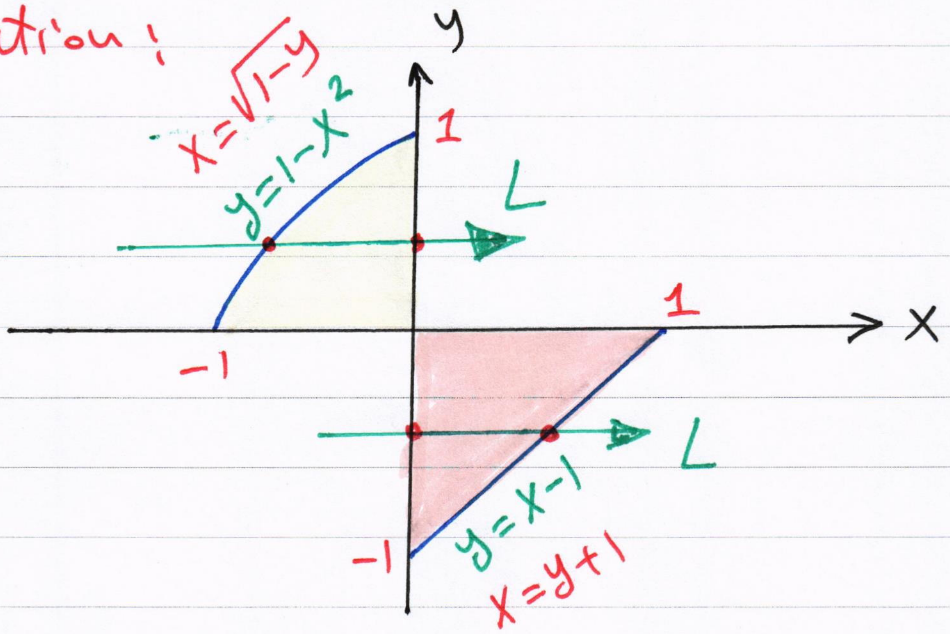
$$\int_{y=0}^{y=4}\int_{x=\frac{y^2-6}{2}}^{x=y+1} f(x,y)\,dx\,dy$$

$$=\int_{x=-3}^{x=1}\int_{y=0}^{y=\sqrt{2x+6}} f(x,y)\,dy\,dx+\int_{x=1}^{x=5}\int_{y=x-1}^{y=\sqrt{2x+6}} f(x,y)\,dy\,dx$$

EX 7: Revers the order of the following integration:

$$\int_{-1}^{0}\int_{0}^{1-x^2} f(x,y)\,dy\,dx + \int_{0}^{1}\int_{x-1}^{0} f(x,y)\,dy\,dx$$

Solution:

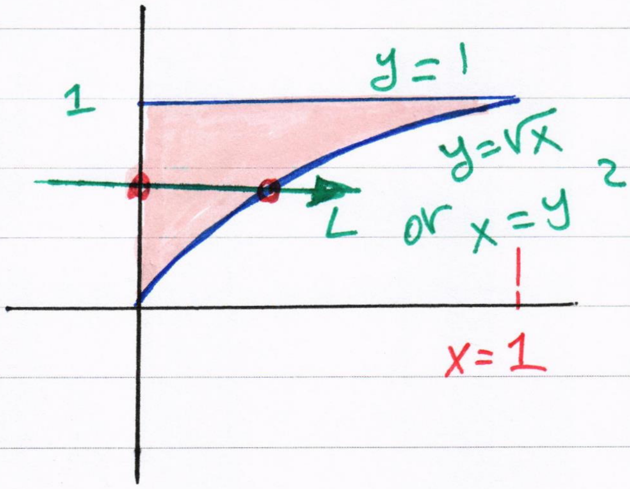


$$\int_{y=1}^{y=0}\int_{x=0}^{x=y+1} f(x,y)\,dx\,dy + \int_{y=0}^{y=+1}\int_{x=\sqrt{1-y}}^{x=0} f(x,y)\,dx\,dy$$

Ex 8: Evaluate $\int_{0}^{1}\int_{\sqrt{x}}^{1} e^{y^3}\, dy\, dx$

solution: مشتقة الدالة غير موجودة لذلك نعكس التكامل

$$\int_{y=0}^{y=1}\int_{x=0}^{x=y^2} e^{y^3}\, dx\, dy$$



$$= \int_{0}^{1} y^2 e^{y^3}\, dy$$

$$= \frac{1}{3}\int_{0}^{1} 3y^2 e^{y^3}\, dy$$

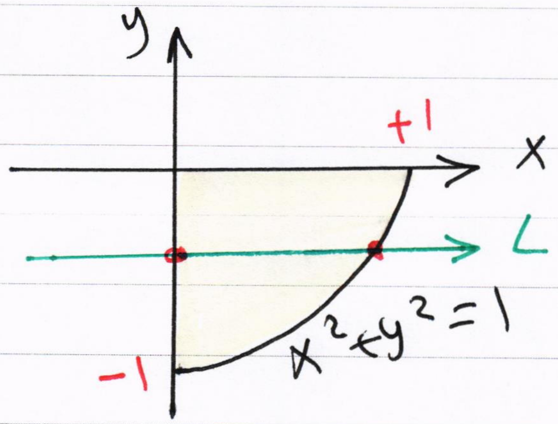
$$= \frac{1}{3}\left[e^{y^3}\right]_0^1$$

$$= \frac{1}{3}\left[e^1 - 1\right]$$

Ex (9): Evaluate the iterated integral

$$\int_{0}^{1}\int_{-\sqrt{1-x^2}}^{0} 2x\cos\left(y-\frac{y^3}{3}\right)dy\,dx$$

solution:



مشتقة cos غير موجودة لذلك نعكس حدود التكامل

$$\int_{x=0}^{x=1}\int_{y=-\sqrt{1-x^2}}^{y=0} 2x\cos\left(y-\frac{y^3}{3}\right)dy\,dx$$

$$=\int_{y=-1}^{y=0}\int_{x=0}^{x=\sqrt{1-y^2}} 2x\cos\left(y-\frac{y^3}{3}\right)dx\,dy$$

$$= \int_{y=-1}^{y=0} \left[x^2 \cos\left(y - \frac{y^3}{3}\right) \right]_0^{\sqrt{1-y^2}} dy$$

$$= \int_{-1}^{0} (1-y^2) \cos\left(y - \frac{y^3}{3}\right) dy$$

$$= \left[\sin\left(y - \frac{y^3}{3}\right) \right]_{-1}^{0}$$

$$= \sin 0 - \sin\left(-1 - \left(-\frac{1}{3}\right)\right)$$

$$= -\sin\left(-\frac{2}{3}\right)$$