# Lecture 2-Hydraulic / Pump

## 2.1 Basic Hydraulic Systems

Regardless of its function and design, every hydraulic system has a minimum number of basic components in addition to a means through which the fluid is transmitted. A basic system consists of (as in fig.2.1):

1. Pump
2. Reservoir,
3. Directional valve
4. Check valve
5. Pressure relieve valve
6. Selector valve
7. Actuator
8. Filter.

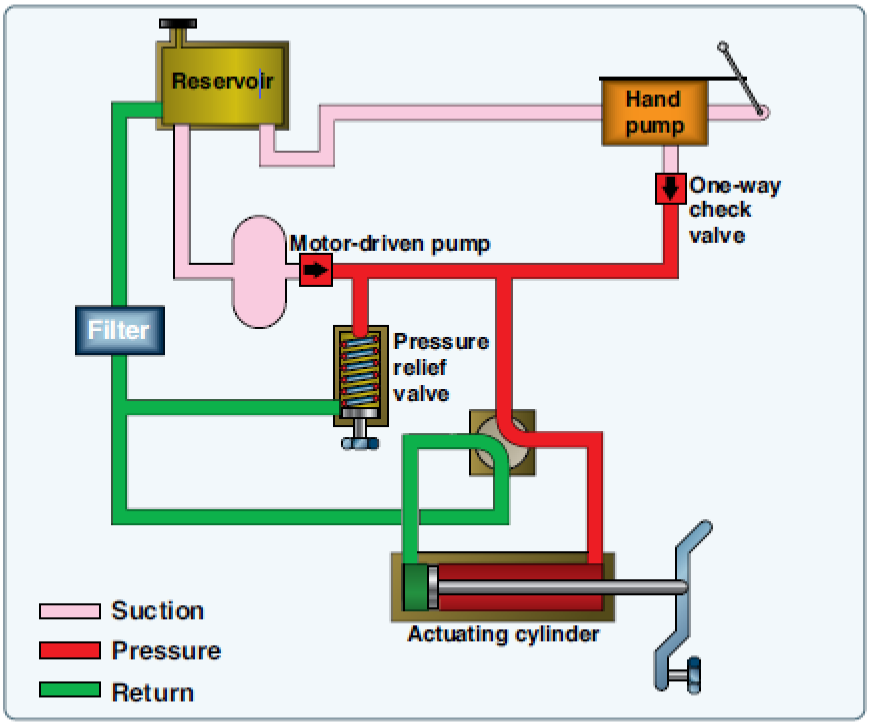



Figure 2.1: Basic hydraulic system.

## 2.2 Pump

The function of a pump is to convert mechanical energy into hydraulic energy. It is the heart of any hydraulic system because it generates the force necessary to move the load. Mechanical energy is delivered to the pump using a prime mover such as an electric motor. The hydraulic pump takes hydraulic fluid (mostly some oil) from the storage tank and delivers it to the rest of the hydraulic circuit.

Pumps can be broadly listed under two categories:

1. Non-positive displacement pumps and

2. Positive displacement pumps.

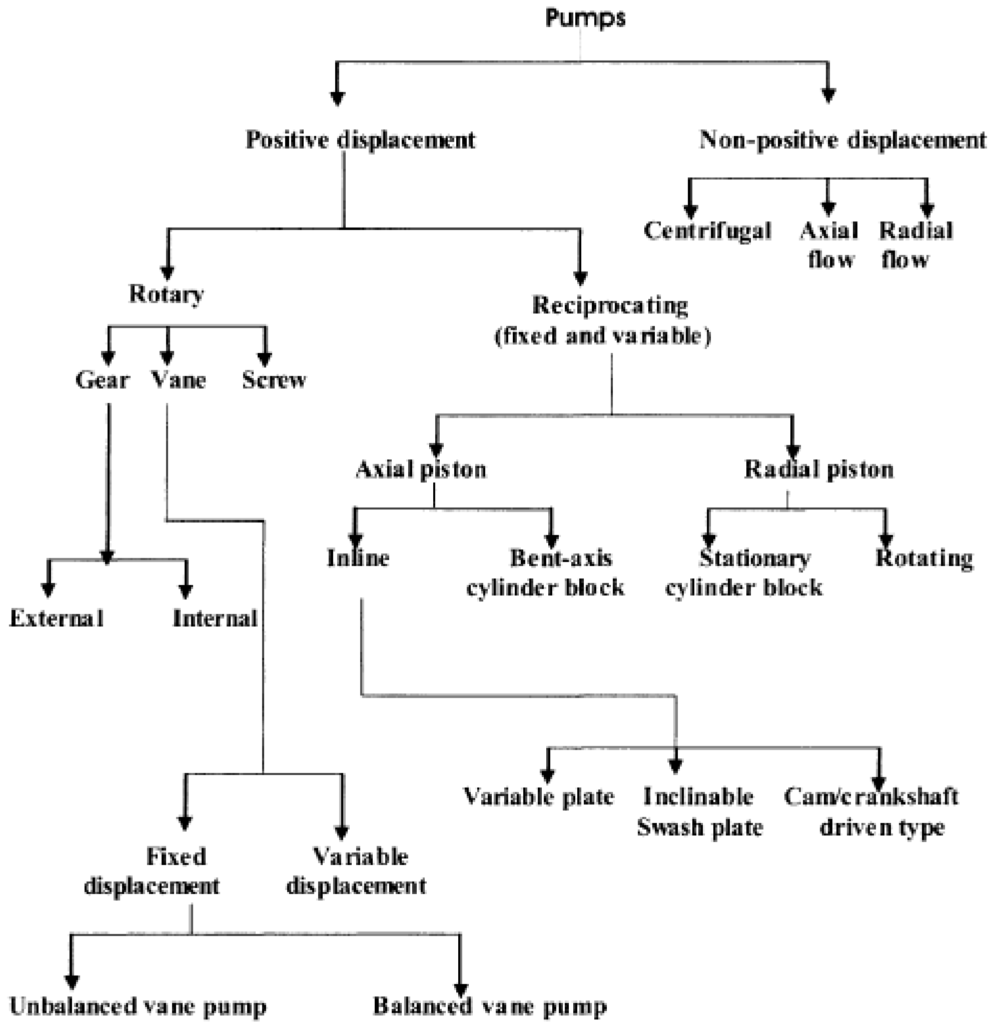



Figure 2.2: Classification of pumps type

### 2.2.1- Non-positive displacement pumps

Non-positive displacement pumps are primarily velocity-type units that have a great deal of clearance between rotating and stationary parts. These pumps can not withstanding high pressures and generally used for low-pressure and high-volume flow applications Normally their maximum pressure capacity is limited to 20-30 kgf/cm$^2$. They are primarily used for transporting fluids from one location to the other and find little use in the hydraulic or fluid power industry. The fluid motion is generated due to rotating propeller. These pumps provide a smooth and continuous flow but the flow output decreases with increase in system resistance (load). The flow output decreases because some of the fluid slip back at higher resistance.

Examples of these pumps are the centrifugal and axial (propeller) pumps. In a centrifugal pump, a simple sketch of which is illustrated in Figure 2.3, rotational inertia is imparted to the fluid. Centrifugal pumps are not self-priming and must be positioned below the fluid level.

### 2.2.1.1 Principle of operation

The fluid from the inlet port enters at the centre of the impeller. The rotating impeller imparts centrifugal force to the fluid and causes it to move radially outward. This results in the fluid being forced through the outlet discharge port of the housing. The tips of the impeller blades merely move through the fluid while the rotational speed maintains the fluid pressure corresponding to the centrifugal force established. Centrifugal pumps are generally used in pumping stations, for delivering water to homes and factories. The advantages of non-positive displacement pumps are:

1. Low initial cost and minimum maintenance
2. Simplicity of operation and high reliability
3. Capable of handling any type of fluid, for example sludge and slurries

The disadvantages are as follows:

1. Non-displacement pumps are not self-priming and hence they must be positioned below the fluid level.
2. Discharge is a function of output resistance.
3. Low volumetric efficiency.

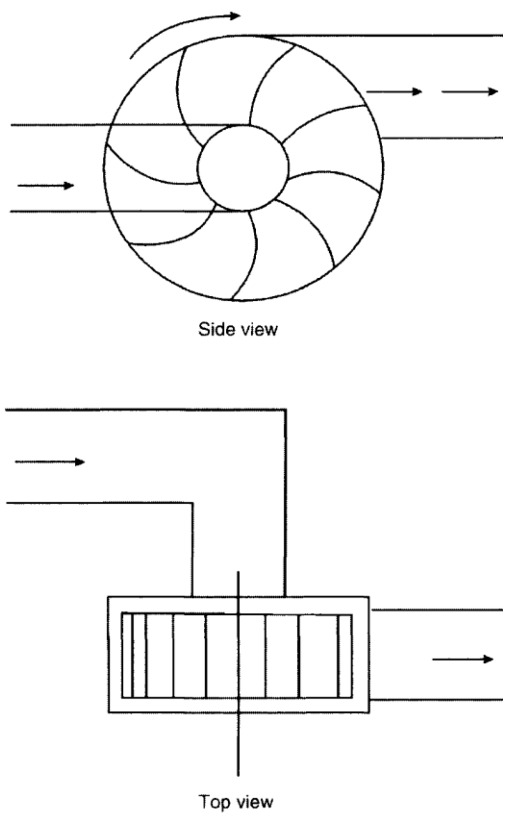



Figure 2.3: centrifugal pump

### 2.2.2 Positive Displacement Pumps

Positive displacement pumps, in contrast, have very little slips, are self-priming and pump against very high pressures, but their volumetric capacity is low. Positive displacement pumps have a very close clearance between rotating and stationary parts and hence are self-priming. Positive displacement pumps eject a fixed amount of fluid into the hydraulic system per revolution of the pump shaft. Such pumps are capable of overcoming the pressure resulting from mechanical loads on the system as well as the resistance of flow due to friction.

This equipment must always be protected by relief valves to prevent damage to the pump or system

#### 2.2.2.1 Principle of operation

The positive displacement hydraulic pump basically performs two functions:

- First, it creates a partial vacuum at the pump inlet port. This vacuum enables atmospheric pressure to force the fluid from the reservoir into the pump.
- Second, the mechanical action of the pump traps this fluid within the pumping cavities, transports it through the pump and forces it into the hydraulic system.

All pumps operate by creating a partial vacuum at the intake, and a mechanical force at the outlet that induces flow. This action can be best described by reference to a simple piston pump shown in Fig.2.4.

1. As the piston moves to the left, a partial vacuum is created in the pump chamber that holds the outlet valve in place against its seat and induces flow from the reservoir that is at a higher (atmospheric) pressure. As this flow is produced, the inlet valve is temporarily displaced by the force of fluid, permitting the flow into the pump chamber (suction stroke).

2. When the piston moves to the right, the resistance at the valves causes an immediate increase in the pressure that forces the inlet valve against its seat and opens the outlet valve thereby permitting the fluid to flow into the system. If the outlet port opens directly to the atmosphere, the only pressure developed is the one required to open the outlet valve (delivery stroke).

### 2.2.3 Gear-Type Power Pump

Gear pumps as the name suggests make use of the principle of two gears in mesh in order to generate pumping action. They are compact, relatively inexpensive and have few moving parts. Gear pumps are further classified as:

1. External gear pumps
2. Internal gear pumps

3. Lobe pumps and
4. Ge-rotor pumps. (meaning: Generated rotor)
5. Screw pump.

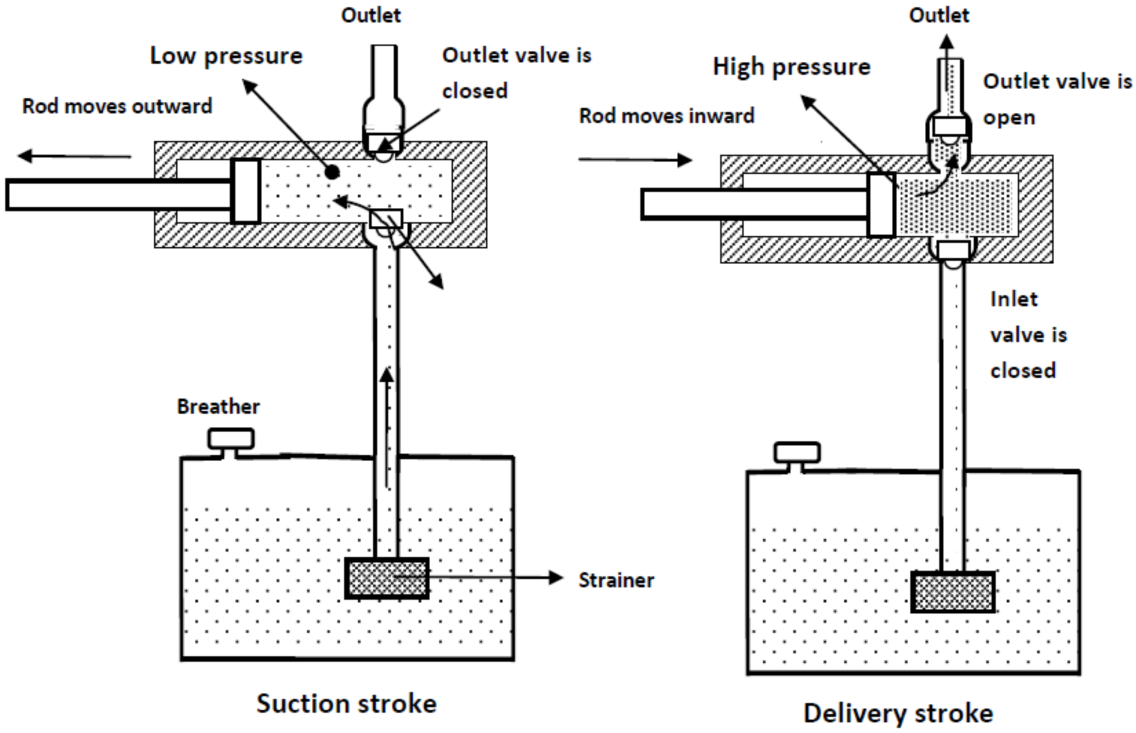
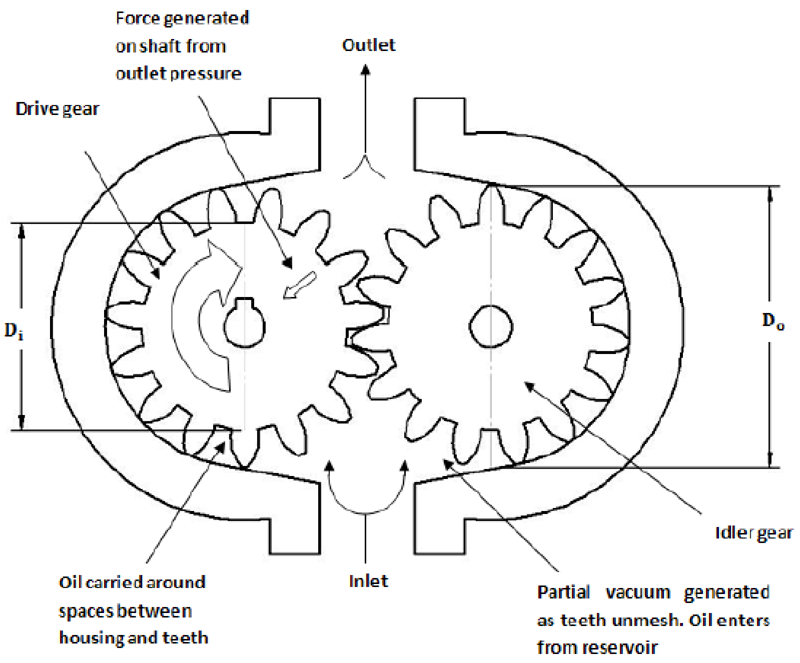


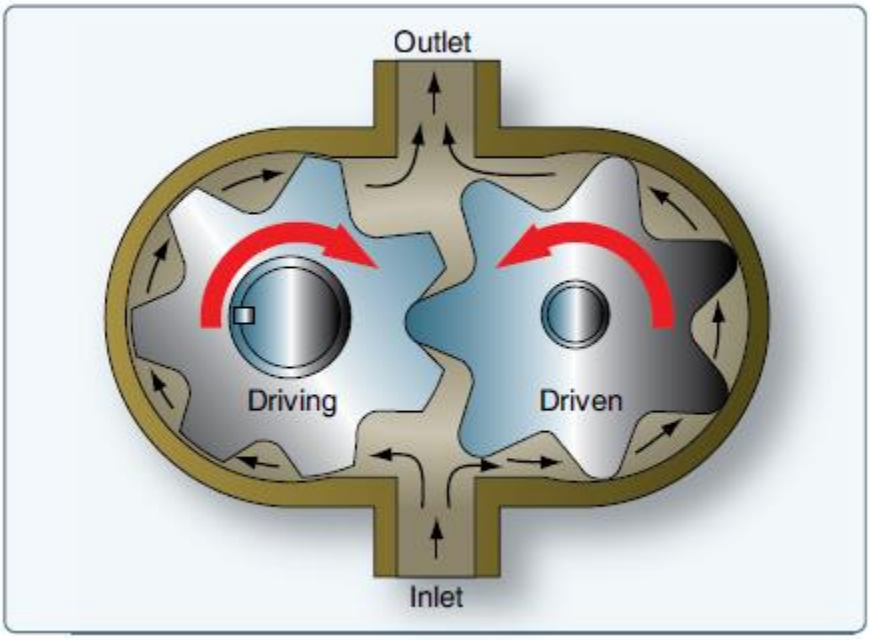

Figure 2.4 Illustration of pumping theory

### 2.2.3.1 External gear pumps

External gear pumps are the most popular hydraulic pumps in low-pressure ranges due to their long operating life, high efficiency and low cost. They are generally used in a simple machine. The most common form of external gear pump is shown in Figs. 2.5a and b It consist of a pump housing in which a pair of precisely machined meshing gears runs with minimal radial and axial clearance. One of the gears, called a driver, is driven by a prime mover. The driver drives another gear called a follower. As the teeth of the two gears separate, the fluid from the pump inlet gets trapped between the rotating gear cavities and pump housing. The trapped fluid is then carried around the periphery of the pump casing and delivered to outlet port. The teeth of precisely meshed gears provide almost a perfect seal between the pump inlet and the pump outlet. When the outlet flow is resisted, pressure in the pump outlet chamber builds up rapidly and forces the gear diagonally outward against the pump inlet. When the system pressure increases, imbalance occurs. This imbalance increases mechanical friction and the bearing load of the two gears. Hence, the gear pumps are operated to the maximum pressure rating stated by the manufacturer.




(a)




(b)

Figure 2.5: External gear pump (a) schematic (b) mechanism of operation

### 2.2.3.2 Advantages and disadvantages of gear pumps

The advantages are as follows:

1.They are self-priming.
2.They give constant delivery for a given speed.

3. They are compact and light in weight.
4. Volumetric efficiency is high.

The disadvantages are as follows:

1. The liquid to be pumped must be clean, otherwise it will damage pump.
2. Variable speed drives are required to change the delivery.
3. If they run dry, parts can be damaged because the fluid to be pumped is used as lubricant.

**2.2.3.3 Theoretical flow rate of an external gear pump**

Volume displacement is:

$$V_D = \frac{\pi}{4}(D_o^2 - D_i^2)L$$

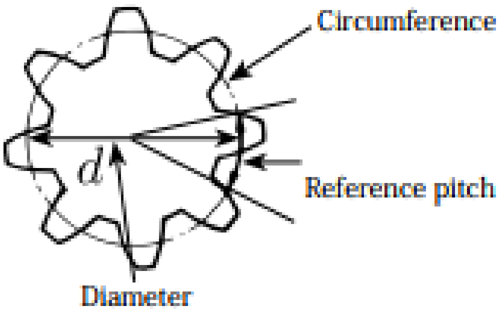
$$D_i = D_o - 2(\text{Addendum} + \text{Dendendum})$$

Theoretical discharge is:

$$Q_T\ (\text{m}^3/\text{min}) = V_D\ (\text{m}^3/\text{rev}) \times N\ (\text{rev/min})$$

If the gear is specified by its module and number of teeth, then the theoretical discharge can be found by:

$$Q_T = 2\pi L m^2 N\left[z + \left(1 + \frac{\pi^2 \cos^2 20}{12}\right)\right] \text{m}^3/\text{min}$$




$m = \frac{d}{z}$ (Module = $\frac{\text{Reference diameter}}{\text{Number of teeth}}$ )

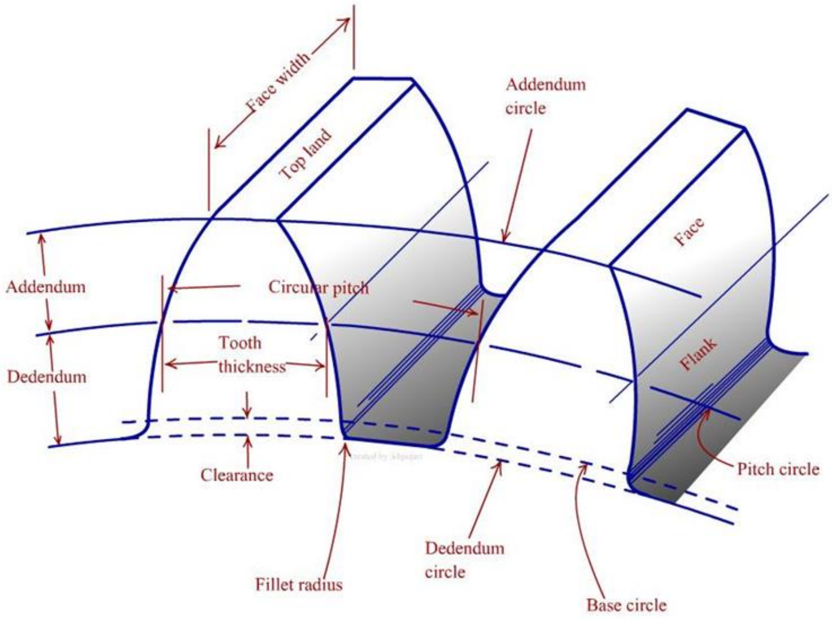



Figure 2.6: Terminology of spur gears

Where

*D*o =the outside diameter of gear teeth
*D*i= the inside diameter of gear teeth
*L* = the width of gear teeth
*N* = the speed of pump in RPM
*V*D = the displacement of pump in m/rev
*M* = module of gear
*z* =number of gear teeth
α = pressure angle

### 2.2.3.2 Internal Gear Pumps

Internal gear pump is another variation of the basic gear pump. Figure 2.7 illustrates clearly, the internal construction and operation of an internal gear pump. The design consists of an internal gear, a regular spur gear, a crescent-shaped seal and an external housing. As power is applied to either gear, the motion of the gears draws the fluid from the suction, and forces it around both the sides of the crescent seal. This acts as a seal between the suction and discharge ports. When the teeth mesh on the side opposite to the crescent seal, the fluid is forced out through the discharge port of the pump. Similar to the external gear pump, internal gear pumps also have an in-built safety relief valve.

Figure 2.7 Schematic diagram of internal gear pump operation

#### 2.2.3.3 Lobe pump

The lobe pump is yet another variation of the basic gear pump. This pump operates in a fashion quite similar to that of an external gear pump, but unlike external gear pumps, the gears in these pumps are replaced with lobes which usually consist of three teeth. Figure 2.8 shows the operation of a lobe pump. Unlike the external gear pumps, both the lobes are driven externally so that they do not actually make contact with each other. They are quieter than the other gear pumps. Due to the smaller number of mating elements, the lobe pump will show a greater amount of pulsation. However, its volumetric displacement is generally greater than other types of gear pumps. Although these pumps have a low-pressure rating, they are well-suited for applications involving shear-sensitive fluids.

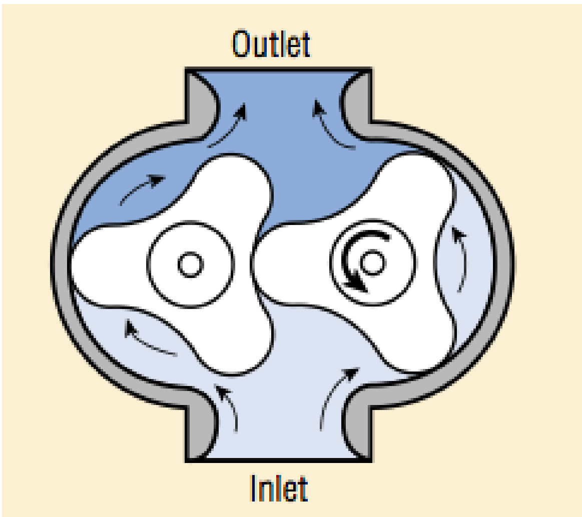



Figure 2.8 Schematic diagram of Lobe pump operation

#### 2.2.3.4 Gerotor pump

The gerotor internal-gear pump consists of a pair of gears which are always in sliding contact. The internal gear has one more tooth than the gerotor gear. Both gears rotate in the same direction as in fig.2.9. Oil is drawn into the chamber where the teeth are separating, and is ejected when the teeth start to mesh again. The seal is provided by the sliding contact.

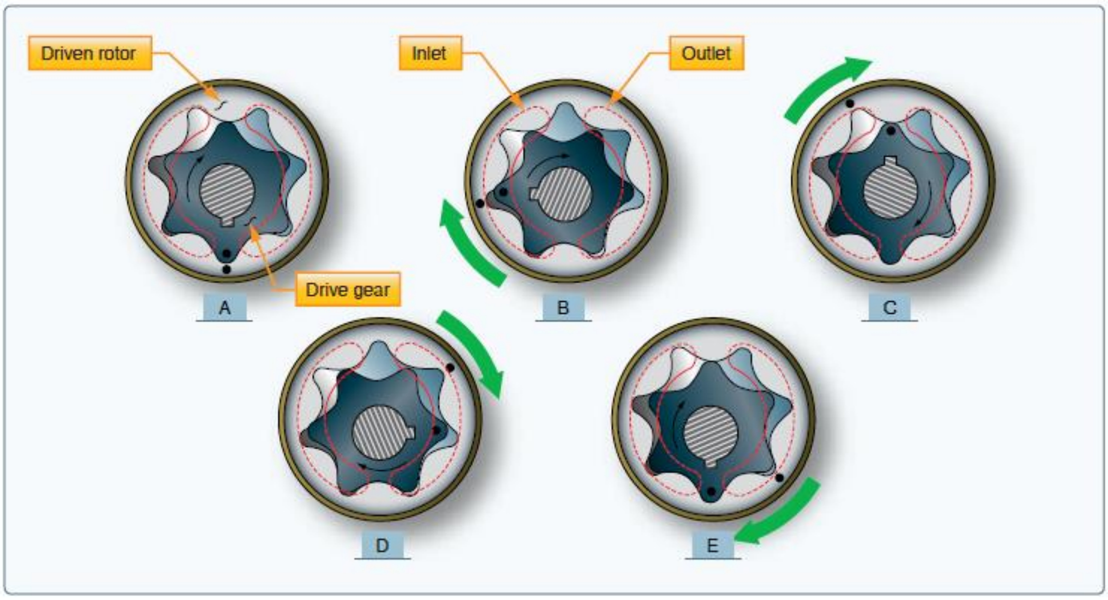



Figure 2.8 Schematic diagram of Gerotor pump operation

**Example 2.1**: The inlet to a hydraulic pump is 0.6 m below the top surface of an oil reservoir. If the specific gravity of the oil used is 0.86, determine the static pressure at the pump inlet.

$$\text{Pressure} = \rho g h$$

The density of water is 1 $g/cm^3$ or 1000 $kg/m^3$.

Therefore, the density of oil is $0.86 \times 1\ g/cm^3$ or $860\ kg/m^3$.

Pressure at the pump inlet is

$$P = 860 \times 0.6 kg/m^2 = 516 kg/m^2 = 0.0516 kg/cm^2 = 0.0516 \times 0.981 bar$$
$$= 0.0506 bar$$

(Note: $1 kg/cm^2 = 0.981 bar$.)

**Example 2.2:** A hydraulic pump delivers 12 L of fluid per minute against a pressure of 200 bar. (a) Calculate the hydraulic power. (b) If the overall pump efficiency is 60%, what size of electric motor would be needed to drive the pump?

Solution:

(a) Hydraulic power is given by

$$\text{Hydraulic power (kW)} = 12\ \text{L/min} \times \frac{200\ \text{(bar)}}{600} = 4\ \text{kW}$$

(b) We have

$$\text{Electric motor power (power input)} = \frac{\text{Hydraulic power}}{\text{Overall efficiency}}$$

Substituting we get

$$\text{Electric motor power (power input)} = \frac{4}{0.6} = 6.67kW$$

$$\text{Electric motor power} = \frac{4}{0.6} = 6.67\,\text{kW}$$

**Example 2.3:**

A gear pump has an outside diameter of 80mm, inside diameter of 55mm and a width of 25mm. If the actual pump flow is 1600 RPM and the rated pressure is 95 LPM what is the volumetric displacement and theoretical discharge.

**Solution:** We have

Outside diameter $D_o = 80$ mm

Inside diameter $D_i = 55$mm

Width $d = 25$mm

Speed of pump $N = 1600$ RPM

Actual flow rate = 95 LPM

Now

$$Q_A = 95\,\text{LPM} = 95 \times 10^{-3}\,\text{m}^3/\text{min}$$

$$V_D = \frac{\pi}{4} \times (D_o^2 - D_i^2) \times L$$

$$V_D = \frac{\pi}{4} \times (0.080^2 - 0.055^2) \times 0.025 = 6.627 \times 10^{-5} m^3 / rev$$

Theoretical flow rate

$$Q_T = \frac{\pi}{4} \times (D_o^2 - D_i^2) \times L \times N$$

$$= \frac{\pi}{4} \times (0.080^2 - 0.055^2) \times 0.025 \times 1600$$

$$= 0.106\,\text{m}^3/\text{min}$$

**Example 2.4:** Calculate the theoretical delivery of a gear pump. Module of the gear teeth is 6mm and width of gear teeth is 25mm. Number of teeth on driver gear is 18 and pressure angle of the gear is 20. Pump speed is 1000 RPM. Volumetric efficiency is 90%.

Solution: If the gear is specified by its module and number of teeth, then the theoretical discharge can be found by:

$$Q_T = 2\pi L m^2 N\left[z + \left(1 + \frac{\pi^2 \cos\alpha}{12}\right)\right] \text{ m}^3/\text{min}$$

$$= 2\pi(0.025)(6\times10^{-3})^2 \times 1000 \times \left[18 + \left(1 + \frac{\pi^2 \cos^2 20}{12}\right)\right] \text{ m}^3/\text{min}$$

$$= 0.1118 \text{ m}^3/\text{min}$$

**Example 2.5**: Calculate the theoretical delivery of a gear pump. Module of the gear teeth is 6mm and width of gear teeth is 65mm. Number of teeth on driver gear is 16 and pressure angle of the gear is 20. Pump speed is 1600 RPM. Outer diameter of gear is 108 mm and Dedendum circle diameter is 81 mm. Volumetric efficiency is 88% at 7 MPa.

Solution: If the gear is specified by its module and number of teeth, then the theoretical discharge can be found by

$$Q_T = 2\pi L m^2 N\left[z + \left(1 + \frac{\pi^2 \cos^2 20}{12}\right)\right] \text{ m}^3/\text{min}$$

$$= 2\pi(0.065)(6\times10^{-3})^2 \times 1600 \times \left[16 + \left(1 + \frac{\pi^2 0.939^2}{12}\right)\right] \text{ m}^3/\text{min}$$

$$= 0.416 \text{ m}^3/\text{min}$$

**Alternatively we can use**

$$V_D = \frac{\pi}{4} \times (D_o^2 - D_i^2) \times L$$

$$Q_T = \frac{\pi}{4} \times (0.108^2 - 0.081^2) \times 0.065 \times 1600 = 0.416 \text{ m}^3/\text{rev}$$

### 2.2.3.4 Screw pump

A schematic diagram of a screw pump is shown in Fig 2.9. A two-screw pump consists of two parallel rotors with inter-meshing threads rotating in a closely machined casing. The driving screw and driven screw are connected by means of timing gears. When the screws turn, the space between the threads is divided into compartments. As the screws rotate, the inlet side of the pump is flooded with hydraulic fluid because of partial vacuum. When the screws turn in normal rotation, the fluid contained in these compartments is pushed uniformly along the axis toward the centre of the pump, where the compartments discharge the fluid. Here the fluid does not rotate but moves linearly as a nut on threads. Thus, there are no pulsations at a higher speed; it is a very quiet operating.

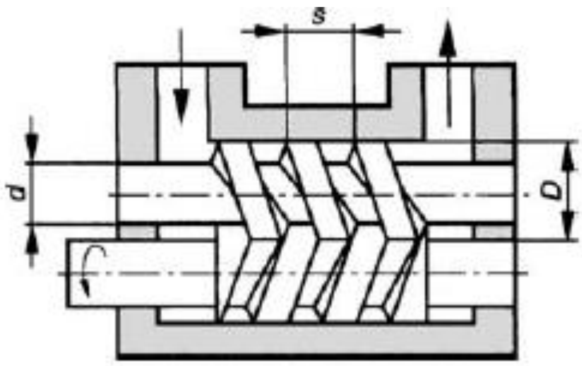



Figure 2.9: A twin-gear screw pump

The following is an expression for the geometric volume of a twin-gear screw pump:

$$V_g = \frac{\pi}{4}(D^2 - d^2)s - D^2\left(\frac{\alpha}{2} - \frac{\sin 2\alpha}{2}\right)s$$

with $\cos(\alpha) = \frac{D+d}{2D}$

**2.2.3.5 Advantages and disadvantages of screw pump**

1. The advantages are as follows:
2. They are self-priming and more reliable.
3. They are quite due to rolling action of screw spindles.
4. They can handle liquids containing gases and vapor.
5. They have long service life.

The disadvantages are as follows:

1. They are bulky and heavy.
2. They are sensitive to viscosity changes of the fluid.
3. They have low volumetric and mechanical efficiencies.
4. Manufacturing cost of precision screw is high.

**2.2.3.5 Vane Pumps**

A vane pump (Fig2.10) has a series of vanes that slide back and forth in slots. There are springs in these slots that push the vanes out until the tip contacts the cam ring. (Some designs port pressurized fluid into the slots to force the vanes out.) A chamber is formed between adjacent vanes and the cam ring. As the rotor turns, the chamber decreases in size. Fluid flows into this chamber when it is a maximum size and exits during some $\Delta\theta$ of rotation when it is a minimum size. This change in chamber size provides the *pumping action.*

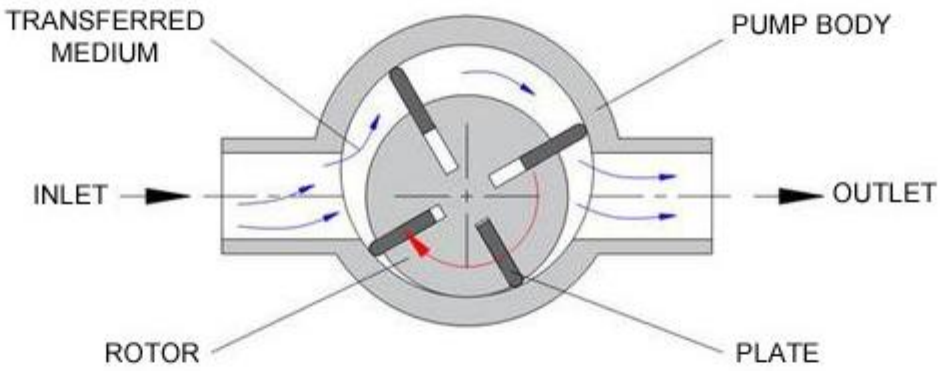



Figure 2.10 Schematic diagram of Unbalanced vane pump

Schematic diagram of variable displacement vane pump is shown in Fig.2.11. Variable displacement feature can be brought into vane pumps by varying eccentricity between the rotor and the cam ring. Here in this pump, the stator ring is held against a spring loaded piston. The system pressure acts directly through a hydraulic piston on the right side. This forces the cam ring against a spring-loaded piston on the left side. If the discharge pressure is large enough, it overcomes the compensated spring force and shifts the cam ring to the left.

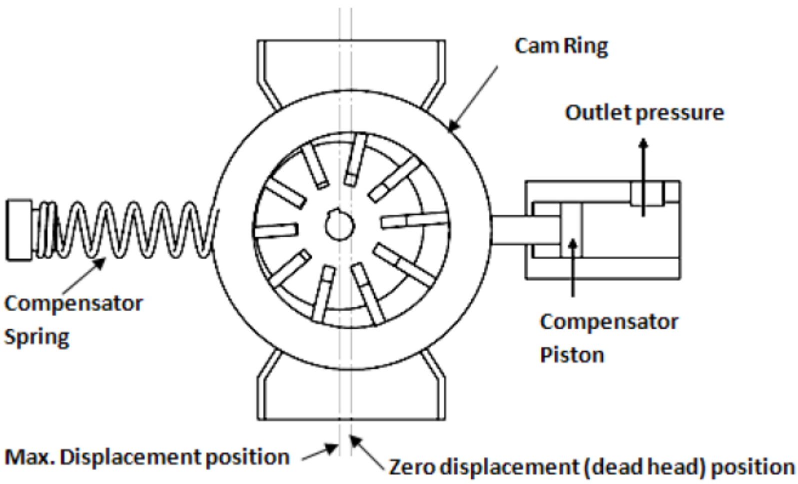



Figure 2.11: Operation of a variable displacement vane pump

The advantages of vane pumps are as follows:

1. Vane pumps are self-priming, robust and supply constant delivery at a given speed.
2. They provide uniform discharge with negligible pulsations.
3. Their vanes are self-compensating for wear and vanes can be replaced easily.
4. These pumps do not require check valves.
5. They are light in weight and compact.

6. They can handle liquids containing vapors and gases.
7. Volumetric and overall efficiencies are high.
8. Discharge is less sensitive to changes in viscosity and pressure variations.

The disadvantages of vane pumps are as follows:

1. Relief valves are required to protect the pump in case of sudden closure of delivery.
2. They are not suitable for abrasive liquids.
3. They require good seals.
4. They require good filtration systems and foreign particle can severely damage pump.

### 2.2.3.6 The Theoretical Discharge of Vane Pumps

Let $D_C$ be the diameter of a cam ring in m, $D_R$ the diameter of rotor in m, $L$ the width of rotor in m, $e$ the eccentricity in m, $V_D$ the pump volume displacement in m3/rev and $e_{max}$ the maximum possible eccentricity in m.

$$e_{max} = \frac{D_C - D_R}{2}$$

The maximum value of eccentricity produces the maximum volumetric displacement

$$V_{D(max)} = \frac{\pi}{4}(D_C^2 - D_R^2)L$$

$$V_{D(max)} = \frac{\pi}{4}(D_C - D_R)(D_C + D_R)L$$

$$V_{D(max)} = \frac{\pi}{4}(D_C + D_R) \times 2e_{max}L$$

The actual volumetric displacement occurs when $e_{max} = e$. Hence,

$$V_{D(max)} = \frac{\pi}{2}(D_C + D_R)e\,L \text{ m}^3\text{/rev}$$

When the pump rotates at $N$ rev/min (RPM), the quality of discharge by the vane pump is given by

$$Q_T = V_D N$$

Theoretical discharge

$$Q_T = \frac{\pi}{2}(D_C + D_R)e\,L \ \text{m}^3\text{/min}$$

**Example 2.6**

A vane pump has a rotor diameter of 63.5 mm, a cam ring diameter of 88.9 mm and a vane width of 50.8 mm. What must be eccentricity for it to have a volumetric displacement of 115cm$^3$?

**Solution:** Volumetric displacement is

$$V_D = \pi\left(\frac{D_C + D_R}{2}\right)Le$$

where$D_c$is the diameter of the cam ring, $D_R$ is the diameter of the rotor, $e$ is the eccentricity and $L$ is the width of the vane pump. So we have

$$115\times10^{-6} = \pi\times\frac{0.0889+0.0635}{2}\times e\times 0.0508$$

Therefore eccentricity

$$e = 9.456\times10^{-3}\text{ m} = 9.456\text{ mm}$$

### 2.2.3.7 Axial piston pump

Axial piston pumps convert rotary motion of an input shaft to an axial reciprocating motion of the pistons. They in turn are categorized as:

(a) Bent-axis-type piston pumps and
(b) Swash plate-type inline piston pumps.

These two types are discussed separately below.

**(a) Bent-axis-type piston pumps**

In these pumps, the reciprocating action of the pistons is obtained by bending the axis of the cylinder block so that it rotates at an angle different than that of the drive shaft. The cylinder block is turned by the drive shaft through a universal link. The centreline of the cylinder block is set at an offset angle, relative to the centerline of the drive shaft. The cylinder block contains a number of pistons along its periphery. These piston rods are connected to the drive shaft flange by ball-and-socket joints. These pistons are forced in and out of their bores as the distance between the drive shaft flange and the cylinder block changes. A universal link connects the block to the drive shaft, to provide alignment and a positive drive. Figure 2.12 shows a bent-axis-type piston pump. The volumetric displacement of the pump varies with the offset angle $\theta$ There is no flow when the cylinder block centerline is parallel to the drive shaft centerline. $\theta$ can vary from 0° to 30°. Fixed displacement units are usually provided with 23° or 30° offset angles.

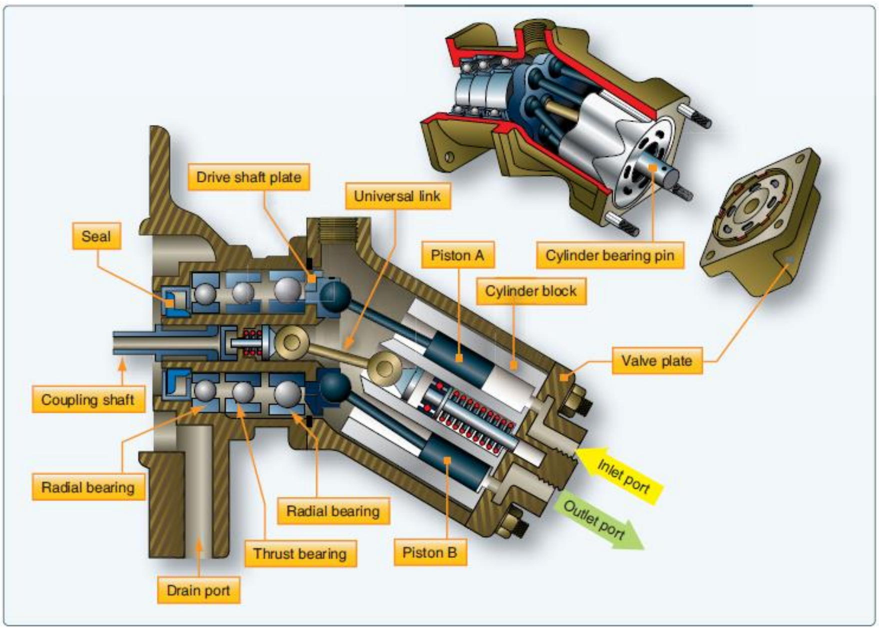
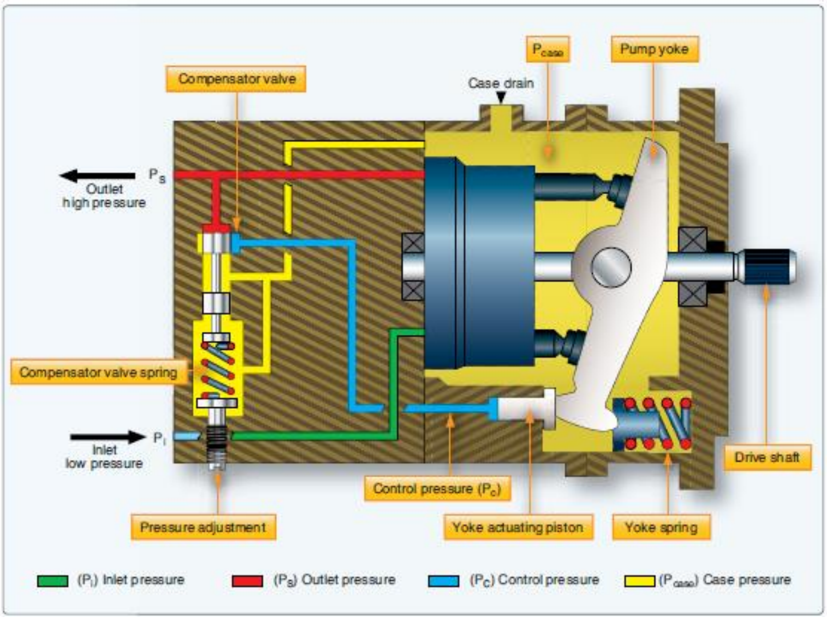


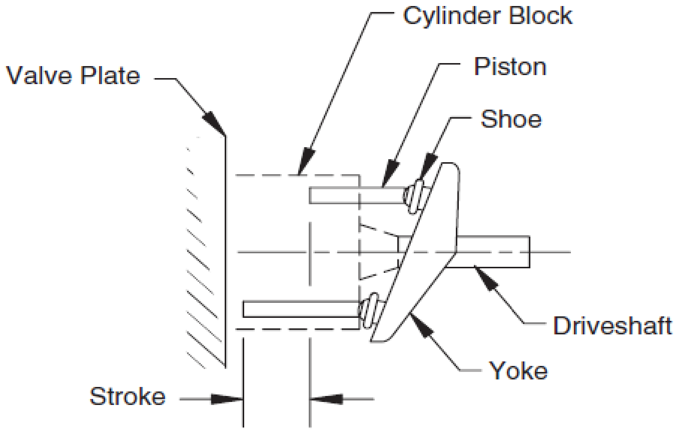

Figure 2.12: Bent axis piston pump

**(b) Swash-Plate-Type Piston Pump**

Schematic diagram of swash plate type piston pump is shown in Fig. 2.13a and b. In this type, the cylinder block and drive shaft are located on the same centreline. The pistons are connected to a shoe plate that bears against an angled swash plate. As the cylinder rotates, the pistons reciprocate because the piston shoes follow the angled surface of the swash plate. The outlet and inlet ports are located in the valve plate so that the pistons pass the inlet as they are being pulled out and pass the outlet as they are being forced back in. This type of pump can also be designed to have a variable displacement capability. The maximum swash plate angle is limited to 17.5° by construction. Figure 2.14 describes the motion of one piston during a single rotation of the cylinder block.




**(a)**




**(b)**

Figure 2.13 (a), (b) Schematic diagram of Swash-Plate-Type Piston Pump

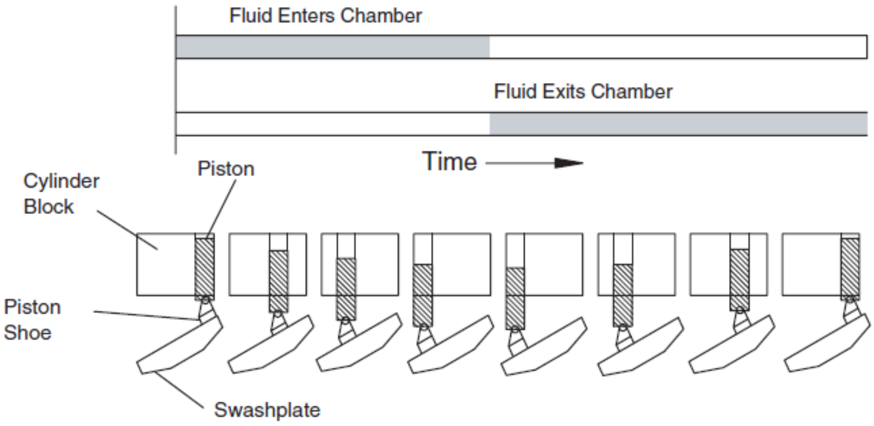



Figure 2.14: Schematic illustrating the motion of one piston during a single rotation of the cylinder block

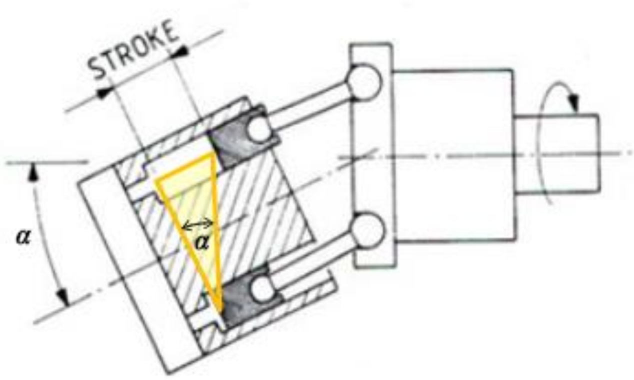



Figure 2.15: stroke displacement according to angle of Bent axis piston pump

α: offset angle, °

S: piston stroke, m

*D:* the piston circle diameter

*Y:* the number of pistons

*A:* the piston area in $m^2$

*N:* the piston speed in RPM

$Q_T$: the theoretical flow rate in $m^3$/min.

$$tan\alpha = \frac{S}{D}$$

$$S = Dtan(\alpha)$$

$$V_D = YAS = YADtan(\alpha)$$

$$Q_T = V_D N = DANY \tan(\alpha)$$

**Example 2.7:**

What is the theoretical flow rate from a fixed-displacement axial piston pump with a nine-bore cylinder operating at 2000 RPM? Each bore has a diameter of 15 mm and stroke is 20 mm.

Theoretical flow rate is given by

$$Q_T = \text{Volume} \times \text{RPM} \times \text{Number of pistons}$$

$$= \frac{\pi}{4} \times D^2 \times L \times N \times n$$

$$= \frac{\pi}{4} \times 0.015^2 \times 0.02 \times \frac{2000}{60} \times 9$$

$$= 10.6 \times 10^{-3} \text{m}^3/\text{s}$$

### 2.2.4 Pump Performance

The performance of a pump is a function of the precision of its manufacture. An ideal pump is one having zero clearance between all mating parts. Because this is not possible, working clearances should be as small as possible while maintaining proper oil films for lubrication between rubbing parts. The performance of a pump is determined by the following efficiencies:

1- **Volumetric efficiency ($\eta_v$):** It is the ratio of actual flow rate of the pump to the theoretical flow rate of the pump. This is expressed as follows:

$$\text{Volumetric efficiency } (\eta_v) = \frac{\text{Actual flow rate of the pump}}{\text{Theoretical flow rate of the pump}}$$

$$= \frac{Q_A}{Q_T}$$

Volumetric efficiency ($\eta_v$) indicates the amount of leakage that takes place within the pump.This is due to manufacture tolerances and flexing of the pump casing under designed pressure operating conditions.

For gear pumps, $\eta_v = 80\%–90\%$.

For vane pumps, $\eta_v = 92\%$.

For piston pumps, $\eta_v = 90\%–98\%$.

2- **Mechanical efficiency ($\eta_m$):** Itis the ratio of the pump output power assuming no leakage to actual power delivered to the pump:

$$\text{Mechanical efficiency } (\eta_m) = \frac{\text{Pump output power assuming no leakages}}{\text{Actual power delivered to the pump}}$$

Mechanical efficiency ($\eta_m$) indicates the amount of energy losses that occur for reasons other than leakage. This includes friction in bearings and between mating parts. This includes the energy losses due to fluid turbulence. Mechanical efficiencies are about 90%–95%. We also have the relation.

$$\eta_m = \frac{p Q_T}{T_A N}$$

Where

$p$ is the pump discharge pressure in Pa or N/m2,

$Q_T$ is the theoretical flow rate of the pump in m3/s,

$T_A$ is the actual torque delivered to the pump in Nm and

$N$ is the speed of the pump in rad/s.

It ($\eta_m$) can also be computed in terms of torque as follows:

$$\eta_m = \frac{\text{Theoretical torque required to operate the pump}}{\text{Actual torque delivered to the pump}}$$

$$= \frac{T_T}{T_A}$$

The theoretical torque ($T_T$) required to operate the pump is the torque that would be required if there were no leakage.

The theoretical torque ($T_T$) is determined as follows

$$T_T = \frac{V_D}{2\pi} \times P(Pressure)$$

Where

$T_T$ is in N.m

$V_D$ is in $m^3$

$P$ is in Pa

$$T_A = \frac{P(Power)}{N}\left[\frac{N.m/s}{\frac{rad}{s}}\right] = N.m$$

3- **Overall efficiency ( $\eta_o$):** It is defined as the ratio of actual power delivered by the pump to actual power delivered to the pump :

$$\text{Overall efficiency } (\eta_o) = \frac{\text{Actual power delivered by the pump}}{\text{Actual power delivered to the pump}}$$

Overall efficiency ( $\eta_o$ ) considers all energy losses and can be represented mathematically as follows:

$$\text{Overall efficiency } (\eta_o) = \eta_v \eta_m$$

$$\Rightarrow \eta_o = \frac{Q_A}{Q_T} \times \frac{pQ_T}{T_A N}$$

**Example 2.8:**

A gear pump has an outside diameter of 82.6 mm, inside diameter of 57.2 mm and a width of 25.4 mm. If the actual pump flow is 1800 RPM and the rated pressure is 0.00183 what is the volumetric efficiency?

**Solution:** We have

Outside diameter $D_o = 82.6$ mm

Inside diameter $D_i = 57.2$ mm

Width $d = 25.4$ mm

Speed of pump $N = 1800$ RPM

Actual flow rate = 0.00183 $m^3/s$

Theoretical flow rate

$$Q_T = \frac{\pi}{4} \times (D_o^2 - D_i^2) \times d \times \frac{N}{60}$$

$$= \frac{\pi}{4} \times (0.0826^2 - 0.0572^2) \times 0.0254 \times \frac{1800}{60}$$

$$= 2.125 \times 10^{-3}$$

Volumetric efficiency is

$$\eta_v = \frac{0.00183}{2.125 \times 10^{-3}} \times 100 = 86.11\%$$

**Example 2.9:**

A pump having a volumetric efficiency of 96% delivers 29 LPM of oil at 1000 RPM. What is the volumetric displacement of the pump?

Volumetric efficiency of the pump $\eta_v = 96\%$

Discharge of the pump = 29 LPM

Speed of pump $N = 1000$ rpm

Now

$$\eta_v = \frac{\text{Actual flow rate of the pump}}{\text{Theoritical flow rate of the pump}} = \frac{Q_A}{Q_T}$$

$$\Rightarrow 0.96 = \frac{29}{Q_T}$$

$$\Rightarrow Q_T = 30.208 \text{ LPM}$$

Volumetric displacement

$$V_D = \frac{Q_T}{N} = \frac{30.208 \times 10^{-3} \times 60}{60 \times 1000}$$

$$= 30.208 \times 10^{-6} \text{ m}^3/\text{rev} = 0.0302 \text{ L/rev}$$

**Example 2.9:**

A positive displacement pump has an overall efficiency of 88% and a volumetric efficiency of 92%. What is the mechanical efficiency?

**Solution:** Theoverall efficiency is

$$\eta_o = \eta_m \times \eta_v$$

$$\Rightarrow \eta_m = \frac{\eta_o}{\eta_v} = \frac{88}{92} \times 100 = 95.7\%$$

**Example 2.10:**

Determine the overall efficiency of a pump driven by a 10 HP prime mover if the pump delivers fluid at 40 LPM at a pressure of 10 MPa.

**Solution:**

$$\text{Output power} = pQ$$

$$= 10 \times 10^6 \text{ N/m}^2 \times 40 \text{ L/min} \times \frac{\text{m}^3/\text{s}}{1000 \text{ L/s}} \times \frac{1 \text{ min}}{60 \text{ s}}$$

$$= 6670 \text{ W}$$

$$\text{Input power} = 10 \text{HP} \times \frac{746 \text{ W}}{1 \text{ HP}} = 7460 \text{ W}$$

Now

$$\eta_o = \frac{\text{Pump output power}}{\text{Pump input power}}$$

$$= \frac{6670}{7460} = 0.894 = 89.4\%$$

**Example 2.11:**

How much hydraulic power would a pump produce when operating at 140 bar and delivering 0.001 $m^3$/s of oil? What power rated electric motor would be selected to drive this pump if its overall efficiency is 85%?

**Solution:**

Operating pressure of the pump = 140 bar

Flow rate $Q = 0.001 m^3/s$. Now

$$\text{Power of pump} = \text{Pressure} \times \text{Flow rate}$$
$$= 140 \times 10^5 \times 0.001$$
$$= 14 \text{ kW}$$

Overall efficiency of pump $\eta_o = 85\%$

Power to be supplied is

$$\frac{\text{Power of pump}}{\eta_o} = \frac{14 \text{kW}}{0.85} = 16.47 \text{kW}$$

.

**Example 2.12:**

A pump has a displacement volume of 98.4 cm3. It delivers 0.0152 $m^3/s$ of oil at 1000 RPM and 70 bar. If the prime mover input torque is 124.3 Nm. What is the overall efficiency of pump? What is the theoretical torque required to operate the pump?

**Solution:**

Volumetric discharge = 98.4 $cm^3$

Theoretical discharge is

$$Q_T = V_D \times \frac{N}{60} = 98.4 \times \frac{1000}{60} = 1.64 \times 10^{-3} \text{ m}^3/\text{s}$$

Volumetric efficiency is

$$\eta_v = \frac{1.52 \times 10^{-3}}{1.64 \times 10^{-3}} \times 100 = 92.68 \%$$

Overall efficiency is

$$\eta_o = \frac{Q_A \times \text{pressure}}{T \times \omega} = \frac{1.52 \times 10^{-3} \times 70 \times 10^5 \times 60}{124.3 \times 2 \times 1000 \times \pi} \times 100 = 81.74\%$$

The mechanical efficiency is

$$\eta_{\text{mechanical}} = \frac{\eta_{\text{overall}}}{\eta_{\text{volumetric}}} = \frac{81.74}{92.78} = 88.2$$

Now

Theoretical torque = Actual torque × $\eta_{\text{mechanical}}$ = 124.3 × 0.882 = 109.6 Nm

Note: Mechanical efficiency can also be calculated as

$$\eta_m = \frac{pQ_T}{T\omega}$$
$$= \frac{70 \times 10^5 \text{ N/m}^2 \times 0.00164 \text{ m}^3/\text{s}}{124.3 \text{ (N m)} \times \frac{1000}{60} \times 2\pi \text{ rad/s}}$$
$$= 0.882 = 88.2\%$$