Data needed for modelling

Data on Travel Behaviour

- Zonal data
- Network data
- Data from other models
- E.g. regional model as input/constraint for an urban model
- OD-matrix trucks from a freight transport model
- Date for modelling travel behaviour
- Data for modelling travel choice behaviour

Data sources

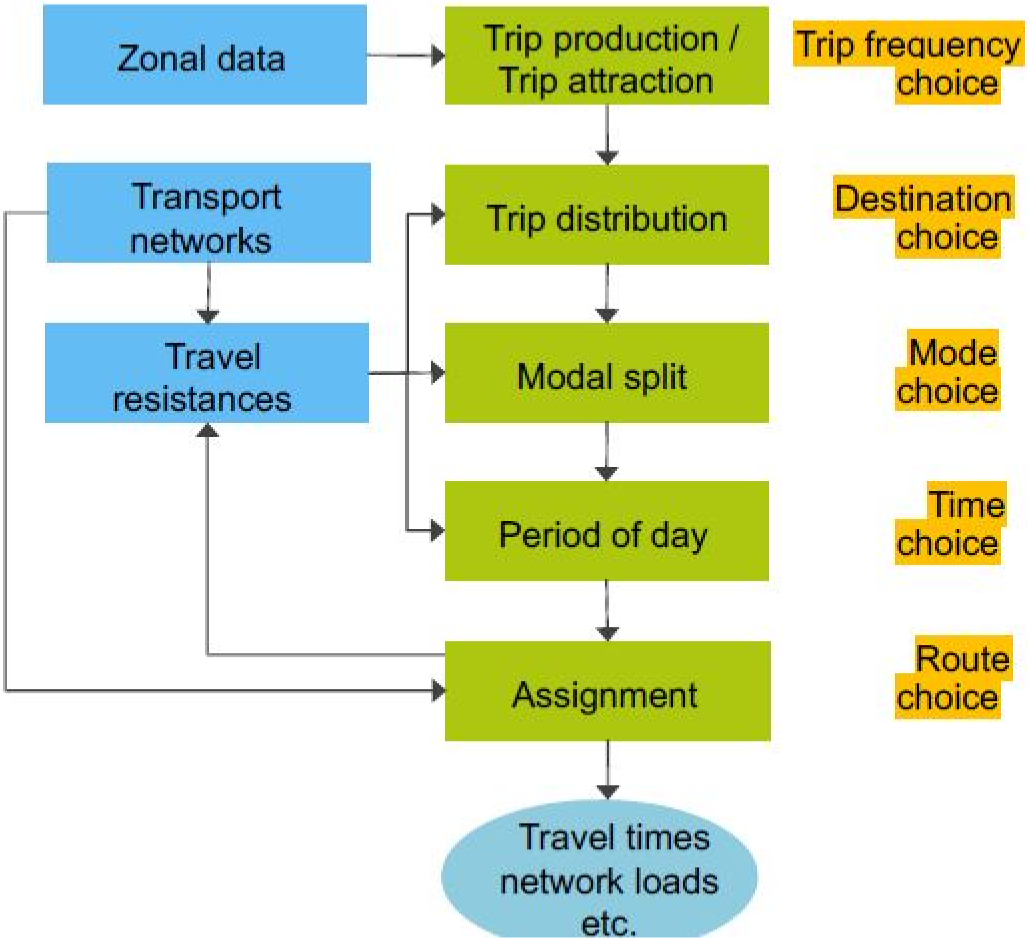
- Traffic/Passenger counts
- Road
- Public transport
- Surveys
- Roadside
- Public transport
- License plate
- Household
- New data sources
- Cell phones
- Route planners
- Chip cards

Counts versus surveys

Counting seems simple

- In practice quite a difference in quality
- Limited number of locations
- Just numbers, no information on traveller
- Surveys focus on travellers
- Road side surveys or PT surveys are still limited
- Limited number of locations
- Household (or person) survey are most informative

Framework for transport modelling



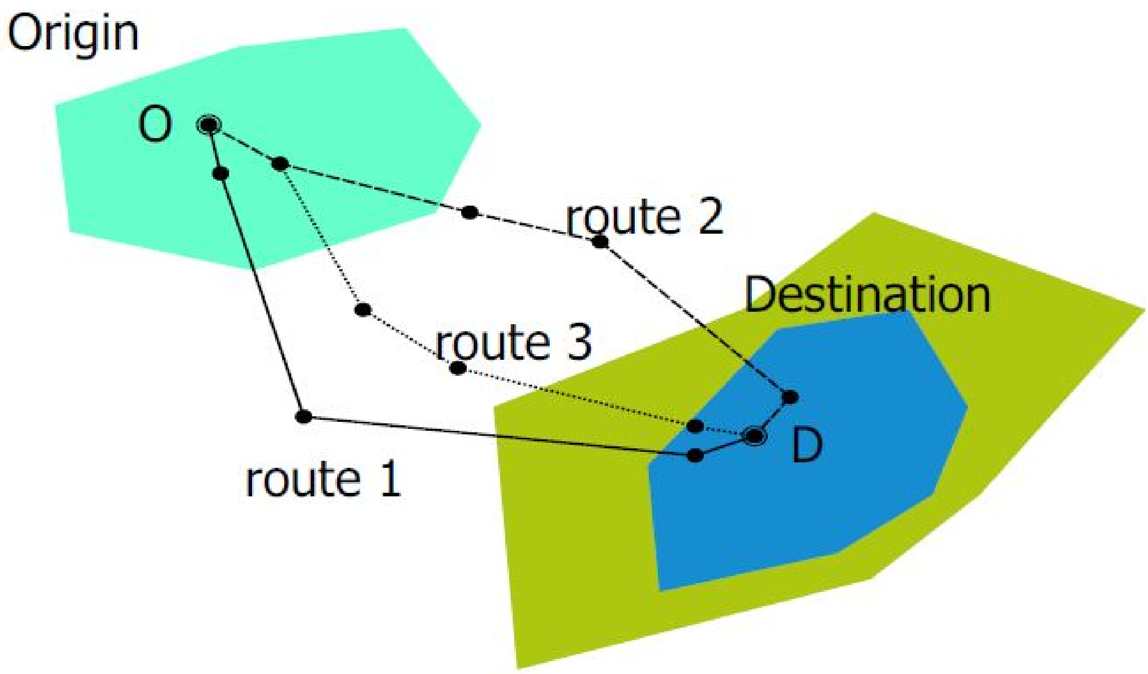
Key building block of transport models

All kind of choices

- Trip choice (stay/go)
- Destination choice
- Mode choice
- Time-of-day choice
- Route choice
- Departure time choice
- Move choice (stay/move)
- Location choice

Discrete choice modelling is used in other disciplines as well, e.g. marketing

Example route choice



• Model to describe choice behaviour in situations where people have to choose from a set of distinct alternatives
• Key: individuals only pick one alternative

Key elements for decision making

- Decision maker: individual person or a group of people
- Alternatives: nonempty set of feasible and known alternatives to the decision makers
- Attributes of alternatives
- Decision rule

Decision rule

- Utility Theory: majority of choice models in transportation are based on the utility maximization assumption
- Travellers act rationally
- Travellers have well defined preferences
- Maximize the utility Uj of choosing alternative j

Random utility models (RUM)

The individuals are assumed to select the alternative with the highest utility

• Inconsistencies in choice behaviour are assumed to be a result of observational deficiencies on the part of the analyst

• The utilities are unknown to the analyst. Thus, they are treated as random variables

$$P(i|C)=Pr(U_i \geq U_j,\ \forall j \in C); \quad i,j: \text{alternatives},\ C: \text{choice set}$$

$$U_i = V_i + \varepsilon_i$$

V_i : systematic component of the utility

ε_i : random part of the utility

Basic case (binary choice)

Example Mode choice

$$\text{Car:} \quad U_c = \theta_1 T_c + \varepsilon_c$$
$$\text{Transit:} \quad U_t = \theta_1 T_t + \varepsilon_t$$

Where T_c is the travel time with car and T_t the travel time with transit

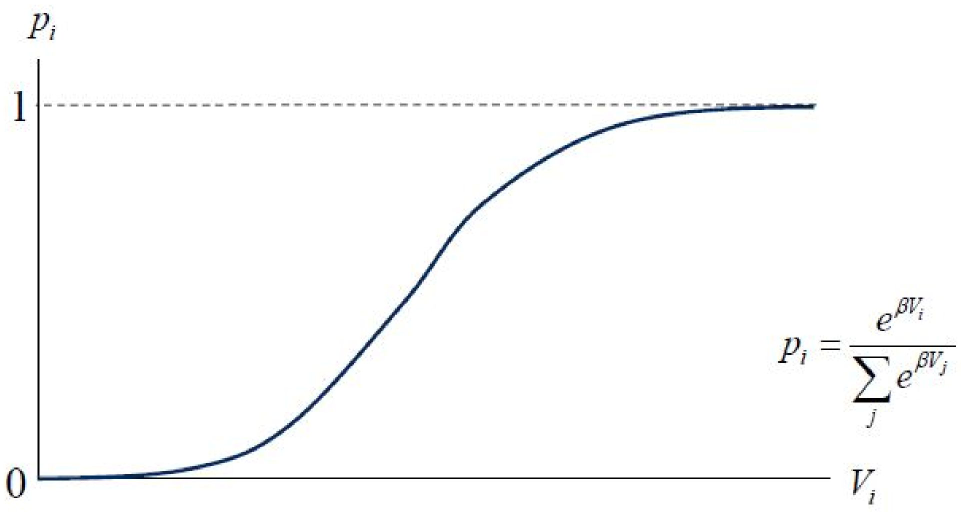
$$\begin{aligned} P(c \mid \{c,t\}) &= P(U_c \geq U_t) \\ &= P(\theta_1 T_c + \varepsilon_c \geq \theta_1 T_t + \varepsilon_t) \\ &= P(\theta_1 T_c - \theta_1 T_t \geq \varepsilon_t - \varepsilon_c) \\ &= P(\theta_1 (T_c - T_t) \geq \varepsilon) \end{aligned}$$

Logit model

- Binary case: $$P\left(c \mid \{c,t\}\right) = \frac{1}{1+e^{-\beta(V_c - V_t)}} = \frac{e^{\beta V_c}}{e^{\beta V_c} + e^{\beta V_t}}$$

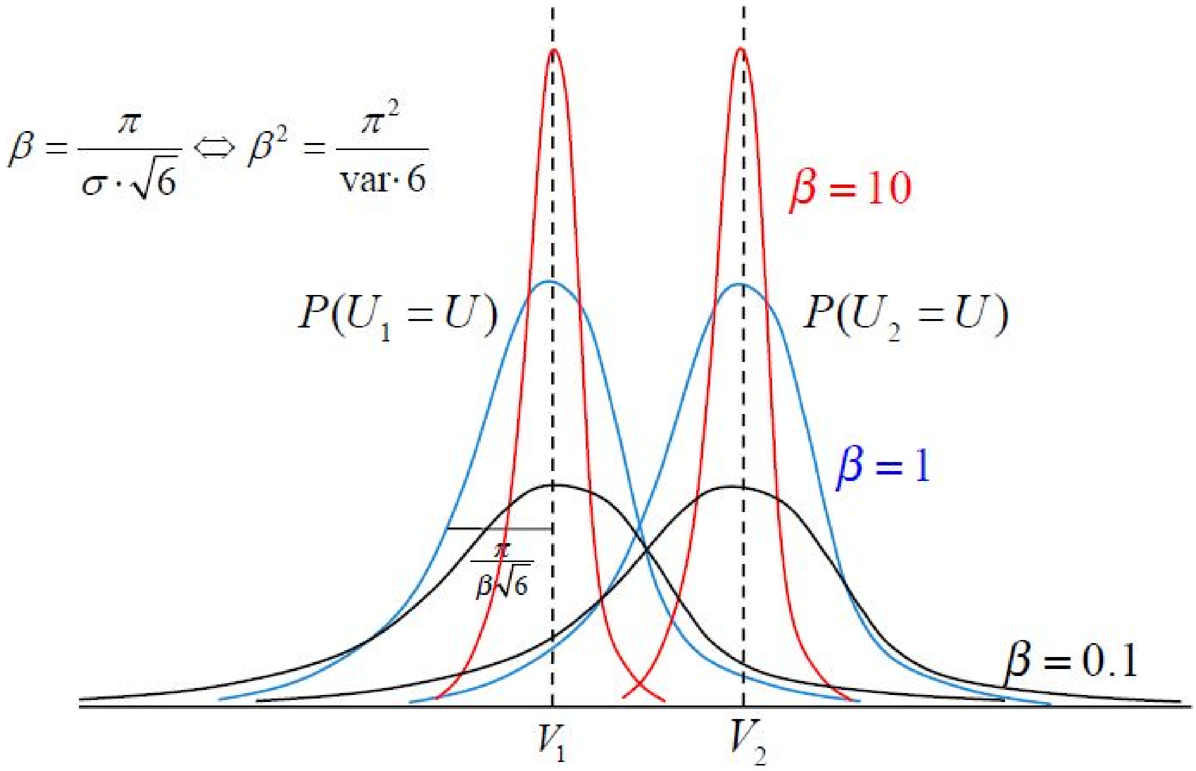
- Note that difference is decisive!
- Parameter β describes sensitivity for differences:
  - β is zero: not sensitive
  - β is large: very sensitive ("all or nothing")
- Multinomial case: $$P\left(i \mid \{alt_1, \ldots, alt_n\}\right) = \frac{e^{\beta V_i}}{\sum_{j=1}^{n} e^{\beta V_j}}$$

Shape of logit function

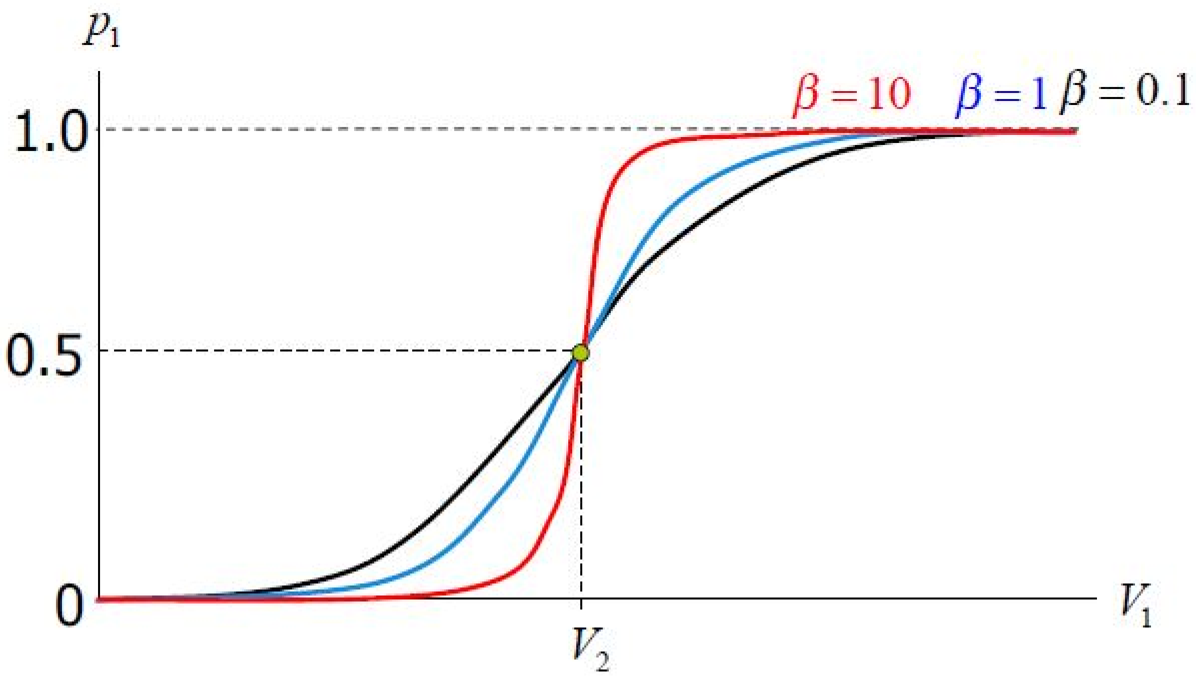


p_i = probability for choosing alternative *i*

Scale parameter and distribution



Impact of the scale parameter



The lower the scale parameter , the higher the variance or ‘spread’ in the choice proportions and vice versa.

Application of the Logit model
Example mode choice

#	Time car	Time transit	Choice
1	52.9	24.4	T
2	14.1	28.5	T
3	14.1	86.9	C
…	…	…	…
10	95.0	43.5	T

$V_t = ASC + \theta_1 T_t$ $\quad V_c = \theta_1 T_c$

$ASC = 0.5$ $\quad \theta_1 = -0.1$

$V_{c,2} = -0.1 * 14.1 = -1.41$

$V_{t,2} = 0.5 - 0.1 * 28.5 = -2.35$

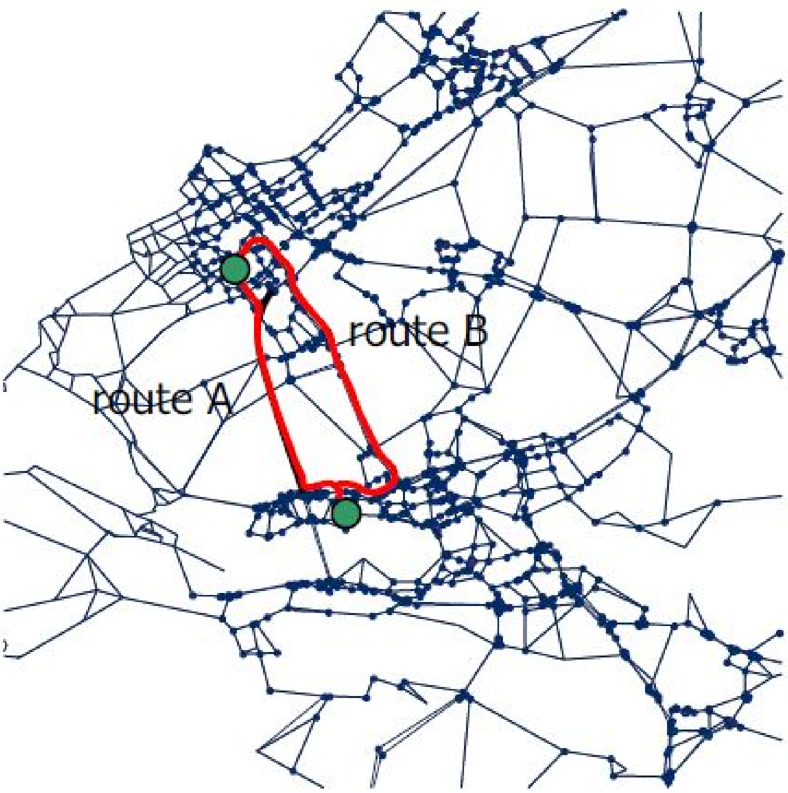
The V values are meaningless!

They make sense only as interpretation of the utility function

Probability of individual 2 to choose transit:

Assume: $\beta = 1$

$$P_{t,2} = \frac{e^{-2.35}}{e^{-1.41} + e^{-2.35}} = 0.28$$



Extension A4

Schiedam - Den Haag
route A: 20 min. + € 2
route B: 35 min.

(dis)utility function:

$V_i = -(time + 5 * toll)\ [min]$

$\beta = 0.1$

$P_A = 62\%$

Where do the parameters come from?

You need data on actual choice behaviour Chosen alternative:
• Non-chosen alternatives
• Including the (possibly) relevant attributes
Typical data collection methods are:
• Revealed preference (i.e. observed behaviour)
• Stated preference of Stated choice
Search for the best model by specifying, estimating and assessing utility specifications:
• Using special software, e.g. ALOGIT, NLOGIT or BIOGEME
• Using statistical tests and travel behaviour theory

Estimation of choice models

- What are the best values for the parameters, e.g. ASC and θ_1?
- Single observation: maximise probability chosen alternative (bit trivial, just define ASC)
- Two observations: maximise probability of observing both choices simultaneously, e.g. max: $P_1(T)*P_2(T)$
- Set of observations: max: $P_1(T)*P_2(T)*P_3(C)\ldots P_{10}(T)$
- Likelihood maximisation or, for numerical reasons, Log-likelihood maximisation

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Scale and utility parameters

- When estimating a model you determine the best value for $\beta\theta$
- In practice it is thus impossible to identify what the value of β or θ is
- Solution in practice is setting β (or one of the θ's) equal to 1
- This identification problem makes it difficult to compare parameters of different models
- Solution here is to compare ratio's of parameters, e.g. $\beta\theta_t/\beta\theta_c$ (=Value of time)

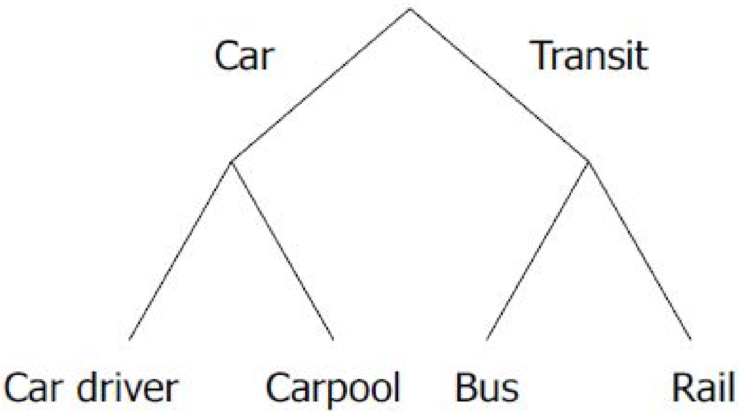
Some comments on the standard logit model

- Logit is commonly used, but isn't perfect
- Logit is sensitive for differences between utilities, independent of the absolute value of the utility
- How to take constraints into account?
- What to do if alternatives are not independent?
  - Route overlap
  - Red/Blue bus problem

Nested logit
Red and blue bus problem

• Assume a simple mode choice problem: car versus bus, e.g. 75% car and 25% bus
• A new company enters having identical buses, except for the colour (i.e. blue instead of red), and having an identical schedule. So now we have 3 modes: car, red bus, blue bus.
• What is the share of car now?
1. Still 75%
2. Decreases to 60% (i.e. 0.75/(0.75+0.25+0.25))
3. Other

Typical example



Nests k
Scale parameter β

Alternatives i
Scale parameter λ_k

$$P_i = \frac{e^{\beta V_i}}{\sum_j e^{\beta V_j}} \Longrightarrow P(i,k) = P(i \mid k)P(k) = \frac{e^{\lambda_k V_{i|k}}}{\sum_{j \in k} e^{\lambda_k V_{j|k}}} \cdot \frac{e^{\beta V_k}}{\sum_{l \in K} e^{\beta V_l}}$$

Decomposition in two logits

Split utility in two parts:

- variables describing attributes for nests (aggregate level): W_k
- variables describing attributes within nest: Y_j

$$U_i = W_k + Y_i + \varepsilon_i \quad i \in B_k$$

Probability alternative is product of probability of alternative within nest and probability of nest

$$P_i = P_{i|B_k} P_{B_k}$$

<u>Decomposition in two logits Resulting formulas</u>

$$P_{B_k} = \frac{e^{\beta \cdot (W_k + I_k)}}{\sum_{l=1}^{K} e^{\beta \cdot (W_l + I_l)}}$$

$$P_{i|B_k} = \frac{e^{\lambda_k \cdot Y_i}}{\sum_{j \in B_k} e^{\lambda_k \cdot Y_j}}$$

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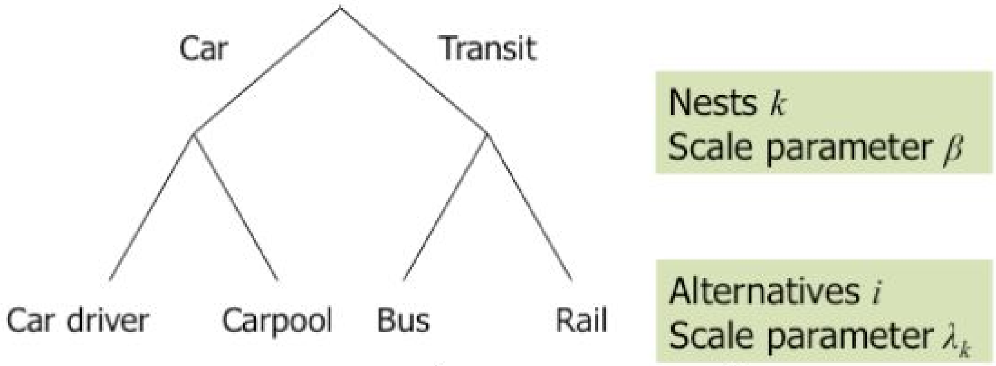
Typical conditions for nested logit



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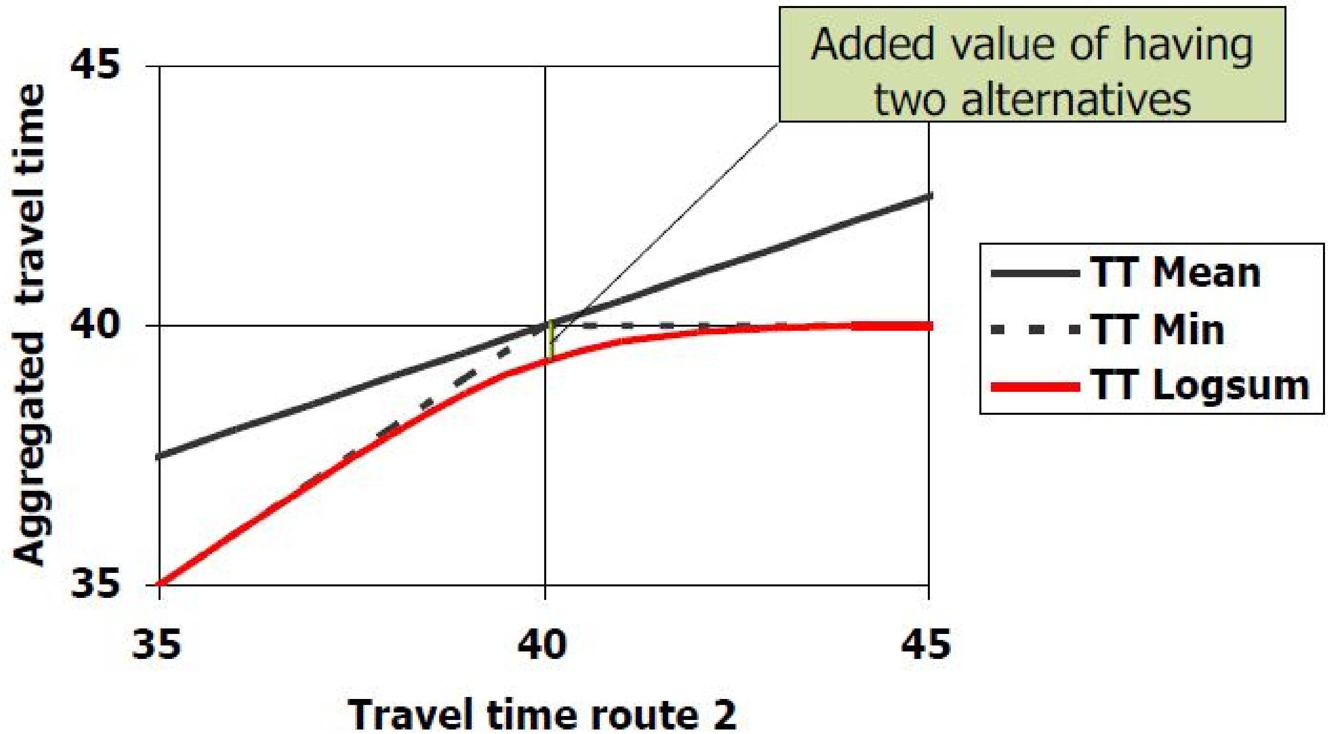
Define the parameter

$$\mu_k = \frac{\beta}{\lambda_k}$$

- It is required that $\mu_k \leq 1$
- Note that if $\mu_k = 1$ this expression collapses to the standard logit model
- If $\mu_k \rightarrow 0$, the nest is reduced to the alternative having the highest utility, i.e. the other alternatives in the nest have no additional value

Example route choice with 2 routes

Travel time route 1 is 40 minutes, travel time route 2 varies



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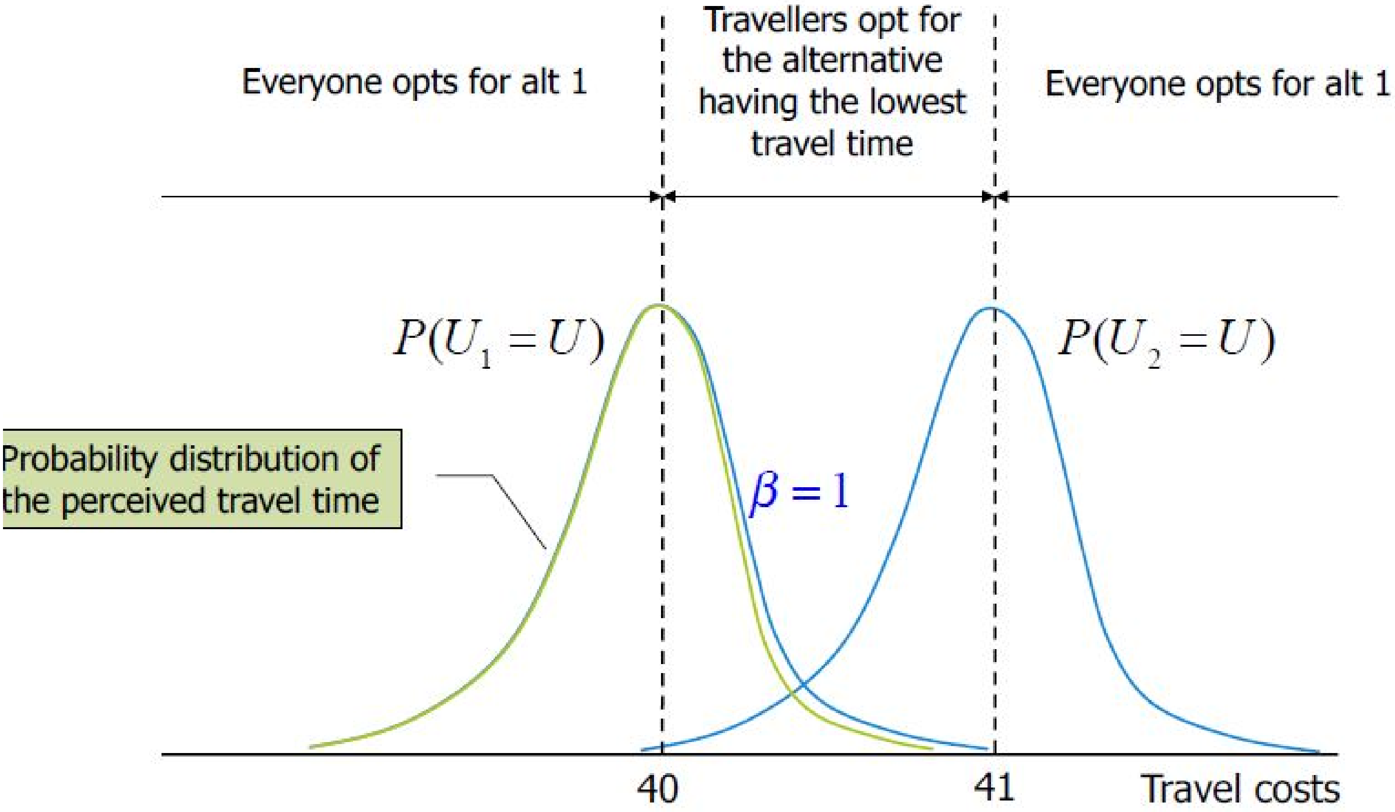

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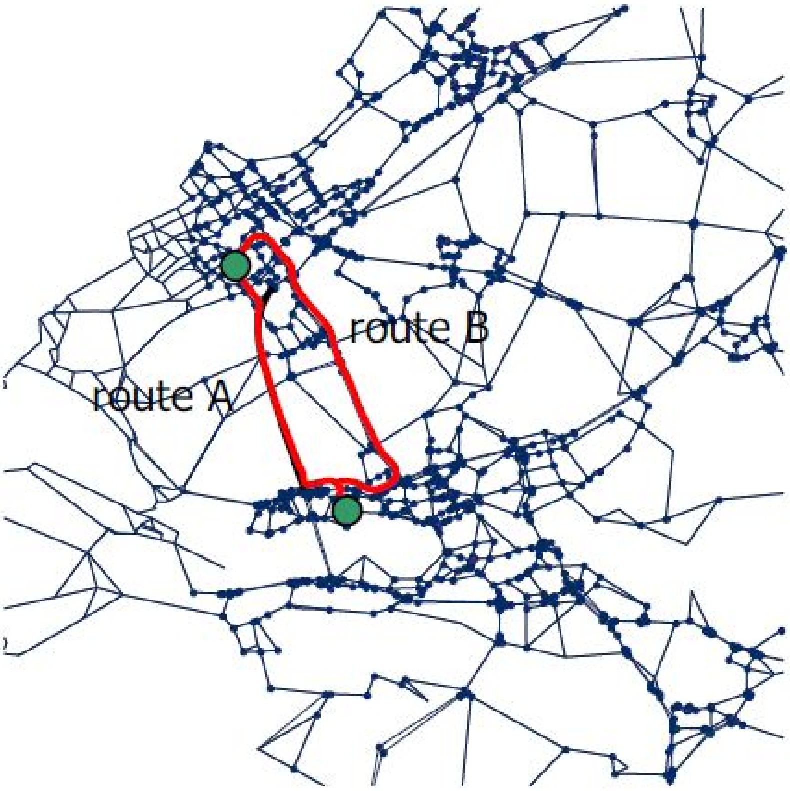

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• Nesting accounts for (unobserved) similarities within nests: mix of correlation, simultaneousness and hierarchy
  • It does not necessarily imply a sequential order of choices!
• Special application/interpretation: Conditional choice:
• Choice for alternative given choice for nest
• Lower level choice options are part of higher level utility

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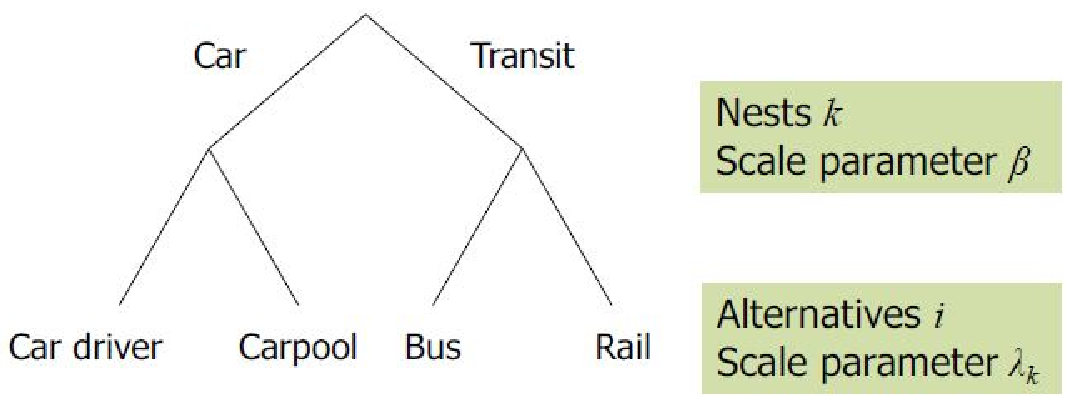

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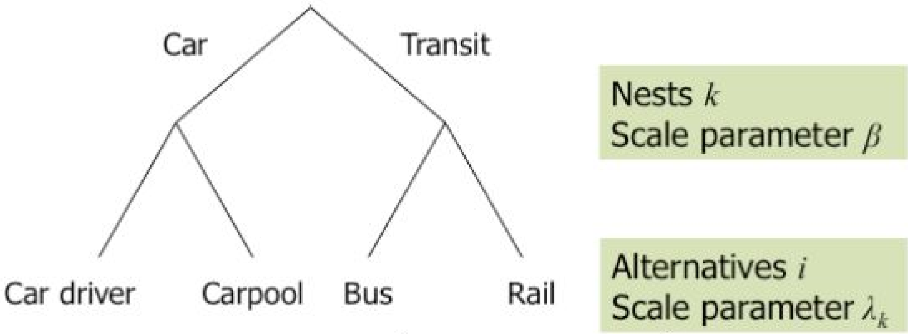

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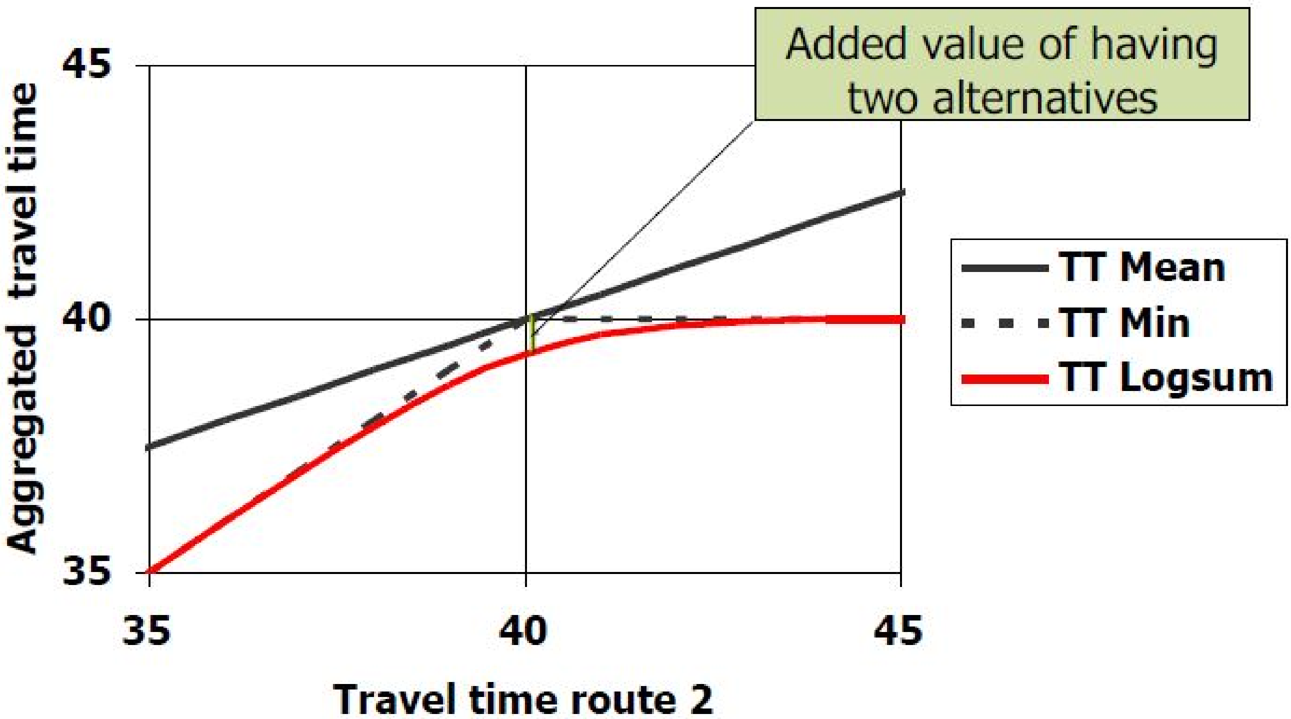

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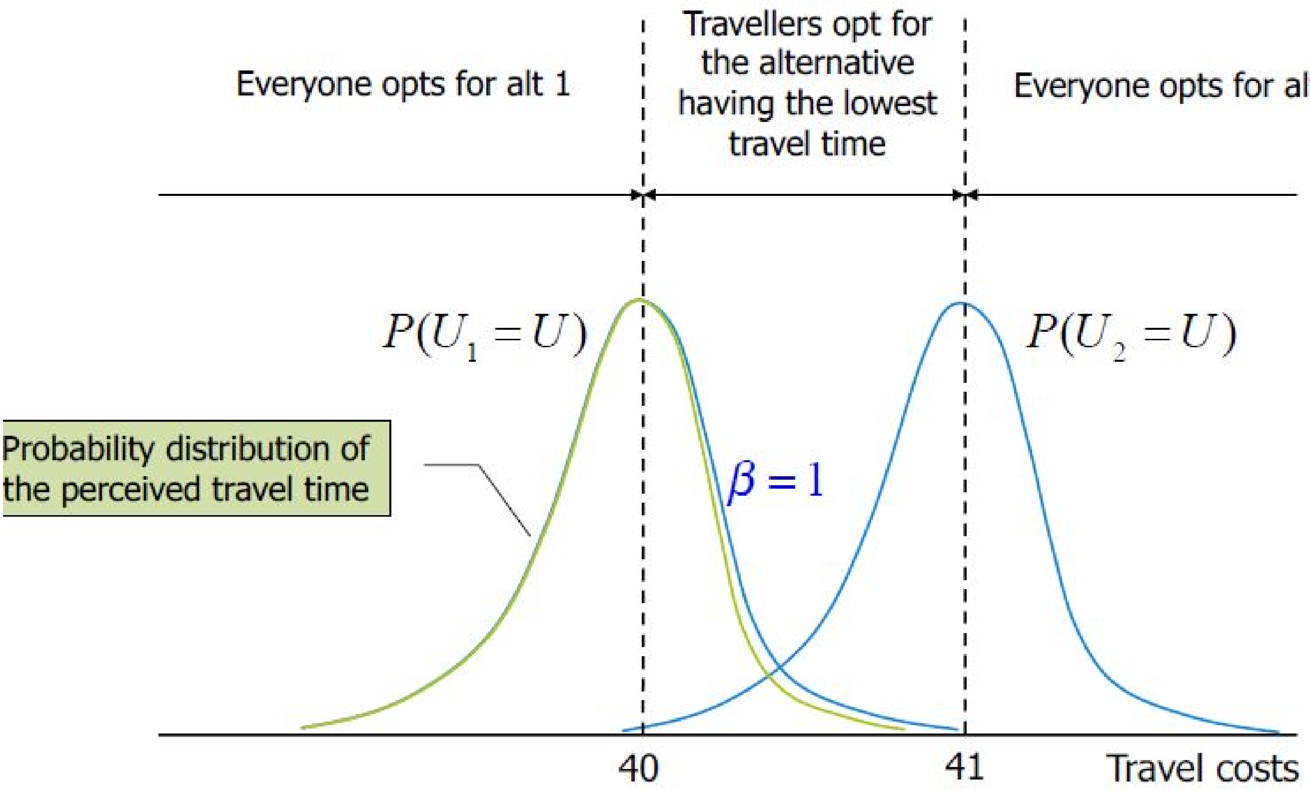

Nested logit: to conclude

Nested logit modelling proved to be a powerful tool for travel behaviour modelling

- Limitations: an alternative can only be allocated to a specific nest
- Possible extensions:
- Cross-nested logit
- Generalised nested logit
- Network GEV (Generalised Extreme Value)