Greenshield Model (linear relationship)

The first steady-state speed-density model was introduced by Greenshields, who proposed a linear relationship between density and speed as follows:

$$u = u_f - \left(\frac{u_f}{K_j}\right) * k$$

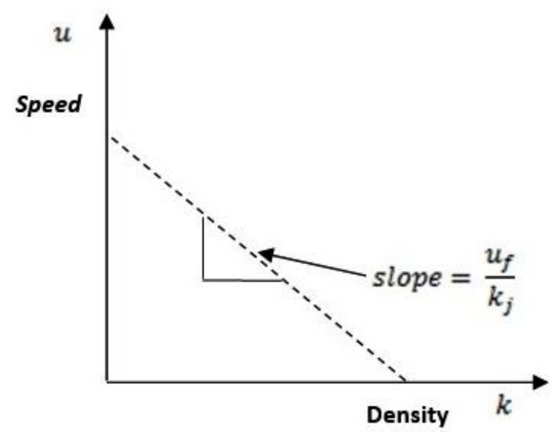
$Where$:

$u = speed\ at\ any\ time$

$u_f = free - flow\ speed$

$k = density\ at\ that\ instant$

$k_j = maximum\ density$



In these equations, as the flow increases, density increases and speed decreases. At optimum density, flow becomes maximum at $u = \frac{u_f}{2}$ and $k = \frac{k_j}{2}$

Greenberg Model

A second early model was suggested by Greenberg (1959). This model shows a logarithmic relationship as follows:

$$u = c \ln\left(\frac{k}{k_j}\right)$$

Where:

$u = speed\ at\ any\ time$

$c = a\ constant\ (optimum\ speed)$

$k = density\ at\ that\ instant$

$k_j = maximum\ density$

Flow-density relationship

$$q = u_f\, k - \frac{u_f}{k_j}\, k^2$$

Where:

$q = flow\ at\ any\ time$

$u_f = free\ flow\ speed$

$k = density\ at\ that\ instant$

$k_j = maximum\ density$

It can be concluded from this equation that maximum flow occurs at density equals to $\left(\frac{k_j}{2}\right)$

Flow-speed relationship

$$q = k_j\, u - \frac{k_j}{u_f}\, u^2$$

Where:

$q = flow\ at\ any\ time$

$u = speed\ at\ any\ time$

$u_f = free\ flow\ speed$

$k_j = maximum\ density$

It can be concluded from this equation that maximum flow occurs at speed equals to $\left(\frac{u_f}{2}\right)$.

Since $q = u * k$, therefore $q_{max} = \frac{k_j u_f}{4}$

Ex: Two platoons of cars are timed over a distance of 0.5 Km. their flows are recorded. The first group is timed at 40 seconds; with the flow at 1350 veh/hr. the second group take 45 second, with a flow of 1800 veh/hr. Determine the maximum flow of the traffic stream?

Solution:

Group 1 has an average speed of 45 Km/hr

Group 2 has an average speed of 40 Km/hr

Group 1 has density value =1350/45=30 veh/Km

Group 2 has density value = 1800/40=45 veh/Km

To get the consequent relationship between speed and density based on the above two results, use co-ordinate geometry:

$y-y_1 = m (x-x_1)$

Where

$m= (y_1-y_2)/(x_1-x_2)$

y= speed

x=density

The slope, m, of the line joining the above two results= -5/15 = -1/3

y-45 = -1/3 (x-30)

y+ x/3 = 45+10

y + x/3 =55

Examining the boundary conditions:

Free flow speed = 55 Km/hr

Jam density= 165 veh/Km

Max flow = 55x 165/4 =2269 veh/hr

- The linear relationships [Greenshields] between Speed and Density is?

$u = u_f - \frac{u_f}{k_j} k$	$u = k_j - \frac{u_f}{k_j} k$	$u = k_j - \frac{u_f}{k_j} k_j$	$u = u_f - \frac{u_f}{k_j} k_j$

-The relationship between flow and density is?

$q = u * k - \frac{u}{k_j} k^3$	$q = u * k - \frac{u}{k_j} k^2$	$\boldsymbol{q = u_f * k - \frac{u_f}{k_j} k^2}$	$q = k_j * k - \frac{u}{k_j} k^2$

-What is the model of Greenberg?

$\boldsymbol{u = C * ln(\frac{k}{k_j})}$	$u = u_f - \frac{u_f}{k_j} k$	$u = u_f * k - \frac{u_f}{k_j} k$	$u = C * k * \ln(\frac{k_j}{k})$

-Get the formula of maximum capacity of roadway

$q_{max} = \frac{u_f * k_j}{2}$	$q = \frac{u_f * k_j}{3}$	$\boldsymbol{q_{max} = \frac{u_f * k_j}{4}}$	$q = \frac{u_f * k_j}{5}$

- The speed at which drivers are observed to operate their vehicles during free flow condition?

a. Design speed	b. Operating speed	c. Lower speed	d. Pace speed