# Divergence Test

$$\lim_{n \to \infty} a_n \neq 0$$

$$\sum a_n \to \text{Diver}$$

# Geometric Series

$$\sum_{n=1}^{\infty} a\boxed{r}^{n-1}$$

$|r| < 1$

$\hookrightarrow$ Converge

$|r| \geq 1 \longrightarrow$

# p - Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

$p > 1 \longrightarrow$ Converge

$p \leq 1 \longrightarrow$ Diver

# Telescoping Series

$$\sum a_n = 1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \frac{1}{4} + \ldots a_n$$

$$\sum_{n=1}^{\infty} a_n = \lim_{n \to \infty} S_n = L \to \text{Converge}$$

# Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1 \rightarrow \text{Converges}$$

$$> 1 \,/\, \infty \rightarrow \text{Diverge}$$

# Root Test

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1 \rightarrow \text{Converges}$$

$$> 1$$

# Limit Comparison Test

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L$$

if $\sum a_n \to C$, $\sum b_n \to C$

if $\sum a_n \to D$, $\sum$

# Alternating Series Test

$$\sum_{n=1}^{\infty} (-1)^n a_n \longrightarrow \text{Converge}$$

1. $\lim_{n \to \infty} a_n = \boxed{0}$

2. $a_{n+1} \leq a_n$

decreasing so a sub n has to be equal to
a greater

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$$\sum_{n=1}^{\infty} \boxed{\frac{2n^2+5}{7n^2-4}} \xrightarrow[\frac{2}{7}]{\lim\limits_{n\to\infty} a_n \neq 0} \text{Divergent}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(2n^2+5)\ d/dn}{(7n^2-4)\ d/dn}$$

$$\lim_{n\to\infty} \frac{4n\ d/dn}{14n\ d/dn} = \lim_{n\to\infty} \frac{4}{14} = \boxed{\frac{2}{7}}$$

to the divergence s and that's it for
this problem

$$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{n^5} = \sum_{n=1}^{\infty} n^{-14/3} = \sum_{n=1}^{\infty} \frac{1}{n^{\boxed{14/3}}}$$

$$p = 14/3$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{\boxed{p}}} \qquad p > 1 \;\Rightarrow\; \text{converges}$$

and for the P series if P is greater
than 1 then the series converges

$$\sum_{n=1}^{\infty} 5\left(\frac{1}{4}\right)^{n-1}$$

$$\boxed{r = 1/4}$$

$$5\left(\frac{1}{4}\right)^{0} = 5$$

$|r| < 1 \rightarrow \boxed{\text{converges}}$

$|r| \geq 1 \rightarrow$ diverges

$$\sum_{n=1}^{\infty} ar^{n-1}$$

$$S = \frac{a_1}{1-r}$$

$$S = \frac{(5)4}{(1-1/4)4} = 20$$

the bottom by 4 5 times 4 is 20 and then
distribute

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} = \frac{-1}{\sqrt{1}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \dots$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0 \checkmark$$

$$a_{n+1} \le a_n$$

$$\frac{1}{\sqrt{n+1}} \le \frac{1}{\sqrt{n}}$$

the next term is going to be 1 over the
square root of n plus 1 so the

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} = \frac{-1}{\sqrt{1}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \dots$$

$$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \left| \frac{(-1)^n}{\sqrt{n}} \right| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

$$= \sum_{n=}^{\infty} \boxed{\frac{1}{n^{1/2}}} \rightarrow \frac{1}{n^p} \qquad p = 1/2$$

$$p \leq$$

greater than 1 but because P is less than or

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$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

$$\lim_{n\to\infty} \frac{1}{n(n+1)} = \lim_{n\to\infty} \frac{(1)^{1/n^2}}{(n^2+n)^{1/n^2}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n^2}}{1+1/n} = \frac{0}{1+0} = 0$$

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$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left[ \frac{1}{n} - \frac{1}{n+1} \right]$$

$$\left[ \frac{1}{\cancel{n(n+1)}} = \frac{A}{\cancel{n}} + \frac{B}{\cancel{n+1}} \right] \cancel{n}\cancel{(n+1)}$$

$$1 = A(n+1) + Bn$$

$$n=0 \qquad 1 = A(0+1)$$

$$A = 1$$

$$n=-1 \qquad 1 = B(-1)$$

$$B = -1$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left[ \frac{1}{n} - \frac{1}{n+1} \right]$$

$$= \overbrace{\left[ \frac{1}{1} - \cancel{\frac{1}{2}} \right]}^{a_1} + \overbrace{\left[ \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} \right]}^{a_2} + \overbrace{\left[ \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} \right]}^{a_3} \cancel{+} \cdots$$

$$+ \overbrace{\left[ \frac{1}{n-1} - \frac{1}{n} \right]}^{a_{n-1}} + \overbrace{\left[ \frac{1}{n} - \frac{1}{n+1} \right]}^{a_n} = S_n$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left[\frac{1}{n} - \frac{1}{n+1}\right]$$

$$= \underbrace{\left[1 - \cancel{\frac{1}{2}}\right]}_{a_1} + \underbrace{\left[\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right]}_{a_2} + \underbrace{\left[\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right]}_{a_3} + \cancel{\ldots}$$

$$+ \underbrace{\left[\cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n}}\right]}_{a_{n-1}} + \underbrace{\left[\cancel{\frac{1}{n}} - \frac{1}{n+1}\right]}_{a_n} = S_n$$

$$S_n = 1 - {}^{1}$$

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$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} =$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} 1 - \frac{1}{n+1}$$

$$= 1 - \lim_{n\to\infty} \frac{1}{n+1} = 1 - 0 = 1$$

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$$\sum_{n=1}^{\infty} \underbrace{\boxed{\frac{1}{n^2+4}}}_{a_n} \leq \sum_{n=1}^{\infty} \underbrace{\boxed{\frac{1}{n^2}}}_{b_n}$$

$p = 2$

Converges

$p > 1.$

$$\sum_{n=1}^{\infty} \underbrace{\boxed{\frac{1}{n^2+4}}}_{a_n} \le \sum_{n=1}^{\infty} \underbrace{\boxed{\frac{1}{n^2}}}_{b_n}$$

Converge

$p = 2$

Converges

$p > 1$

problem
so this is

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$$\sum_{n=3}^{\infty} \frac{1}{\sqrt{n-2}}$$

$$\lim_{n \to \infty} \frac{1}{\sqrt{n-2}} = 0$$

limit will go to 0 and so based on a
divergence

$$\sum_{n=3}^{\infty} \frac{1}{\sqrt{n-2}}$$

$$\sum_{n=3}^{\infty} \frac{1}{\sqrt{n}}$$

$$\downarrow$$

$$\frac{1}{n^{1/2}}$$

Diverges $\leftarrow$ $p = 1/2$

$p \leq 1$

now which series is bigger is it

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$$\sum_{n=1}^{\infty} \boxed{\frac{1}{n^2+4}} \le \sum_{n=1}^{\infty} \boxed{\frac{1}{n^2}}$$

$a_n$ $\qquad$ $b_n$

$p=2$

Converge

Converges

$p>1$

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$$\sum_{n=3}^{\infty} \underbrace{\boxed{\frac{1}{\sqrt{n-2}}}}_{b_n} \geq \sum_{n=3}^{\infty} \boxed{\frac{1}{\sqrt{n}}} \; a_n$$

$$\downarrow$$

$$\frac{1}{n^{1/2}}$$

$$p = 1/2$$

$$\text{Diverges} \leftarrow p \leq 1$$

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^3+2}$$

$$\lim_{n \to \infty} \frac{a_n}{b_n} = L$$

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^3}$$

$$\sum \frac{n^{1/2}}{n^3}$$

$$\sum n^{-2.5}$$

$$\sum \frac{1}{n^{2.5}}$$

$$p = 2.5 > 1$$

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$$\sum_{n=1}^{\infty} \boxed{\frac{\sqrt{n}}{n^3+2}} \leftarrow a_n \qquad \rightarrow C$$

$$\sum_{n=1}^{\infty} \boxed{\frac{\sqrt{n}}{n^3}} \rightarrow \boxed{C} \qquad \leftarrow b_n$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \left[\frac{\cancel{\sqrt{n}}}{n^3+2} \cdot \frac{n^3}{\cancel{\sqrt{n}}}\right]$$

$$\lim_{n\to\infty} \left[\frac{(n^3)\,1/n^3}{(n^3+2)\,1/n^3}\right] = \lim_{n\to\infty} \frac{1}{1+2\cdot\frac{1}{n^3}}$$

$$= \frac{1}{1+2(0)} = \frac{1}{1} = \boxed{1}$$

$$\sum_{n=1}^{\infty}\left[\frac{3n^2-9}{7n^2+4}\right]^n$$

$$\lim_{n\to\infty}\sqrt[n]{|a_n|}\quad\begin{array}{l} <1 \rightarrow C \\ >1 \rightarrow D \\ =1 \rightarrow I \end{array}$$

$$\sum_{n=1}^{\infty} \left[\frac{3n^2-9}{7n^2+4}\right]^n \longrightarrow \frac{3}{7} < 1$$

converg

$$\lim_{n\to\infty} \left(\left[\frac{3n^2-9}{7n^2+4}\right]^{\cancel{n}}\right)^{\cancel{1/n}}$$

$$\frac{6}{14} = \boxed{\frac{3}{7}}$$

$$\lim_{n\to\infty} \left[\frac{3n^2-9}{7n^2+4}\right] \begin{matrix} d/dn \\ d/dn \end{matrix} = \lim_{n\to\infty} \frac{6n \; d/dn}{14n \; d/dn}$$

$$\sum_{n=1}^{\infty} \frac{2^n}{n!}$$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \quad \begin{aligned} &< 1 \longrightarrow C \\ &> 1 \longrightarrow D \\ &= 1 \longrightarrow I \end{aligned}$$

$$\sum_{n=1}^{\infty} \frac{2^n}{n!} \quad a_n = \frac{2^n}{n!} \quad a_{n+1} = \frac{2^{n+1}}{(n+1)!}$$

converges

$$\lim_{n\to\infty}\left[\frac{2^{n+1}}{(n+1)!}\cdot\frac{n!}{2^n}\right] = \boxed{0} < 1$$

$$= \lim_{n\to\infty}\left[\frac{\cancel{2^n}\cdot 2^1}{(n+1)\cancel{n!}}\cdot\frac{\cancel{n!}}{\cancel{2^n}}\right] = \lim_{n\to\infty}\frac{2}{n+1}$$