1.2 Application of Differentiation

Theorem: 1. if $\acute{f}(x) > 0$ on (a, b) then f is increase on (a, b).
2. if $\acute{f}(x) < 0$ on (a, b) then f is decrease on (a, b).

Ex: On which intervals is the function $f(x) = x^3 - 3x^2 + 1$ increase, decrease.

$$\acute{f}(x) = 3x^2 - 6x$$
$$= 3x(x-2) = 0$$

either $x=0$ or $x=2$

$+++++$ ------------ $+++++++$ sign of $\acute{f}(x)$

$\nearrow$ 0 $\searrow$ 2 $\nearrow$

So $f(x)$ is **increase** on $(-\infty, 0) \cup (2, \infty)$
and **decrease** on $(0, 2)$

Def.: a function $f(x)$ is **concave up** on (a, b) if $f''(x) > 0$
and **concave down** on (a, b) if $f''(x) < 0$

Ex: find the intervals where the fun. $f(x) = x^3 - 3x^2 + 1$ concave up, concave down

$$\acute{f}(x) = 3x^2 - 6x$$
$$f''(x) = 6x - 6 = 6(x-1) = 0$$
$$\downarrow$$

equal to zero at $x=1$

-------------- 1 $++++++++$ sign of $f''(x)$

f concave up on $(1, \infty)$ and concave down on $(-\infty, 1)$

Def.: Let f be a continuous function on $[a, b]$ and f changes direction of concavity at x_o then $(x_o, f(x_o))$ called an **inflection point** of f.
or $(x_o, f(x_o))$ is an **inflection point** of f if $f''(x) = 0$

Ex: Find the location of all inflection points of $f(x) = x^4 - 8x^2 + 16$

$$\acute{f}(x) = 4x^3 - 16x$$

$f''(x) = 12x^2 - 16 = 4(3x^2 - 4) = 0$

$\downarrow$

$x = \pm\frac{2}{\sqrt{3}}$

$\left(\frac{2}{\sqrt{3}}, 7.1\right)$ *and* $\left(-\frac{2}{\sqrt{3}}, 7.1\right)$ *are* ***inflection points***

Ex: Find the intervals in which the function $f(x) = x^{1/3}(x+4)$ *Is* ***increasing , decreasing , concave up , concave down*** *,and* ***inflection point.***

$f(x) = x^{1/3}(x+4)$

$f'(x) = x^{1/3}(1) + (x+4)\left(\frac{1}{3}\right)x^{-2/3}$

$f'(x) = x^{1/3} + \frac{(x+4)}{3x^{2/3}}$

$= \frac{4x+4}{3x^{2/3}} = \frac{4(x+1)}{3x^{2/3}}$

------------------ +++ +++++++++ *sign of* $f'(x)$

-1 0

f increase on (-1, 0) , $(0, \infty)$

f decrease on $(-\infty, -1)$

$f'(x) = \frac{4}{3}(x+1)x^{-2/3}$

$f''(x) = \frac{4}{3}\left[(x+1)\left(\frac{-2}{3}x^{-5/3}\right) + x^{-2/3}\right]$

$= \frac{4(x-2)}{9\,x^{5/3}} = 0 \quad \rightarrow \quad x = 2$

++++++++++++ -------- +++++++ *sign of* $f''(x)$

0 2

f concave up on $(-\infty, 0)$ *,* $(2, \infty)$*,concave down on (0, 2)*

(2 , 7.6) is the inflection point

1.3 Sketching Graphs of polynomials and Rational functions

Ex: Sketch the graph of the curve $f(x) = x^3 - 3x + 2$

$f'(x) = 3x^2 - 3 = 3(x-1)(x+1) = 0$

x=1 *x= -1*

+++++ ---------------- ++++++++ *sign of* $f'(x)$

-1 1

f increase on $(-\infty,-1)$, $(1,\infty)$

f decrease on (-1, 1)

$f(-1) = 4$ $\rightarrow (-1,4)$ *is relative maximum*

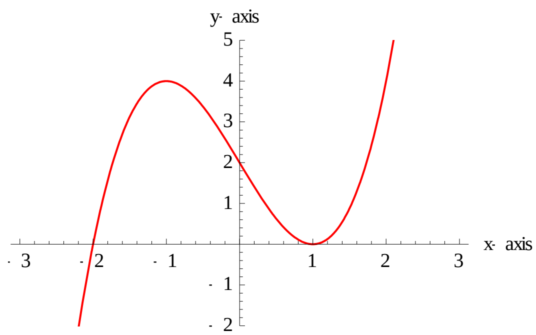
$f(1) = 0$ $\rightarrow (1,0)$ *is relative minimum*

$f''(x) = 6x = 0$ $\rightarrow$ $x = 0$ $\rightarrow (0,2)$ is inflection point

---------------- +++++++++ sign of $f''(x)$

0

f is concave up on $(0,\infty)$ **and** concave down on $(-\infty,0)$



Ex: sketch the graph of $f(x) = \frac{x^2}{x^2-1}$

$x^2 - 1 = 0$

(x-1)(x+1) = 0 $\rightarrow$ $\boldsymbol{x = 1}$ *and* $\boldsymbol{x = -1}$ are **vertical asymptote**.

$\lim_{x\to\infty} \frac{x^2}{x^2-1} = \lim_{x\to\infty} \frac{1}{1-\frac{1}{x^2}} = 1$ ***y* =*1*** is **Horizontal asymptote**

$$f(x) = \frac{x^2}{x^2-1}$$

$$f'(x) = \frac{(x^2-1).(2x) - (x^2).(2x)}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2} = 0 \rightarrow x = 0$$

++++ ++++ ------- ---------- sign of $f'(x)$

-1 0 1

f increase on $(-\infty,-1)$, $(-1,0)$

f decrease on (0, 1) , (1 , ∞)

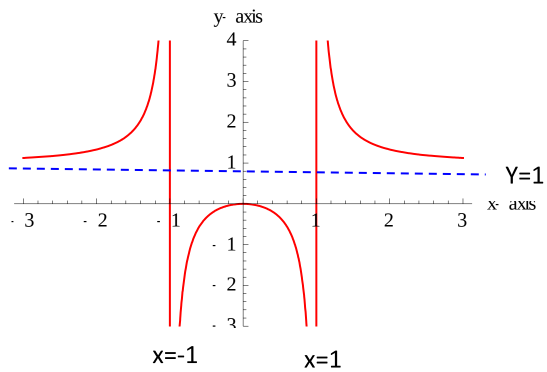
$f'(x) = \frac{-2x}{(x^2-1)^2}$

$$f''(x) = \frac{(x^2-1)^2(-2)-(-2x)(2((x^2-1)(2x))}{(x^2-1)^4} = \frac{6(x^2+1)}{(x^2-1)^3} \neq 0$$

+++++ ------------ ++++++ sign of $f''(x)$

-1 1

f is concave up on $(-\infty,-1),(1,\infty)$ **and** concave down on $(-1,1)$



Ex: Sketch the graph of the rational function $f(x) = \frac{4-x^3}{x^2}$

$x^2 = 0 \quad \rightarrow \ x = 0$ **is** the **vertical asymptote**

There is **no** Horizontal asymptote

$$f(x) = \frac{4-x^3}{x^2} = -x + \frac{4}{x^2}$$

y= -x *is the* ***oblique asymptote***

$$\begin{array}{r} -x \\ x^2\overline{)\,-x^3-4} \\ \mp x^3 \\ \hline +4 \end{array}$$

$$\acute{f}(x) = \frac{(x^2)(-3x^2)-(4-x^3)(2x)}{x^4}$$

$$= \frac{-3x^4-8x+2x^4}{x^4} = \frac{-x^4-8x}{x^4} = \frac{-(x^3+8)}{x^3}$$

$$= \frac{-(x+2)(x^2-2x+4)}{x^3} = 0 \quad \rightarrow \quad x = -2$$

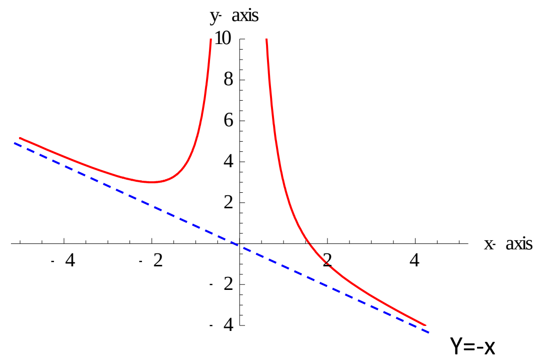
------------ +++++++ ------------- sign of $\acute{f}(x)$

-2 0

f decrease on $(-\infty,-2)$, $(0 , \infty)$

f increase on (-2, 0)

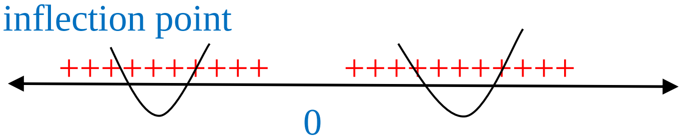
(-2 , 3) is relative minimum



$$f'(x) = \frac{-(x^3+8)}{x^3}$$

$$f''(x) = \frac{(x^3)(-3x^2) + (x^3+8)(3x^2)}{x^6} = \frac{24}{x^4} \neq 0$$

There is no inflection point



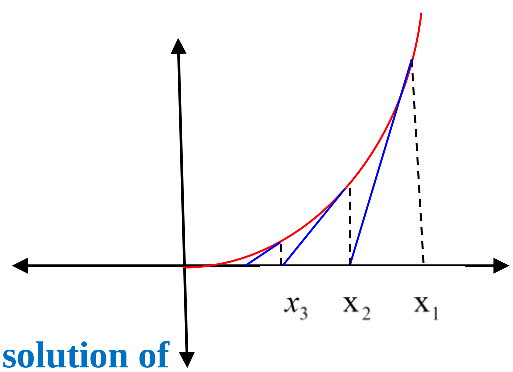
sign of $f''(x)$

f is concave up on $(-\infty, 0), (0, \infty)$

1.4 Newton method:

used to find the roots of the function f(x) Numerically.

$$\boldsymbol{x_{n+1} = x_n - \frac{f(x)}{f'(x)}}$$



Ex: use newton method to approximate the solution of

$$x^3 - x - 1 = 0$$

X	-2	-1	0	1	2	3
f(x)	-	-	-	-	+	+

$f(1)=-1$ and $f(2)=5$
the sign is vary for the function between $x=1$ and $x=2$ so the root lie between them ,choose $\boldsymbol{x_1 = 1.5}$

let $f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

$$\boldsymbol{x_{n+1} = x_n} - \frac{x^3{}_n - x_n - 1}{3x^2{}_n - 1}$$

$x_2 = 1.5 - \frac{(1.5)^3 - (1.5) - 1}{3(1.5)^2 - 1} = 1.34782609$

$\boldsymbol{x_3} = 1.34782609 - \frac{(1.34782609)^3 - (1.34782609) - 1}{3(1.34782609)^2 - 1} = 1.32520040$

$\vdots$

$x_4 = 1.32471817$

$x_5 = 1.32471796$

$x_6 = 1.32471796$

No need to continue because we reached the accuracy **so**

$$\boldsymbol{x} \approx 1.32471796$$