Synthetic Unit Hydrograph الهيدروغراف القياسي المصطنع

The most of Catchments especially those which are at remote locations, have not adequate information about rainfall and the resulting flood hydrograph, or normally scanty. Therefore to construct the UH for such area, empirical equations or curves were used to derive it but of regional validity.

The unit hydrograph derived from empirical correlations is known as synthetic hydrograph. Many methods are availables, but two of well-known of these are currently used in the world. The first is the Natural Resources Conservation service (NRCS) which previously called US-SCS (Soil Conservations Service), the other method is Snyder's. The Snyder's method is considered herein.

Snyder Method

Snyder in 1988, developed a set of empirical equations for synthetic hydrograph which recieved some modifications in many countries. The most important characteristic affecting the hydrograph is basin Lag (Time Lag T_L) measured from centroid of excess rainfall (ER), and the centriod of DRH. For practical purposes, the Lag time is measured from centriod of ER to peak of DRH and it is defined as the mean time of travel of water from all parts of the watershed to the outlet during a given storm يعرف T_L بأنه المعدل الزمني الذي تستغرقه كتل الماء من جميع اجزاء الساكبة الى المخرج خلال عاصفة مطرية.

Synder defined the basin Lag as the time interval from mid-point of rainfall Excess to the peak of unit hydrograph, and formulated in equation as:

$$t_p = C_1 C_t (L L_c)^{0.3} \quad \text{------------} \quad (7)$$

t_p = basin Lag (hr)

L = basin length measured from basin divide to the gauging station (Km, mile)

L_c = distance from centroid of watershed to the gauging station (km, mile)

C_t = regional constant including storage effects and slope (C_t is ranged from $1.8 \rightarrow 2.2$)

C_1 = conversion constant of units
($C_1 = \frac{3}{4}$ for metric system, $C_1 = 1$ for customary units)

i.e,
$$\underset{hr}{t_p} = \frac{3}{4} C_t (L_{km} \, L_{c\,km})^{0.3}$$

$$\underset{hr}{t_p} = C_t (L_{mile} \, L_{c\,mile})^{0.3}$$

Snyder adopted standard duration tr (hr) as;

$$\underset{(hr)}{t_r} = \frac{t_p}{5.5} \quad \text{------------} \quad (8)$$

The peak discharge Q_p for a unit hydrograph of standard duration tr is;

$$Q_p = \frac{C_2 \, C_p \, A_{km^2}}{t_{p\,(hr)}} \quad \text{------------} \quad (9)$$

C_p = peak discharge coefficient ranged from $0.56 \rightarrow 0.69$

C_2 = Conversion constant of units :-
($C_2 = 2.78$ for 1 cm depth, A (km²), Q m³/sec)
($C_2 = 645$ for 1 inch depth, A mile², Q ft³/sec)

i.e
$$\underset{m^3/sec}{Q_p} = \frac{2.78 \, C_p . A}{t_p}$$ for metric system

If non-standard duration of rainfall, t_R is adopted (instead of t_r), thus, the modified basin Lag, t_p' is given by;

$$t_p' = t_p + \frac{t_R - t_r}{4}$$

$$\text{or; } t_p' = \frac{21}{22} t_p + \frac{t_R}{4} \qquad (10)$$

t_p' = basin Lag (hr) for duration of excess rainfall of t_R (hr).

Note:

eq (9) for t_p' is; $Q_p' = \frac{C_2 C_p A}{t_p'}$, and when

$t_R = t_r \Rightarrow t_p' = t_p \Rightarrow Q_p' = Q_p$.

The time base T_B is given by Snyder as;

$$\underset{(hr)}{T_B} = 72 + 3 t_p' \qquad (11)$$

It is suitable for large catchment or moderate say > 500 km²

$$T_B = 5\left(t_p' + \frac{t_R}{2}\right) \qquad (12)$$

Suitable for small catchments.

For standard duration ($t_R = t_r$) $T_B = \frac{60}{11} t_p \qquad (13)$

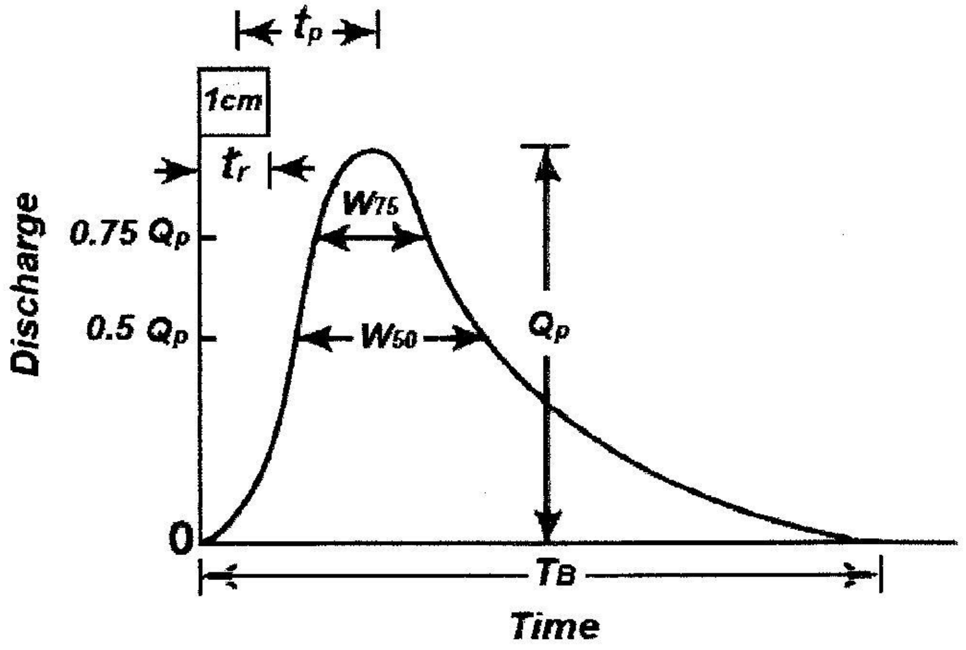


Fig. 21—Elements of a Synthetic Unit Hydrograph

To assist the sketching of unit hydrograph, the width of units (of course time) at 50% and 75% of the peak discharge have been found by US Army corps of Engineers as; see Fig (21).

$$\left.\begin{aligned} W_{50} &= \frac{2.144}{q^{1.08}} \\ W_{75} &= \frac{4}{7} W_{50} \end{aligned}\right\} \text{———} (14)$$

In which $q = Q_P/A$ = peak discharge per unit Area $(m^3/s/Km^2)$

Alternatively, the more reliable value of T_B is derived from the fact that the area under UH is equivalent to direct runoff of 1cm assuming triangular shape yields;

$$T_B = \frac{\left(\frac{50}{9}\right)}{q} = \frac{5.56}{q} \text{———} (15)$$

If the area under curve of UH approximated by trapezoidal segment, it yields (as shown in Fig 22)

$$1\,cm = \frac{1}{A}\left[\frac{W_{50}+T_B}{2} * 0.5 Q_p + \frac{W_{50}+W_{75}}{2} * 0.25 Q_p + \frac{W_{75}}{2} * 0.25 Q_p\right]\left(\frac{hr}{Km^2}\cdot\frac{m^3}{Sec}\right) * 0.36$$

subs. W_{50} & W_{75} from eq (14) and approximate $q^{1.08} \approx q$; yields

$$T_B = \frac{6.67}{q} \text{———} (16)$$

i.e.: $T_B = \left(\frac{60}{9}\right)/q$

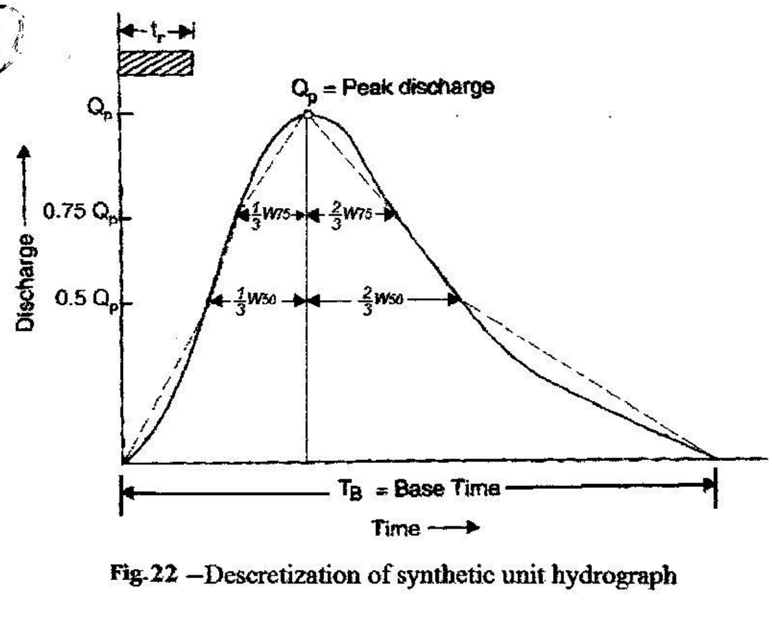


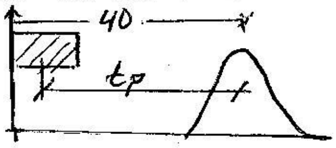
Fig.22 –Descretization of synthetic unit hydrograph

Example 11:

From the topographical map of drainage basin, the following quantities are measured: $A = 3480\ km^2$, $L = 235\ Km$, $L_c = 120\ Km$. The 12hr UH derived for the basin has a peak discharge of $155\ m^3/s$ occurring at 40 hrs. Determine the coefficients C_t & C_p for the synthetic Hydrograph.

Solution:

Note: If duration is given, it is mostly $t_R \neq t_r$ and must be checked.



$$t_R = 12hr,\ t_p' = 40 - \frac{12}{2} = 34\ hr$$

$$t_p' = t_p + \frac{t_R - t_r}{4} \Rightarrow t_p' = t_p + \frac{t_R}{4} - \frac{t_p}{4(5.5)}$$

$$t_p = \left(34 - \frac{12}{4}\right)\frac{22}{12} = 32.48\ h \Rightarrow t_r = \frac{t_p}{5.5} = 5.90\ hr.$$

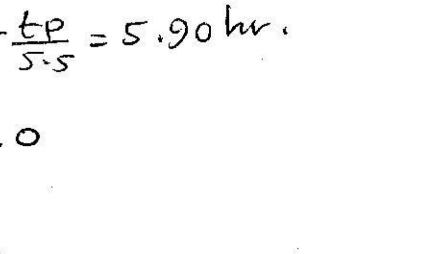
$$t_p = \frac{3}{4} C_t (235 \times 120)^{0.3} = 32.48 \Rightarrow C_t = 2.0$$

$$Q_p' = \frac{2.78 C_p A}{t_p'} \Rightarrow 155 = \frac{2.78\ C_p (3480)}{34}$$

thus; $C_p = 0.545$

Example 12: Derive a 3-hr synthetic unit hydrograph of a basin with Area $= 3000\ km^2$, Length of main stream $= 120\ Km$, distance from centroid of the basin to the outlet $= 63\ Km$, $C_t = 2.10$, $C_p = 0.64$

$$t_p = \frac{3}{4}(2.1)(120 \times 64)^{0.3} = 23.06\ hr$$



$$t_r = \frac{t_p}{5.5} = \frac{23.06}{5.5} = 4.19\ hr$$

$t_R = 3\ hr$ (given)

$$t_p' = t_p + \frac{t_R - t_r}{4} = 23.06 + \frac{3 - 4.19}{4} = 22.76\ hr$$

$$Q_p' = \frac{2.78(0.64)(3000)}{22.76} = 234.52\ m^3/sec$$

$$T_B = \frac{6.67}{Q/A} = \frac{6.67(3000)}{234.52} = 85.3\ hr$$

$$W_{50} = \frac{2.144}{Q/A} = 27\ hr$$

$$W_{75} = \frac{4}{7}(27) = 15.67\ hr$$

(ملاحظات مهمة)

1. إذا كان لديك على مقطع عرضي لنهر فيه (n) من محطات القياس فيتوجب حساب التصريف في (n-2) من المقاطع الصغيرة ، لأن حسابها في أكثر من ذلك يعني قياس السرع عند الضفاف وهذا غير مقبول مطلقا.

2. لاحظ أن التصريف الكلي يجب إيجاده بجمع التصاريف الجزئية وليس الاكتفاء بإيجاد هذه التصاريف دون تدوين نتيجة التصريف.

3. السرع والتصاريف تعامل لثلاث مراتب بعد الفارزة. لان وحداته يسيره أي ($م^3$/ثا) .

4. الهيدروغراف يجب أن يبدأ بصفر وينتهي بصفر ، أليس كذلك ؟ فلماذا نهمل الصفر الأخير !!! كما أن على الطالب التحقق (Checking) من النتائج عند إجراء حسابات الهيدروغراف.

5. عند إجراء عملية التزحيف (shifting) عند حساب الهيدروغراف في أي من الطرق المعروفة ، يجب تمديد عمود الزمن ليتناسب مع التزحيف وقيمه المقابلة.

Derivations:

$$Q_p = C_p \frac{\text{Volume}}{\text{time}}$$

If Area is in *mil²* and depth in *inch* (unit depth), Q_p in *ft³/sec*, and time in *hour*, thus;

$$Q_{p\,(ft^3/sec)} = C_p \frac{A \times (5280)^2\,ft^2 \times \frac{\text{depth(1 inch)}}{12}}{t_p \times (3600)}$$

$$Q_{p\,(ft^3/sec)} = 645\,C_p \frac{A_{(mil^2)}}{t_{p\,(hr)}} \qquad \text{for unit depth 1 – inch.}$$

So as for Area is in *km²* and depth in *cm* (unit depth), Q_p in *m³/s*, and time in *hour*, thus;

$$Q_{p\,(m^3/s)} = C_p \frac{A \times (1000)^2\,m^2 \times \frac{\text{depth(1cm)}}{100}}{t_p \times (3600)}$$

$$Q_{p\,(m^3/s)} = 2.78 C_p \frac{A_{(Km^2)}}{t_{p\,(hr)}} \qquad \text{for unit depth 1 – cm.}$$

$$W_{50(hr)} = \frac{770}{q^{1.08}} \qquad q \text{ is } \frac{Q_p}{A} \text{ measured in } \frac{ft^3/s}{mil^2} \quad \text{for 1 – inch depth.}$$

Hence, the dimension of the factor (770) is: $hr \times \left[\frac{ft^3/s}{mil^2 \times inch}\right]^{1.08}$

The factor (770) can be reduced im metric system as:

$$= 770 \times hr \times \left[\frac{ft^3}{s} \times \frac{m^3}{(3.281)^3 \times ft^3} \times \frac{1}{mil^2} \times \frac{mil^2}{(1.609)^2 \times km^2} \times \frac{1}{inch} \times \frac{1 \times inch}{2.54 \times cm}\right]^{1.08}$$

$$= 770 \times \left[\frac{1}{(3.281)^3} \times \frac{1}{(1.609)^2} \times \frac{1}{2.54}\right]^{1.08} hr \times \left[\frac{m^3/s}{km^2 \times cm}\right]^{1.08}$$

$$= \left[\frac{770}{(232.2547)^{1.08}}\right] hr \times \left[\frac{m^3/s}{km^2 \times cm}\right]^{1.08} = 2.144 \text{ of units} : hr \times \left[\frac{m^3/s}{km^2 \times cm}\right]^{1.08}$$

And thus;

$$W_{50(hr)} = \frac{2.144}{q^{1.08}} \qquad q \text{ is } \frac{Q_p}{A} \text{ measured in } \frac{m^3/s}{km^2} \quad \text{for 1 – cm depth.}$$

The factor (5.87) is followed wrongly in the text, through replacing the right factor 2.144, since it was derived for metric system but with 1-inch not 1-cm.

١-٤٠, 5.87

Snyder conversions

Basic formulae:

$$Q_{p\ (m^3/s)} = 2.78 C_p \frac{A_{(Km^2)}}{t_{p\ (hr)}}$$ for unit depth (1- cm).

$$Q_{p\ (ft^3/sec)} = 645\, C_p \frac{A_{(mil^2)}}{t_{p\ (hr)}}$$ for unit depth (1- inch).

C_p is ranged from 0.56 to 0.69 for both systems of units .

$$t_{p\ (hr)} = \frac{3}{4} C_t \left[L_{(km)} L_{c\ (km)} \right]^{0.3}$$

$$t_{p\ (hr)} = C_t \left[L_{(mil)} L_{c\ (mil)} \right]^{0.3}$$

C_t is ranged from 1.8 to 2.2 for both systems of units .

$$w_{50(hr)} = \frac{770}{q^{1.08}} \quad q \text{ is in } \frac{ft^3/s}{mil^2} \quad \text{for 1-inch depth.}$$

$$w_{50(hr)} = \frac{2.144}{q^{1.08}} \quad q \text{ is in } \frac{m^3/s}{km^2} \quad \text{for 1-cm depth.}$$

$$w_{75(hr)} = \frac{w_{50}}{1.75}$$ for both above equations.
