

Depending on the figure below; use the following data for all Questions:

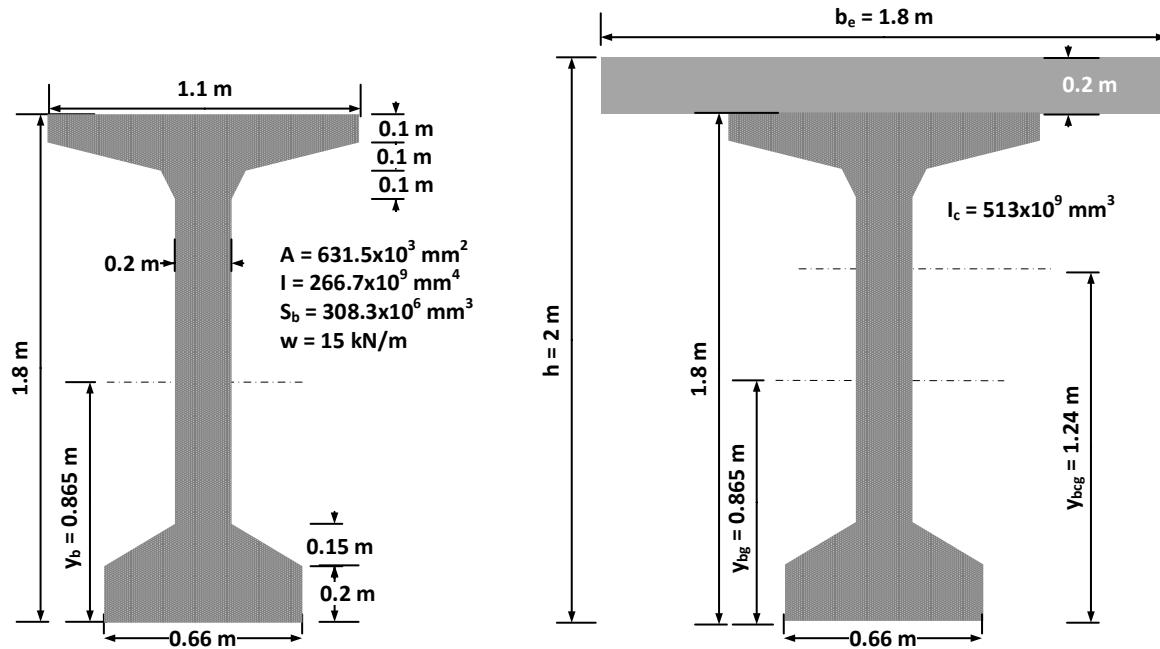
Effective length (L) = 30 m

Concrete deck: (f'_c) = 28 MPa

Concrete beam: (f'_{ci}) = 35 MPa and (f'_c) = 42 MPa

Prestressing steel: (ϕ_p) = 12.7 mm, (A_p) = 98.7 mm², (f_{py}) = 1674 MPa and (f_{pu}) = 1860 MPa

Losses expected in prestressing = 0.2



Q.1: Determine the required number of strands for the girder to carry (M_{LL+IM}) = 2700 kN.m, (M_{DC1}) = 3000 kN.m, (M_{DC2}) = 360 kN.m and overlay weight (w_{DW}) = 3.2 kN/m.

Q.2: Check the allowability of stresses in girder section in-service stage of the bridge life.

Q.3: Check the flexural strength of the girder during the bridge life.

Q.1 Sol:

$$M_{DW} = w_{DW} \cdot L^2 / 8 = 3.2 \times 30^2 / 8 = 360 \text{ kN.m}$$

$$S_{bcg} = I_c / y_{bcg} = 513 \times 10^9 / 1240 = 413.7 \times 10^6 \text{ mm}^3$$

$$\begin{aligned} f_{bot} &= \frac{M_{DC1}}{S_{bg}} + \frac{M_{DC2} + M_{DW} + 0.8M_{(LL+IM)}}{S_{bcg}} \\ &= \frac{3000}{308.3} + \frac{360 + 360 + 0.8 \times 2700}{413.7} = 16.69 \text{ MPa} \end{aligned}$$

$$f_t = 0.50\sqrt{f'_c} = 0.5 \times \sqrt{42} = 3.24 \text{ MPa}$$

$$f_{c,pe} = f_{bot} - f_t = 16.69 - 3.24 = 13.45 \text{ MPa}$$

Try $y_{bp} = 165 \text{ mm}$

$$e = y_{bg} - y_{bp} = 865 - 165 = 700 \text{ mm}$$

$$f_{c,pe} = \frac{P_e}{A_g} + \frac{P_e \cdot e_c}{S_{bg}}$$

$$13.45 = \frac{P_e}{631.5 \times 10^3} + \frac{P_e \times 0.7}{308.3 \times 10^3}$$

$$\rightarrow P_e = 3489.84 \text{ kN}$$

$$f_{pi} = 0.75 f_{pu} = 0.75 \times 1860 = 1395 \text{ MPa}$$

$$P_{i,p} = A_p \cdot f_{pi} = 98.7 \times 1395 = 137.68 \text{ kN}$$

$$R = 1 - \text{losses} = 1 - 0.2 = 0.8$$

$$P_{e,p} = R \cdot P_{i,p} = 0.8 \times 137.68 = 110.29 \text{ kN}$$

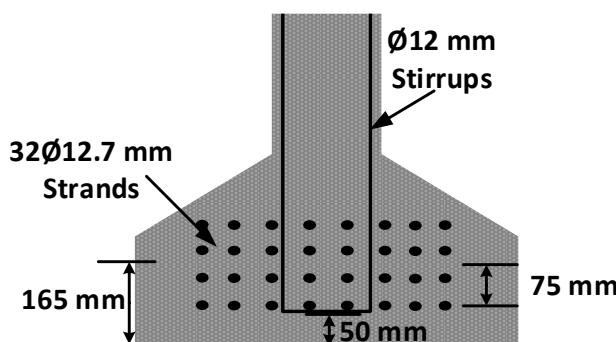
$$N_p = P_e / P_{e,p} = 3489.84 / 110.29 = 32 \text{ strands}$$

Try to use 4 layers of 8 strands in each one with $s = 50 \text{ mm}$

$$y_{bp} = 165 \text{ mm} \rightarrow e_c = 700 \text{ mm} \therefore \text{OK}$$

$$c_b = 165 - 75 - 12.7/2 - 12 = 71.75 \text{ mm} > 50 \text{ mm} \therefore \text{OK}$$

$$c_s = (660 - 8 \times 12.7 - 7 \times 50 - 2 \times 12) / 2 = 92.2 \text{ mm} > 50 \text{ mm} \therefore \text{OK}$$



Q.2 Sol:

$$f_c = 0.45 f'_c = 0.45 \times 42 = 18.9 \text{ MPa}$$

$$f_t = 3.24 \text{ MPa}$$

At midspan P_e , $M_{D,nc}$, $M_{D,c}$, M_{DW} and $M_{(LL+IM)}$.

$$P_i = N_p \cdot P_{i,p} = 32 \times 137.68 = 4405.76 \text{ kN}$$

$$P_e = R \cdot P_i = 0.8 \times 4405.76 = 3524.61 \text{ kN}$$

$$S_{tg} = I_g / y_{tg} = 266.7 \times 10^9 / 935 = 285.24 \times 10^6 \text{ mm}^3$$

$$S_{tcg} = I_c / y_{tcg} = 513 \times 10^9 / 560 = 916.07 \times 10^6 \text{ mm}^3$$

$$\begin{aligned} f_{top} &= -\frac{P_e}{A_g} + \frac{P_e \cdot e}{S_{tg}} - \frac{M_{DC1}}{S_{tg}} - \frac{M_{DC2} + M_{DW} + 0.8M_{(LL+IM)}}{S_{tcg}} \\ &= -\frac{3524.61}{631.5} + \frac{3524.61 \times 0.7}{285.24} - \frac{2700}{285.24} - \frac{2880}{916.07} \\ &= -5.58 + 8.65 - 9.47 - 3.14 = -9.54 \text{ MPa} < 18.9 \text{ MPa} \therefore \text{OK} \end{aligned}$$

$$\begin{aligned} f_{bot} &= -\frac{P_e}{A_g} - \frac{P_e \cdot e}{S_{bg}} + \frac{M_{DC1}}{S_{bg}} + \frac{M_{DC2} + M_{DW} + 0.8M_{(LL+IM)}}{S_{bcg}} \\ &= -\frac{3524.61}{631.5} - \frac{3524.61 \times 0.7}{308.3} + \frac{2700}{308.3} + \frac{2880}{413.7} \\ &= -5.58 - 8 + 8.76 + 6.96 = 2.14 \text{ MPa} < 3.24 \text{ MPa} \therefore \text{OK} \end{aligned}$$

At ends P_e load only.

$$f_{top} = -\frac{P_e}{A_g} + \frac{P_e \cdot e}{S_{tg}} = -5.58 + 8.65 = 3.07 \text{ MPa} < 3.24 \text{ MPa} \therefore \text{OK}$$

$$f_{bot} = -\frac{P_e}{A_g} - \frac{P_e \cdot e}{S_{bg}} = -5.58 - 8 = -13.58 \text{ MPa} < 18.9 \text{ MPa} \therefore \text{OK}$$

Q.3 Sol:

$$M_{DC} = M_{DC1} + M_{DC2} = 3000 + 360 = 3360 \text{ kN.m}$$

$$\begin{aligned}M_u &= \eta_i [1.25M_{DC} + 1.50M_{DW} + 1.75M_{LL+IM}] \\&= 1.0[1.25 \times 3360 + 1.50 \times 360 + 1.75 \times 2700] = 9465 \text{ kN.m}\end{aligned}$$

$$A_{ps} = N_p \cdot A_p = 32 \times 98.7 = 3158.4 \text{ mm}^2$$

$$0.5f_{pu} = 0.5 \times 1860 = 930 \text{ MPa}$$

$$f_{pe} = R \cdot f_{pi} = 0.8 \times 1395 = 1116 \text{ MPa} > 0.5f_{pu} = 930 \text{ MPa} \therefore \text{OK}$$

$$d_{ps} = h - y_{bp} = 2000 - 165 = 1835 \text{ mm}$$

$$\beta_1 = 0.85 - 0.05(f'_c - 28)/7 = 0.85 - 0.05(42 - 28)/7 = 0.75$$

$$k = 2(1.04 - f_{py}/f_{pu}) = 2(1.04 - 0.9) = 0.28$$

$$c = \frac{A_{ps} \cdot f_{pu}}{0.85f'_c \cdot \beta_1 \cdot b_e + k \cdot A_{ps} \cdot f_{pu} / d_{ps}}$$
$$= \frac{3158.4 \times 1860}{0.85 \times 42 \times 0.75 \times 1800 + 0.28 \times 3158.4 \times 1860 / 1835}$$

$$c = 119.67 \text{ mm} < 200 \text{ mm} \rightarrow \text{rectangular section}$$

$$f_{ps} = f_{pu}(1 - k \cdot c / d_{ps}) = 1860(1 - 0.28 \times 119.67 / 1835) = 1826 \text{ MPa}$$

$$a = \beta_1 \cdot c = 0.75 \times 119.67 = 89.75 \text{ mm}$$

$$M_n = A_{ps} \cdot f_{ps} (d_{ps} - a/2) = 3158.4 \times 1826 (1835 - 89.75/2) = 10324 \text{ kN.m}$$

$$\phi = 1.0 \rightarrow M_r = M_n$$

$$M_r = 10324 \text{ kN.m} > M_u = 9465 \text{ kN.m} \therefore \text{OK}$$