Ex. 2: Use information mentioned in **Ex. 1** to design the exterior beams if the concrete strength (f_c') for deck slab = 35 MPa.

Sol:

Determination of Composite Section Properties:

$$b_f = S/2 + w_o = 2400/2 + 900 = 2100\, mm$$

$$E_c = 0.043 K_1 \Upsilon_c^{1.5} \sqrt{f_c'}$$

$$n = E_{c,d}/E_{c,g} = \sqrt{35/35} = 1.0$$

$$b_e = n.b_f = 2100\, mm$$

$$A_{d,tr} = b_e.h_d = 2100\ x\ 200 = 420x10^3\ mm^2$$

$$I_{d,tr} = b_e.h^3/12 = 2100\ x\ 200^3/12 = 1.4x10^9\ mm^4$$

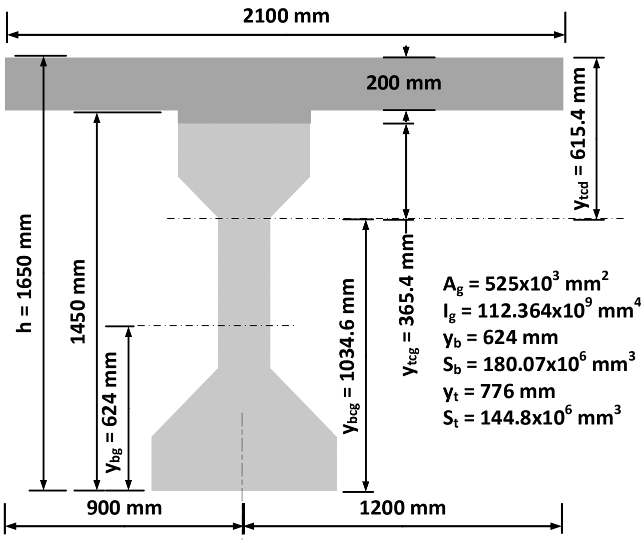
$$h = h_d + h_h + h_g = 200 + 50 + 1400 = 1650\, mm$$

Component	A mm^2	y_t mm	$A.y_t$ mm^3	y_{tc} mm	$I_\circ$ mm^4	$d = (y_t - y_{tc})$ mm	$A.d^2$ mm^4	$I_\circ + A.d^2$ mm^4
Deck	$420x10^3$	100	$42x10^6$	615.4	$1.4x10^9$	▯ 515.4	$111.568x10^9$	$112.968x10^9$
Girder	$525x10^3$	1026	$538.65x10^6$		$112.364x10^9$	410.6	$88.511x10^9$	$200.875x10^9$
$\sum$	$945x10^3$		$581.58x10^6$					$313.843x10^9$

$$y_{tcd} = \sum(A.y_t)/\sum A = 581.58x10^6/945x10^3 = 615.4\, mm$$

$$y_{tcg} = y_{tcd} - h_d - h_h = 615.4 - 200 - 50 = 365.4\, mm$$

$$y_{bcg} = h - y_{tcd} = 1650 - 615.4 = 1034.6\, mm$$



$$I_c = \sum(I_\circ + A.d^2) = 313.843x10^9\ mm^4$$

$$S_{tcd} = I_c/(n.y_{tcd}) = 313.843x10^9/(615.4) = 509.98x10^6\ mm^3$$

$$S_{tcg} = I_c/y_{tcg} = 313.843x10^9/365.4 = 858.9x10^6\ mm^3$$

$$S_{bcg} = I_c/y_{bcg} = 313.843x10^9/1034.6 = 303.35x10^6\ mm^3$$

Determination of Unfactored Loads:

Force effects from unfactored composite (dead) loads:

$$w_d = h_d\ x\ b_f\ x\ \Upsilon_c = 0.2\ x\ 2.1\ x\ 24 = 10.08\ kN/m$$

$$w_{iws} = h_{iws}\ x\ b_f\ x\ \Upsilon_c = 0.02\ x\ 2.1\ x\ 24 = 1.01\ kN/m$$

$w_h = 0.6\ kN/m$

$w_g = 12.6\ kN/m$

$b_{dia} = (S - b_w)/2 = (2.4 - 0.2)/2 = 1.1\ m$

$DL_{dia} = (b\ x\ d\ x\ t)_{dia}\ x\ \Upsilon_c = 1.1\ x\ 0.6\ x\ 0.3\ x\ 24 = 4.75\ kN$

$w_{dia} = N_{dia} x\ DL_{dia}/L = 2\ x\ 4.75/24 = 0.4\ kN/m$

$w_{DC1} = w_{D,nc} = 10.08 + 1.01 + 0.6 + 12.6 + 0.4 = 24.69\ kN/m$

$M_{DC1} = w_{DC1}L^2/8 = 24.69\ x\ 24^2/8 = 1777.68\ kN.m$

Force effects from unfactored composite (dead and live) loads:

$M_{DC2} = 180\ kN.m$

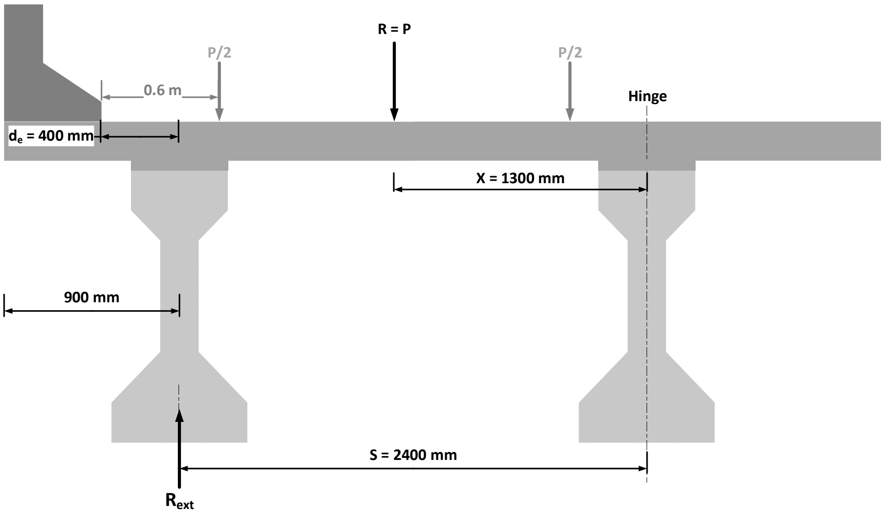
$M_{DW} = 115.2\ kN.m$

$M_{Ln} = 669.6\ kN.m$

$M_{Tr} = 1570.15\ kN.m$

Live load distribution factors:

$-0.3 \le d_e \le 1.7$ $\qquad d_e = 0.4\text{ m} \ \therefore \text{OK}$



$R_{ext} = X/S = 1300/2400 = 0.542$

$DFM_{se} = m.R_{ext} = 1.2\ x\ 0.542 = 0.65$

$DFM_{me} = e_M.DFM_{mi}$

$e_M = 0.77 + d_e/2800 = 0.77 + 400/2800 = 0.913$

$DFM_{mi} = 0.709$

$DFM_{me} = 0913\ x\ 0.709 = 0.647$

$\rightarrow DFM_{ext} = 0.65$

$M_{LL+IM} = DFM_{ext}[(1 + IM)M_{Tr} + M_{Ln}]$

$= 0.65[1.33\ x\ 1570.15 + 669.6] = 1792.64\ kN.m$

Determination of Required Effective Prestress Load

$$f_{bot} = \frac{M_{DC1}}{S_{bg}} + \frac{M_{DC2} + M_{DW} + 0.8M_{(LL+IM)}}{S_{bcg}}$$

$$= \frac{1777.68}{180.07} + \frac{180 + 115.2 + 0.8\ x\ 1955.35}{303.97} = 15.99\ MPa$$

$f_t = 2.95\ MPa < f_{bot} = 15.99\ MPa$ ∴ prestress is required

$f_{c,pe} = f_{bot} - f_t = 15.99 - 2.95 = 13.04\ MPa$

$e_c = 504\ mm$

$$f_{c,pe} = \frac{P_e}{A_g} + \frac{P_e.e_c}{S_{bg}} \rightarrow 13.04 = \frac{P_e}{525x10^3} + \frac{P_e\ x\ 504}{180.07x10^6} \rightarrow P_e = 2772.3\ kN$$

Determination of Required Number of Strands

$f_{pi} = 0.75 f_{pu} = 1395\ MPa$

$\emptyset_p = 12.7\ mm \rightarrow A_p = 98.7\ mm^2$

$P_{i,p} = A_p.f_{pi} = 137.68\ kN$

$R = 0.8$

$P_{e,p} = R.P_{i,p} = 110.29\ kN$

$N_p = P_e/P_{e,p} = 2772.3\ /110.29 = 25.14$ strands

use $N_p = 27$ strands

Distribution of Strands

use 3 layers of 9 strands in each one with $s = 50\ mm$

$e_c = 504\ mm \therefore OK$

$c_b = 51.65\ mm \therefore OK$

$c_s = 92.85\ mm \therefore OK$

Check of Concrete Stresses

At-Release Stage

$f_{ti} = 0.63\sqrt{f'_{ci}} = 3.45\ MPa$

$f_{ci} = 0.6 f'_{ci} = 18\ MPa$

At midspan P_i and M_g.

$f_{i,top} = -0.41\ MPa < 18\ MPa \therefore OK$

$f_{i,bot} = -12.44\ MPa < 18\ MPa \therefore OK$

At ends P_i load only.

$f_{i,top} = 5.86\ MPa > 3.45\ MPa \therefore NOK$

$N_{dp} = 12$ strands

$P_{i,eff.} = 2065.2\ kN$

$\rightarrow f_{i,top} = 3.26\ MPa < 3.45\ MPa \therefore OK$

$\rightarrow f_{i,bot} = -9.71\ MPa < 18\ MPa \therefore OK$

Check if tensile reinforcement is needed at top of beam

$f_{ti} = 0.25\sqrt{f'_{ci}} = 1.37\ MPa < f_{i,top} = 3.26\ MPa$

∴ provide 4$\emptyset$25 mm at top of each end of the girder

In-Service Stage

$f_c = 0.45 f'_c = 15.75\ MPa$

$f_t = 0.50\sqrt{f'_c} = 2.96\ MPa$

At midspan P_e, $M_{D,nc}$, $M_{D,c}$, M_{DW} and $M_{(LL+IM)}$.

$$f_{top} = -\frac{P_e}{A_g} + \frac{P_e.e}{S_{tg}} - \frac{M_{DC1}}{S_{tg}} - \frac{M_{DC2} + M_{DW} + 0.8M_{(LL+IM)}}{S_{tcg}}$$

$$= -5.66 + 10.35 - \frac{1777.68}{144.8} - \frac{1859.48}{858.9} = -9.75\ MPa < 15.75\ MPa \quad \therefore OK$$

$$f_{bot} = -\frac{P_e}{A_g} - \frac{P_e.e}{S_{bg}} + \frac{M_{DC1}}{S_{bg}} + \frac{M_{DC2} + M_{DW} + 0.8M_{(LL+IM)}}{S_{bcg}}$$

$$= -5.66 - 8.37 + \frac{1777.68}{180.07} + \frac{1859.48}{303.35} = 1.97\ MPa < 2.96\ MPa \quad \therefore OK$$

At ends P_e load only.

$\because P_e < P_i \rightarrow P_{e,eff.} < P_{i,eff.}$ $\therefore$ no need to check

Check of Flexural Strength

Strength I limit State (Factored Moments):

$$M_{DC} = M_{DC1} + M_{DC2} = 1777.68 + 180 = 1957.68\ kN.m$$

$$M_u = \eta_i[1.25M_{DC} + 1.50M_{DW} + 1.75M_{LL+IM}]$$

$$= 1.0[1.25\ x\ 1957.68 + 1.50\ x 115.2 + 1.75\ x\ 1955.35] \cong 6041.8\ kN.m$$

$$A_{ps} = N_p.A_p = 2664.9\ mm^2$$

$$f_{pe} = R.f_{pi} = 1116\ MPa > 0.5f_{pu} = 930\ MPa \ \therefore OK$$

$$d_{ps} = h - y_{bp} = 1530\ mm$$

$$\beta_1 = 0.8$$

$$k = 0.28$$

$$c = \frac{A_{ps}.f_{pu}}{0.85f_c'.\beta_1.b_e + k.A_{ps}.f_{pu}/d_{ps}} = \frac{4.96x10^6}{0.85\ x\ 35\ x\ 0.8\ x\ 2100 + 907.11}$$

$c = 97.4\ mm < 200\ mm \rightarrow$ rectangular section

$$f_{ps} = f_{pu}(1 - k.c/d_{ps}) = 1827.55\ MPa$$

$$a = \beta_1.c = 0.8\ x\ 97.4 = 77.92\ mm$$

$$M_n = A_{ps}.f_{ps}(d_{ps} - a/2) = 4.87x10^6(1530 - 77.92/2) = 7261.72\ kN.m$$

$$\phi = 1.0$$

$$M_r = 7261.72\ kN.m > M_u = 6041.8\ kN.m \quad \therefore OK$$

Check of Minimum Reinforcement

$$f_r = 0.63\sqrt{f_c'} = 3.73\ MPa$$

$$P_e = A_{ps}.f_{pe} = 2974\ kN$$

$$f_{c,pe} = \frac{P_e}{A_g} + \frac{P_e.e_c}{S_{bg}} = 13.99\ MPa$$

$$M_{cr} = (f_r + f_{c,pe} - M_{D,nc}/S_{bg})S_{bcg}$$

$$= (3.73 + 13.99 - 1777.68/180.07)\ x\ 303.35x10^6 = 2380.64\ kN.m$$

$$M_{cr} \geq f_r.S_{bcg} = 3.73\ x\ 303.35x10^6 = 1131.5\ kN.m \ \therefore OK$$

$$1.2M_{cr} = 1.2\ x\ 2380.64 = 2856.77\ kN/m$$

$1.33M_u = 1.33\ x\ 6041.8 = 8035.6\ kN.m > 1.2M_{cr}\ \therefore OK$

$M_r = 7261.72\ kN.m > 1.2M_{cr} = 2856.77\ kN.m \quad \therefore OK$

Design of Shear

Location of the Critical Section:

$d_v = 1453.74\ mm$

$x = d_v + 0.5w_b \cong 1.6\ m$

Calculation of Shear Force at the Critical Section:

$w_{DC} = w_{DC1} + w_{DC2} = 24.69 + 2.5 = 27.19\ kN/m$

$V_{DC} = w_{DC}(0.5L - x) = 27.19(10.4) = 282.78\ kN$

$V_{DW} = w_{DW}(0.5L - x) = 16.64\ kN$

$V_{Ln} = w_{Ln}(0.5L - x) = 96.72\ kN$

$V_{Tr} = 264.81\ kN$

Live Load Distribution Factor for Shear:

$DFV_{se} = DFM_{se} = 0.65$

$DFV_{me} = e_V.DFV_{mi}$

$e_V = 0.60 + d_e/3000$

$\quad = 0.60 + 400/3000 = 0.733$

$DFV_{mi} = 0.817$

$DFV_{me} = 0.733\ x\ 0.817 = 0.6$

$\rightarrow DFV_{int} = 0.65$

$V_{LL+IM} = DFV_{ext}[(1 + IM)V_{Tr} + V_{Ln}]$

$\quad = 0.65[1.33\ x\ 246.81 + 96.72] = 276.24\ kN$

Strength I limit State (Factored Shear):

$V_u = \eta_i[1.25V_{DC} + 1.50V_{DW} + 1.75V_{LL+IM}]$

$\quad = 1.0[1.25\ x\ 282.78 + 1.5\ x\ 16.64 + 1.75\ x\ 276.24] = 861.86\ kN$

Check the adequacy of the section for shear resistance:

$V_n = 0.25f_c'.b_v.d_v = 2544\ kN$

$\phi V_n = 2289.6\ kN > V_u = 861.86\ kN \rightarrow$ the section is adequate

$V_c = 0.166\sqrt{f_c'}.b_v.d_v = 285.53\ kN$

$\phi V_c = 257\ kN < V_u \ \rightarrow\ A_v$ is required

$V_s = (V_u - \phi V_c)/\phi = (861.86 - 257)/0.9 = 672.07\ kN$

Details of shear reinforcement:

$v_u = V_u/\emptyset b_v.d_v = 861.86\ x10^3/261.67\ x10^3 = 3.294\ MPa$

$0.125f_c' = 4.375\ MPa > v_u$

$s_{max} = 600\ mm$

$\emptyset_v = 12\ mm \ \rightarrow A_v = 226\ mm^2$

$s = A_v.f_y.d_v/V_s = 137.99x10^6/(672.07x10^3) = 205.13\ mm$

use Ø12 @ 150 mm o.c. stirrups

∴ Reinforcement of the interior T-girders is adequate for the exterior T-girders