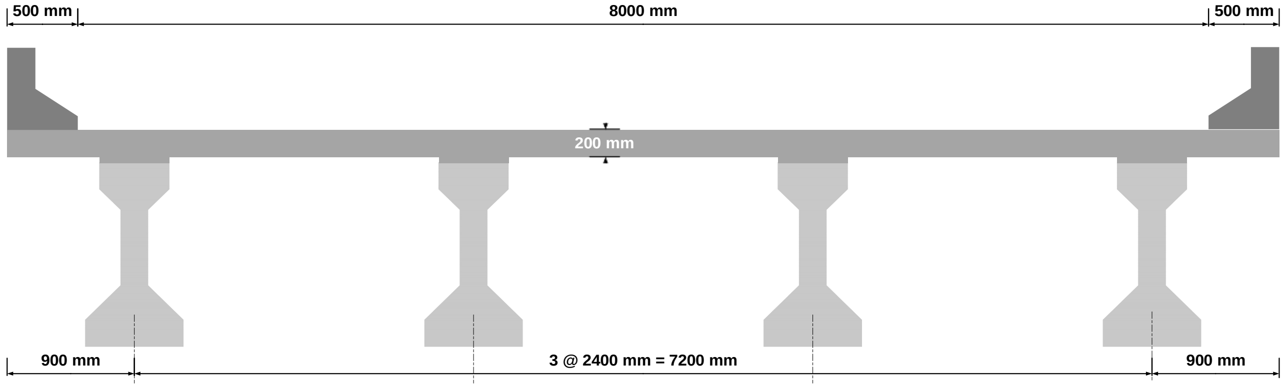
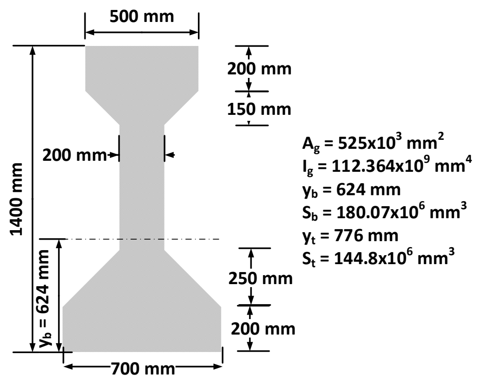
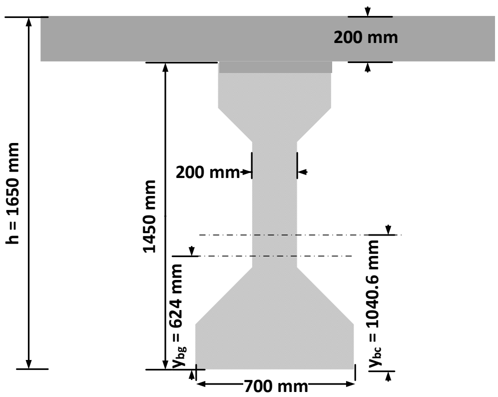
Design of Prestressed Girders

Ex. 1: Design the interior beams for the bridge of single span (L) = 24 m on total width of 9 m shown below to carry standard HL-93 load in addition to distributed weight of overlay (FWS) = 0.8 kN/m^2, integral concrete wearing surface of 20 mm thick, intermediate diaphragms of 0.3 m thickness with 0.6 m depth at 1/3 points and New Jersey-type barriers of 5 kN/m each one. The beams must be pretensioned precast concrete supporting cast-in-place deck slab. The design data are: for beams; concrete strength at prestress transfer (f'_{ci}) = 30 MPa and at service (f'_c) = 35 MPa, for deck slab; concrete strength (f'_c) = 28 MPa, for reinforcing steel; yield strength of (f_y) = 420 MPa. Also, the prestressing steel is low-relaxation strands of yield strength of (f_{py}) = 1674 MPa, ultimate strength (f_{pu}) = 1860 MPa and 20% losses expected.



Section





Sol:

Determination of Composite Section Properties:

$$b_f = S = 2400\ mm$$

$$E_c = 0.043 K_1 \Upsilon_c^{1.5} \sqrt{f'_c}$$

$$n = E_{c,d}/E_{c,g} = \sqrt{28/35} = 0.8944$$

$$b_e = n.b_f = 0.8944\ x\ 2400 = 2146.6\ mm$$

$$A_{d,tr} = b_e.h_d = 2146.6\ x\ 200 = 429.3x10^3\ mm^2$$

$$I_{d,tr} = b_e.h^3/12 = 2146.6\ x\ 200^3/12 = 1.431x10^9\ mm^4$$

$$h = h_d + h_h + h_g = 200 + 50 + 1400 = 1650\ mm$$

Component	A mm^2	y_t mm	$A.y_t$ mm^3	y_{tc} mm	$I_\circ$ mm^4	$d = (y_t - y_{tc})$ mm	$A.d^2$ mm^4	$I_\circ + A.d^2$ mm^4
Deck	$429.3x10^3$	100	$42.93x10^6$	609.4	$1.431x10^9$	− 509.4	$111.398x10^9$	$112.829x10^9$
Girder	$525x10^3$	1026	$538.65x10^6$		$112.364x10^9$	416.6	$91.117x10^9$	$203.482x10^9$
$\sum$	$954.3x10^3$		$581.58x10^6$					$316.311x10^9$

$y_{tcd} = \sum(A.y_t)/\sum A = 581.58x10^6/954.3x10^3 = 609.4\,mm$

$y_{tcg} = y_{tcd} - h_d - h_h = 609.4 - 200 - 50 = 359.4\,mm$

$y_{bcg} = h - y_{tcd} = 1650 - 609.4 = 1040.6\,mm$

$I_c = \sum(I_\circ + A.d^2) = 316.311x10^9\,mm^4$

$S_{tcd} = I_c/(n.y_{tcd}) = 316.311x10^9/(0.8944\,x\,609.4) = 580.34x10^6\,mm^3$

$S_{tcg} = I_c/y_{tcg} = 316.311x10^9/359.4 = 880.11x10^6\,mm^3$

$S_{bcg} = I_c/y_{bcg} = 316.311x10^9/1040.6 = 303.97x10^6\,mm^3$

Determination of Unfactored Loads:

Force effects from unfactored composite (dead) loads:

$w_d = h_d\,x\,b_f\,x\,\Upsilon_c = 0.2\,x\,2.4\,x\,24 = 11.52\,kN/m$

$w_{iws} = h_{iws}\,x\,b_f\,x\,\Upsilon_c = 0.02\,x\,2.4\,x\,24 = 1.152\,kN/m$

$w_h = h_h\,x\,b_h\,x\,\Upsilon_c = 0.05\,x\,0.5\,x\,24 = 0.6\,kN/m$

$w_g = A_g\,x\,\Upsilon_c = 0.525\,x\,24 = 12.6\,kN/m$

$b_{dia} = S - b_w = 2.4 - 0.2 = 2.2\,m$

$DL_{dia} = (b\,x\,d\,x\,t)_{dia}\,x\,\Upsilon_c = 2.2\,x\,0.6\,x\,0.3\,x\,24 = 9.504\,kN$

$w_{dia} = N_{dia} x\,DL_{dia}/L = 2\,x\,9.504/24 = 0.792\,kN/m$

$w_{DC1} = w_{D,nc} = 11.52 + 1.152 + 0.6 + 12.6 + 0.792 = 26.67\,kN/m$

$M_{DC1} = w_{DC1}L^2/8 = 26.67\,x\,24^2/8 = 1920.24\,kN.m$

Force effects from unfactored composite (dead and live) loads:

$w_{DC2} = w_{D,c} = 2w_{ba}/N_g = 2\,x\,5/4 = 2.5\,kN/m$

$M_{DC2} = w_{DC2}.L^2/8 = 2.5\,x\,24^2/8 = 180\,kN.m$

$w_{DW} = w_{DW} = w_{fws}\,x\,w/N_g = 0.8\,x\,8/4 = 1.6\,kN/m$

$M_{DW} = w_{DW}.L^2/8 = 1.6\,x\,24^2/8 = 115.2\,kN.m$

$w_{Ln} = 9.3\,kN/m$

$M_{Ln} = w_{Ln}.L^2/8 = 9.3\,x\,24^2/8 = 669.6\,kN.m$

$M_{Tr} = 1570.15\,kN.m$

Live load distribution factors:

$N_g \geq 4$ $\quad N_g = 4\ \therefore OK$

$6 \leq L \leq 73$ $\quad L = 24\,m\ \therefore OK$

$1.1 \leq S \leq 4.9$ $\quad S = 2.4\,m\ \therefore OK$

$110 \leq h_d \leq 300$ $\quad h_d = 200\,mm\ \therefore OK$

$n = E_{c,g}/E_{c,d} = \sqrt{35/28} = 1.118$

$I_g = 112.364x10^9\,mm^4$

$A_g = 525x10^3\ mm^2$

$e_g = y_{tg} + h_h + h_d/2 = 776 + 50 + 200/2 = 926\ mm$

$K_g = n(I_g + A_g.e_g{}^2) = 1.118(112.364x10^9 + 525x10^3\ x\ 926^2)$

$= 628.919x10^9\ mm^4$

$4x10^9 \leq K_g \leq 3x10^{12} \qquad K_g = 628.919x10^9\ mm^4\ \therefore OK$

Thus, the cross section satisfies the design stipulations

$w = 9 - 2(0.5) = 8\ m\ \rightarrow N_L = 2$

$\therefore$ check both DF_{si} and DF_{mi}

Live Load Distribution Factor for Moment:

$DFM_{si} = 0.06 + (S/4300)^{0.4}.(S/L)^{0.3}.\left(K_g/L.h_d{}^3\right)^{0.1}$

$= 0.06 + (2.4/4.3)^{0.4}.(2.4/24)^{0.3}.(0.62892/24x0.2^3)^{0.1} = 0.507$

$DFM_{mi} = 0.075 + (S/2900)^{0.6}.(S/L)^{0.2}.\left(K_g/L.h_d{}^3\right)^{0.1}$

$= 0.075 + (2.4/2.9)^{0.6}.(2.4/24)^{0.2}.(0.62892/24x0.2^3)^{0.1} = 0.709$

$\rightarrow DFM_{int} = 0.709$

$IM = 0.33$

$M_{LL+IM} = DFM_{int}[(1 + IM)M_{Tr} + M_{Ln}]$

$= 0.709[1.33\ x\ 1570.15 + 669.6] = 1955.35\ kN.m$

Determination of Required Effective Prestress Load

$$f_{bot} = \frac{M_{DC1}}{S_{bg}} + \frac{M_{DC2} + M_{DW} + 0.8M_{(LL+IM)}}{S_{bcg}}$$

$$= \frac{1920.24x10^6}{180.07x10^6} + \frac{180x10^6 + 115.2x10^6 + 0.8\ x\ 1955.35x10^6}{303.97x10^6} = 16.79\ MPa$$

$f_t = 0.50\sqrt{f_c'} = 0.5\ x\ \sqrt{35} = 2.95\ MPa$

$\because f_{bot} = 16.79\ MPa > f_t = 2.95\ MPa\ \rightarrow\ \therefore$prestress is required

$f_{c,pe} = f_{bot} - f_t = 16.79 - 2.95 = 13.84\ MPa$

Assume $y_{bp} = 0.085h_g = 0.085\ x\ 1400 \cong 120\ mm$

$e_c = y_{bg} - y_{bp} = 624 - 120 = 504\ mm$

$$f_{c,pe} = \frac{P_e}{A_g} + \frac{P_e.e_c}{S_{bg}} \rightarrow 13.84 = \frac{P_e}{525x10^3} + \frac{P_e\ x\ 504}{180.07x10^6} \rightarrow P_e = 2942.39\ kN$$

Determination of Required Number of Strands

$f_{pi} = 0.75f_{pu} = 0.75\ x\ 1860 = 1395\ MPa$

Try $\emptyset_p = 12.7\ mm\ \rightarrow\ A_p = 98.7\ mm^2$

$P_{i,p} = A_p.f_{pi} = 98.7\ x\ 1395 = 137.68\ kN$

$R = 1 - losses = 1 - 0.2 = 0.8$

$P_{e,p} = R.P_{i,p} = 0.8\ x\ 137.86 = 110.29\ kN$ [one strand provision]

$N_p = P_e/P_{e,p} = 2942.39/110.29 = 26.68$ say 27 strands