مرحله رابعة
ست سناء

Specific Humidity (qv): is the ratio of the densities of vapor and moist air.

$$qv = \frac{\rho v}{\rho a} \qquad \text{..........................(1)}$$

Daltons law states that the pressure exerted by a gas (its vapor pressure) is independent on the pressure of other gas, the vapor pressure of the water vapor is given by ideal gas law as:

$e = \rho v * Rv * T$ ....(2)
T =Absolute Temp. (K^0)
Rv=Gas constant for water vapor.
ρv=The density of water vapor.

If the total pressure exerted by the moist air (P)
P-e is the partial pressure due to dry air.

$P - e = \rho d * Rd * T$ ....(3)
ρd= the density of dry air.
Rd=Gas constant of dry air =287(J/Kg.K^0).
T= absolute Temp. (K^0).

The density of moist air is (ρa)= $\rho d + \rho v$
The gas constant of water vapor :
Rv=Rd/0.622 ....(4)
Where (0.622) is the ratio of molecular weight of water vapor to the average molecular weight of dry air.

Combining eqs.(2)and (3):

$$P = \left[\rho_d + \left(\frac{\rho v}{0.622}\right)\right] R_d \times T \qquad \text{..........................(5)}$$

The specific humidity qv is approximated by :

$$q_v = 0.622\frac{e}{P} \qquad \text{..........................(6)}$$

Also eq. (5) can be rewritten in terms of the gas constant for the moist air (Ra) as:

$P = \rho a * Ra * T$ ....(7) (T) in kalvins.

The relation ship between the gas constants for moist and dry air is given by:

Ra=Rd (1+0.608qv)
Ra=287(1+0.608qv) ....(8)

For a given air temperature there is maximum moisture content the air can hold, the vapor pressure is called the saturation vapor pressure at this vapor pressure. The rates of evaporation and condensation are equal:

تركيز

$$es = 611\exp\left[\frac{17.27T}{237.3+T}\right] \qquad \text{.....................(9)}$$

es is in Pascal. (N/m^2)
T is degree Celsius.

The gradient Δ =(des/dT) of the saturated vapor pressure curve is found by differentiating eq.(9):

$$\Delta = \frac{4097es}{(237.3+T)^2} \qquad \text{.........................(10)}$$

Where Δ is the gradient in pascal per degree Celsius.

$$\text{Relative humidity } R_h = \frac{e}{es} \qquad \text{......................(11)}$$

Ex. At a climate station, air pressure is measured as 100Kpa, air temp. was 20C^0, and the wet bulb temp as 16C^0. Calculate the <u>corresponding vapor pressure, relative humidity, specific humidity and air density.</u>

Solution:

$$es = 611\exp\left[\frac{17.27T)}{237.3+T}\right]$$

$$= 611\left[\frac{17.27*20}{237.3+20}\right]$$

=2339 Pa.

$$e = 611\exp\left[\frac{17.27T)}{237.3+T}\right]$$

$$= 611\left[\frac{17.27*16}{237.3+16}\right] = 1819\text{Pa.}$$

$$R_h = \frac{e_w}{e_s} = \left[\frac{1819}{2339}\right] = 0.78 = 78\%$$

$$q_v = 0.622\frac{e}{P} = 0.622\frac{1819}{100*1000} = 0.0113 kg_w / kg_{moist}.$$

Ra=Rd(1+0.608 qv)
=287(1+0.608*0.113)

P = ρa* Ra* T
100*10^3 = ρa*289*(20+273)
ρa=1.18 kg/m^3

Example [2]

Calculate the relative humidity hr ,specific humidity q_v, air density ρ_a and dry air density ρ_d, in a climate station where the air pressure is 100 kpa (standard pressure=760 mmHg equivalent to101.3 kpa), air temperature is 20 C^o , temperature of wet-bulb is 16 C^o , Also set your comment about ρ_a and ρ_d? $\mathcal{R}_d = 287\ J/(Kg.K^o)$.

$$e_s = 4.584\ e^{\left(\frac{17.27T}{237.3+T}\right)}$$

$$e_s|_{20C} = 4.584\ e^{\left(\frac{17.27(20)}{237.3+20}\right)} = 17.55\ mmHg$$

$$e_w|_{16C} = e_s|_{16C} = 4.584\ e^{\left(\frac{17.27(16)}{237.3+16}\right)} = 13.65\ mmHg$$

$$\gamma = \frac{e_w - e}{t - t_w}\ ;\quad 0.49 = \frac{13.65 - e}{20 - 16}\ ;\quad e = 11.69\ mmHg\ ;\quad h_r = \frac{e}{e_s} = \frac{11.69}{17.55} = 66.61\%$$

$$P = 100\ kp = 760 * \frac{100}{101.3} = 750.25\ mmHg$$

$$q_v = 0.622\frac{e}{p} = 0.622\frac{11.69}{750.25} = 0.0097\ \ kg\ of water/kg\ of\ moist$$

$$T = 20 + 273 = 293\ K^o$$

$$\mathcal{R}_a = 287[1 + 0.608(0.0097)] = 288.7\ \ J/(Kg.K^o)$$

$$\rho_a = \frac{P}{\mathcal{R}_a T} = \frac{100\ 000}{288.7(293)} = 1.182\ kg/m^3$$

$$\rho_d = \frac{P}{\mathcal{R}_d T} = \frac{100\ 000}{287(293)} = 1.189\ kg/m^3$$

Thus; $\rho_a < \rho_d$,then the moist air is floating over dry air layer due to buoyancy.

لغرض حساب ea بأستخدام المحرار ذو البصلة الرطبة (المرطاب psycrometer)
حيث يقيس الجهاز درجة الحرارة tw (wet bulb Temp.) وهي الدرجة التي يصل اليها الهواء
عند تبريده بواسطة تبخر الماء ، نستخدم المعادلة ادناه :

$$\frac{e_w - e_a}{t_w - t_a} = \gamma$$

e_w : Saturated vapor press. corresponding to wet-bulb temp.

t_w : wet-bulb temp.

t_a : dry-bulb temp.

e_a : corresponding air vapor press to ta

γ = Psycrometer constant ثابت المرطاب

$\gamma = 6.5 \times 10^{-4}$ Pa / c°

$\gamma = 0.49$ mm Hg / °c

$\gamma = 0.066$ kpa / °c

$\gamma = 0.66$ mb / °c

P_a = atmospheric press.

وحدات قياس الضغط

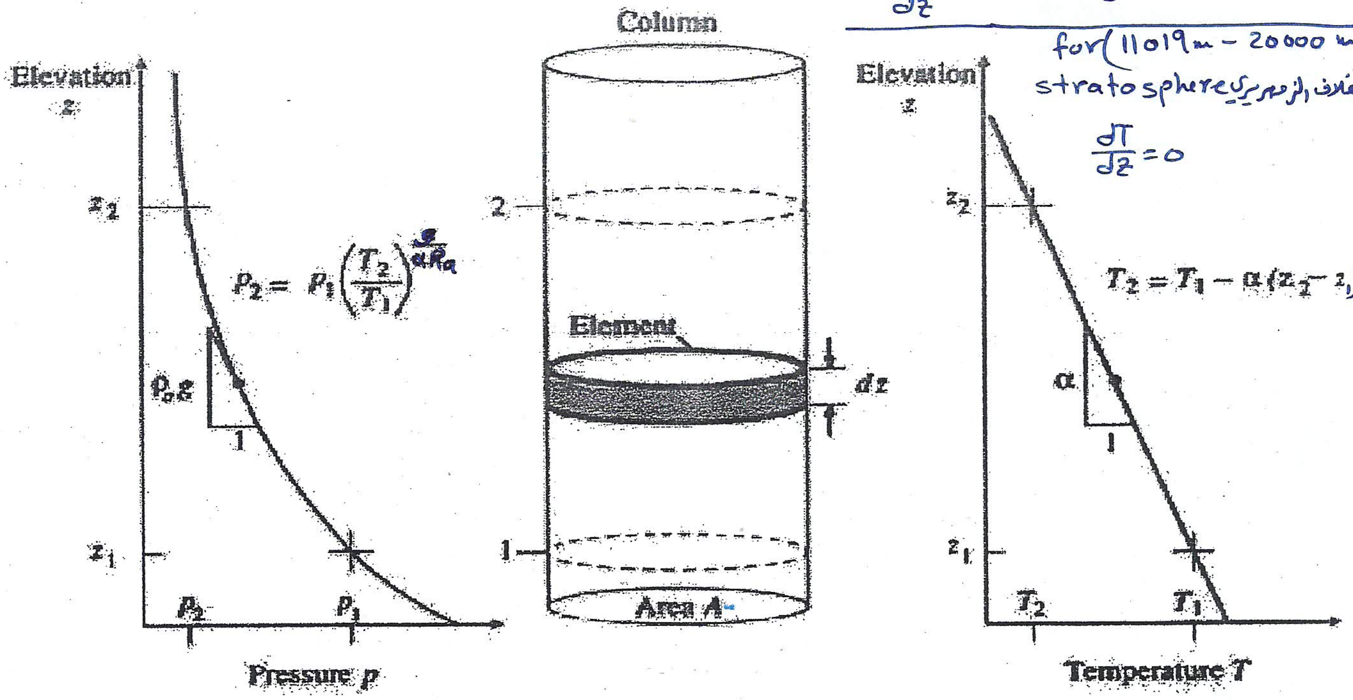
$mb = \frac{1}{10}\ Kpa \Rightarrow mb = 100\ Pa$
$mb = \frac{3}{4}\ mm\ Hg$
$mm\ Hg = \frac{2}{15}\ Kpa$

Water Vapor in a Static Atmospheric Column

$\alpha = -0.0065\ ^\circ c/m$ for
for lower portion of atmosphere
(0m – 11019m) (Tropospher)
$\frac{dT}{dz} = -0.0065$

for (11019m – 20000m)
stratosphere الغلاف الزمهريري
$\frac{dT}{dz} = 0$

يتغير الضغط
مع الارتفاع
حيث يقل كلما
ابتعدنا عن سطح
الارض



Two laws govern the properties of water vapor in a static column:

1- **The ideal gas law:** $P = \rho a * Ra * T$ ....(12)

2- **The hydrostatic pressure law:** $\frac{d_p}{d_z} = -\rho_a g$ ....(13)

The variation of air temp. with altitude is described by:

$$\frac{d_T}{d_z} = -\alpha \qquad ....(14)$$

Where α is the lapse rate.

From eq.(12) $\rho_a = \dfrac{P}{R_a T}$

Substitute in eq.(13)

$$\frac{dp}{dz} = \frac{-\rho g}{R_a T}$$

$$\frac{dp}{P} = (\frac{-g}{R_a T})dz$$

Substitute eq.(14), we can get:-

$$\therefore \frac{dP}{P} = (\frac{-g}{R_a T})(\frac{-dt}{\alpha})$$

$$\therefore \frac{dP}{P} = (\frac{-g}{R_a \alpha})(\frac{dT}{T})$$

And integrating both sides between levels (1) and (2) In the atmosphere gives:

$$Ln\frac{P_2}{P_1} = (\frac{g}{\alpha R_a})Ln\frac{T_2}{T_1}$$

Or

$$P_2 = P_1(\frac{T_2}{T_1})^{g/\alpha R_a} \quad \text{.............................(15)}$$

T : K°

From eq.(14) the temp. variation between altitude Z1 and Z2:

T2=T1-α (Z_2-Z_1) ....(16)

<u>Perceptible Water</u>

The amount of moisture in an atmospheric column is called (perceptible water). Consider an element of height dz in a column of horizontal cross-sectional area A. The <u>mass of air</u> in the element is ρa.A.d_z

(ρ_v)

$q_v = \frac{\rho_v}{\rho_a} \Rightarrow \rho_a = q_v \cdot \rho_v$

The mass of water contained in the air is qv. ρa.A.d_z
The total mass of perceptible water in the column between elevation $z_1$andz_2 is:

$$mp = \int_{z_1}^{z_2} qv.\rho a.A.dz \quad \text{....17}$$

The integral eq.17 is calculated using incremental mass of perceptible water:

$$\Delta mp = qv'.\rho a'.A.\Delta z \quad \text{..........................18}$$

Where:

qv' and $\rho a'$ are the average values of specific humidity and air density over the interval.
The mass increments are summed over the column to give the total perceptible water.

Example. Calculate the precipitable water in a saturated air column 10 km high above 1 m2 of ground surface. The surface pressure is 101.3 kPa, the surface air temperature is 300C, and the lapse rate is 6.5°C/km.

$30^{\circ}c$

Solution:
The increment in elevation is taken as Az = 2 km = 2000 m. For the first increment, at Z1 = 0 m, T1 = 300C = (30 + 273) K = 303 K; at z2 = 2000 m, using a = 6.5°C/km = 0.0065°C/m, $30^{\circ}c$

$$
\begin{aligned}
T_2 &= T_1 - \alpha(z_2 - z_1) \\
&= 30 - 0.0065(2000 - 0) \\
&= 17^{\circ}\mathrm{C} \\
&= (17 + 273)\ \mathrm{K} \\
&= 290\ \mathrm{K}
\end{aligned}
$$

The gas constant Ra can be taken as 287 J/kg-K in this example because its variation with specific humidity is small [see Eq. (.8)].The air pressure at 2000 m is then given by eq. (.15) with
g/aRa = 9.81/(0.0065 x287) = 5.26

$$
\begin{aligned}
p_2 &= p_1\left(\frac{T_2}{T_1}\right)^{g/\alpha R_a} \\
&= 101.3\left(\frac{290}{303}\right)^{5.26} \\
&= 80.4\ \mathrm{kPa}
\end{aligned}
$$

The air density at the ground is then calculated:

$$
\begin{aligned}
\rho_a &= \frac{p}{R_a T} \\
&= \frac{101.3 \times 10^3}{(287 \times 303)} \\
&= 1.16\ \mathrm{kg/m^3}
\end{aligned}
$$

$\rho_a = \frac{80.4 \times 10^3}{287 * 290}$
$= 0.966 \approx 0.97\ kg/m^3$

and a similar calculation yields the air density of 0.97 kg/m^3 at 2000 m. The average density over the 2 km increment is therefore $\bar{\rho}_a = (1.16 + 0.97)/2 = 1.07$ kg/m^3 (see columns 5 and 9).

The saturated vapor pressure at the ground is determined as:

$$e = 611 \exp\left(\frac{17.27T}{237.3 + T}\right)$$

$$= 611 \exp\left(\frac{17.27 \times 30}{237.3 + 30}\right)$$

$$= 4244 \text{ Pa}$$

$$= 4.24 \text{ kPa}$$

The corresponding value at 2000 m where $T = 17°C$, is $e = 1.94$ kPa (column 6).

The specific humidity at the ground surface is calculated

$$q_v = 0.622\frac{e}{p}$$

$$= 0.622 \times \frac{4.24}{101.3}$$

$$= 0.026 \text{ kg/kg}$$

At 2000 m $q_v = 0.015$ kg/kg. The average value of specific humidity over the 2-km increment is therefore $\bar{q}_v = (0.026 + 0.015)/2 = 0.0205$ kg/kg (column 8).

The mass of precipitable water in the first 2-km increment is:

$$\Delta m_p = \bar{q}_v \bar{\rho}_a A \, \Delta z$$

$$= 0.0205 \times 1.07 \times 1 \times 2000$$

$$= 43.7 \text{ kg}$$

The equivalent depth of liquid water is:

$m_p / \rho_w A = 77/(1000 \times 1) = 0.077$ m = 77 mm.

Column	1 Elevation z (km)	2 Temperature (°C)	3 (°K)	4 Air pressure p (kPa)	5 Density ρ_a (kg/m³)	6 Vapor pressure e (kPa)
	0	30	303	101.3	1.16	4.24
	2	17	290	80.4	0.97	1.94
	4	4	277	63.2	0.79	0.81
	6	−9	264	49.1	0.65	0.31
	8	−22	251	37.6	0.52	0.10
	10	−35	238	28.5	0.42	0.03

Column	7 Specific humidity q_v (kg/kg)	8 Average over increment $\bar{q}_v$ (kg/kg)	9 $\bar{\rho}_a$ (kg/m³)	10 Incremental mass Δm (kg)	11 % of total mass
	0.0261				
	0.0150	0.0205	1.07	43.7	57
	0.0080	0.0115	0.88	20.2	26
	0.0039	0.0060	0.72	8.6	11
	0.0017	0.0028	0.59	3.3	4
	0.0007	0.0012	0.47	1.1	2
				$\Sigma =$ 77.0	