Mustansiriyah University
Materials Engineering Department

Applied Engineering Analysis
Course II

Tutor: Ahmed AYASH, Ph.D.

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- An overview of engineering analysis
- Types of differential equations
- A quick review of solving ordinary differential equations (ODEs)
- Boundary and initial conditions

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- Classification and Characteristics of Linear PDEs

- Methods of solving PDEs.
  Combination of variables
  Separation of Variables
  Using Laplace Transformation

- Analysis of engineering systems producing PDEs

Chapter Three: Numerical Methods

This chapter includes using numerical methods to carry out Interpolation, Numerical differentiation, Numerical integration and solving ODEs and PDEs.

Chapter Four: Using Software to solve equations (Algebraic, ODEs and PDEs)

- Microsoft Excel 2013 using Solver
- Matlab
- Maple
- Comsol Multiphysics (Basic) Ver. 5.2

Recommended References

1. Powers, D.L., 2009. *Boundary value problems: and partial differential equations*. Academic Press.

2. Rice, R.G. and Do, D.D., 2012. *Applied mathematics and modelling for chemical engineers*. John Wiley & Sons.

3. Kreyszig, E., 2009. *Advanced engineering mathematics*. 9th edition. John Wiley & Sons.

4. Courant, R. and Hilbert, D., 2008. *Methods of Mathematical Physics: Partial Differential Equations*. John Wiley & Sons.

5. Chapra, S.C. and Canale, R.P., 2010. *Numerical methods for engineers*. Boston: McGraw-Hill Higher Education.

Course Grade

Activity	Percent
Mid-Course Exam	This will count for **40%** of the course grade.
Homework	The total homework assignment score will count for **25%** of the course grade. **Late homework** will not be accepted unless you have a valid reason **and** you arrange it with me **in advance**.
Pop-quizzes	There will be 8 - 10 of them. Quiz questions will refer only to the recently covered material **and** not to the new material. Two or three lowest quiz scores will be dropped, and the remaining scores will count for **25%** of the course grade.
Others	This includes submitting posters about solving equations using software **or** analysis an engineering problems. This will count for **10%** of the course grade.

Chapter One

Introduction

At the end of this chapter, you should be able to:

- Understand the concept of engineering analysis.
- Make analysis for engineering problems.
- Classify the differential equations
- Explain what is meant by boundary and initial conditions (or boundary-value problems).

2. An Overview of Engineering Analysis

Engineering analysis deals with the use of basic physical laws and mathematical skills to describe, study and understand different engineering systems. In most cases, engineering analysis involves translation of physical (engineering) problems to a set of mathematical equations (algebraic equations, differential equations or even empirical equations based on experimental data). Describing systems in this way (i.e. using mathematical formula) is known as "*modelling of engineering systems*".

According to the above description, an engineering analysis of a problem can be represented, simply, using the following diagram.

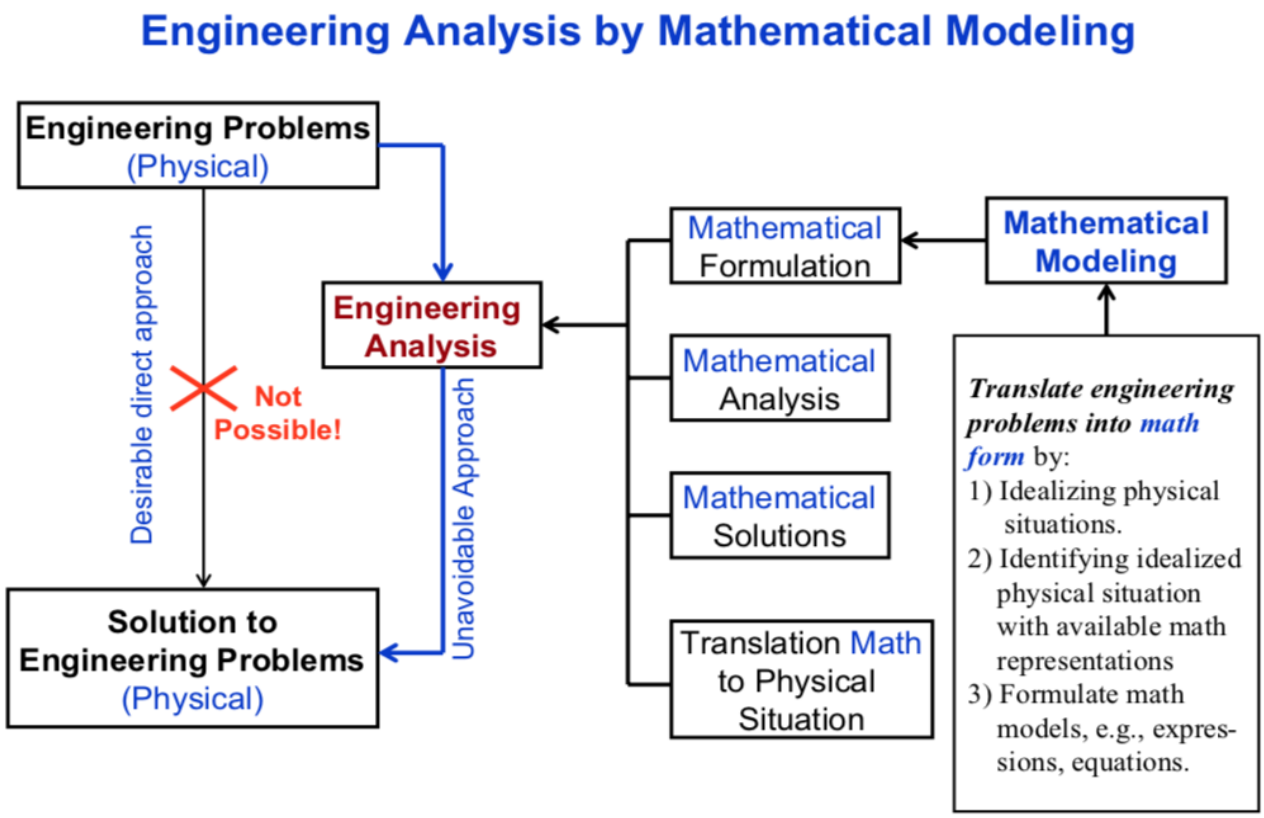


Figure 1. Analysis of an engineering problem.

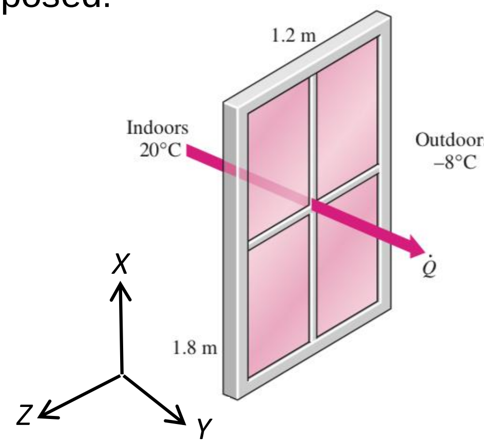
2.1 Example of an engineering analysis of a problem:

A window of a house with 10 cm thickness is maintained at 20°C. Find the temperature distribution through the window when the outdoor air temperature is – 8°C and there is a heat source of 0.8 W/m^3.

Step 1: mathematical Formulation

To simplify the problem, some assumptions need to be proposed:

1. Heat transfer occurs only in the *y*-direction.
2. The process is steady-state.
3. The rate of heat generation is constant.



Under these assumptions, energy balance on an element with dy thickness and *A* area gives:

$$q_{in}\big|_{\text{at } y} + \dot{q}(Ady) = q_{out}\big|_{\text{at } y+dy} \Rightarrow \lim_{dy \to 0} \frac{q_{out}\big|_{\text{at } y+dy} - q_{in}\big|_{\text{at } y}}{dy} = A\dot{q}$$

The limit in the above equation can be replaced by the derivative, thus:

$$\frac{dq}{dy} = A\dot{q} \text{-------------} (1)$$

Step 2: mathematical Analysis

To find the change in temperature with distance in y-direction, Fourier law can be used now in Eq. (1). This gives:

$$-k\frac{d^2T}{dy^2} = \dot{q} \text{-------------} (2)$$

Step 3: mathematical Solution

The solution of Eq. (2) is straightforward and can be written as:

$$T = -\frac{\dot{q}}{K}\frac{y^2}{2} + C_1 y + C_2 \text{--------} (3)$$

where C_1 and C_2 are constant

Step 4: Translate Math to physical situation

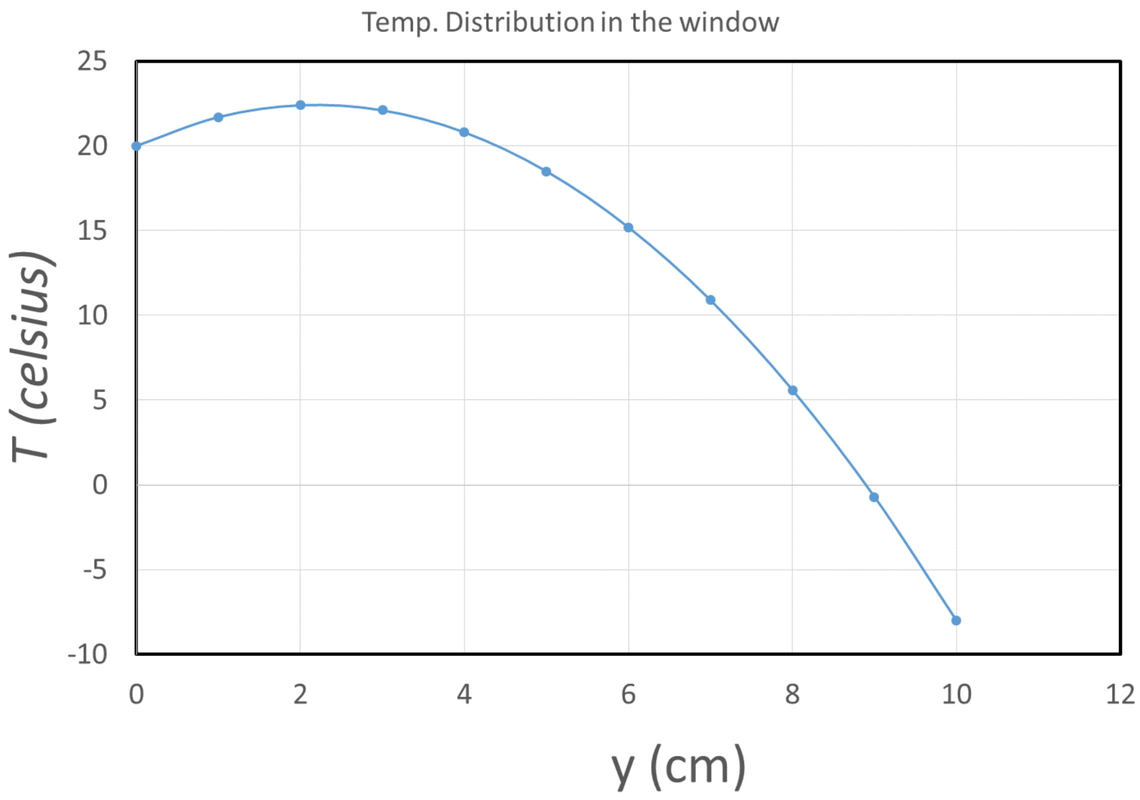
It is necessary to find the value of the constants (C_1 and C_2) in Eq. 3 to be able to determine the temperature distribution in the window. To find C_1 and C_2, the known conditions at the boundary of the window will be used (in some textbooks called boundary conditions). These are:

$$\text{B.C 1: at } y = 0 \quad \rightarrow \quad T = 20\,°\text{C}$$

$$\text{B.C 2: at } y = 10\text{ cm} \quad \rightarrow \quad T = -8\,°\text{C}$$

Now, take K_{glass}= 0.8 W/K and substitute the above conditions in Eq. 3, the temperature distribution can be written as:

$$T = -\frac{y^2}{2} + 2.2y + 20 \quad -------(4)$$



Conclusion: Math (i.e. solving the equations) is the key factor in the analysis process of engineering problems. Since most the practical problems produce differential equations, methods of solving such equations would be a helpful tool for engineers.

2. Types of differential equations

2.1 Ordinary Differential equations (ODEs)

ODE is a differential equation that contains one or more functions of one independent variable and the derivatives of those functions.

ODEs are classified according to their order as:

– *A first order equation* includes a first derivative as its highest derivative.

$$\frac{dy}{dx} + \alpha(x)y = f(x)$$

– *A second order equation* includes a second derivative.

$$\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + y^{\alpha} = 0 \qquad \text{(Lane–Emden equation)}$$

$$\frac{d^2\psi}{dt^2} + \omega^2 \sin\psi = 0 \qquad \text{(Nonlinear Pendulum equation)}$$

$$\frac{d^2y}{dx^2} + ay + by^3 = 0 \qquad \text{(Duffing equation)}$$

$$\frac{d^2y}{dx^2} + a(y^2 - 1)\frac{dy}{dx} + y = 0 \qquad \text{(Van der Pol equation)}$$

– *A high order equation* includes a derivative higher than second order.

$$\frac{d^n y}{dx^n} + \frac{d^{n-1}y}{dx^{n-1}} + \frac{d^{n-2}y}{dx^{n-2}} + \ldots\ldots + f(x)y = 0$$

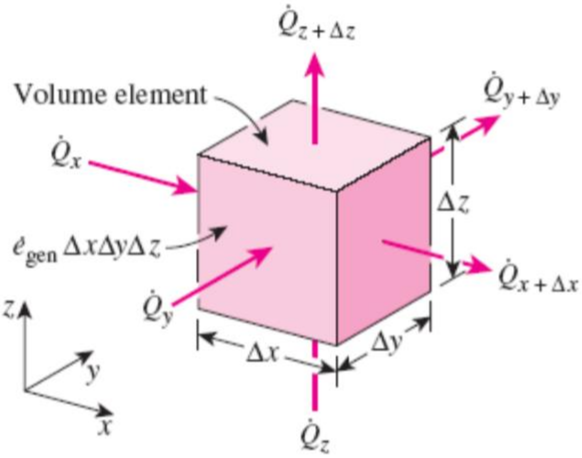
2.2 Partial differential equations (PDEs)

PDE is a differential equation that contains more than one independent variables. (This is in contrast to ordinary differential equations, which deal with functions of a single independent variable.) PDEs can be used to describe a wide variety of phenomena such as sound, heat, electrostatics, electrodynamics, fluid flow, or elasticity.

Some Examples of PDEs:

1) Unsteady-state three dimensional Heat conduction in a solid:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

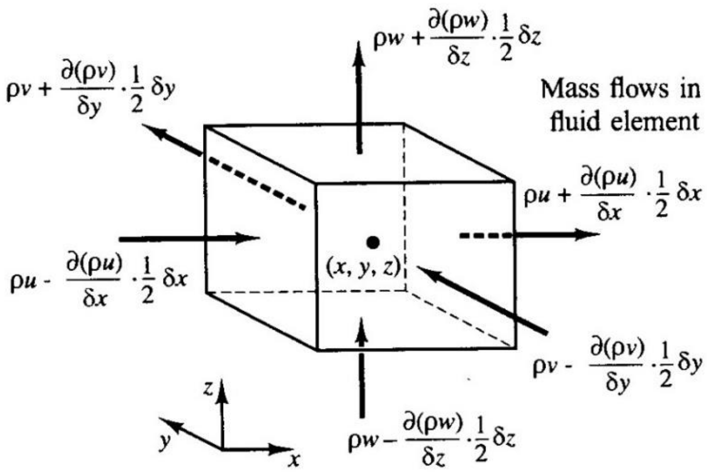


2) Navier-Stokes equations for incompressible Newtonian fluid:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\rho g_x$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\rho g_y$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\rho g_z$$



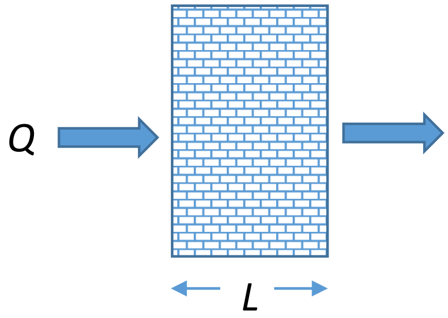
NOTE:

In this course, the focus is on the analytical solution of PDEs. Our principal solution technique will involve separating a partial differential equation into ordinary differential equations. Therefore, you ***must*** review some facts about ordinary differential equations and their solutions.

3. Boundary and Initial Conditions

To solve differential equations, it is always necessary to define the integration constants, exactly as we did in example 2.1 (page 7). This, usually, needs to know the boundary conditions and sometimes the initial conditions for time-dependent problems. For the clarification, unsteady state, one dimensional heat transfer will be taken as an example:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$



Note that, principally, there are five possible types of boundary conditions that may arise:

1. Specific values at boundary; e.g. for $T(x,t)$

   $T(0,3) = 10$ °C and $T(L,3) = 20$ °C

2. Function specified at boundary; e.g. for $T(x,t)$

   $T = f_1(x)$ at $x = 0$, $T = f_2(x)$ at $x = L$; for all t

3. Derivative of function specified at boundary

   $$\frac{\partial T}{\partial x} = 0 \;\; at \;\; x = 0 \;\; ; \;\; -k\frac{\partial T}{\partial x} = Q \;\; at \;\; x = L$$

4. Mixed functions at boundary

   $$-k\frac{\partial T}{\partial x} = h\,(T - T\infty) \;\; at \;\; x = L \;\; ; \text{for all } \; t$$

5. Initial condition which may be written for $T(x,t)$

   $$T(x,0) = f(x) \;\; \text{or a value such as} \; T(x,0) = 5\ ^{\circ}\text{C}$$