Chapter (4)
Additional Analysis Techniques

Here we will study four additional Techniques

- Superposition
- Source transformation
- Thevenin and Norton Theorems
- Maximum power principle

1. <u>Superposition :</u>

<u>Definition</u> :

Whenever a linear circuit is excited by more than one independent source, the total response is the algebraic sum of individual responses

The idea is to activate one independent source at a time to get individual response.

Then add the individual response to get total response

Note:

1. Dependent source are Never deactivated (always active)
2. When an independent voltage source is deactivated, it is set to zero.

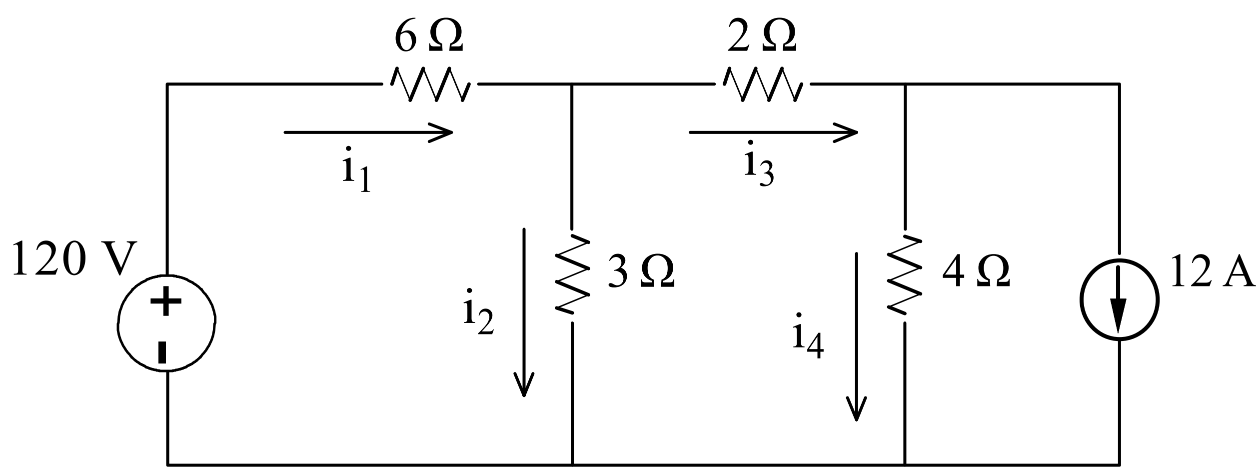
⇨ replaced by short circuit

3. When an independent current source is deactivated, it is set to zero.

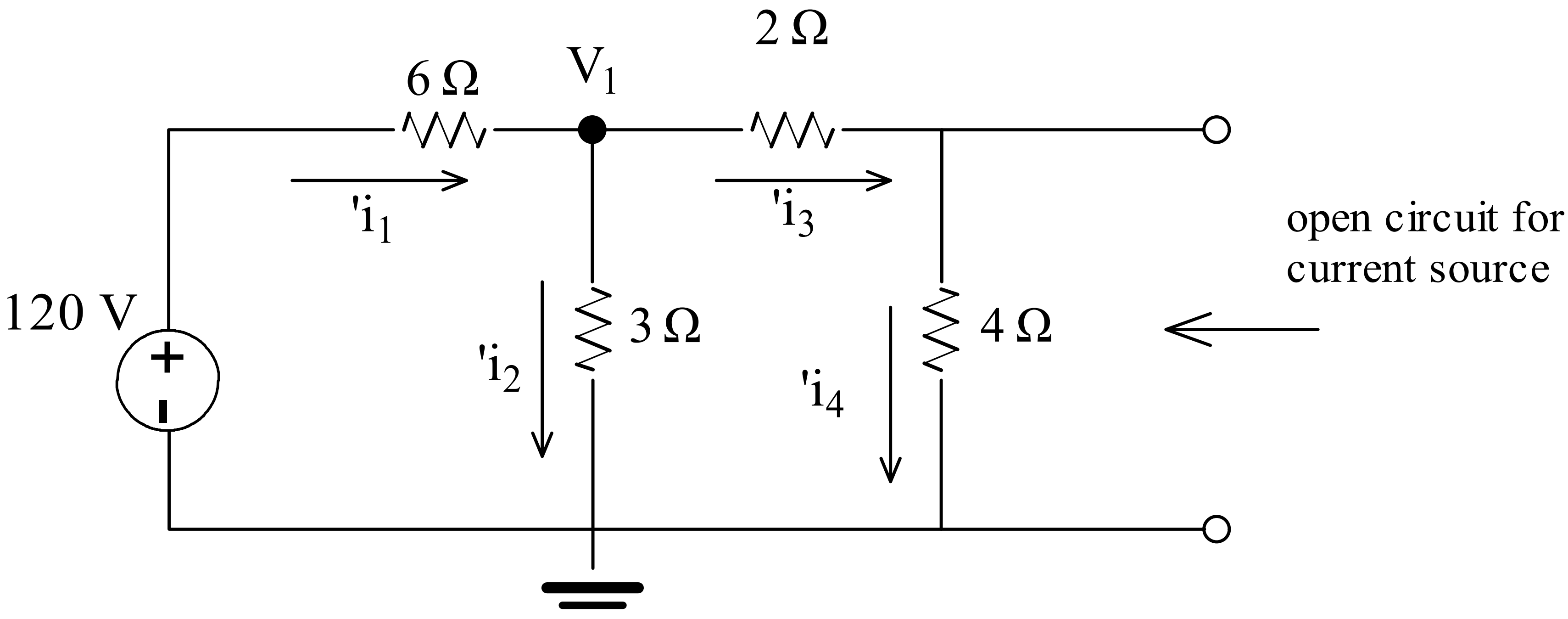
⇨ replaced by open circuit

Example:

Use superposition to find i_1, i_2, i_3, i_4 ?



•Activate independent voltage source 120 V only



•Using KCL at V1 (nodal analysis)

$$'i_1 - 'i_2 - 'i_3 = 0$$

$$\frac{120 - V_1}{6} - \frac{V_1}{3} - \frac{V_1}{2+4} = 0$$

$$20 - V_1\left(\frac{1}{6} + \frac{1}{3} + \frac{1}{6}\right) = 0$$

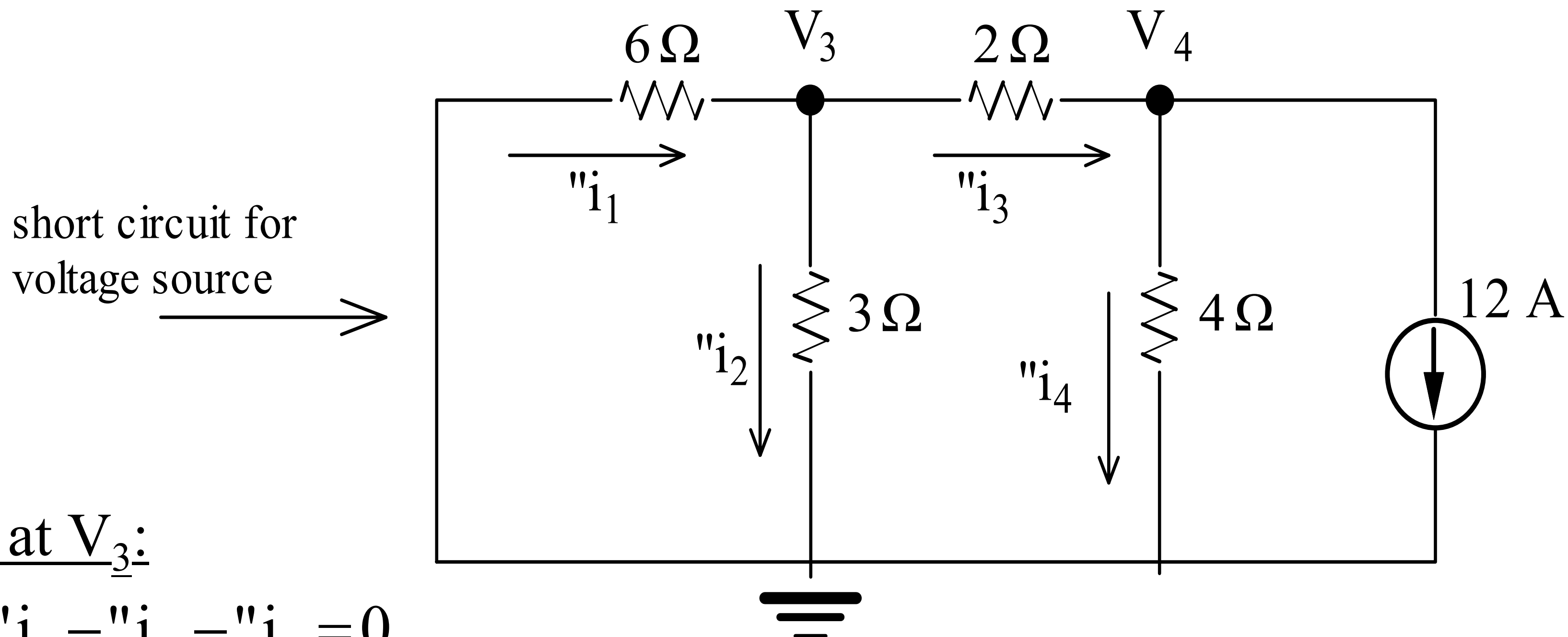
$$\Rightarrow \quad V_1 = 30\ \text{V}$$

$$'i_1 = \frac{120 - V_1}{6} = \frac{90}{6} = 15\ \text{A}$$

$$'i_2 = \frac{V_1}{3} = \frac{30}{3} = 10\ \text{A}$$

$$'i_3 = i_4 = \frac{V_1}{6} = 5\ \text{A}$$

* Activate the independent current source only



KCL at V_3:

$$"i_1 - "i_2 - "i_3 = 0$$

$$\frac{-V_3}{6} - \frac{V_3}{3} - \frac{V_3 - V_4}{2} = 0$$

$$-V_3 - 2\,V_3 - 3\,(V_3 - V\,) = 0$$

$$-6\,V_3 + 3\,V_4 = 0 \qquad \ldots\ldots(1)$$

<u>KCL at V4:</u>

$$"i_3 - "i_4 - 12 = 0$$

$$\frac{V_3 - V_4}{2} - \frac{V_4}{4} - 12 = 0$$

$$2\ V_3 - 2\ V_4 - V_4 = 48$$

$$2\ V_3 - 3\ V_4 = 48 \qquad \ldots\ldots(2)$$

$$V_3 = -12\ V$$

$$V_4 = -24\ V$$

$$''i_1 = \frac{-V_3}{6} = \frac{12}{6} = 2\ A$$

$$''i_2 = \frac{V_3}{3} = \frac{-12}{3} = -4\ A$$

$$''i_3 = \frac{V_3 - V_4}{2} = \frac{-12+24}{2} = 6\ A$$

$$''i_4 = \frac{V_4}{4} = \frac{-24}{4} = -6\ A$$

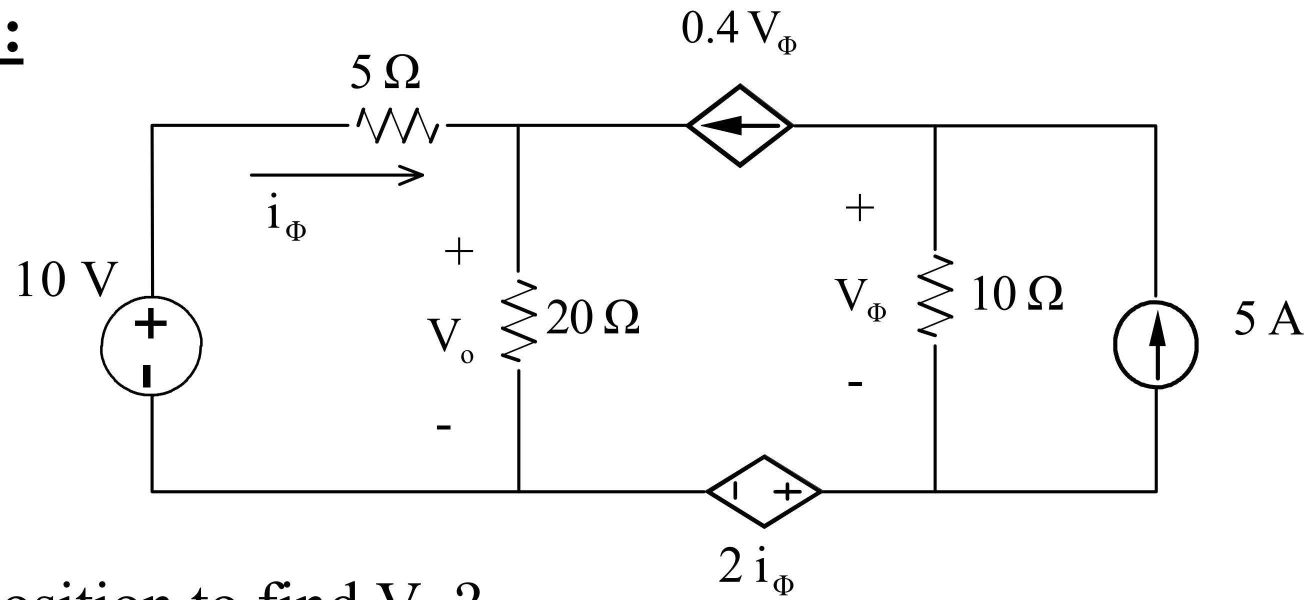
$$i_1 = 'i_1 + ''i_1 = 15 + 2 = 17\ A$$

$$i_2 = 'i_2 + ''i_2 = 10 - 4 = 6\ A$$

$$i_3 = 'i_3 + ''i_3 = 5 + 6 = 11\ A$$

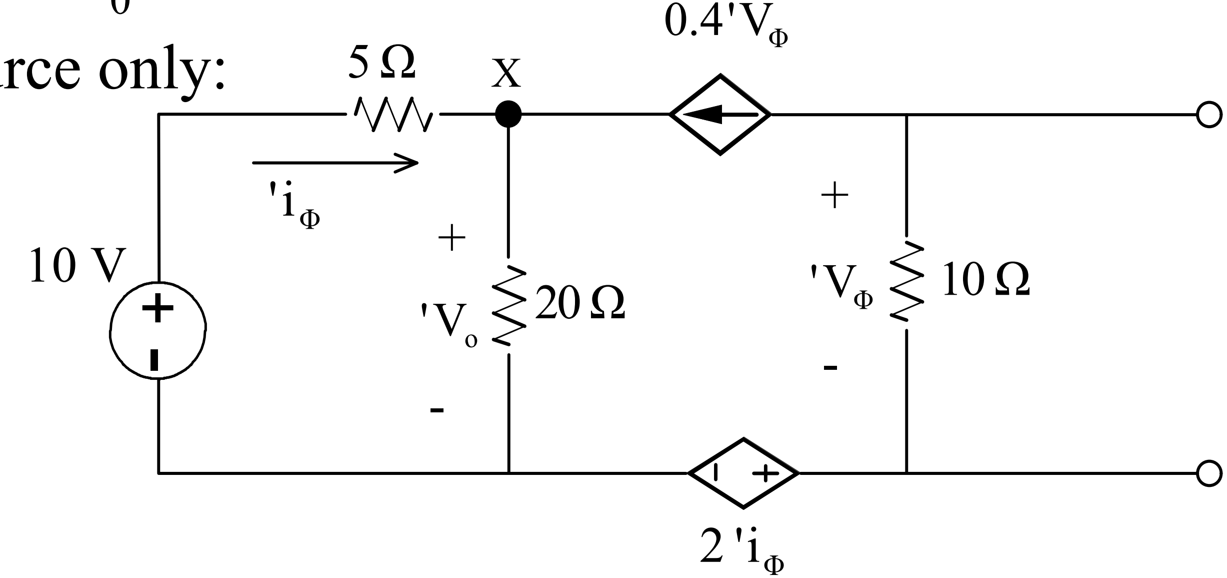
$$i_4 = 'i_4 + ''i_4 = 5 - 6 = -1\ A$$

Example :



Use super position to find V_0 ?

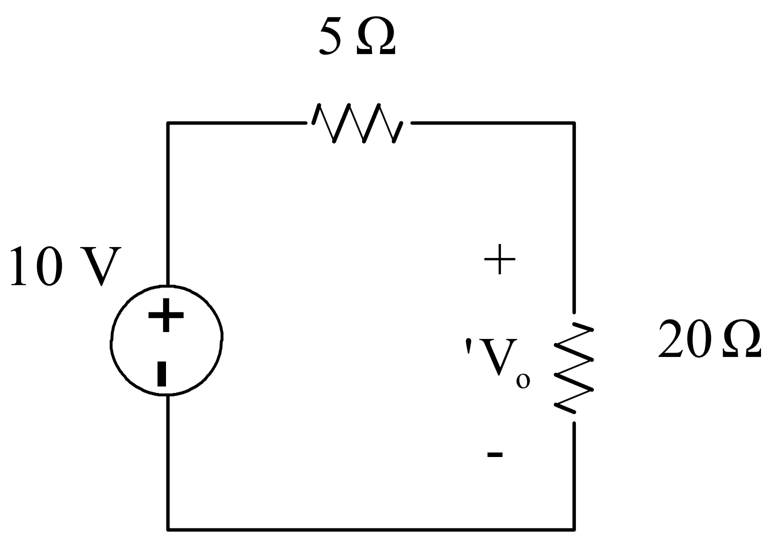
⇨ Activate voltage source only:



$$'V_{\Phi} = 10\,(0.4\,'V_{\Phi}) = 4\,'V_{\Phi}$$

$$'V_{\Phi} = 4\,'V_{\Phi} \quad \Rightarrow \quad 'V_{\Phi} = 0$$

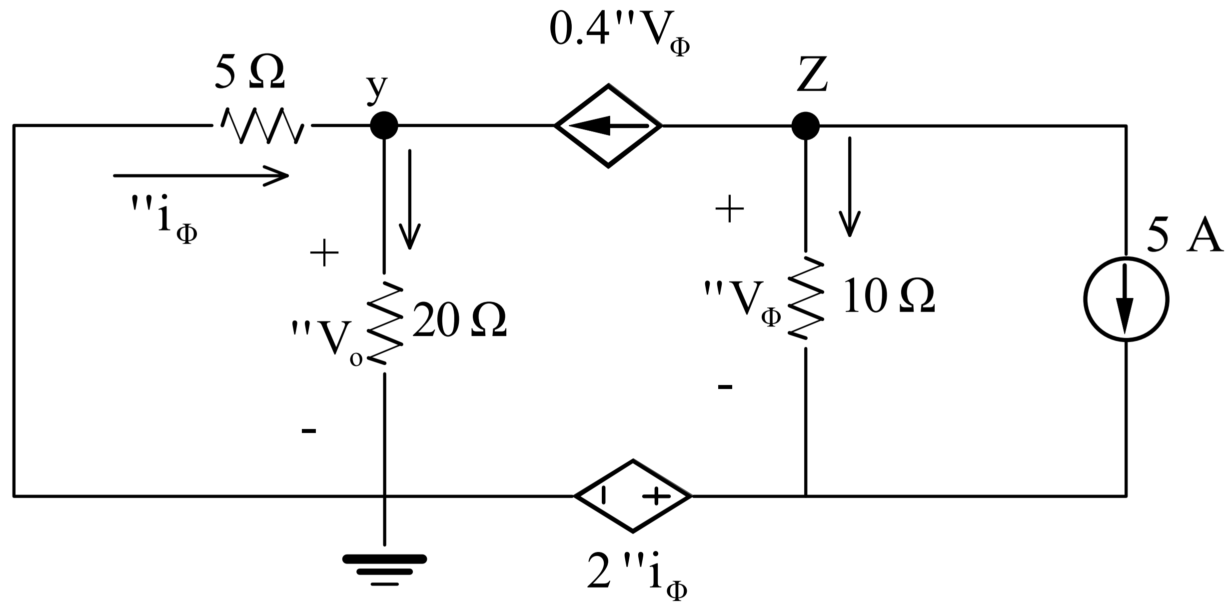
⇨ Dependent current source is open



$$'V_0 = 10\left(\frac{20}{25}\right)$$

$$'V_0 = 8\,V$$

Activate independent current source only:



KCL at node (y):

$$\frac{-''V_0}{5} - \frac{''V_0}{20} + 0.4\,''V_\Phi = 0$$

$$-4\,''V_0 - ''V_0 + 8\,''V_\Phi = 0$$

$$-5\,''V_0 + 8\,''V_\Phi = 0 \qquad \ldots\ldots(1)$$

$$5 + \frac{''V_{\Phi}}{10} + 0.4\,''V_{\Phi} = 0$$

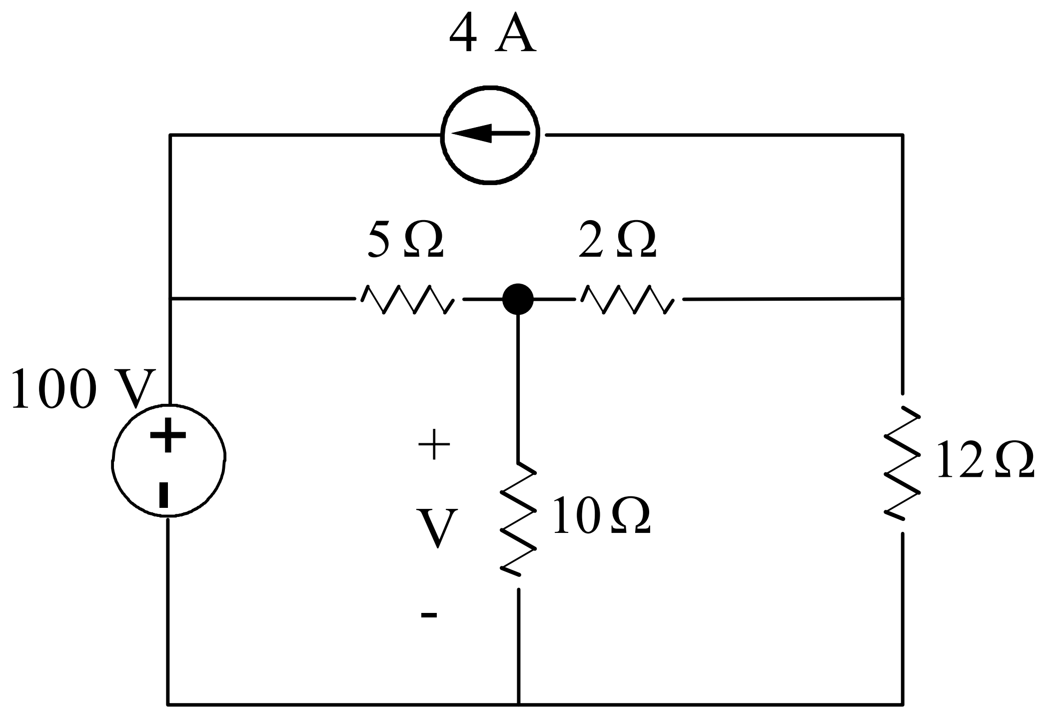
$$0.5\,''V_{\Phi} = -5 \qquad \Rightarrow \qquad ''V_{\Phi} = -10\ \text{V}$$

$$\therefore \quad ''V_0 = \frac{8}{5}\,''V_{\Phi} = \frac{-80}{5} = -16\ \text{V}$$

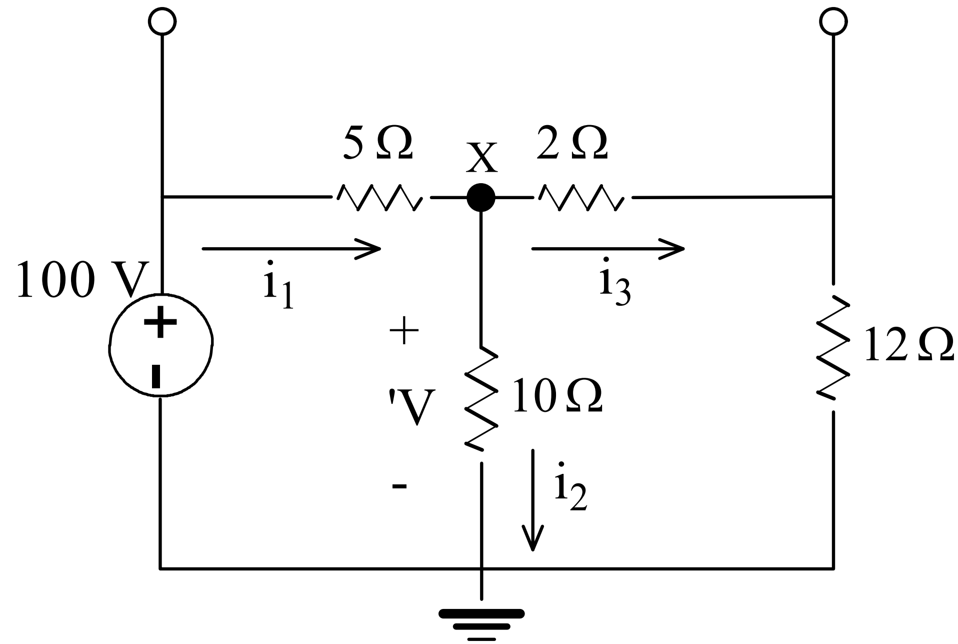
$$V_0 = 'V_0 + ''V_0 = 8 - 16 = -8\ \text{V}$$

Example:

Use superposition to find V ?



Consider the independent source only



Apply KCL at node (x) :

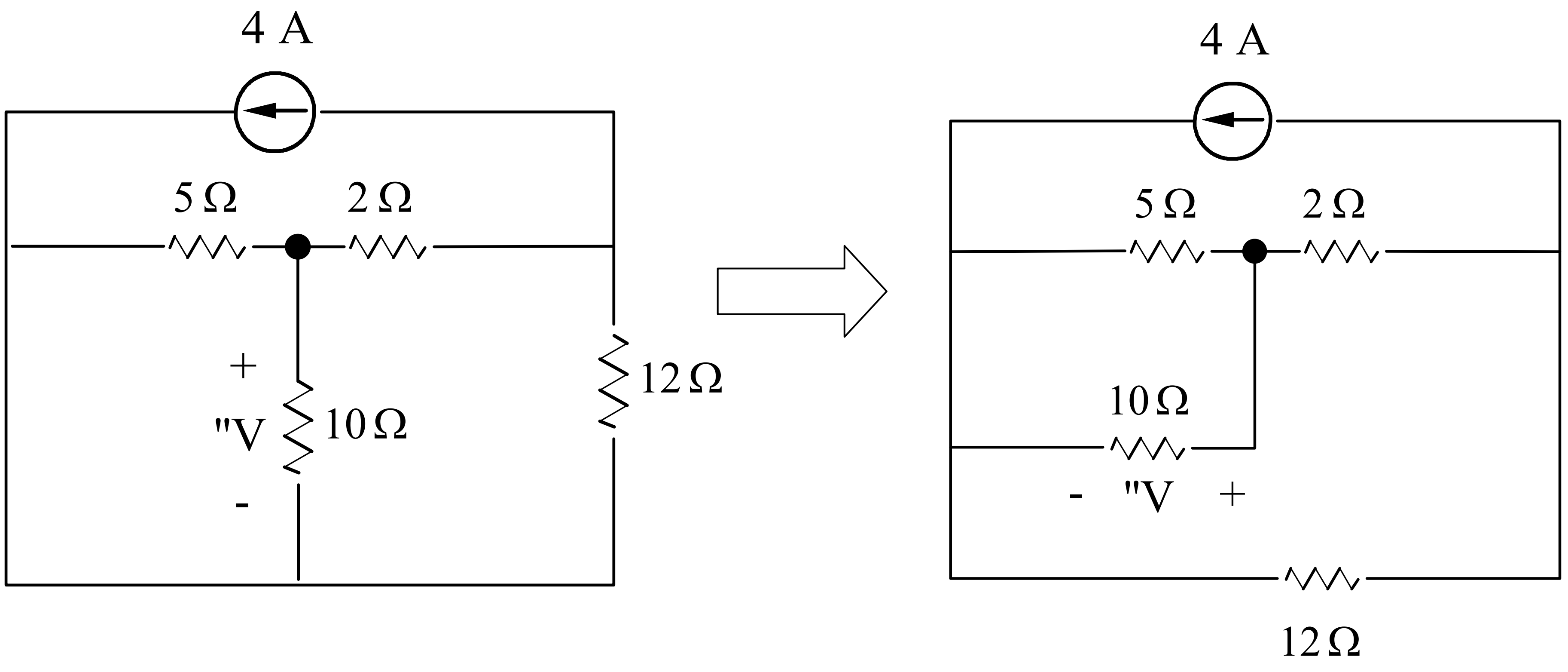
$$i_1 - i_2 - i_3 = 0$$

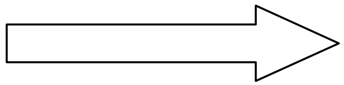
$$\frac{100-'V}{5} - \frac{'V}{10} - \frac{'V}{14} = 0$$

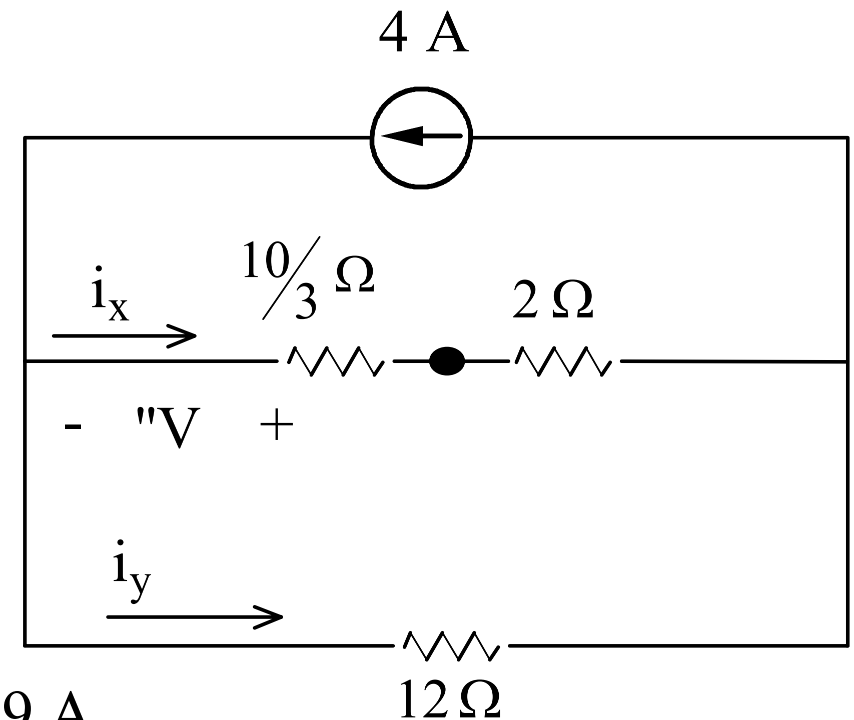
$$22 - \frac{'V}{5} - \frac{'V}{10} - \frac{'V}{14} = 0$$

$$'V\left[\frac{1}{5} + \frac{1}{10} + \frac{1}{14}\right] = 22 \quad \Rightarrow \quad 'V = 59.23\ V$$

Consider the independent source only.







Current divider

$$i_x = 4\text{ A}\left(\frac{12}{12+\frac{10}{3}+2}\right) = 2.769\text{ A}$$

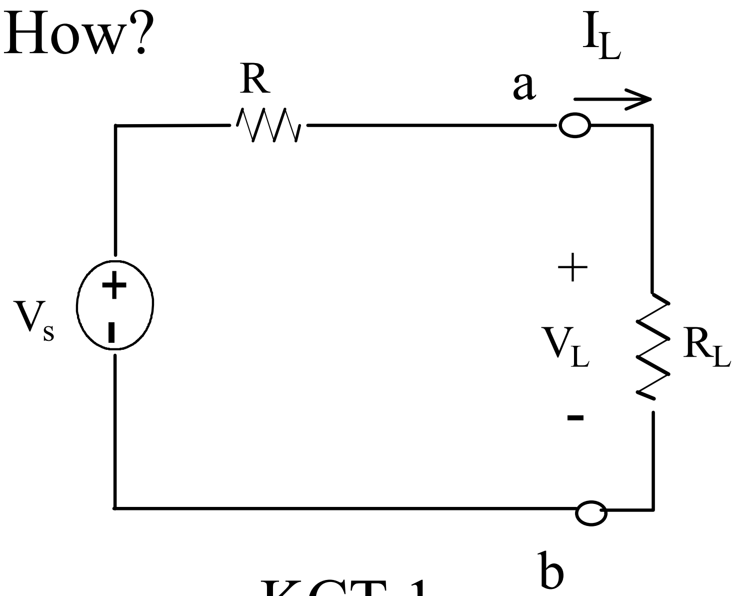
$$''V = -i_x\left(\frac{10}{3}\right) = -9.23\text{ V}$$

$$V = 'V + ''V = 59.23 - 9.23 = 50\text{ V}$$

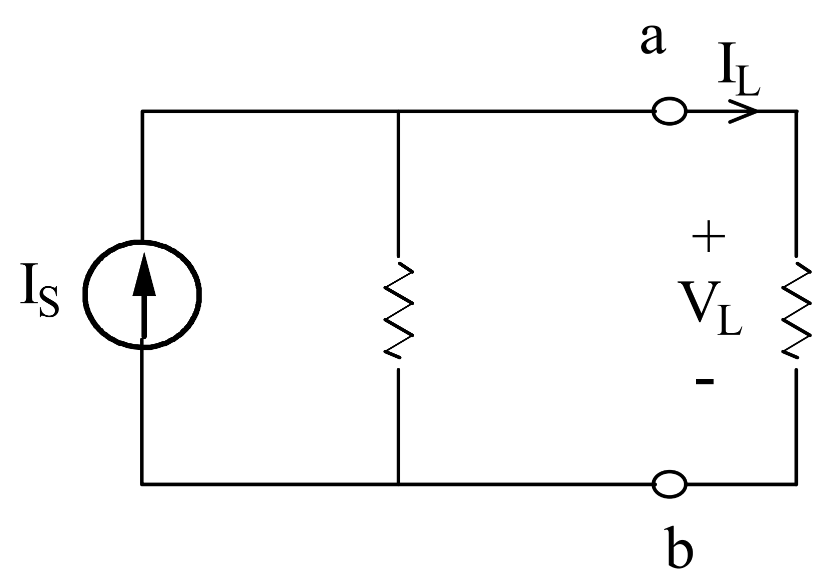
2. Source Transformation:

A transformation that allow a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor or vice versa

How?



KCT 1



KCT 2

We need to find I_s and V_s such that V_L and I_L is the same in both circuits

In KCT 1 ,

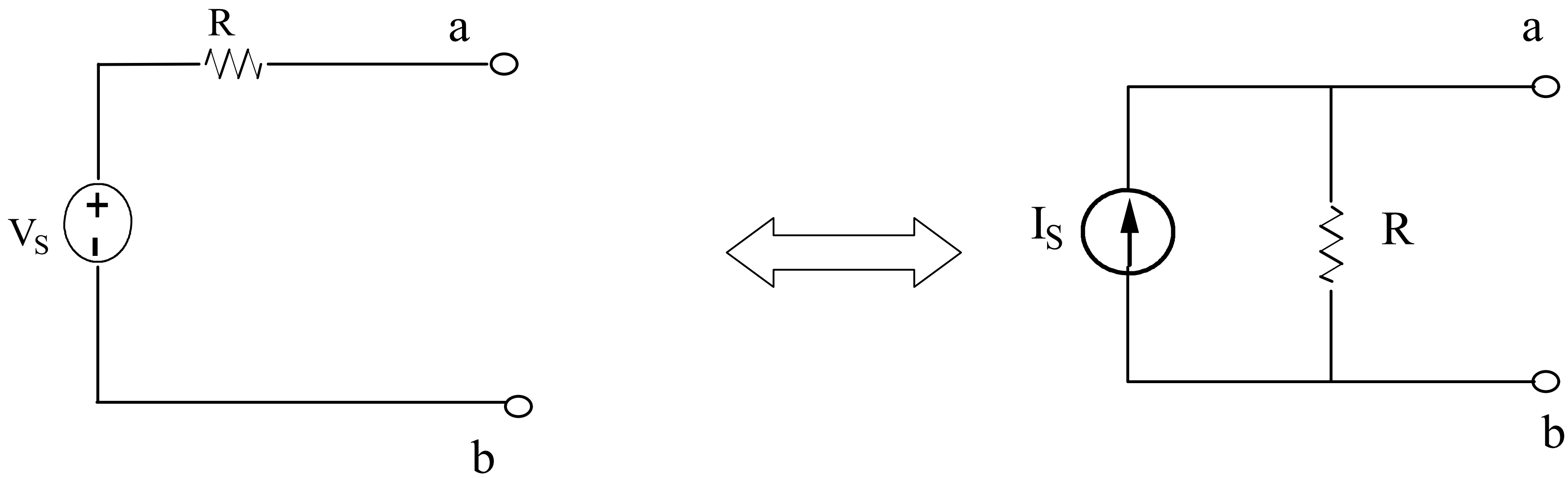
$$I_L = \frac{V_s}{R_L + R}$$

In KCT 2,

$$I_L = \frac{R}{R_L + R} I_s$$

For I_L to be the same , we need

$$V_s = R\ I_s$$

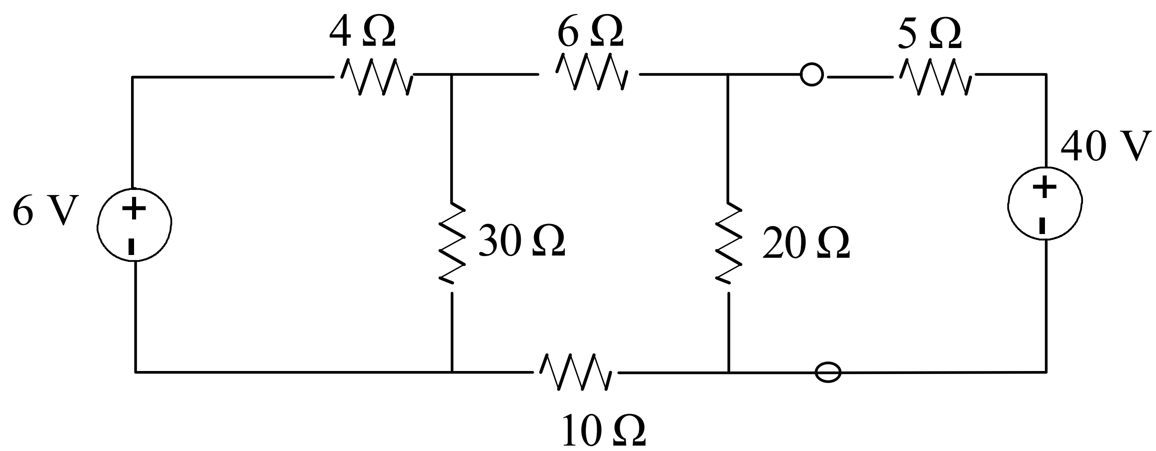


Where

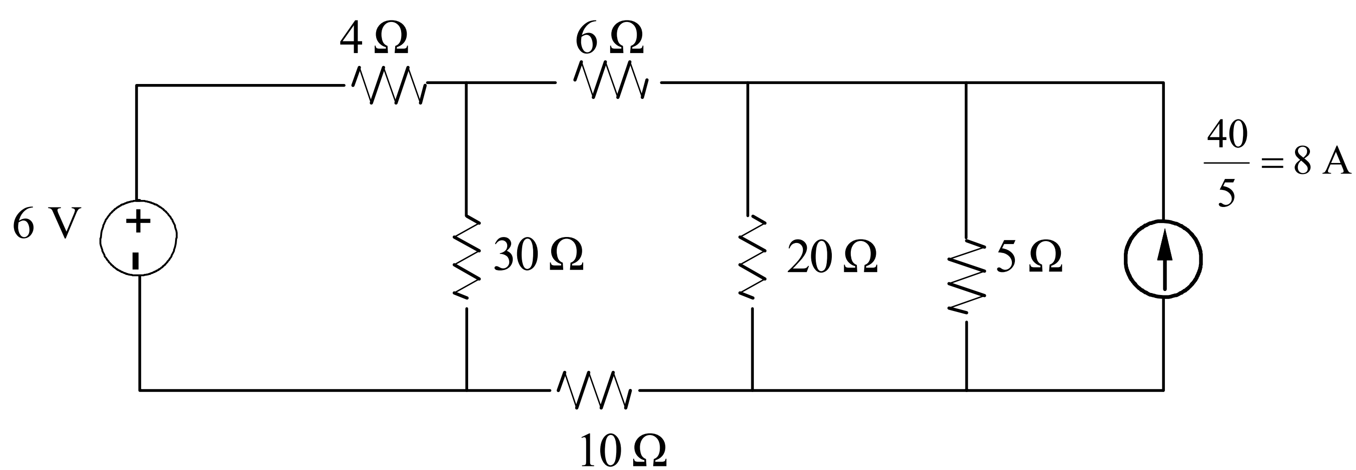
$$V_s = R\ I_s \qquad \text{or} \qquad I_s = \frac{V_s}{R}$$

Example :

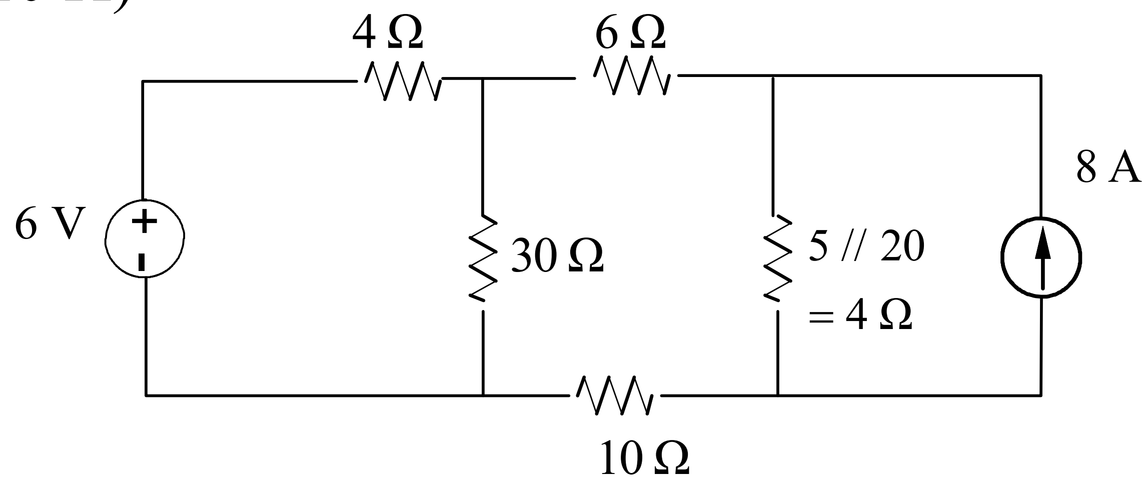
Using source transformation, find the power associated with the 6 V source.



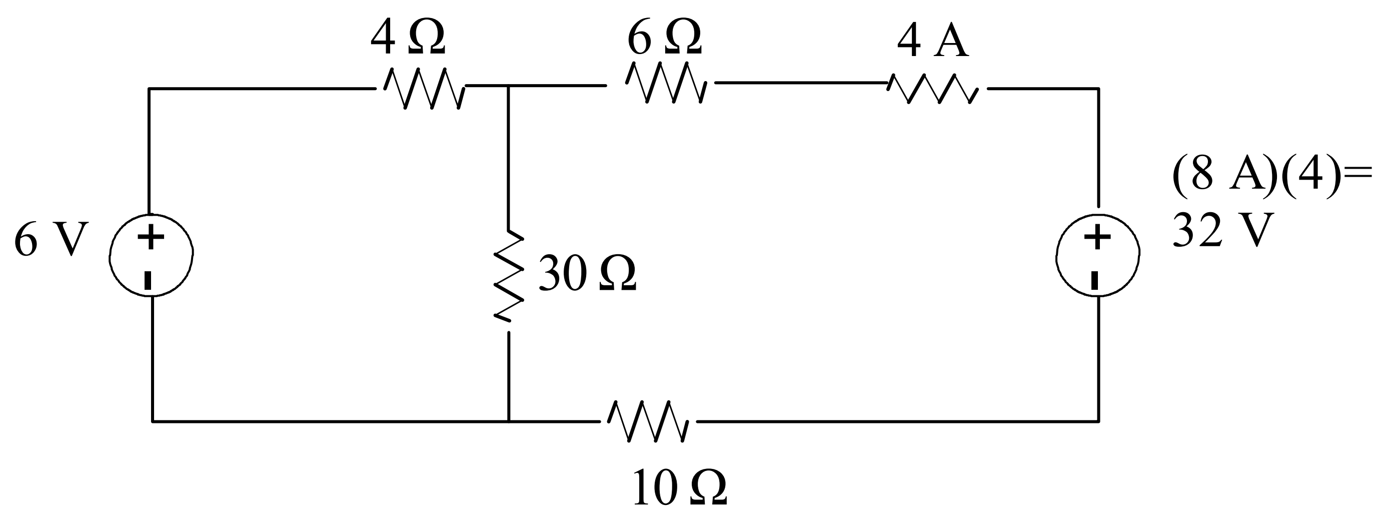
1. Consider the 40 V source in series with (5Ω)



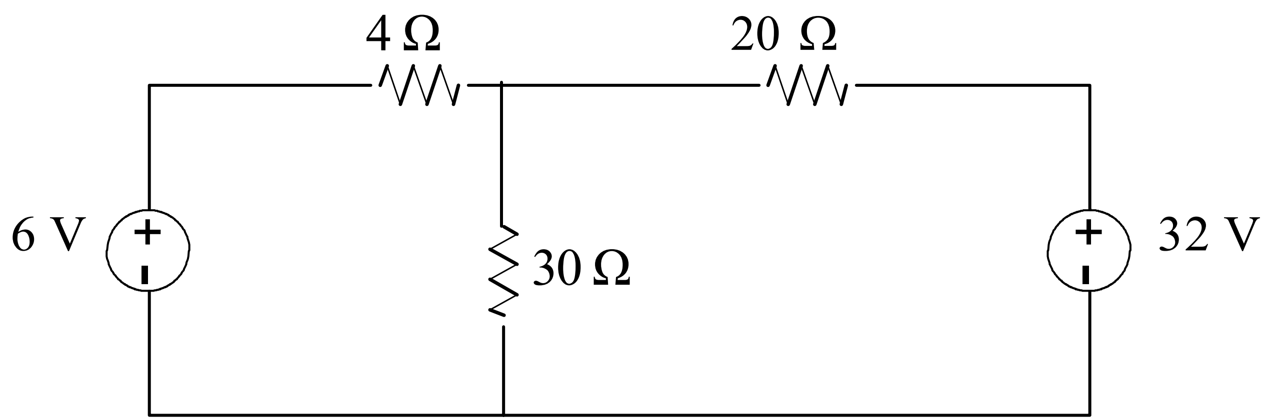
2. Take (5// 20 Ω)



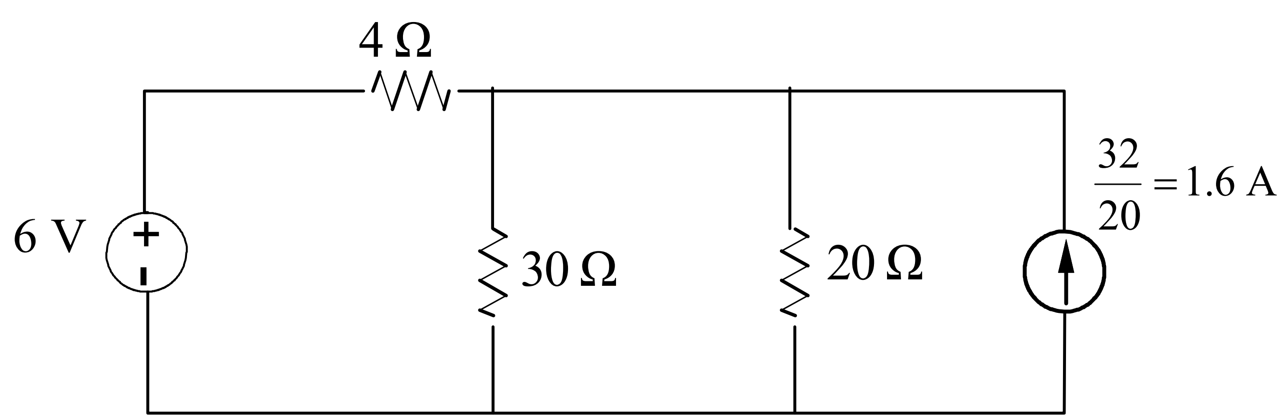
3. Consider 8A in parallel with (4Ω)



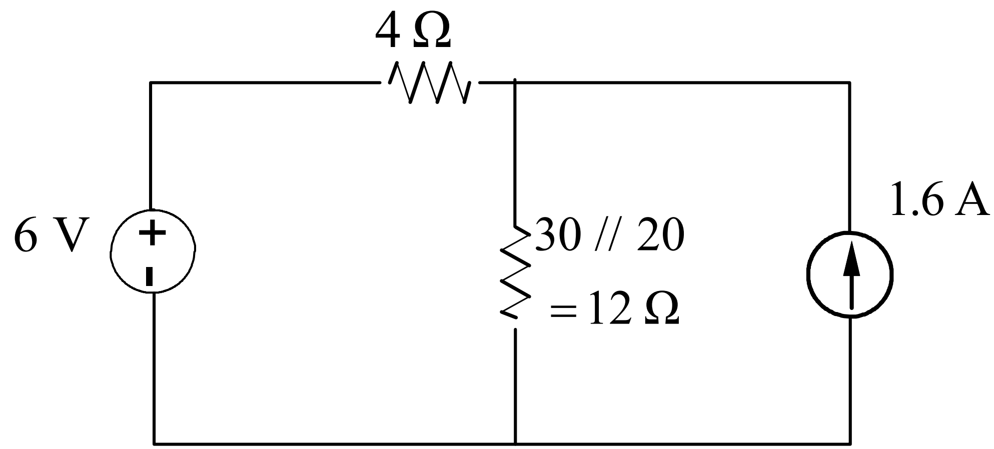
4. Take (4+6+10) in series



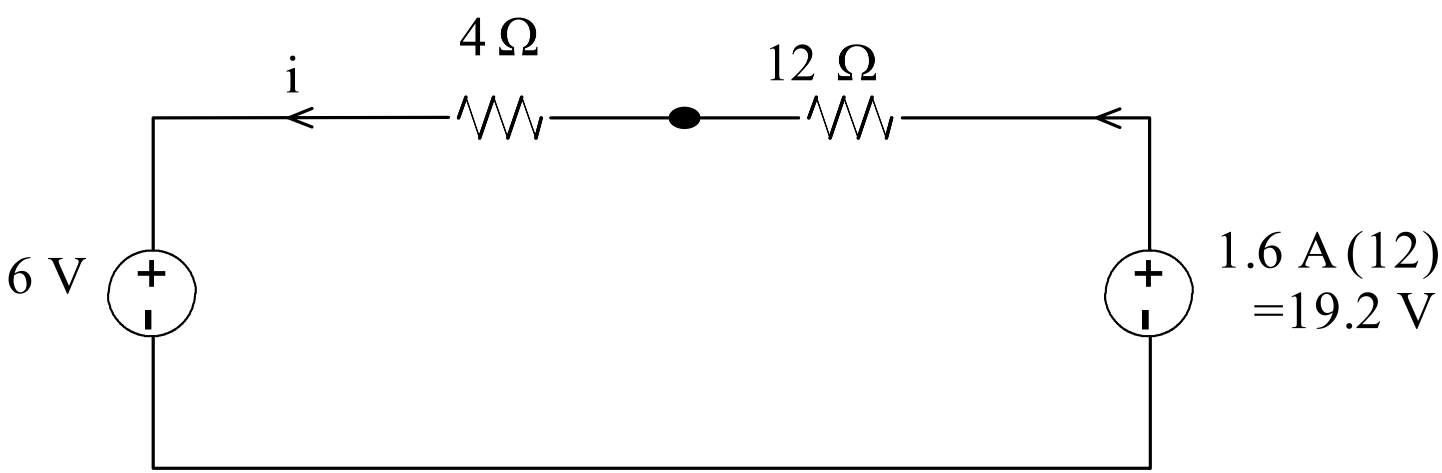
5. Consider 32 V in series with (20Ω)



6. Take (30//20 Ω)



7. Consider 1.6A in parallel with (15 Ω)

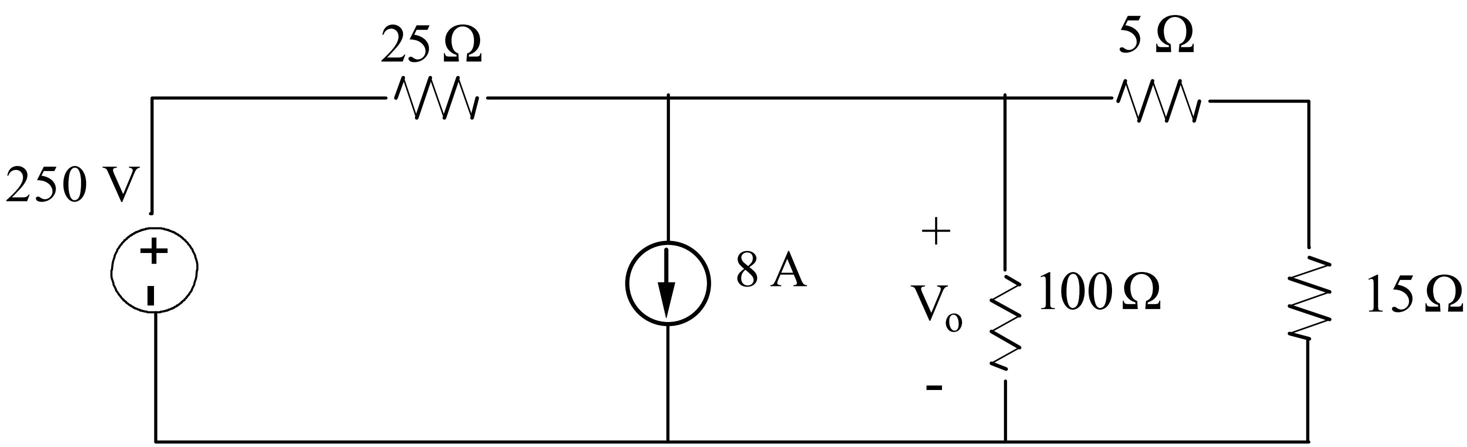


$$i = \frac{19.2 - 6}{4 + 12} = 0.825 \text{ A} \quad \Rightarrow \quad P_{6V} = v\,i = 6\,(0.825)$$

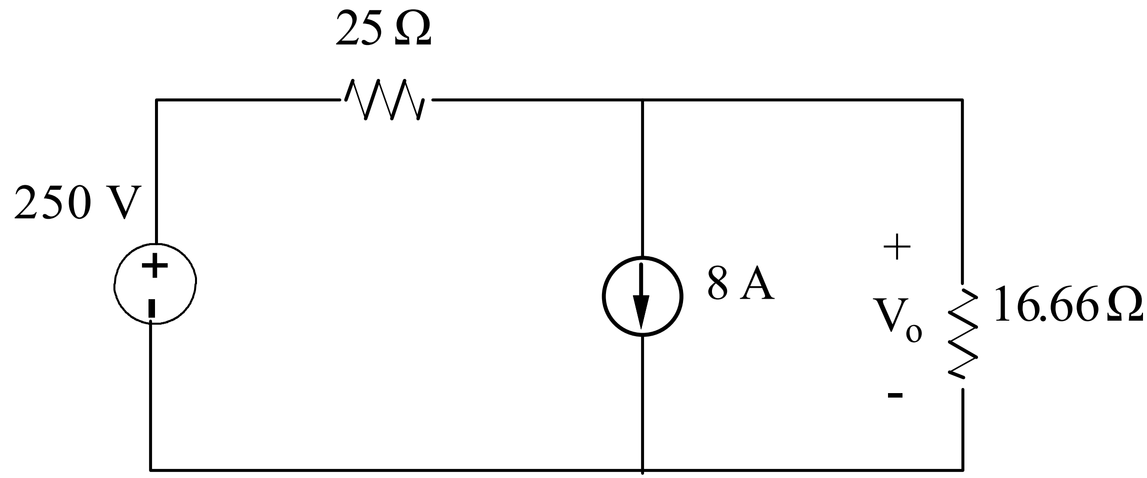
$$P_{6v} = 4.95 \text{ W (absorbing)}$$

Example :

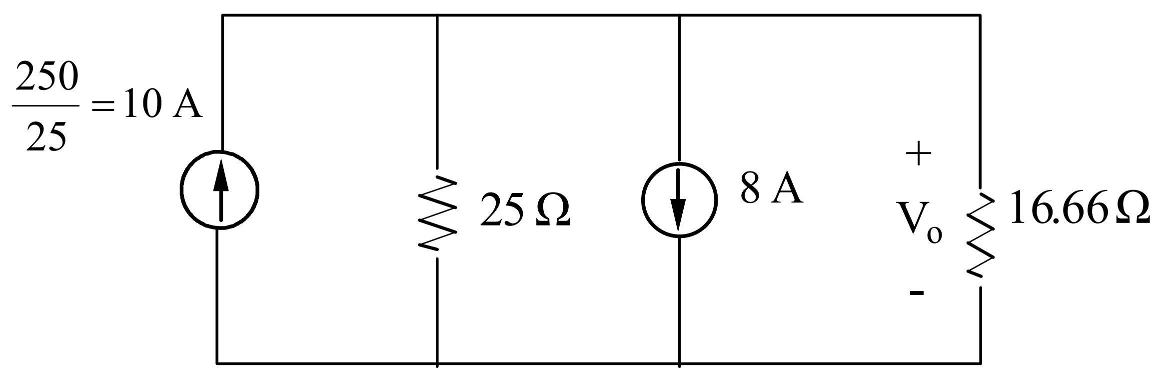
Use source transformation to find V_0



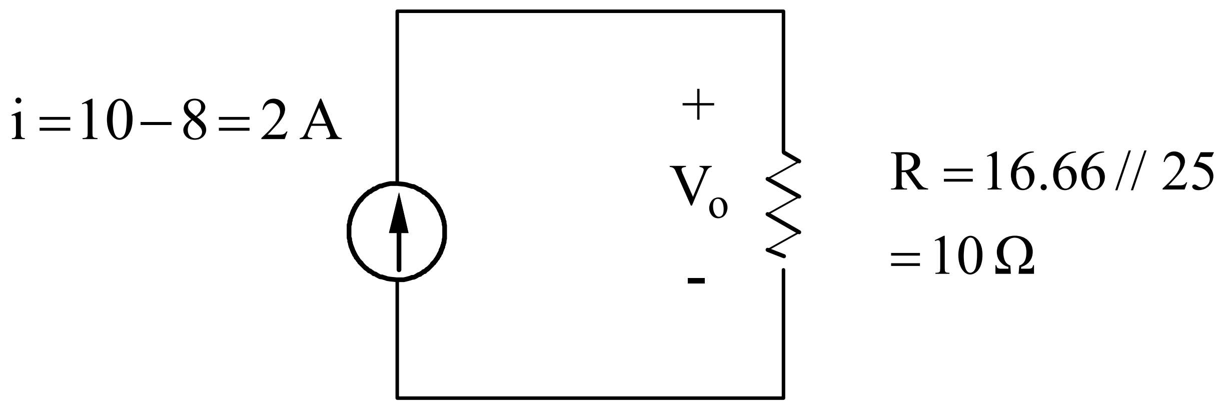
1. Take (5//15)//100 = 6.66 Ω



2. Consider (250 V) in series with (25 Ω)



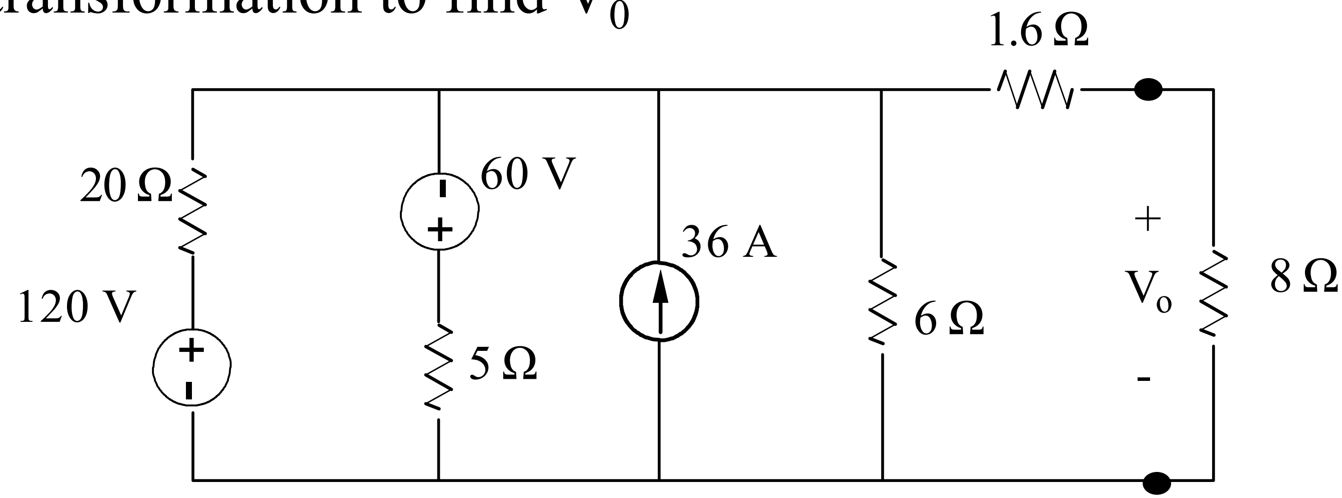
3. Find equivalent

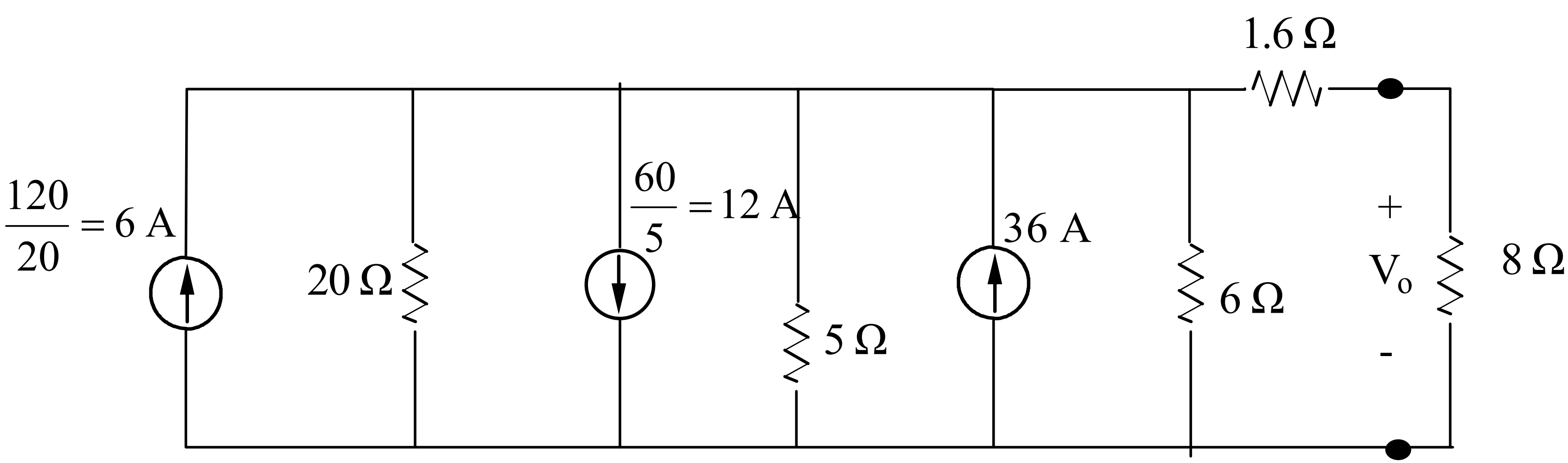


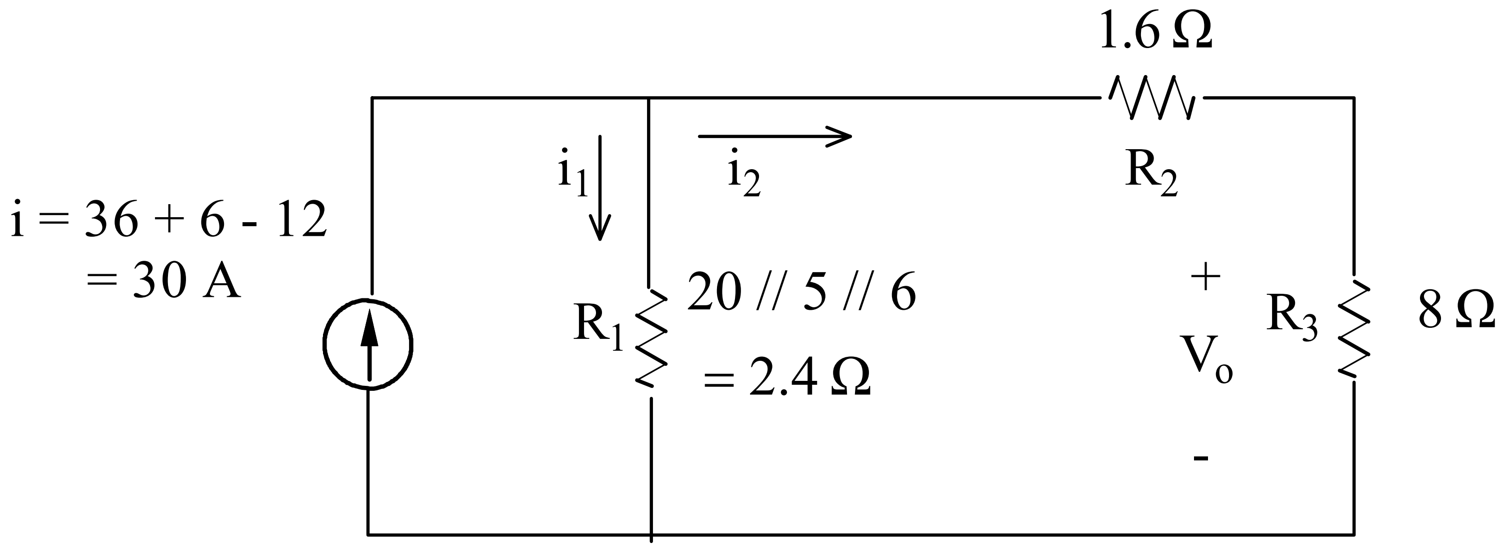
$$V_0 = i\ R = (2\ A)\ (10\ \Omega) = 20\ V$$

Example:

Use source transformation to find V_0



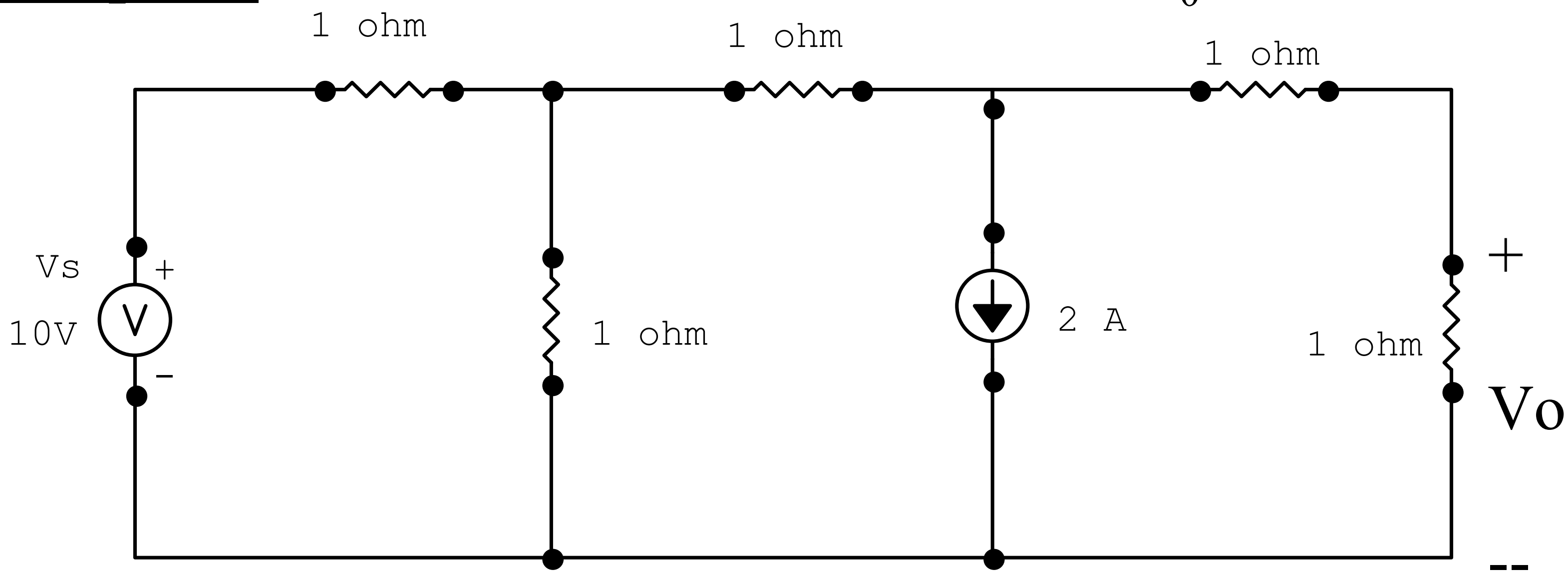




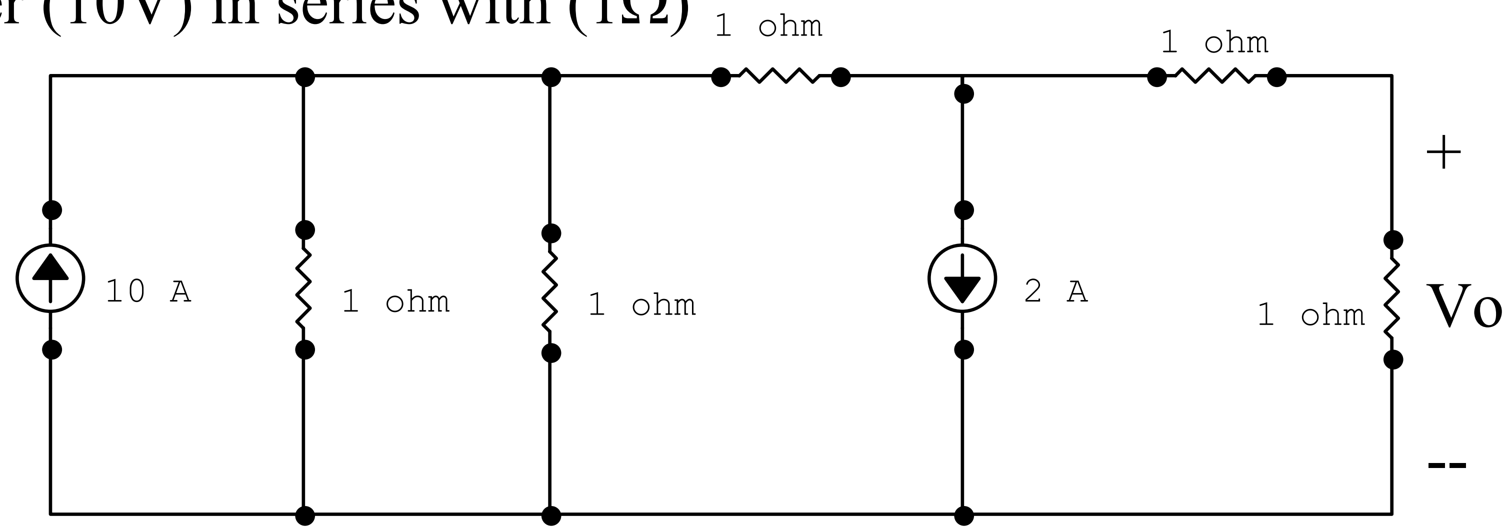
$$i_2 = \frac{R_1}{R_1 + R_2 + R_3}\, i = \frac{2.4}{2.4+1.6+8}(30) = 6\ \text{A}$$

$$V_0 = i_2\ R_3 = (6\ \text{A})(8\ \Omega) = 48\ \text{V}$$

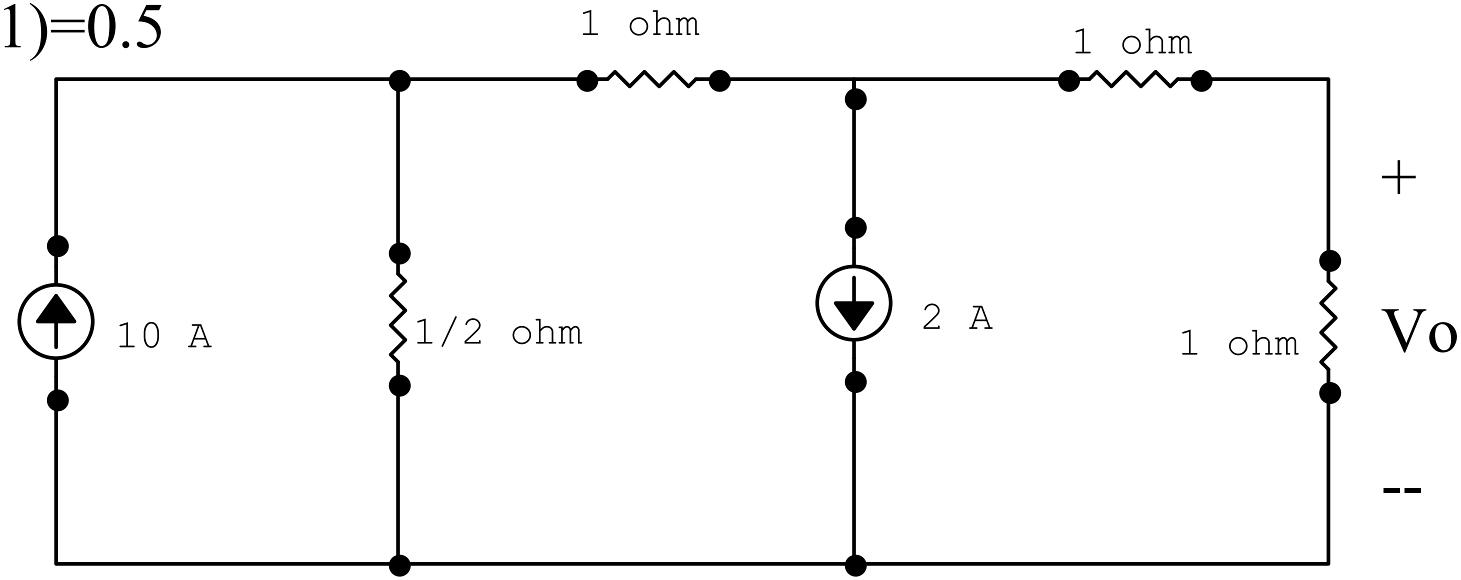
Example : Use source transformation to find V_0?



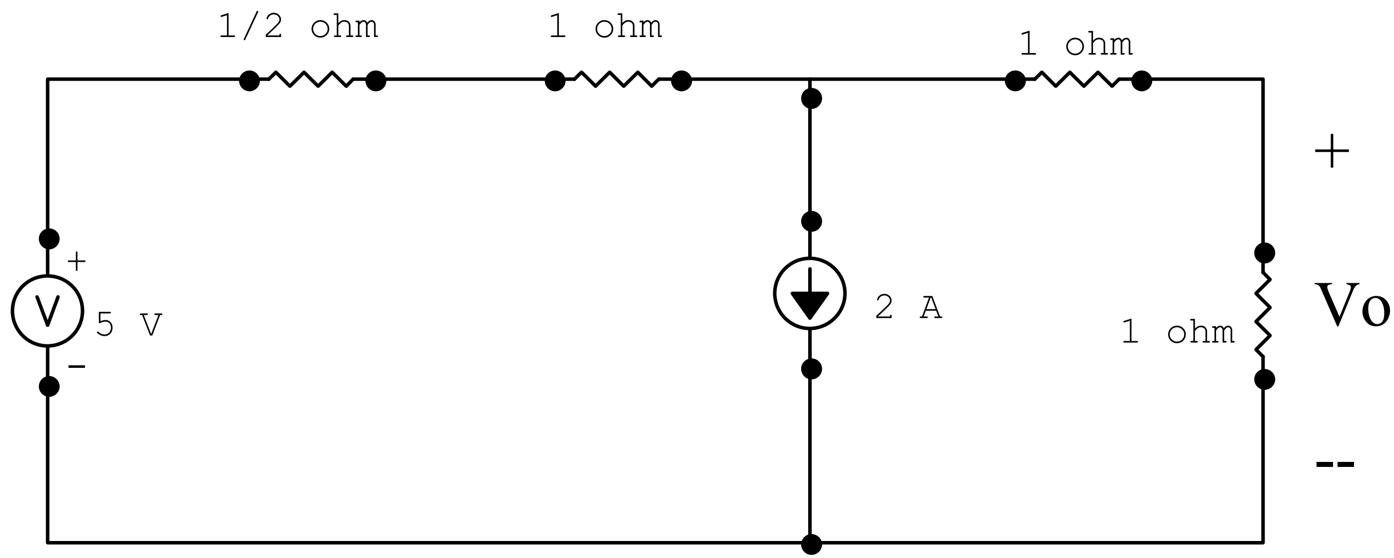
Consider (10V) in series with (1Ω)



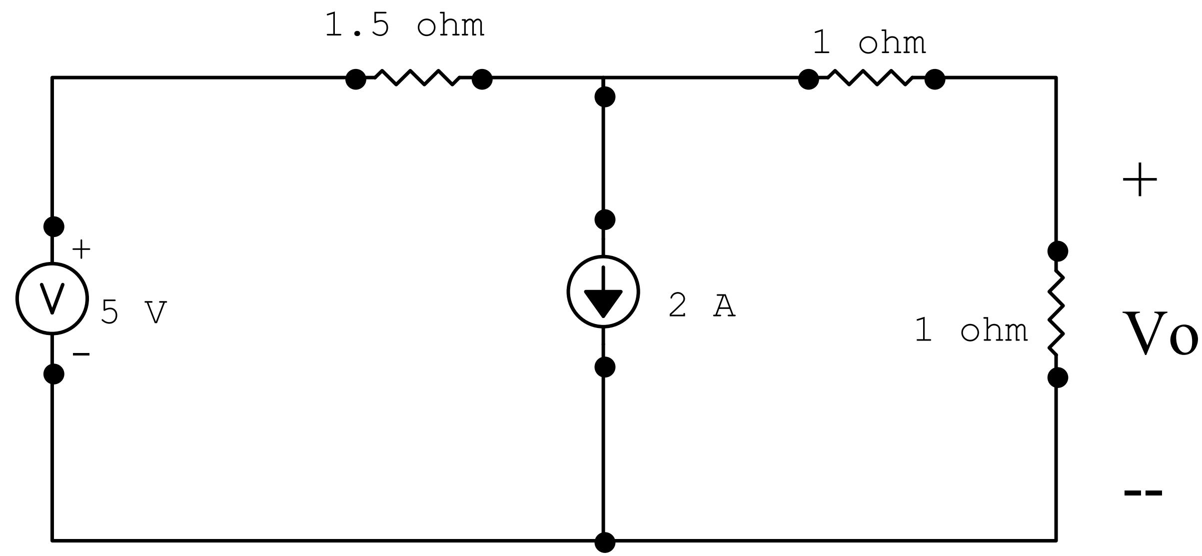
Take (1//1)=0.5



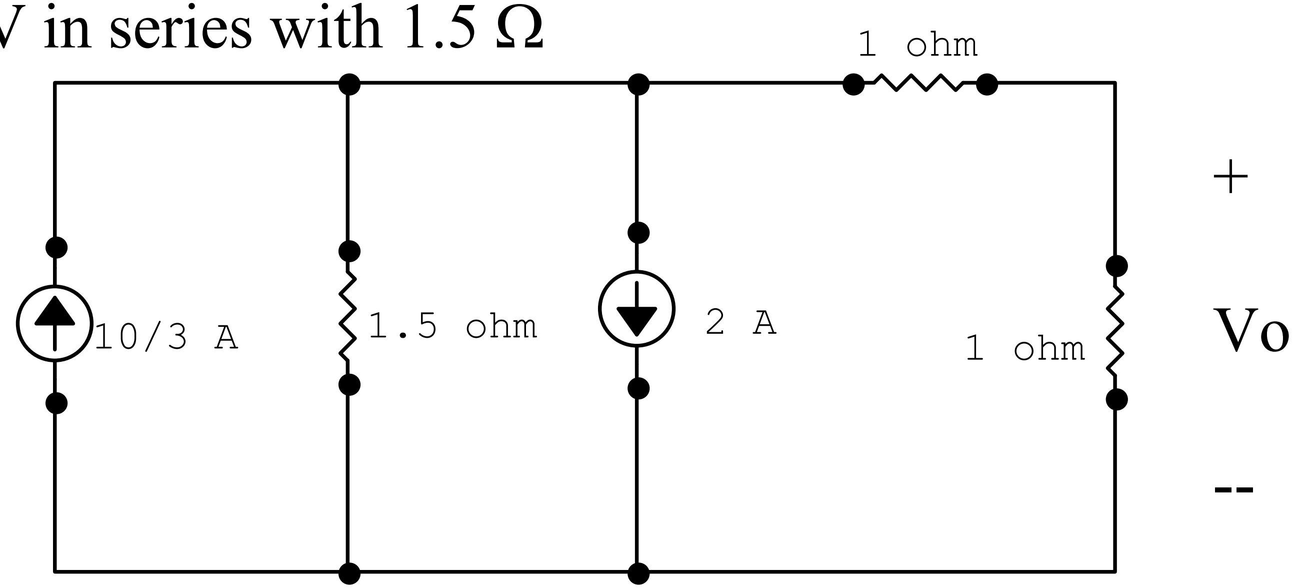
Consider (10A) in parallel with (0.5 Ω)



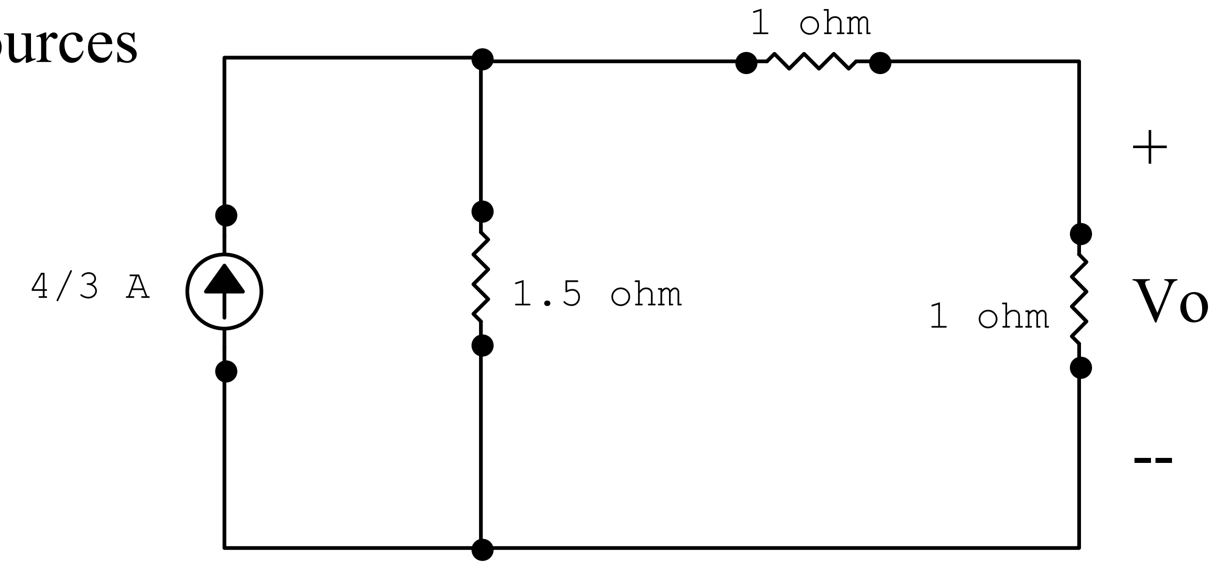
Take 0.5Ω in series with 1 Ω



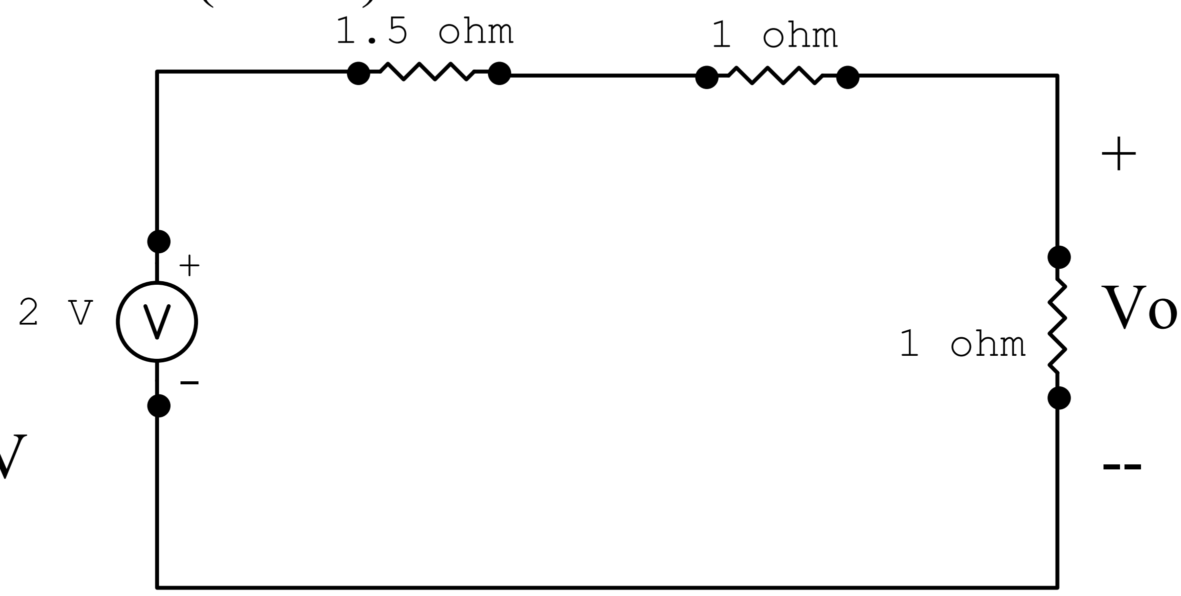
Consider 5V in series with 1.5 Ω



Add the current sources



7. Take (4/3A) in parallel with (3/2 Ω)

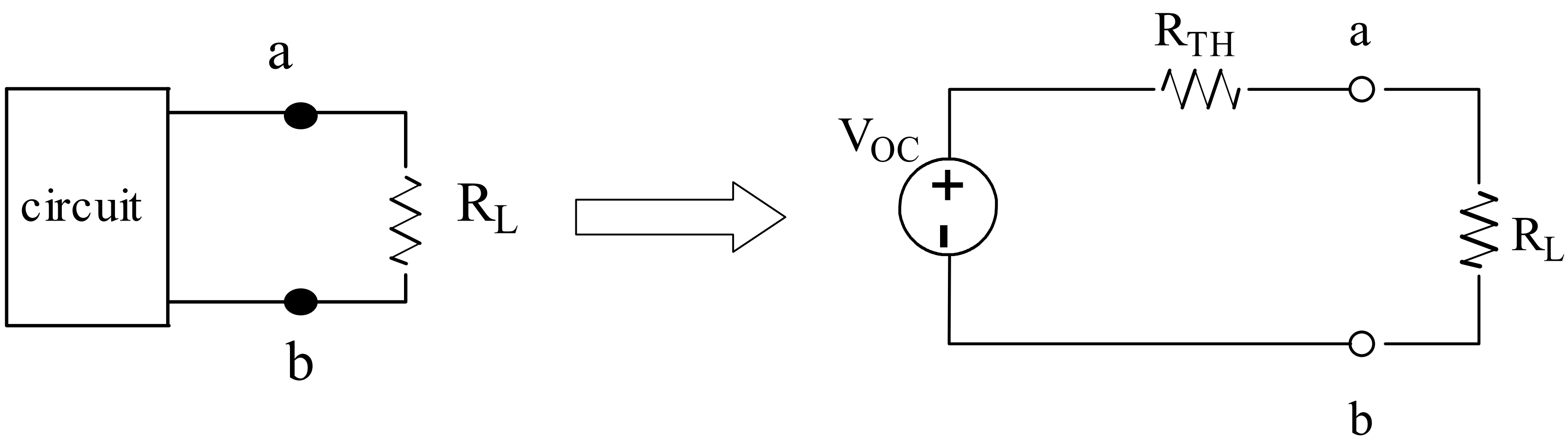


$$V_0 = \frac{1}{1+1+1.5}(2) = 4/7 \text{ V}$$

Thevenin and Norton Theorems

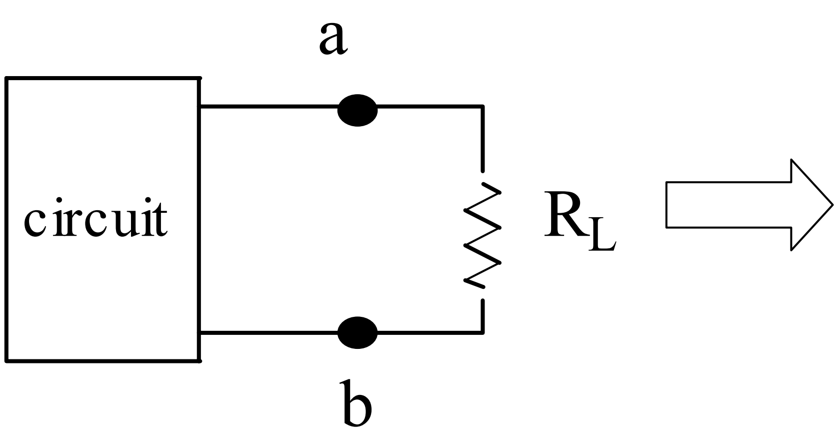
Thevenin Theorem:

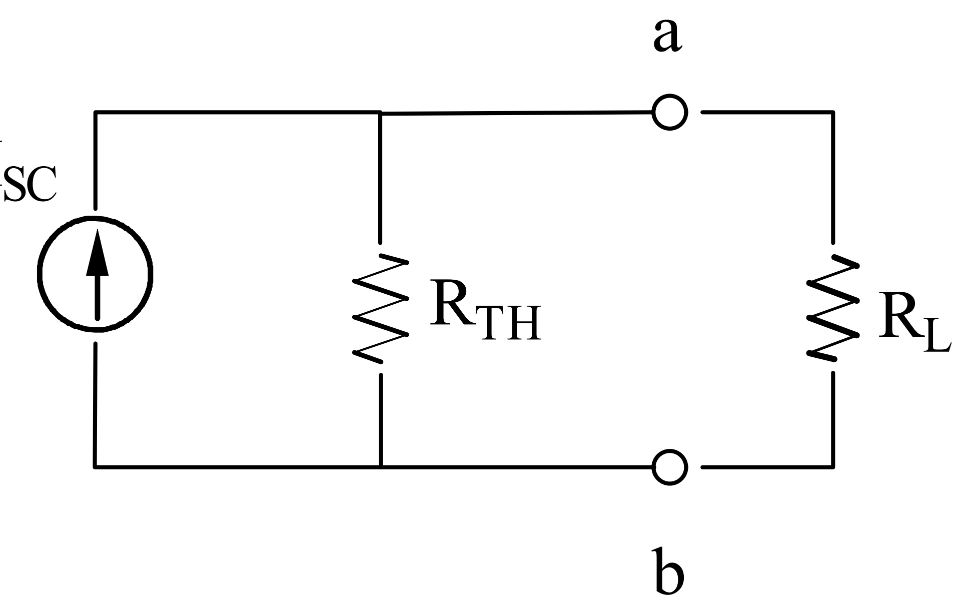
A portion of the circuit at a pair of nodes can be replaced by a voltage source V_{oc} in series with a resistor R_{TH}, where V_{oc} is the open circuit voltage and R_{TH} is the Thevenin's equivalent resistance obtained by considering the open circuit with all independent sources made zero



<u>Norton Theorem :</u>

A portion of the circuit at pair of nodes can be replaced by a current source I_{sc} in a parallel with a resistor R_{TH}. I_{sc} is the short circuit current at the terminals, and R_{TH} is the Thevenin's equivalent resistance





Here we will consider (3) cases :

1. Circuit containing only independent sources.
2. Circuit containing only dependent sources.
3. Circuit containing both independent and dependent sources.

Case (1): Circuit containing only independent sources:

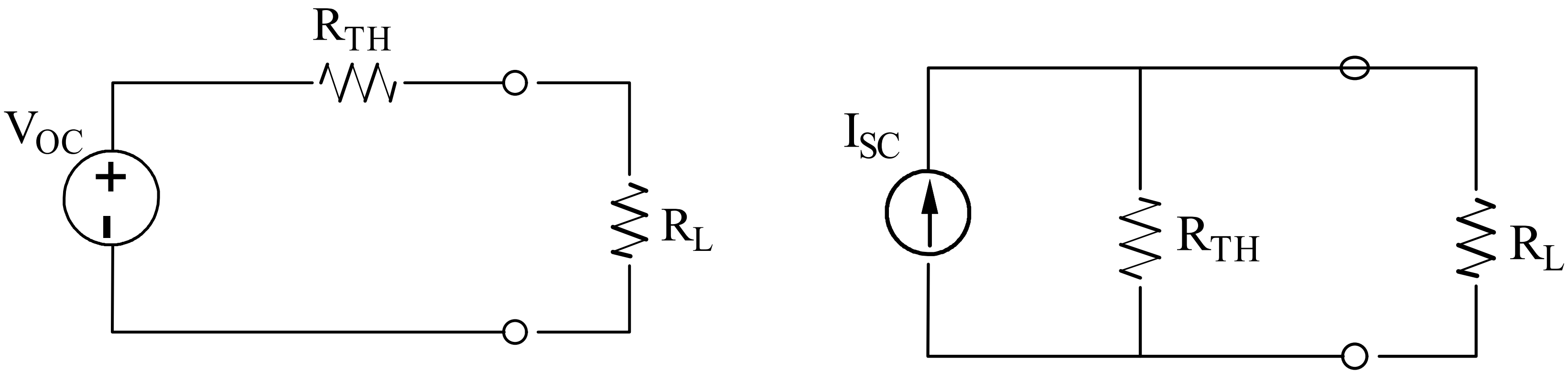
- **Procedure of Thevenin's Theorm:**

a. Find the open circuit voltage at the terminals , Voc.
b. Find the Thevenin's equivalent resistance, RTH at the terminals when all independent sources are zero:
   - Replacing independent voltage sources by short circuit
   - Replacing independent current sources by open circuit

c. Reconnect the load to the Thevenin equivalent circuit

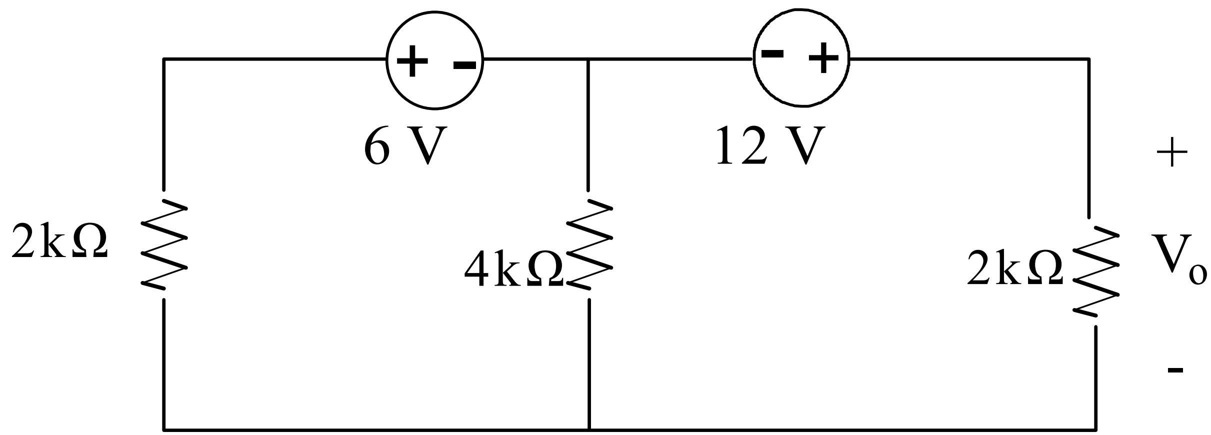
- **Procedure of Norton's Theorm:**

a. Find the short circuit current at the terminals, I_{sc}.
b. Find Thevenin's equivalent resistance, R_{TH} (as before).
c. Reconnect the load to Norton's equivalent circuit.



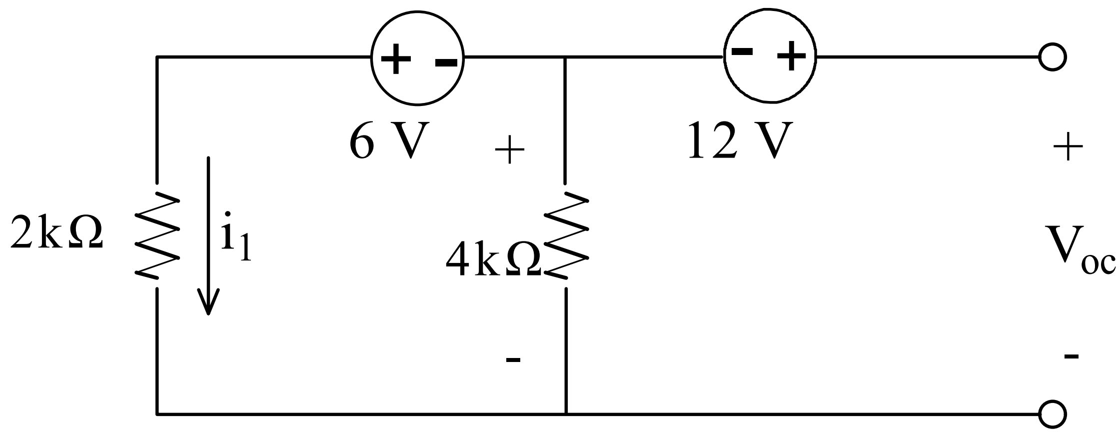
Example :

Use Thevenin's and Norton Theorms to find V_0



Using Thevenin Theorm:

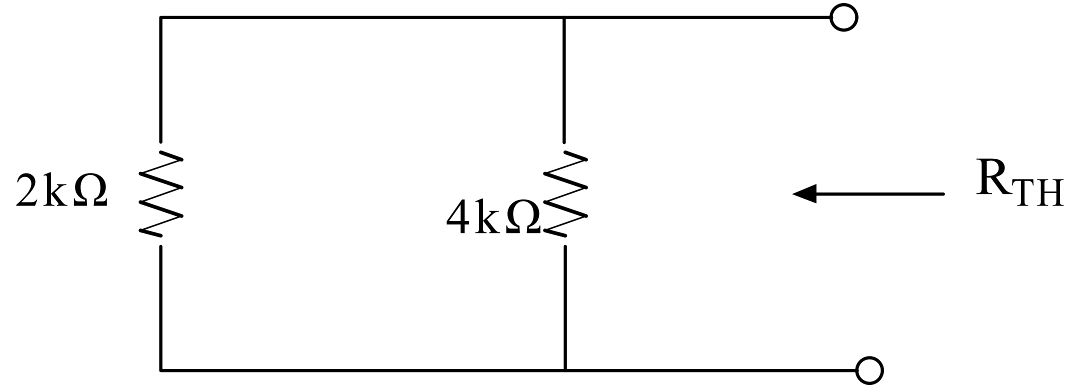
First find V_{OC}:



$$i_1 = \frac{6\ V}{2\ k + 4\ k} = 1\ mA \quad \Rightarrow \quad V_{4k\Omega} = i_1\,(4\,k) = -4\ V$$

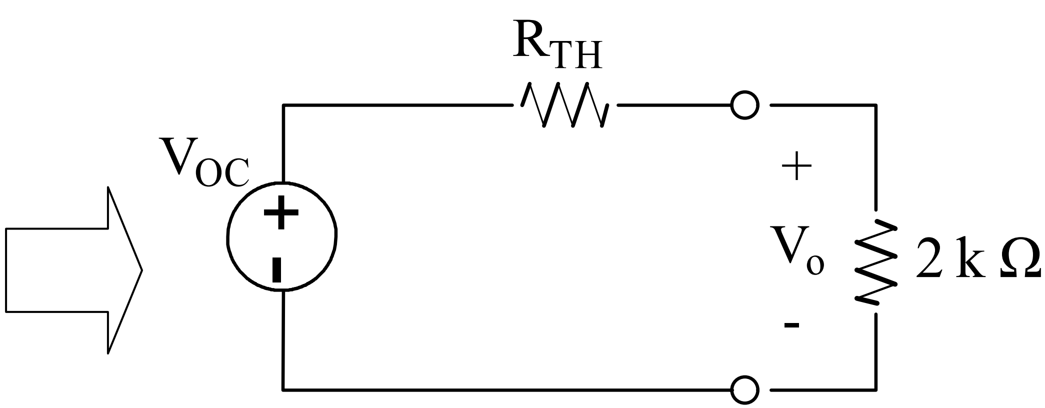
$$V_{oc} = 12\ V - 4\ V = 8\ V$$

Second, find R_{TH}



$R_{TH} = 2k//4k = 4/3\ k\ \Omega$

Thevenin equivalent circuit is



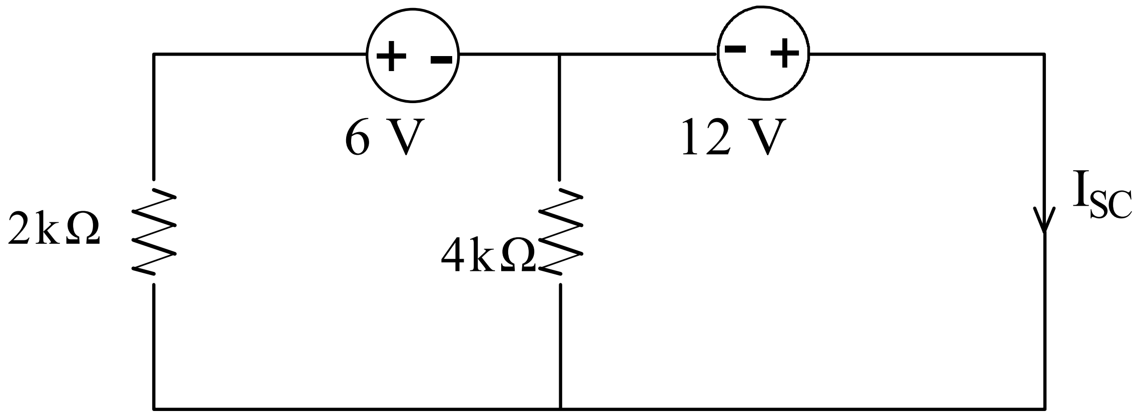
$$V_0 = \frac{2\ k\ \Omega}{2\ k + R_{TH}} V_{oc}$$

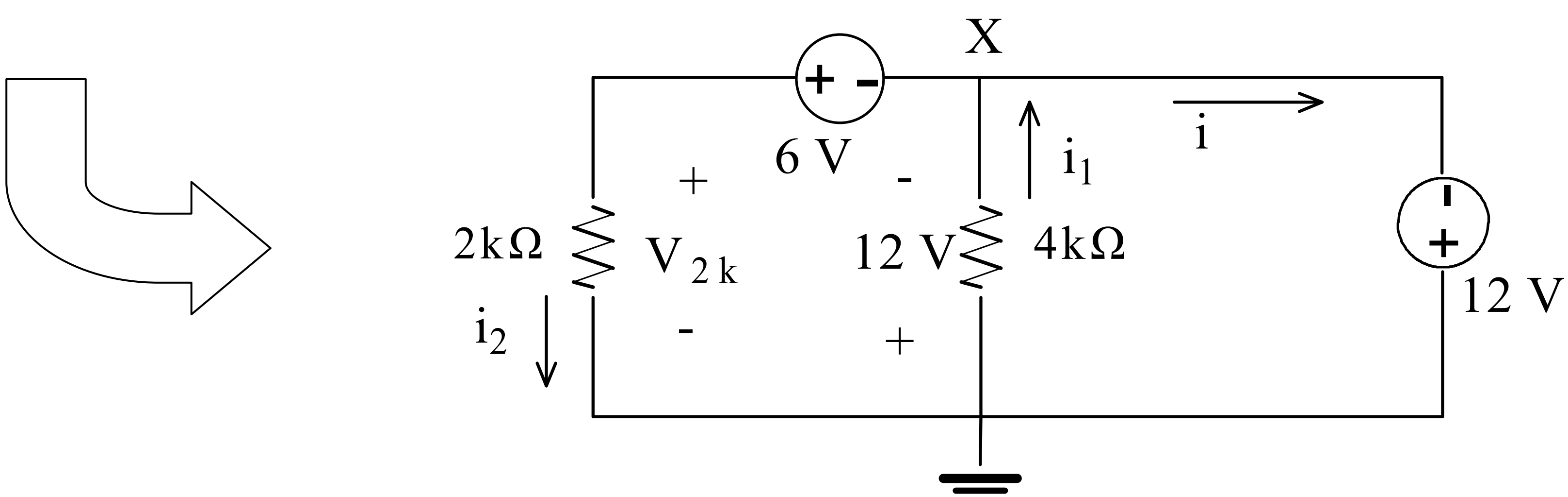
$$= \frac{2\ k}{10/3\ k}(8\ V)$$

$$V_0 = 4.8\ V$$

Using Norton Theorm

First find I_{sc}





$$i_1 = \frac{12\ V}{4\ k} = 3\ mA$$

KVL around outer loop:

$$12 - 6 + V_{2k} = 0 \qquad \Rightarrow \quad V_{2k} = -6\ V$$

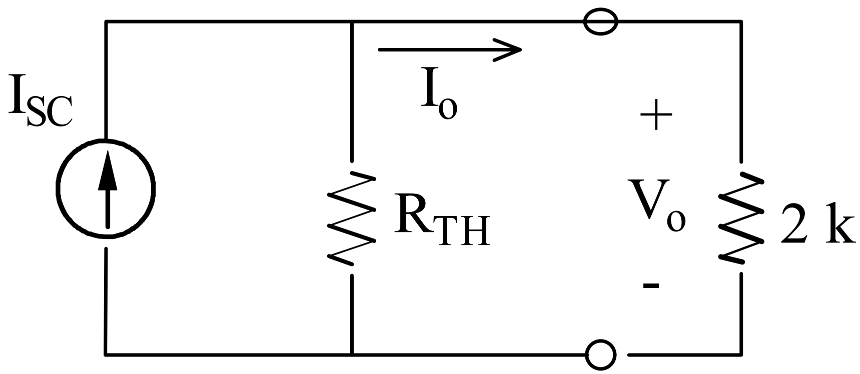
$$i_2 = \frac{V_{2k}}{2\ k} = \frac{-6}{2\ k} = -3\ mA$$

KCL at x :

$$i_1 - i_2 - i = 0$$

$$3\ m + 3\ m - i = 0 \qquad \Rightarrow \quad i = 6\ mA \qquad \Rightarrow I_{sc} = 6\ mA$$

R_{TH} is the same as before:

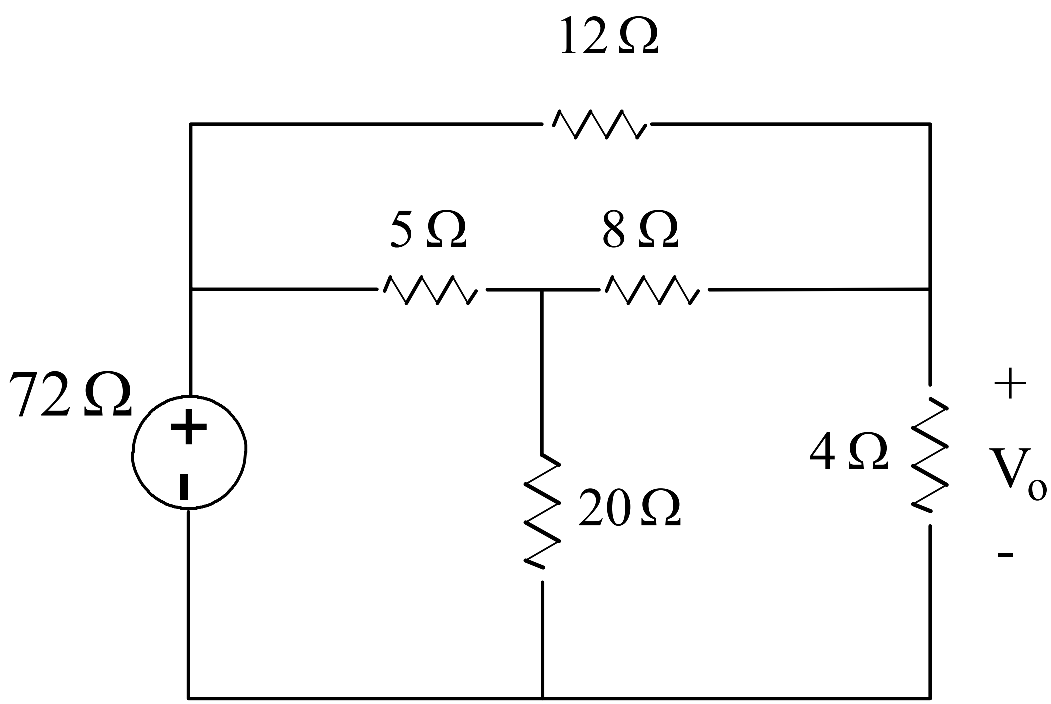


$$I_0 = \frac{R_{TH}}{R_{TH} + 2\text{ k}}(I_{sc}) = \frac{\frac{4}{3}\text{ k}}{\frac{4}{3}\text{ k} + 2\text{ k}}(6\text{ m}) = 2.4\text{ mA}$$

$$V_0 = I_0\,(2\text{ k}) = (2.4\text{ m})\,(2\text{ k}) = 4.8\text{ V}$$

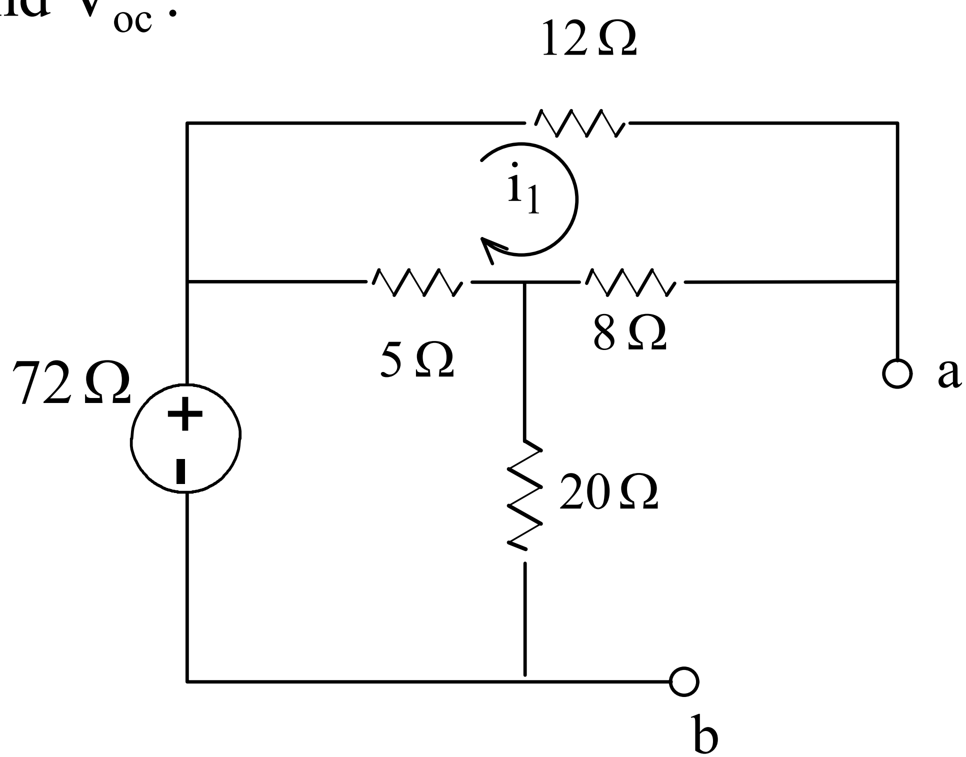
Example :

Use Thevenin and Norton to find V_0



Using Thevenin Theorm:

1. Find V_{oc} :



KVL around the upper loop :

$$12\, i_1 + 8\, i_1 + 5\,(i_1 - i_2) = 0$$

$$25\, i_1 - 5\, i_2 = 0 \qquad \ldots\ldots(1)$$

KCL around lower loop :

$$5\,(i_2 - i_1) + 20\,i_2 = 72$$

$$-5 i_1 + 25\,i_2 = 72 \qquad \ldots\ldots(2)$$

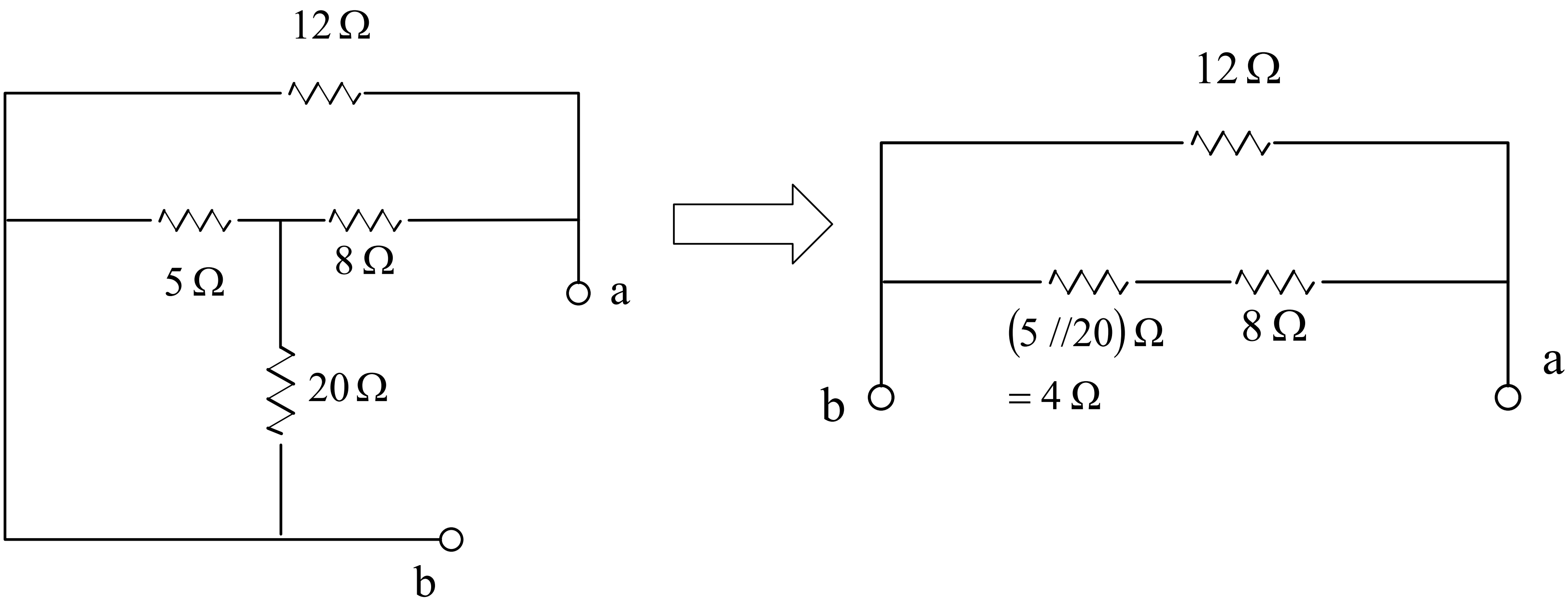
$$i_1 = 0.6\text{ A}, \quad i_2 = 3\text{ A}$$

$$V_{oc} = 8\,i_1 + 20\,i_2$$

$$= 8\,(0.6) + 20\,(3)$$

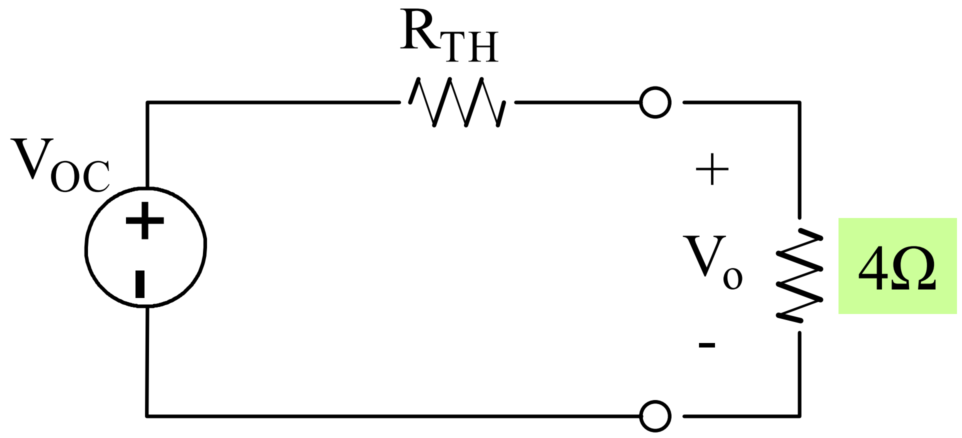
$$V_{oc} = 64.8\text{ V}$$

2. Find R_{TH}



$$R_{TH} = (8+4) \mathbin{//} 12 = 12 \mathbin{//} 12 = 6\Omega$$

3. Reconnect the load :



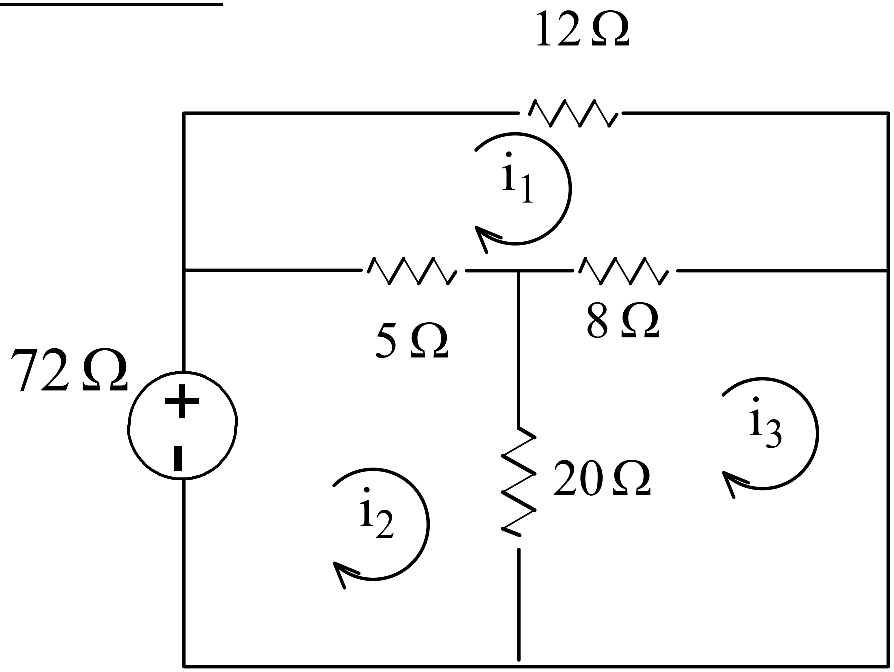
$$V_o = \frac{4}{4 + R_{TH}} V_{oc}$$

$$= \frac{4}{4 + 6} (64.8)$$

$$V_o = 25.92 \text{ V}$$

Using Norton Theorm:

1. Find I_{SC} :



KVL around upper loop :

$$12\,i_1 + 8\,(i_1 - i_3) + 5\,(i_1 - i_2) = 0$$

$$25\,i_1 - 5\,i_2 - 8\,i_3 = 0 \quad \ldots\ldots(1)$$

KVL around lower loop :

$$5\,(i_2 - i_1) + 20\,(i_2 - i_3) = 72$$

$$-5\,i_1 + 25\,i_2 - 20\,i_3 = 72 \qquad \ldots\ldots(2)$$

KVL around right loop :

$$8\,(i_3 - i_1) + 20\,(i_3 - i_2) = 0$$

$$-8\,i_1 - 20\,i_2 + 28\,i_3 = 0 \qquad \ldots\ldots(3)$$

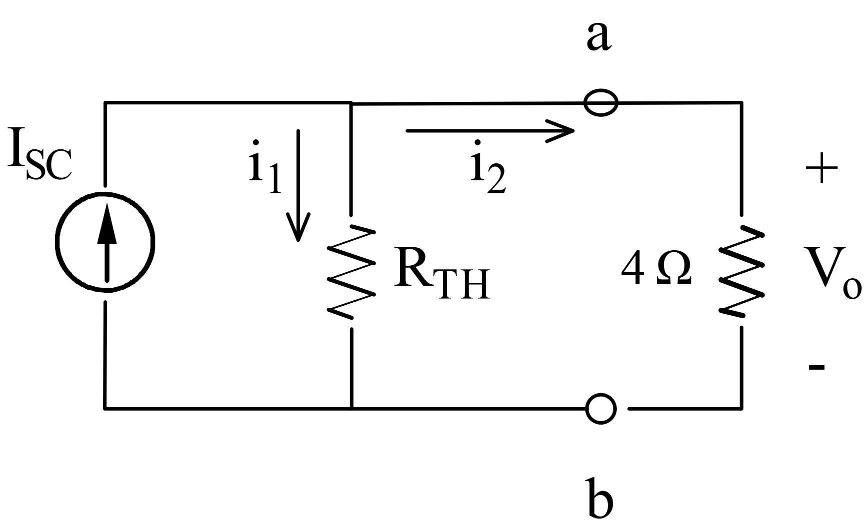
$$i_1 = 6\text{ A}, \quad i_2 = 12.72\text{ A}, \quad i_3 = 10.8\text{ A}$$

$$\Rightarrow \quad I_{SC} = 10.8\text{ A}$$

2. Find R_{TH}

From before , $R_{TH} = 6\ \Omega$

3. Reconnect the load



$$V_o = (4\ \Omega)\, i_2$$

$$= (4\ \Omega)\left(\frac{R_{TH}}{R_{TH}+4}\right) I_{SC}$$

$$V_o = 4\left(\frac{6}{6+4}\right)(10.8) = 25.92\ \text{V}$$

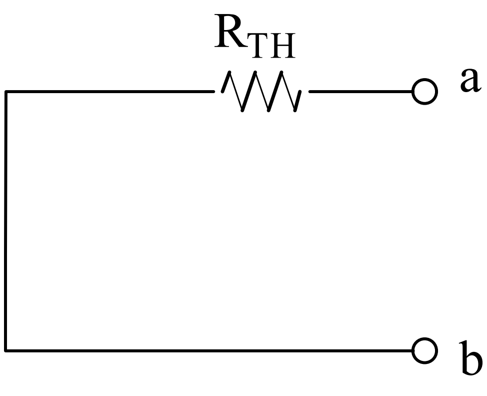
Case(2) : Circuits containing only dependent sources

Here there is NO energy source in the circuit.

- V_{OC} is always zero and I_{SC} is always zero
- So we can only find R_{TH}

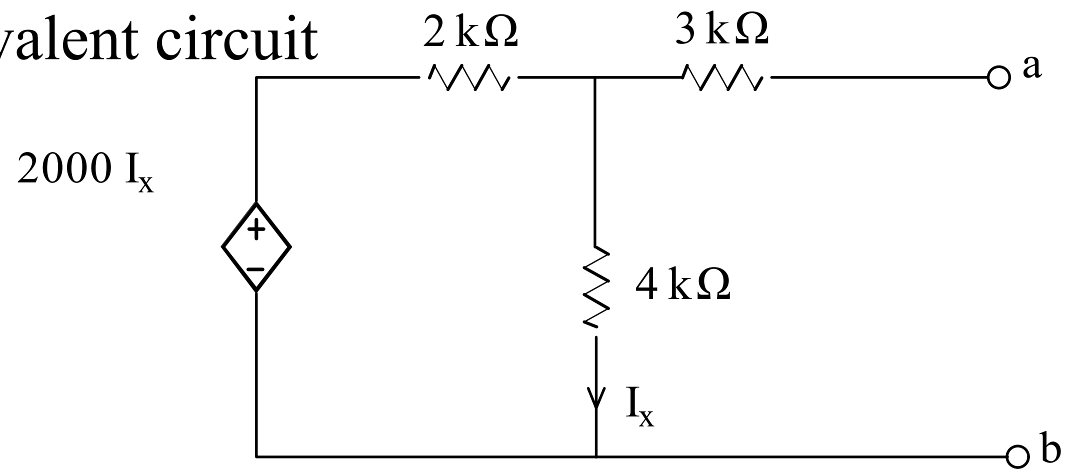
Procedure for finding R_{TH}

1. Connect an independent voltage (or current) source at the terminals ,Vx (or Ix)
2. Find the corresponding current (or voltage) at the terminal , I_o (or V_o)
3. Find $R_{TH} = Vx / I_o$ or $R_{TH} = V_o / Ix$

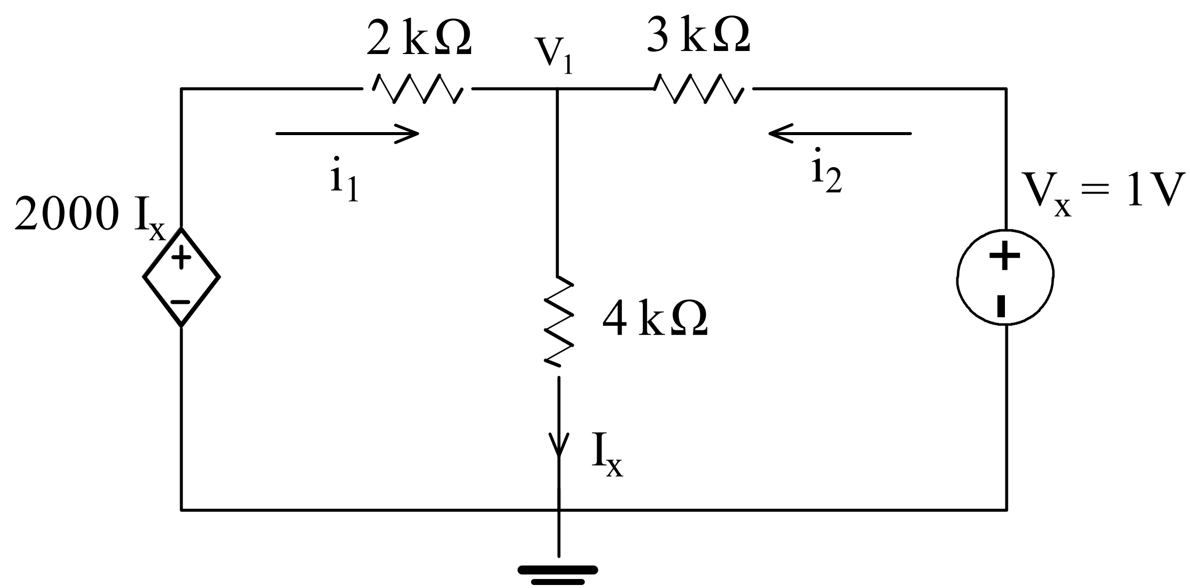


Example:

Find the Thevenin equivalent circuit



1. Apply voltage source at the terminals (Vx=1V)



KCL at node V1 :

$$i_1 + i_2 - I_X = 0$$

$$\frac{2000\ I_X - V_1}{2\ k} + \frac{1 - V_1}{3\ k} - I_X = 0$$

where $\qquad V_1 = (4\ k)\ I_X$

$$\frac{2000\ I_X - 4000\ I_X}{2000} + \frac{1 - 4000\ I_X}{3000} - I_X = 0$$

$$I_X - 2\ I_X + \frac{1}{3\ k} - \frac{4}{3\ k} I_X - I_X = 0$$

$$I_X \left[2 + \frac{4}{3} \right] = \frac{1}{3000}$$

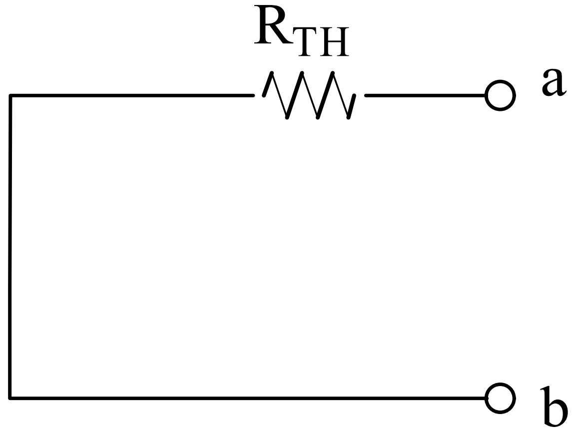
$$I_X = 0.1\ \text{m A}$$

$$i_2 = \frac{V_X - V_1}{3\ k}$$

$$= \frac{V_X - (4\ k)\, I_X}{3\ k} = \frac{1 - (4\ k)(0.1\ mA)}{3\ k}$$

$$i_2 = 0.2\ mA$$

$$R_{TH} = \frac{V_X}{i_2} = \frac{1\ V}{0.2\ mA} = 5\ k\ \Omega$$



Case (3) : Circuits containing both independent and dependent sources

Procedure of Thevenin or Norton Theorms:

a. Find the open circuit voltage and the terminals ,V_{OC}

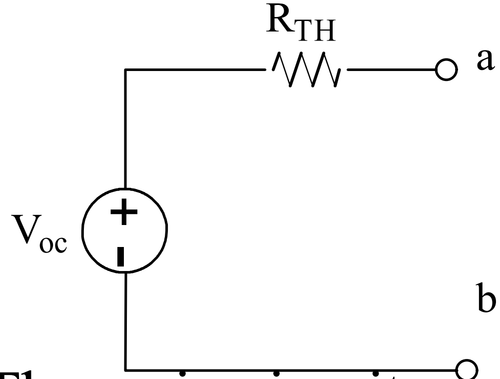
b. Find the short circuit current at the terminals, I_{SC} .

c. Compute $R_{TH} = V_{OC}/I_{SC}$

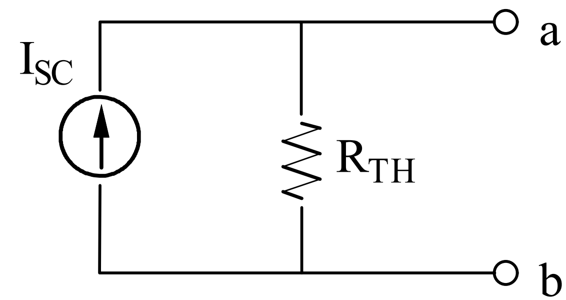
Note :

R_{TH} can not be found as in the case of only independent sources

d. Construct the Thevenin or Norton circuits



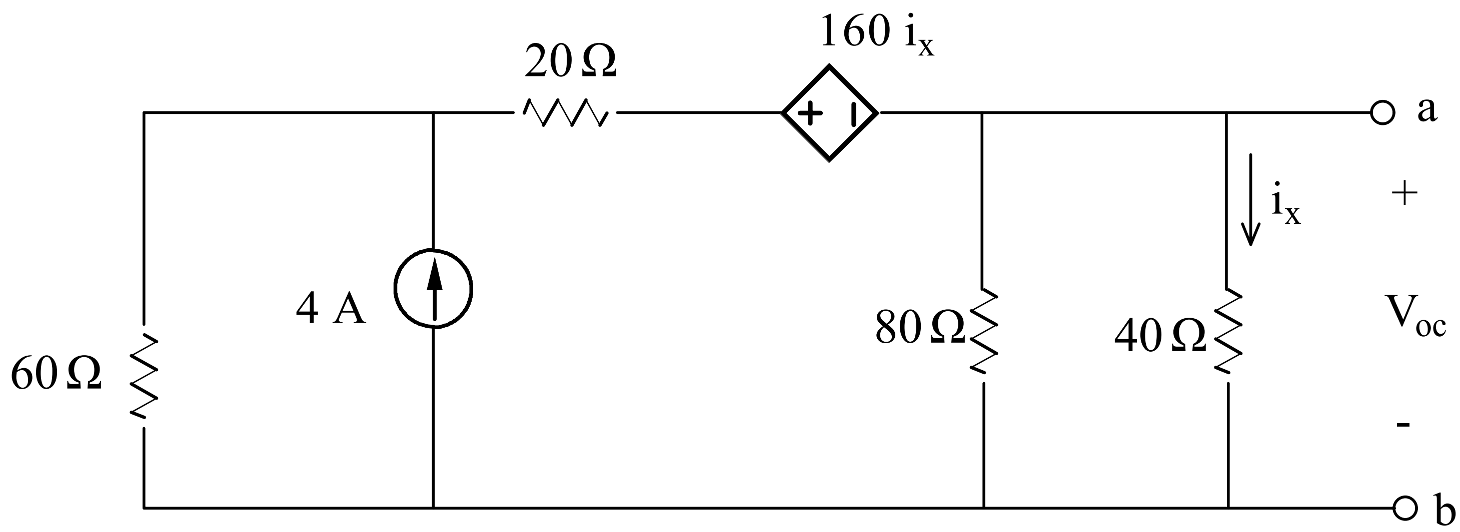
Thevenin circuit



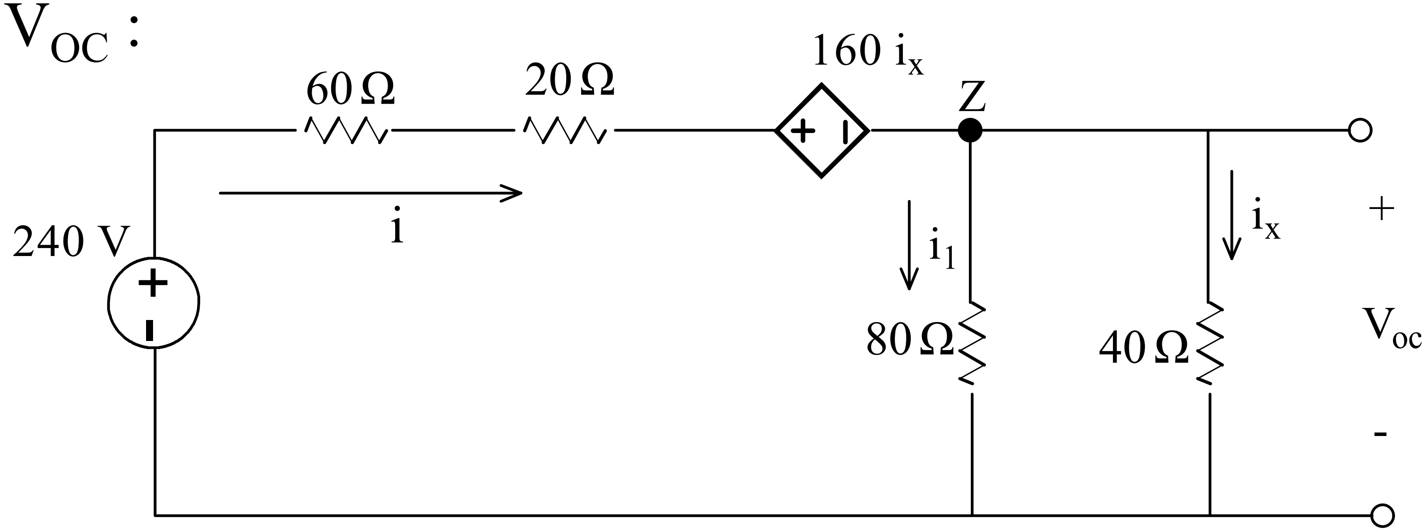
Norton circuit

Example :

Find the Thevenin equivalent circuit with respect to the terminals a, b



1. Find V_{OC} :



KVL around the lift loop :

$$-240+80\,i+160\,i_x+40\,i_x=0$$

$$80\,i+200\,i_x=240 \qquad \ldots\ldots(1)$$

KVL around right loop :

$$80\,i_1=40\,i_x$$

$$2\,i_1-i_x=0 \qquad \ldots\ldots(2)$$

KCL at Z:

$$i-i_1-i_x=0 \qquad \ldots\ldots(3)$$

$$i=1.125\text{ A}$$

$$i_1=0.375\text{ A}$$

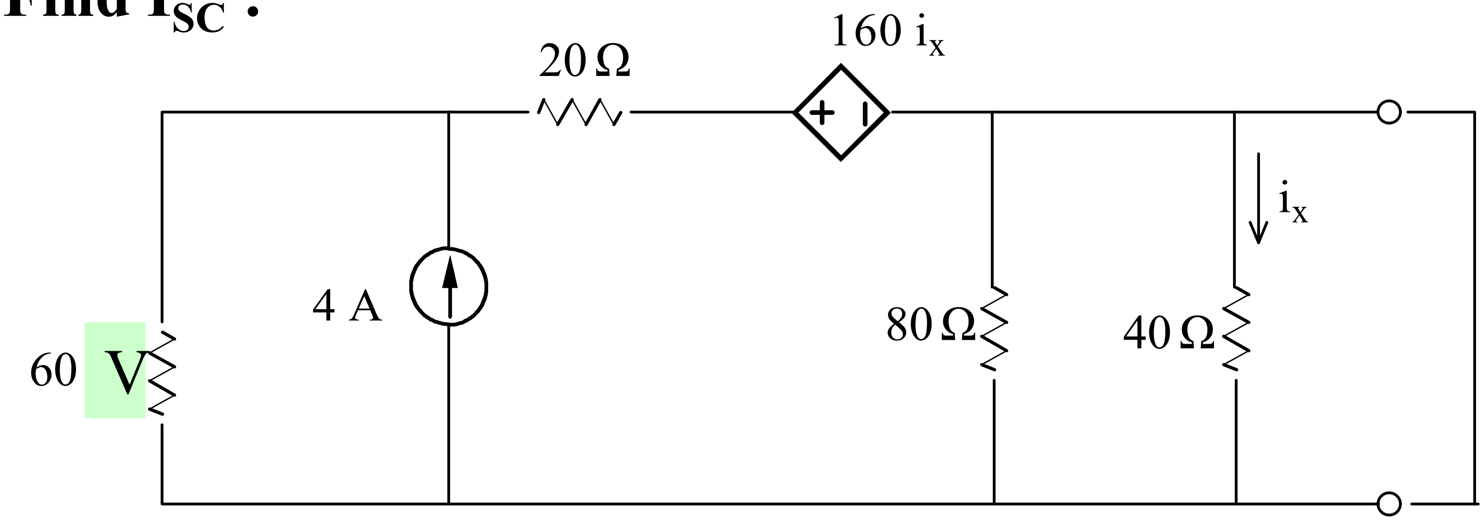
$$i_x=0.75\text{ A}$$

$$\therefore \quad V_{OC} = i_x \,(40\ \Omega)$$

$$= (0.75\text{A})\,(40\ \Omega)$$

$$V_{OC} = 30\ \text{V}$$

2. Find I_{SC} :

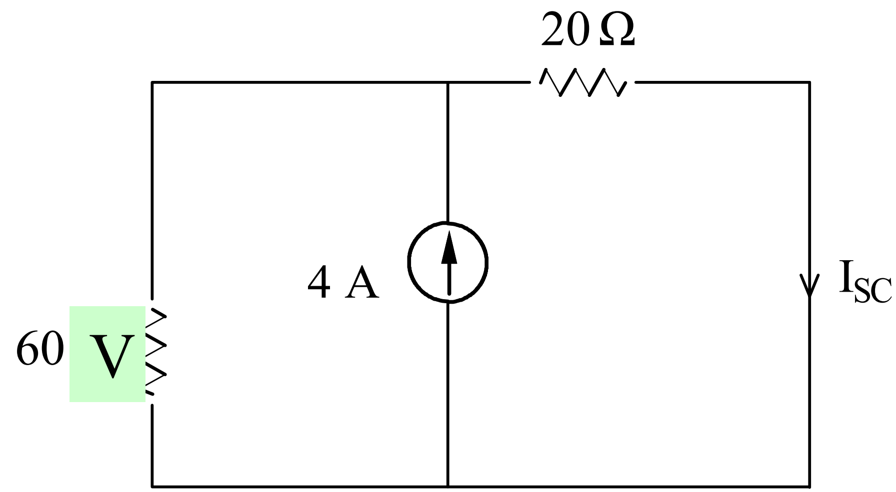


Since we have short circuit , 80 // 40 // 0 = 0

⇨ $i_x = 0$

⇨ $160 i_x$ source is zero

$$I_{SC} = \frac{60}{60+20}(4) = 3\text{ A}$$

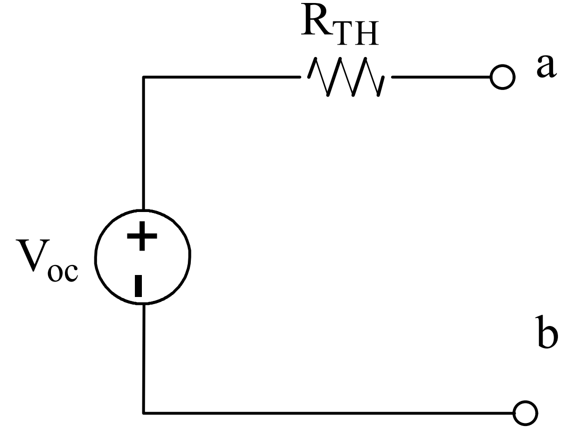


Current divider

3. Find R_{TH}

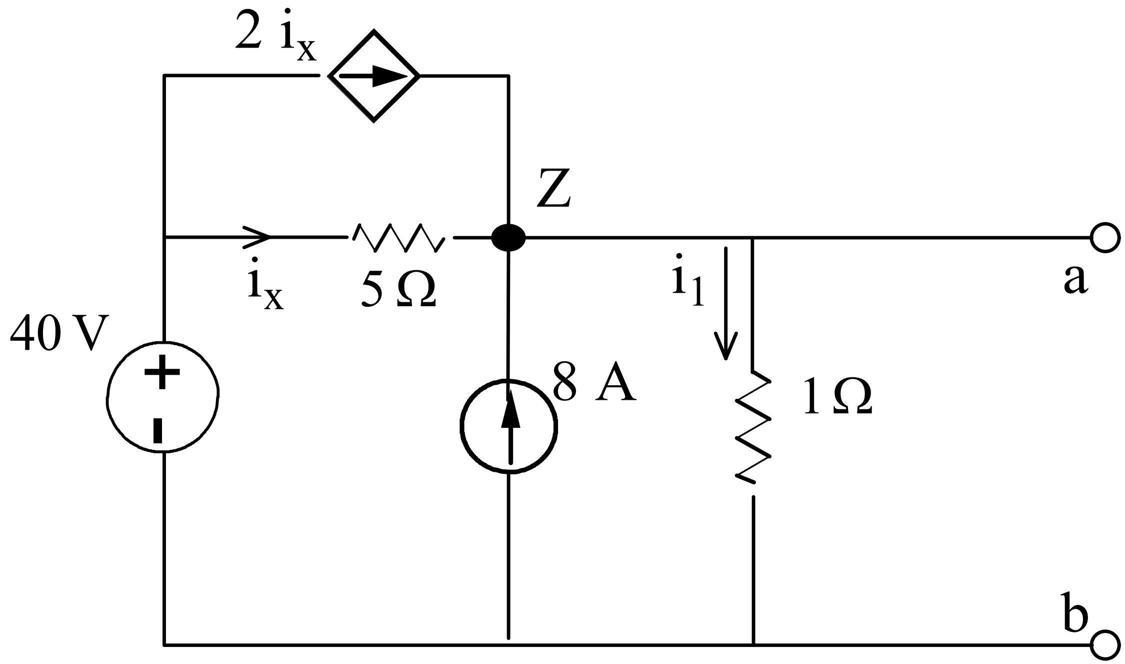
$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{30\text{ V}}{3\text{ A}} = 10\ \Omega$$

4. ⇨



Example :

Use Thevenin theorem to find the Thevenin equivalent circuit with respect to a, b



1. Find V_{OC}

KCL at node z :

$$2\,i_x + i_x + 8 - i_1 = 0$$

$$3\,i_x - i_1 = -8 \qquad \ldots\ldots(1)$$

KVL around outer loop

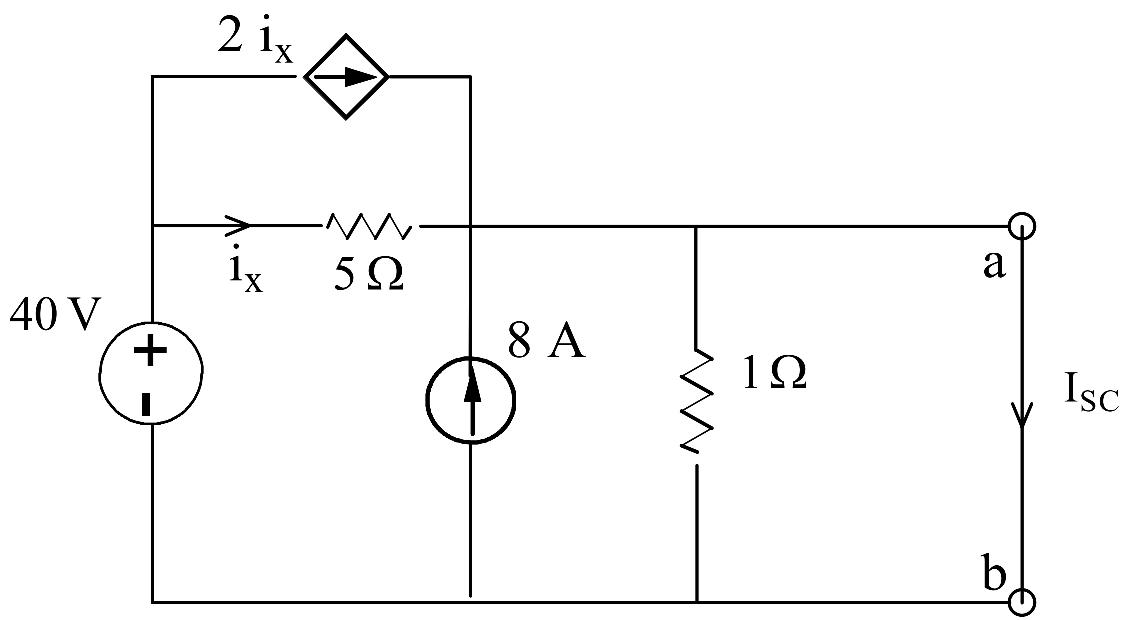
$$-40 + 5\,i_x + 1\,i_1 = 0$$

$$5\,i_x + i_1 = 40 \qquad \ldots\ldots(2)$$

$$\Rightarrow \quad i_x = 4\text{ A} \quad , i_1 = 20\text{ A}$$

$$\Rightarrow \quad V_{OC} = 1\,i_1 = 20\text{ V}$$

Find I_{SC} :



KVL around outer loop :

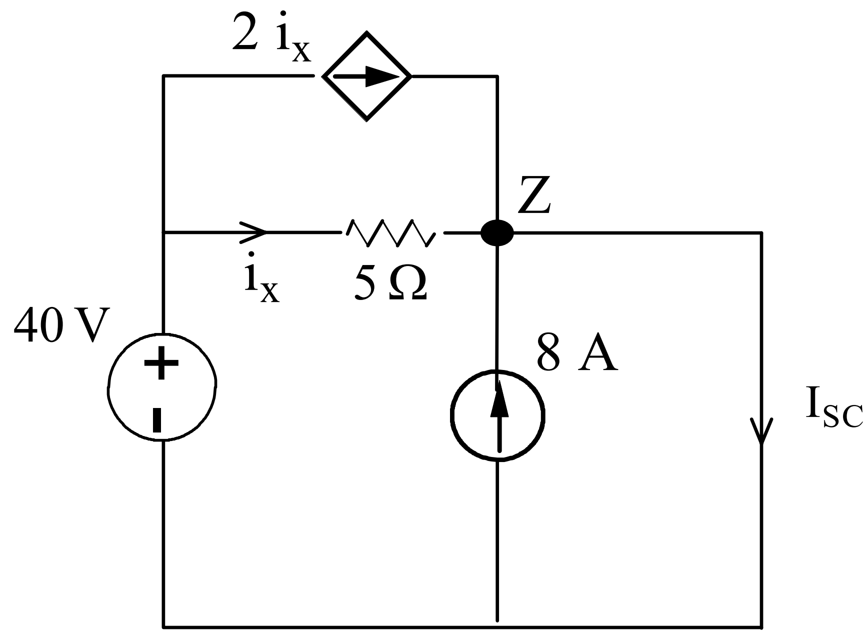
$$-40+5\,i_x=0 \quad \Rightarrow \quad i_x=8\text{ A}$$

KCL at z :

$$2\,i_x+i_x+8=I_{SC}$$

$$3\,i_x+8=I_{SC}$$

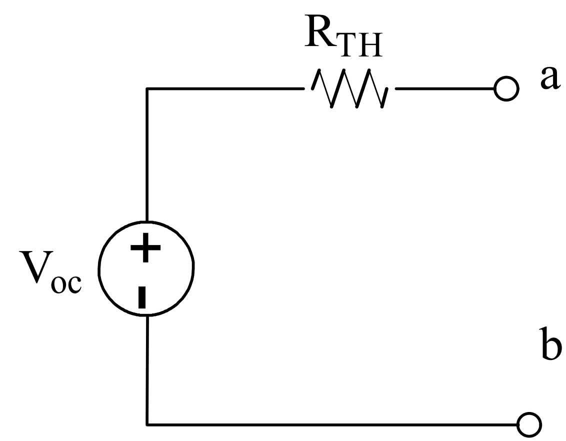
$$\Rightarrow \quad I_{SC}=(3)(8)+8=32\text{ A}$$



3. Find R_{TH} :

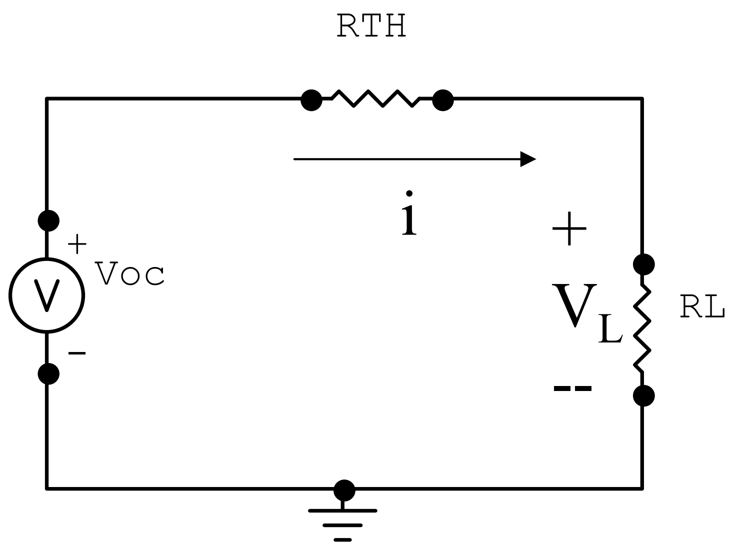
$$R_{TH}=\frac{V_{OC}}{I_{SC}}=\frac{20}{32}=0.625\ \Omega$$

Thevenin equivalent circuit is



4. Maximum Power Transfer

- A technique in which the load is selected to maximize the power transfer.
- This technique is based on the Thevenin equivalent circuit.



$$P_L = v_L\, i = i^2 R_L$$

$$= \left(\frac{V_{OC}}{R_{TH} + R_L}\right)^2 R_L$$

We wish to select R_L to maximize P_L:

Take $$\frac{dP_L}{dR_L} = 0$$

$$\frac{dP_L}{dR_L} = \frac{(R_{TH} + R_L)^2 \left(V_{OC}\right)^2 - R_L \left(V_{OC}\right)^2 \, 2(R_{TH} + R_L)}{\left(R_{TH} + R_L\right)^4} = 0$$

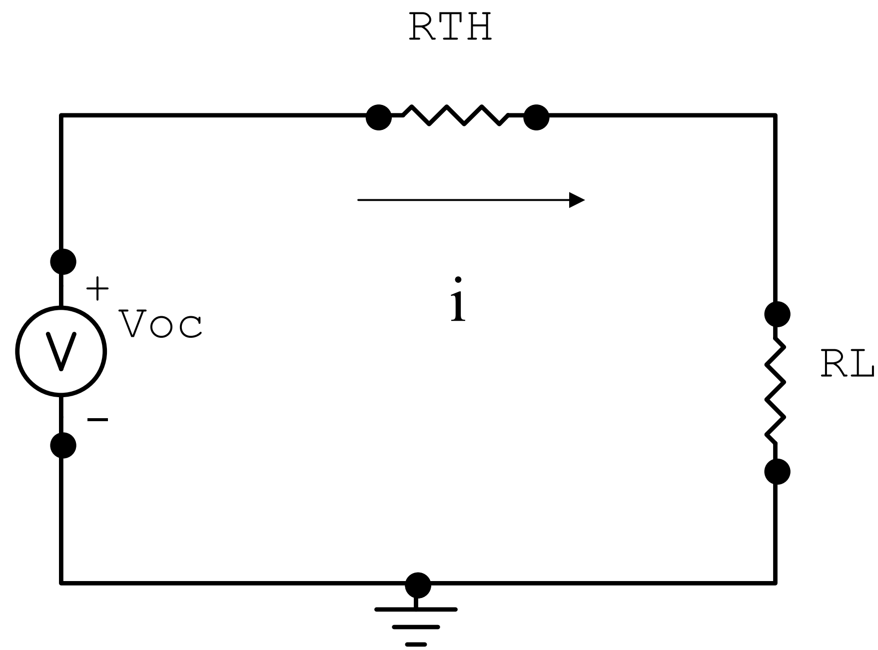
$$\frac{V_{oc}^2 \, (R_{TH} + R_L) \left[(R_{TH} + R_L) - 2\, R_L\right]}{(R_{TH} + R_L)^4} = 0$$

$$\Rightarrow \quad R_{TH} + R_L - 2\, R_L = 0$$

$$\Rightarrow \quad R_{TH} - R_L = 0$$

$$R_L = R_{TH}$$

If $R_L = R_{TH}$, what is the maximum Power Transfer?



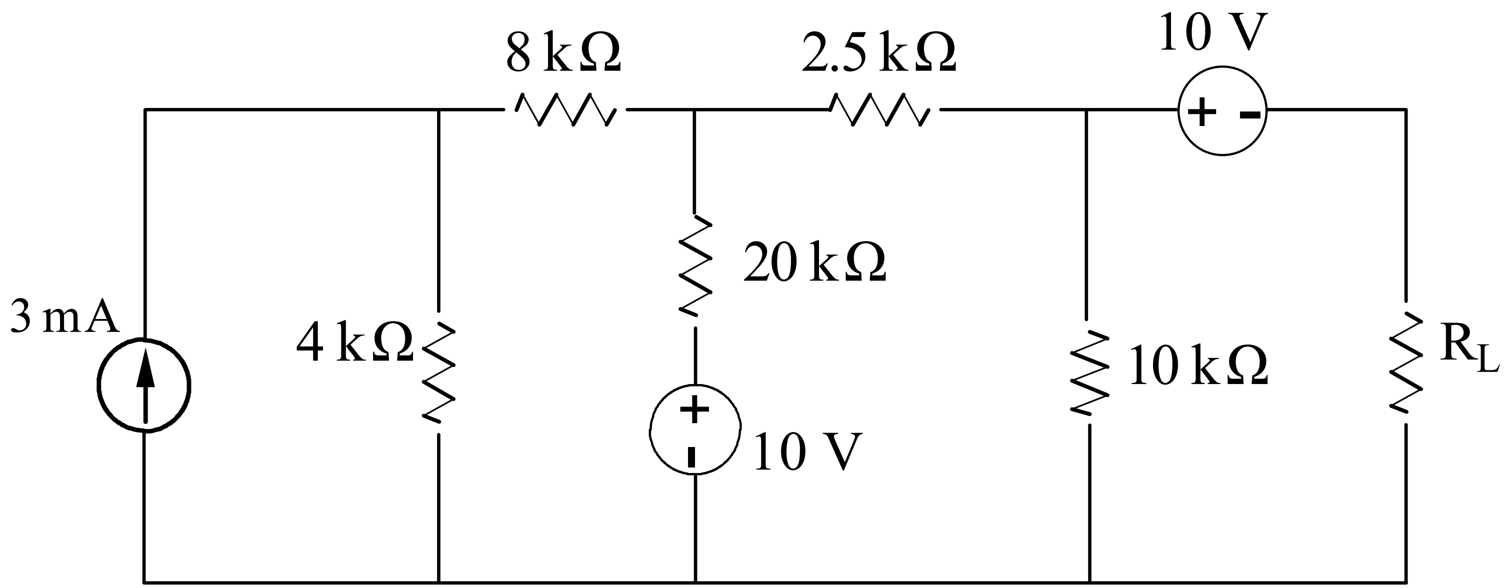
$$P_{L\max} = i^2 R_L$$

$$= \left(\frac{V_{OC}}{2 R_{TH}} \right)^2 R_{TH}$$

$$= \frac{(V_{OC})^2 R_{TH}}{4 R_{TH}^2} = \frac{(V_{OC})^2}{4 R_{TH}}$$

$$P_{L\max} = \frac{V_{OC}^{\ 2}}{4 R_{TH}}$$

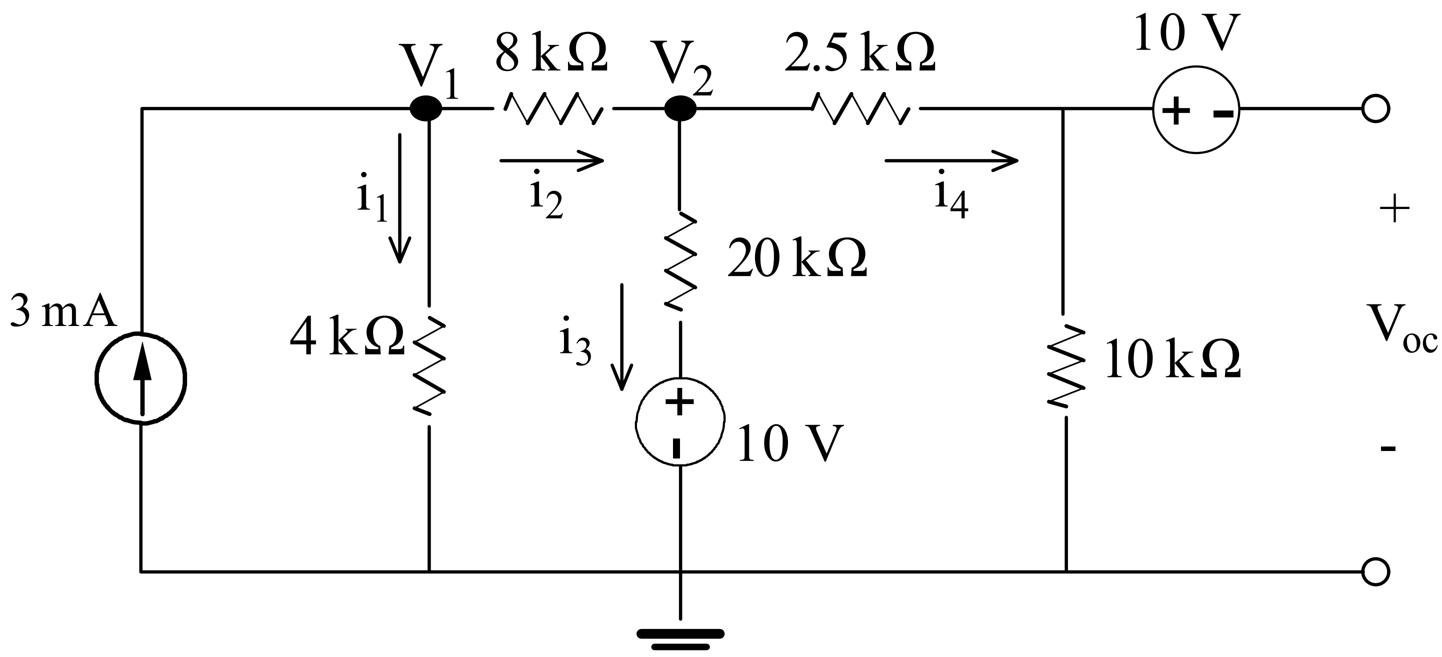
Example:



•Find R_L for maximum Power Transfer ?

•Find the maximum Power transfer to R_L ?

Let's find Thevenin equivalent circuit .



KCL at node V1 :

$$3\ \text{mA} - i_1 - i_2 = 0$$

$$3\ \text{mA} - \frac{V_1}{4\ \text{k}\Omega} - \frac{V_1 - V_2}{8\ \text{k}\Omega} = 0$$

$$V_1\left[\frac{1}{4\text{ k}}+\frac{1}{8\text{ k}}\right]-\left(\frac{1}{8\text{ k}}\right)V_2=3\text{ m}$$

$$0.375\text{ m } V_1-0.125\text{ m } V_2=3\text{ m} \qquad \ldots\ldots(1)$$

KCL at node V2:

$$i_2-i_3-i_4=0$$

$$\frac{V_1-V_2}{8\text{ k}}-\frac{V_2-10}{20\text{ k}}-\frac{V_2}{12.5\text{ k}}=0$$

$$V_1\left(\frac{1}{8\text{ k}}\right)-\left(\frac{1}{8\text{ k}}+\frac{1}{20\text{ k}}+\frac{1}{12.5\text{ k}}\right)V_2=-0.5\text{ m}$$

$$0.125\text{ m } V_1-0.255\text{ m } V_2=-0.5\text{ m} \qquad \ldots\ldots(2)$$

$$V_1=10.34\text{ V}$$

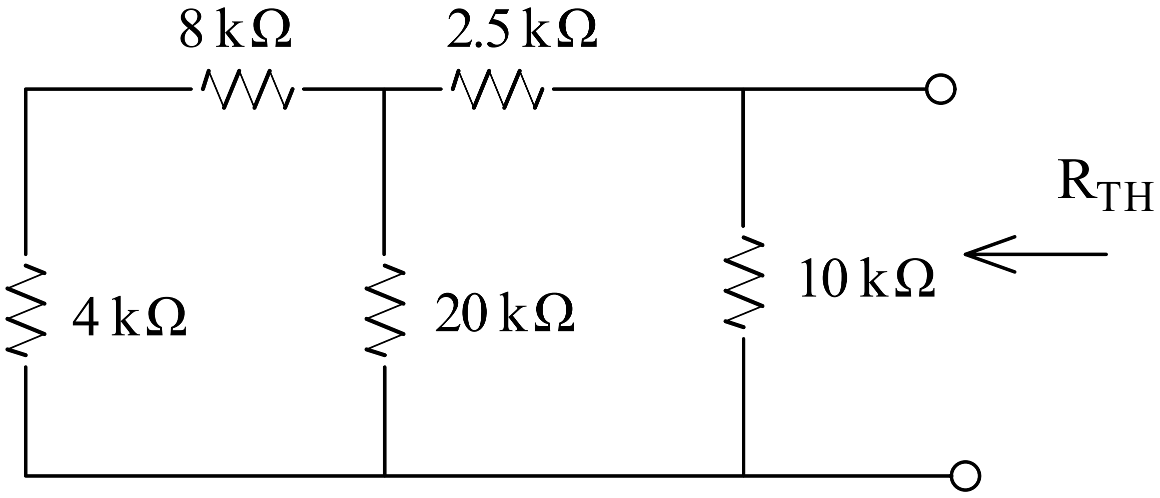
$$V_2=7.03\text{ V}$$

$$V_{OC} = -10 + 10\text{ k } i_4$$

$$= -10 + 10\text{ k}\left(\frac{V2}{12.5\text{ k}}\right) = -10 + \frac{10}{12.5}(7.03)$$

$$V_{OC} = -4.375\text{ V}$$

To find R_{TH} :



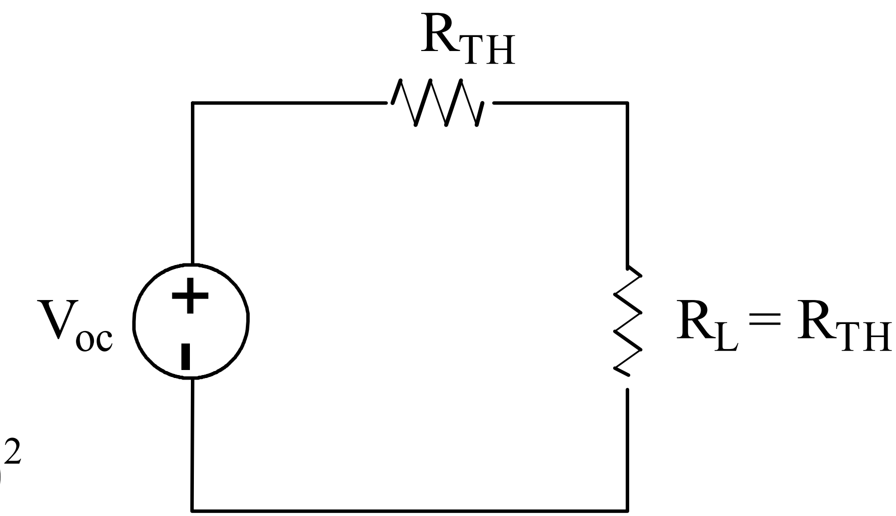
$$R_{TH} = \{ [(8\text{ k} + 4\text{ k}) \,//\, 20\text{ k}] + 2.5\text{ k} \} \,//\, 10\text{ k}$$

$$= [(12\text{ k} \,//\, 20\text{ k}) + 2.5\text{ k}] \,//\, 10\text{ k}$$

$$= (7.5\text{ k} + 2.5\text{ k}) \,//\, 10\text{ k}$$

$$= 10\text{ k} \,//\, 10\text{ k}$$

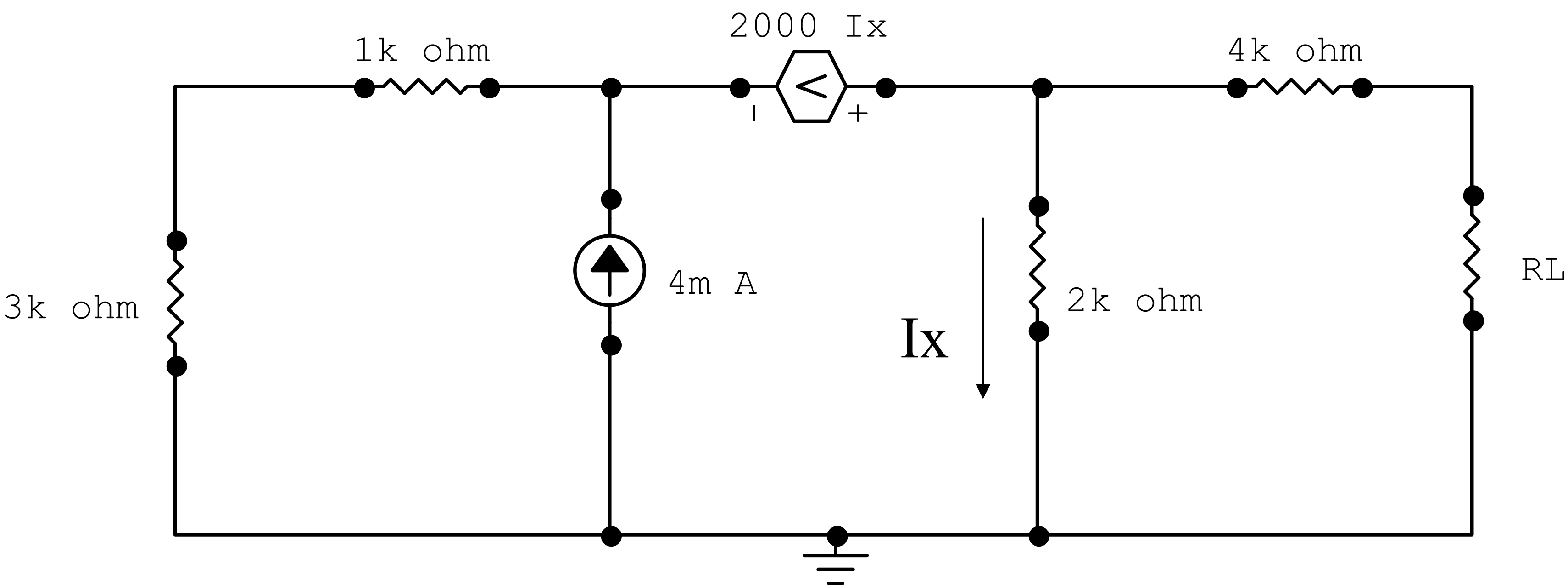
$$R_{TH} = 5\text{ k}\,\Omega$$



$$P_{L\max} = \frac{V_{OC}^{\;2}}{4\,R_{TH}} = \frac{(-4.375)^2}{4\,(5\text{ k})}$$

$$P_{L\max} = 0.957\text{ m W}$$

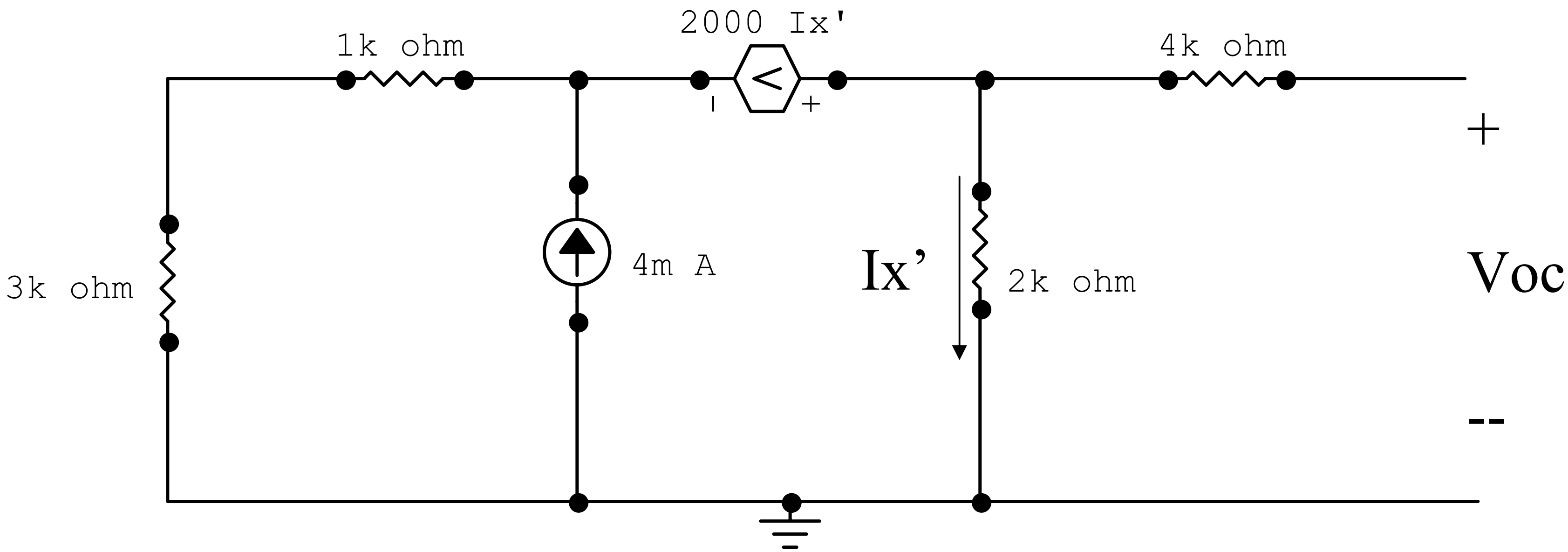
Example :



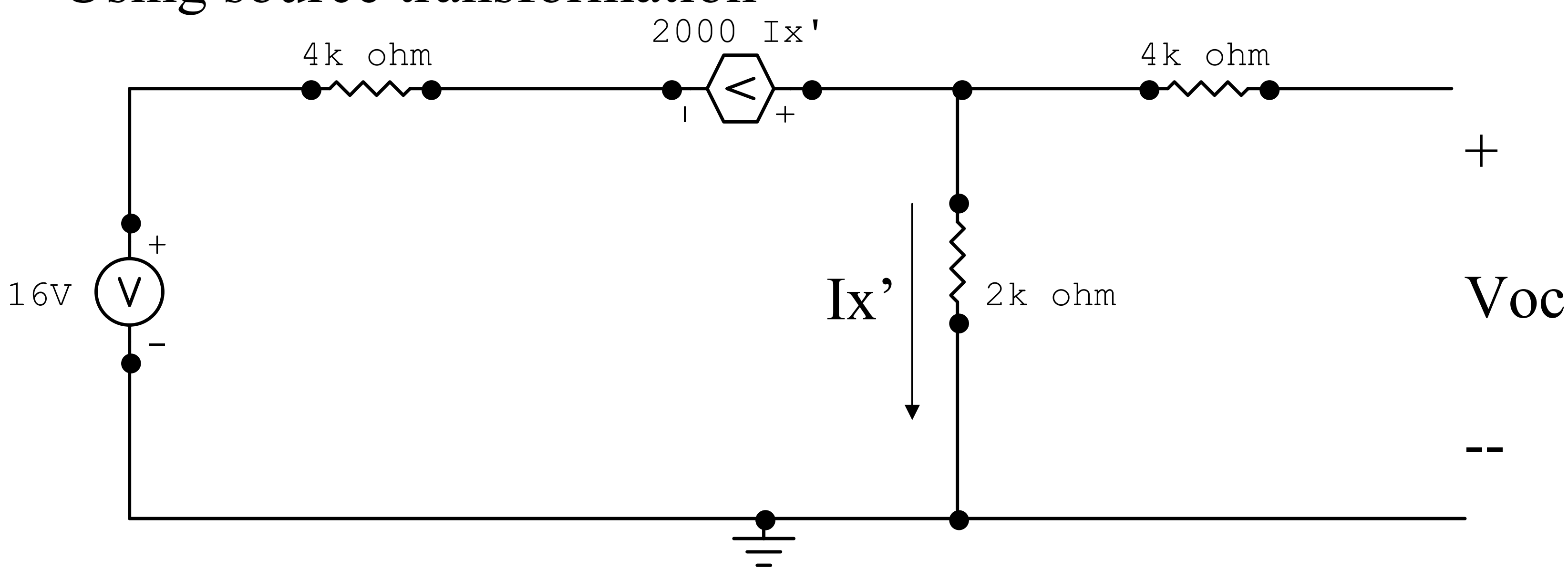
1. Find R_L for maximum Power Transfer?

2. Find max. power transfer to R_L ?

First , find Thevenin equivalent:



Using source transformation



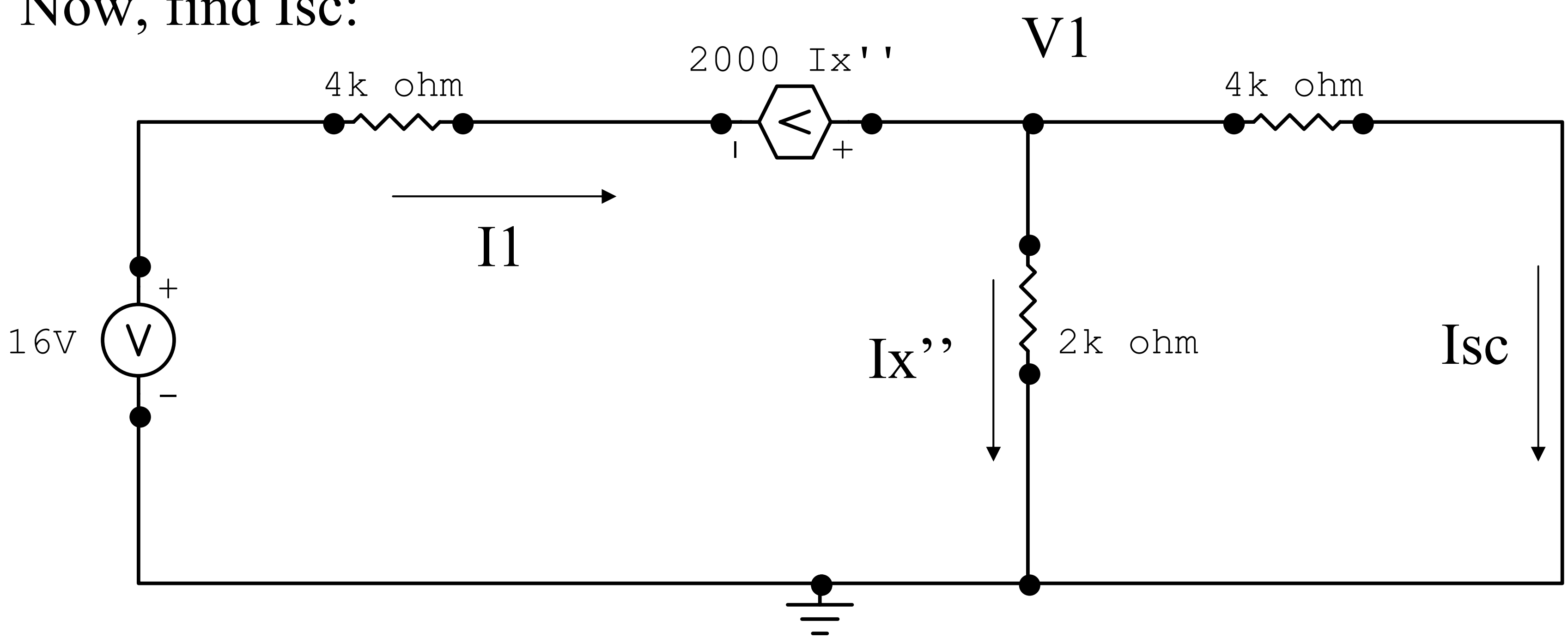
KVL around the loop:

-16 + 4k Ix' - 2k Ix' + 2k Ix'=0

Ix'= 4mA.

Voc= (2kΩ) Ix'= 8V.

Now, find Isc:



KCL at V1:

I1 - Ix'' – Isc = 0

$$\frac{16-(V1-2kIx'')}{4k}-\frac{V1}{2k}-\frac{V1}{4k}=0$$

Where V1=2k Ix''

Hence,

$$\frac{16}{4k}-\frac{V1}{2k}-\frac{V1}{4k}=0$$

Or V1=5.333V

And

$$Isc=\frac{V1}{4k}=1.333mA$$

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{8\,V}{1.333\,mA} = 6\,k\Omega$$

$$P_{L(max)} = \frac{{V_{OC}}^2}{4R_{TH}} = \frac{(8)^2}{4(6\,k)} = \frac{64}{24\,k}$$

$$P_{L(max)} = \frac{8}{3}\,mW$$