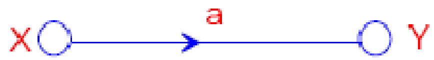
CHAPTER # 4 SIGNAL FLOW GRAPH (SFG)

1. Introduction

For complex control systems, the block diagram reduction technique is cumbersome. An alternative method for determining the relationship between system variables has been developed by ***Mason*** and is based on a signal flow graph. A signal flow graph is a diagram that consists of nodes that are connected by branches. A node is assigned to each variable of interest in the system, and branches are used to relate the different variables. The main advantage for using SFG is that a straight forward procedure is available for finding the transfer function in which it is not necessary to move pickoff point around or to redraw the system several times as with block diagram manipulations.

SFG is a diagram that represents a set of simultaneous linear algebraic equations which describe a system. Let us consider an equation, $y = a\,x$. It may be represented graphically as,

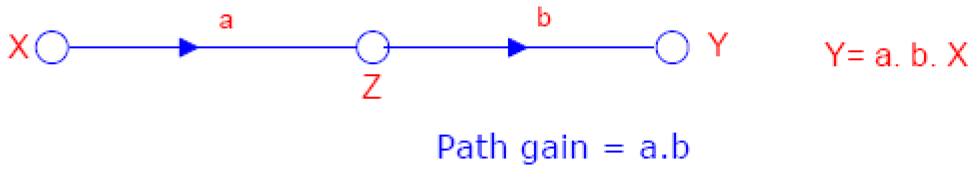


2. Terminology

Node: A junction denoting a variable or a signal.

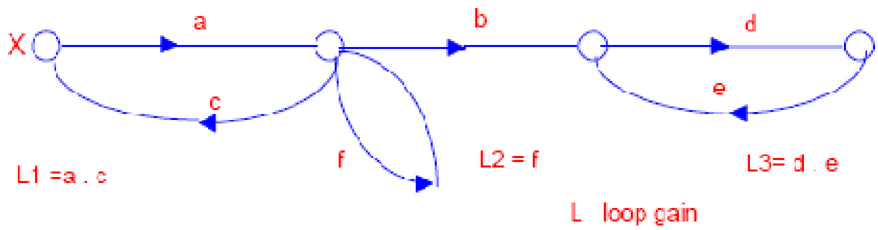
Branch: A unidirectional path that relates the dependency of an input and an output. Relation between variables is written next to the directional arrow.

Path: A branch or a continuous sequence of branches that can be traversed from one node to another



Forward Path: A path from input to output node.

Loop: A closed path that originates at one node and terminates at the same node. Along the path no node is touched twice.



Non-Touching Loops: Loops with no common nodes

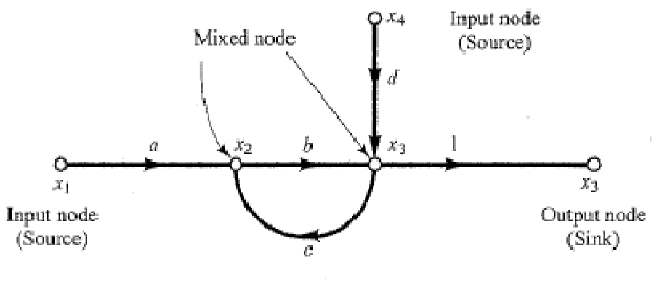
Examples: L1 and L2 are touching loops

L1 and L3 & L2 and L3 are non-touching loops,

Input node (Source): node having only outgoing branches

Output node (Sink): node having only incoming branches

Mixed node: A node that has both incoming and outgoing branches.



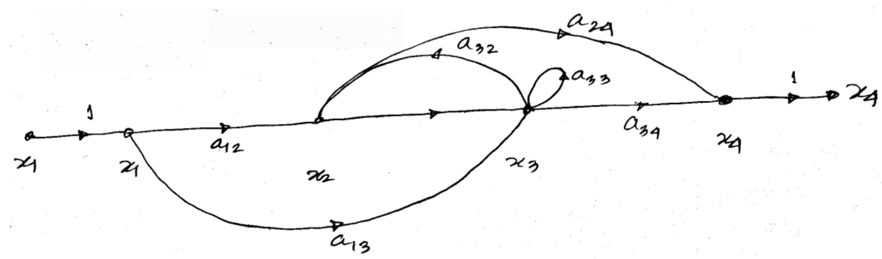
1. Construction of SFG from D.E.

SFG of a system can be constructed from the describing equations:

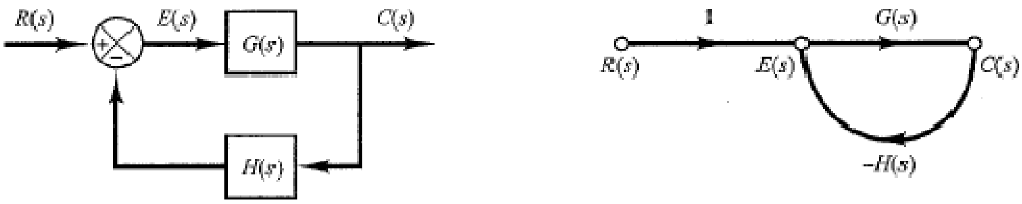
$$x_2 = a_{12} x_1 + a_{32} x_3$$

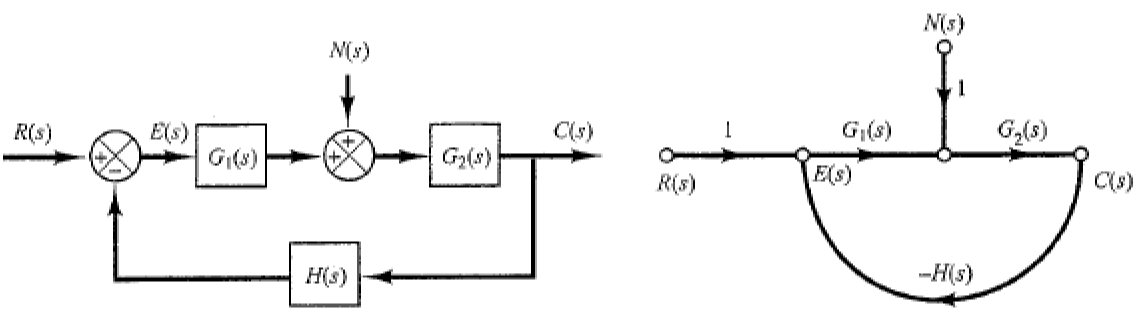
$$x_3 = a_{13} x_1 + a_{23} x_2 + a_{33} x_3$$

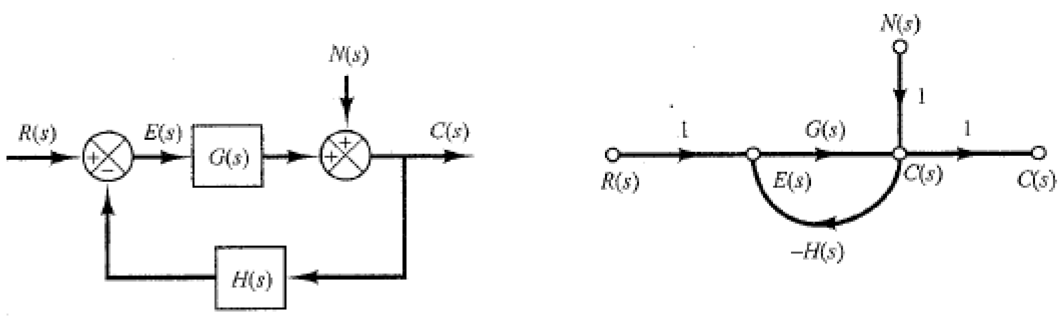
$$x_4 = a_{24} x_4 + a_{34} x_3$$

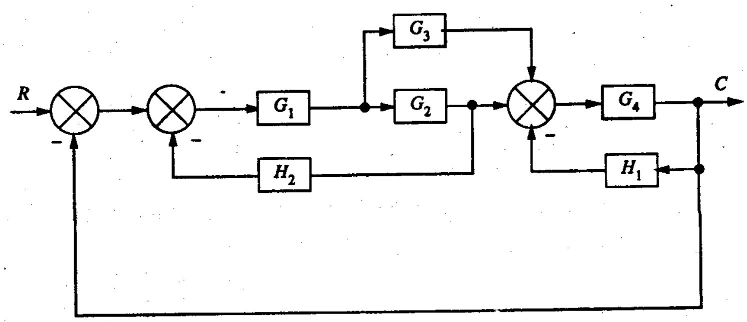


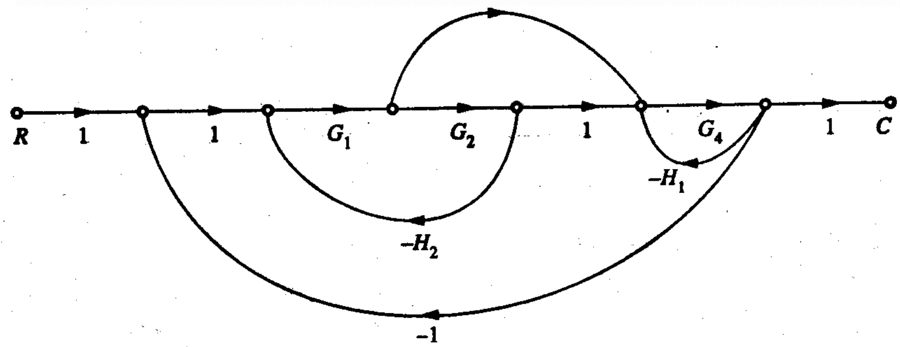
4. SFG from Block Diagram











Mason's Rule

$$T = \sum_{k=1}^{N} \frac{P_k \Delta_k}{\Delta}$$

N Number of paths from input to output

P Gain of the k_{th} path from input to output

Δ Determinant of the graph

Δ_k Cofactor of the k_{th} path

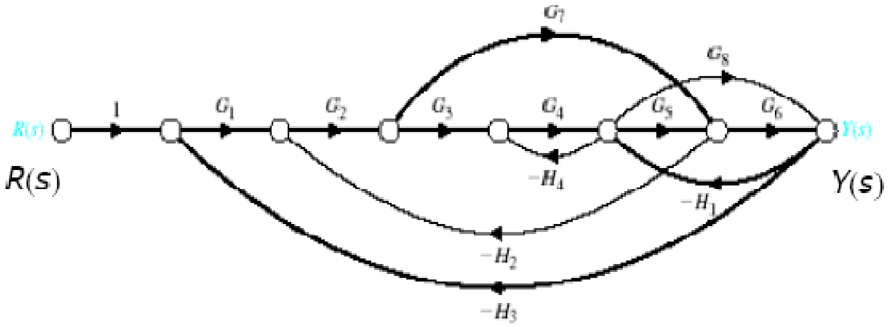
Summation is taken over all possible paths from input to output

$\Delta = 1$ - ($\sum$ all different loop gains)
+ ($\sum$ gain products of all combinations of 2 non-touching loops)
- ($\sum$ gain products of all combinations of 3 non-touching loops)
+ ($\sum$ gain products of all combinations of 4 non-touching loops)
- .....

Δ_k = value of Δ for that part of the SFG not touching the k_{th} forward path

Example 1

Find the T.F. $Y(s)/X(s)$



Forward Paths =3

$P_1 = G_1G_2G_3G_4G_5G_6$; $P_2 = G_1G_2G_7G_6$ $P_3 = G_1G_2G_3G_4G_8$

Feedback loops

$L_1 = -G_2G_3G_4G_5H_2$; $L_2 = -G_5G_6H_1$; $L_3 = -G_8H_1$; $L_4 = -G_7H_2G_2$;
$L_5 = -G_4H_4$; $L_6 = -G_1G_2G_3G_4G_5G_6H_3$; $L_7 = -G_1G_2G_7G_6H_3$
$L_8 = -G_1G_2G_3G_4G_8H_3$

Loop L_5 does not touch loop L_4 or Loop L_7
Loop L_3 does not touch Loop L_4
All other loops touch

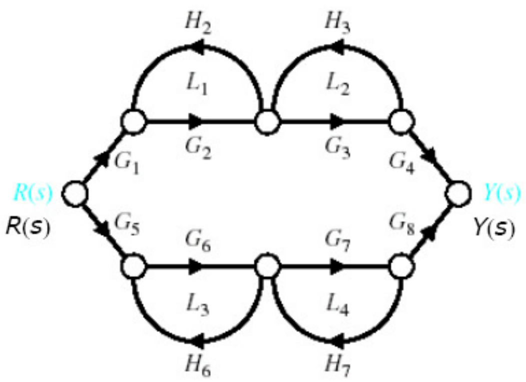
$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8) + (L_5L_7 + L_5L_4 + L_3L_4)$$

$$\Delta_1 = \Delta_3 = 1 ; \Delta_2 = 1 - L_5 = 1 + G_4H_4$$

$$\frac{Y(s)}{R(s)} = \frac{P_1 + P_2\Delta_2 + P_3}{\Delta}$$

Example 2

Find the T.F. $Y(s)/X(s)$



Forward Paths =2

$$P_1 = G_1G_2G_3G_4 \quad ; \quad P_2 = G_5G_6G_7G_8$$

Feedback loops

$$L_1 = G_2H_2 \quad ; \quad L_2 = G_3H_3 \quad ; \quad L_3 = G_6H_6 \; ; \; L_4 = G_7H_7 \; ;$$

Loops L_1 and L_2 do not touch loop L_3 and L_4

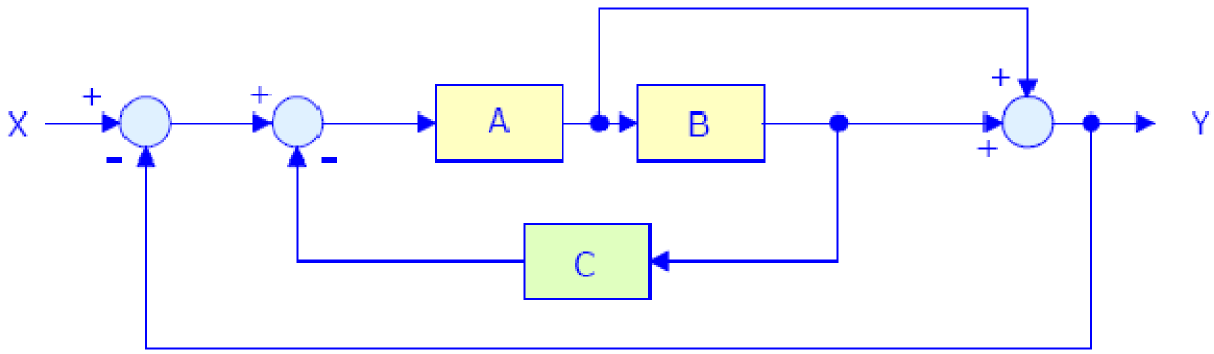
$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4)$$

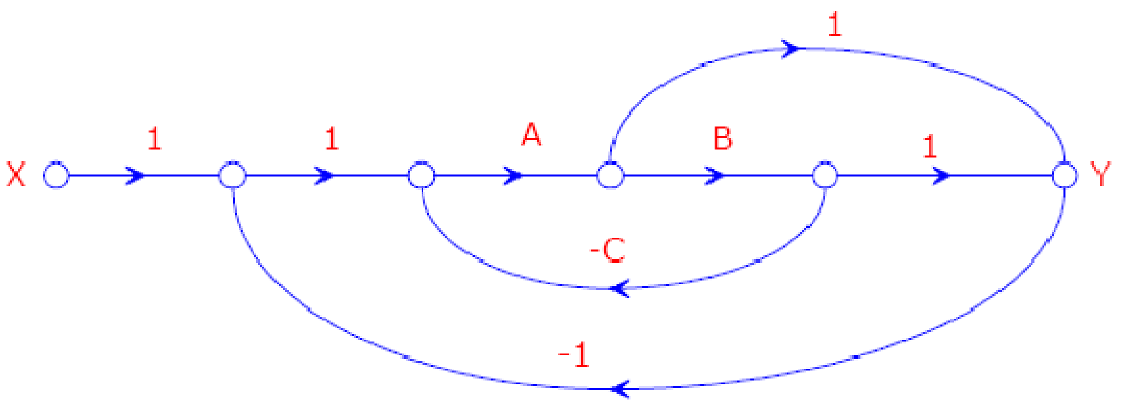
$$\Delta_1 = 1 - (L_3 + L_4) \quad ; \quad \Delta_2 = 1 - (L_1 + L_2)$$

$$\frac{Y(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3G_4(1 - L_3 - L_4) + G_5G_6G_7G_8(1 - L_1 - L_2)}{1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4)}$$

Example 3

Using Mason's Formula, Find the T.F. *Y*(*s*)/*X*(*s*)





$P_1 = AB$; $P_2 = A$

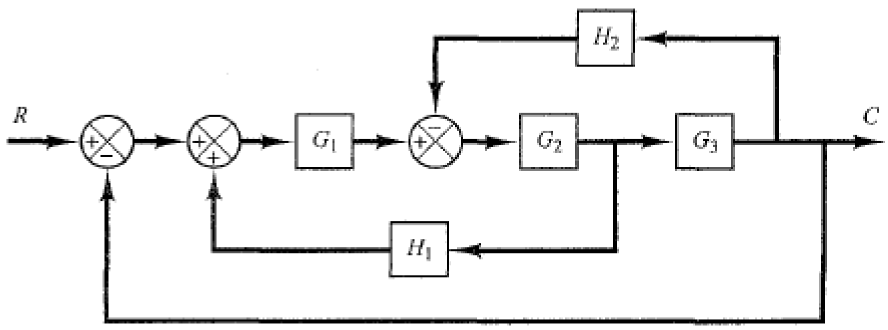
$$\Delta = 1 - (-ABC - AB - A)$$
$$\Delta = 1 + ABC + AB + A$$

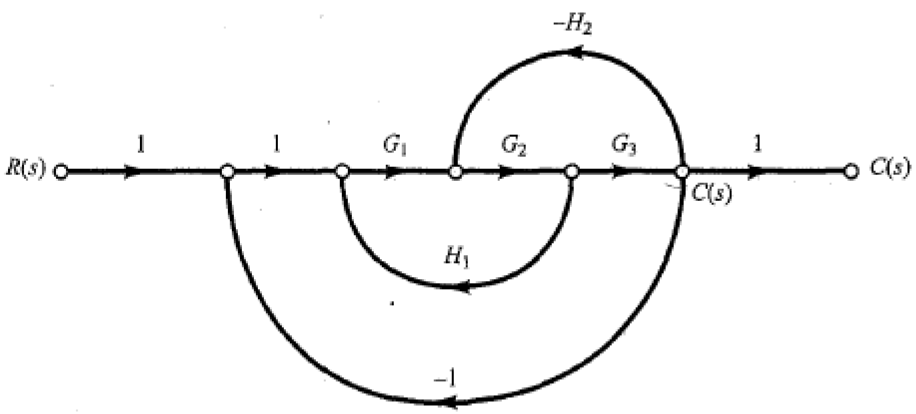
$\Delta_1 = 1$; $\Delta_2 = 1$

$$\frac{Y}{X} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{A(1+B)}{1+ABC+AB+A}$$

Example 4

Using Mason's Formula, Find the T.F. $C(s)/R(s)$





In this system there is only one forward path between the input $R(s)$ and the output $C(s)$. The forward path gain is

$$P_1 = G_1G_2G_3$$

we see that there are three individual loops. The gains of these loops are

$$L_1 = G_1 G_2 H_1$$
$$L_2 = -G_2 G_3 H_2$$
$$L_3 = -G_1 G_2 G_3$$

Note that since all three loops have a common branch, there are no non-touching loops. Hence, the determinant **Δ** is given by

$$\Delta = 1 - (L_1 + L_2 + L_3)$$
$$= 1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3$$

The cofactor Δ_1 of the determinant along the forward path connecting the input node and output node is obtained from **Δ** by removing the loops that touch this path. Since path P_1 touches all three loops, we obtain

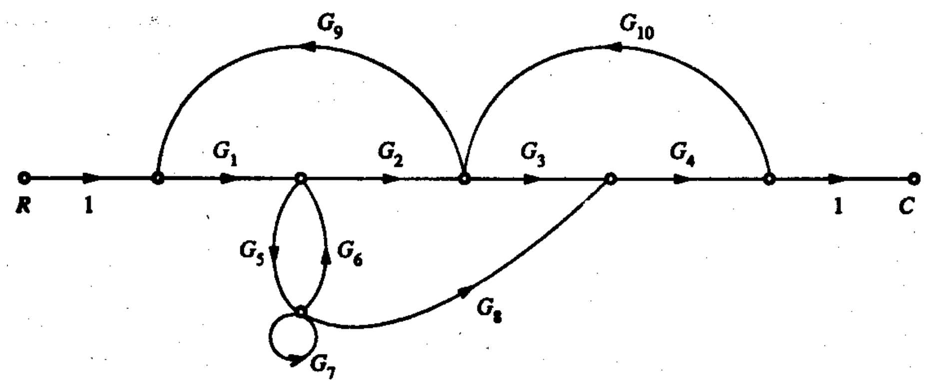
$$\Delta_1 = 1$$

Therefore, the overall gain between the input *R(s)* and the output *C(s),* or the closed-loop transfer function, is given by

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3}$$

Example 5

Using Mason's Formula, Find the T.F. $C(s)/R(s)$

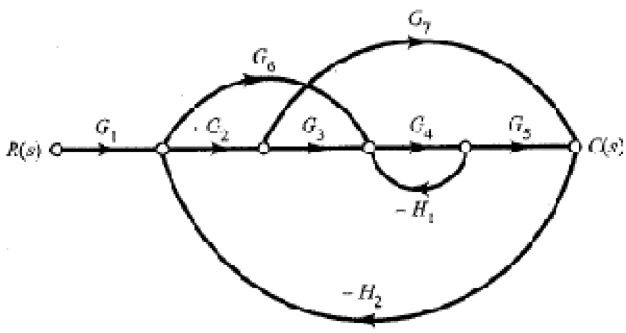


$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 G_3 G_4 (1 - G_7) + G_1 G_5 G_8 G_4}{1 - [G_1 G_2 G_9 + G_3 G_4 G_{10} + G_1 G_5 G_8 G_4 G_{10} G_9 + G_5 G_6 + G_7] + [G_1 G_2 G_9 G_7 + G_3 G_4 G_{10} G_5 G_6 + G_3 G_4 G_{10} G_7]}$$

Example 6

Using Mason's Formula, Find the T.F. $C(s)/R(s)$



In this system, there are three forward paths between the input *R(s)* and the output *C(s)*. The forward path gains are

$$P_1 = G_1G_2G_3G_4G_5$$
$$P_2 = G_1G_6G_4G_5$$
$$P_3 = G_1G_2G_7$$

There are four individual loops, the gains of these loops are

$$L_1 = -G_4H_1$$
$$L_2 = -G_2G_7H_2$$
$$L_3 = -G_6G_4G_5H_2$$
$$L_4 = -G_2G_3G_4G_5H_2$$

Loop L_1 does not touch loop L_2; Hence, the determinant **Δ** is given by

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_1L_2$$

The cofactor Δ_1, is obtained from Δ by removing the loops that touch path *PI*. Therefore, by removing L_1, L_2, L_3, L_4, and L_1, L_2 from Δ equation, we obtain

$$\Delta_1 = 1$$

By the same way $\Delta_2 = 1$

The cofactor $\mathbf{\Delta_3}$ is obtained by removing L_2, L_3, L_4, and L_1, L_2 from Δ Equation, giving

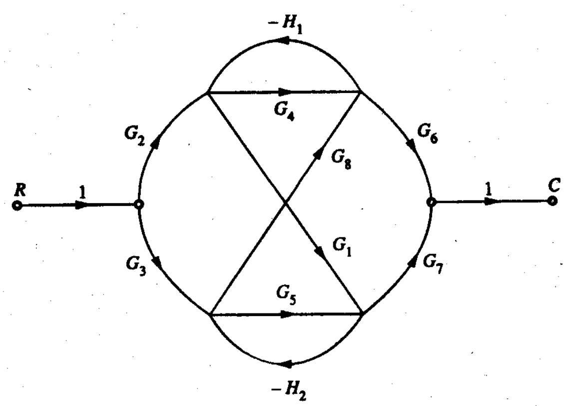
$$\Delta_3 = 1 - L_1$$

The closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4G_5 + G_1G_6G_4G_5 + G_1G_2G_7(1 + G_4H_1)}{1 + G_4H_1 + G_2G_7H_2 + G_6G_4G_5H_2 + G_2G_3G_4G_5H_2 + G_4H_1G_2G_7H_2}$$

Example 7

Consider the control system whose signal flow graph is shown below. Determine the system transfer function using Mason's formula.



* There are **SIX** Forward Paths:

$P_1 = G_2\,G_4\,G_6$

$P_2 = G_3\,G_5\,G_7$

$P_3 = G_2\,G_1 \cdot G_7$

$P_4 = G_3\,G_8\,G_6$

$P_5 = -G_2\,G_1 \cdot H_2\,G_8 \cdot G_6$

$P_6 = -G_3\,G_8\,H_1\,G_1\,G_7$

* There are **THREE** feedback loops:

$P_{11} = -\,H_1\,G_4$

$P_{21} = -\,H_2\,G_5$

$P_{31} = G_1\,H_2\,G_8\,H_1$

* There are **ONE** combination of two-non-touching feedback loops:

$P_{12} = H_1H_2G_4G_5$

$$\Delta = 1 - [-H_1\,G_4 - H_2\,G_5 + G_1\,H_2\,G_8\,H_1] + [H_1\,H_2\,G_4\,G_5]$$
$$= 1 - G_1\,H_2\,G_8\,H_1 + H_2\,G_5 - G_1\,H_2\,G_8\,H_1 + H_1\,H_2\,G_4\,G_5$$

$$\Delta_1 = 1 - (-\,H_2\,G_5) = 1 + H_2\,G_5$$

$$\Delta_2 = 1 - (-H_1 G_4) = 1 + H_1 G_4$$

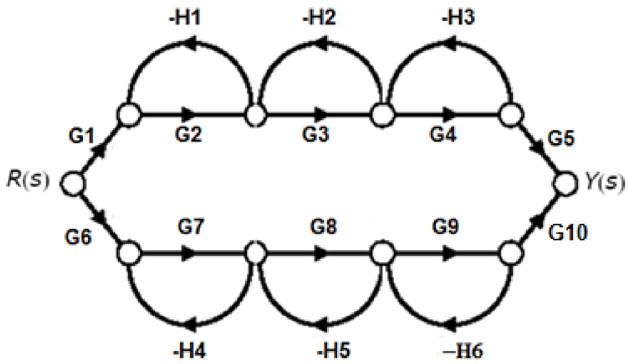
$$\Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$$

Using Mason's Formula, the system Transfer Function is:

$$T = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6}{\Delta}$$

Example 8

For the signal flow graph of a certain control system shown below, find the system characteristic equation.



The characteristic equation obtained from mason's formula is Δ=0

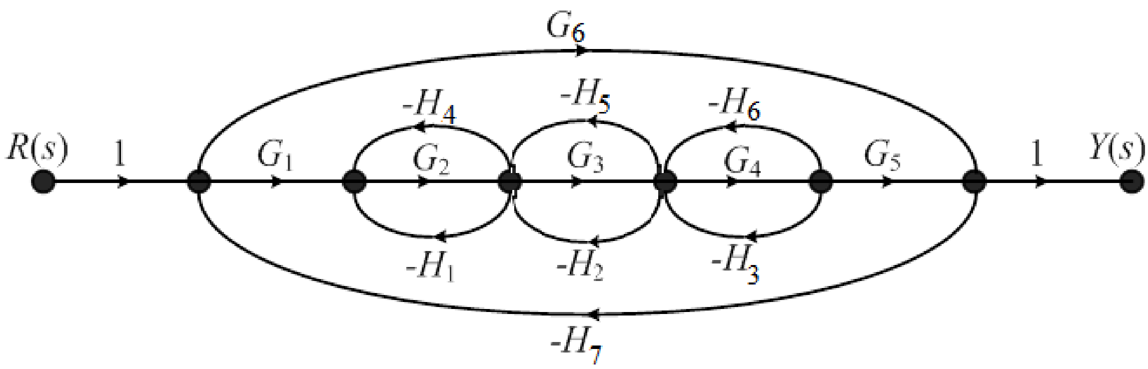
$\Delta = 1$ - ($\sum$ all different loop gains)
+ ($\sum$ gain products of all combinations of 2 non-touching loops)
- ($\sum$ gain products of all combinations of 3 non-touching loops)
+ ($\sum$ gain products of all combinations of 4 non-touching loops)
- .....

Loop Gains	Two non-touching Loops	Three non-touching Loops
$L_1 = -G_2H_1$ $L_2 = -G_3H_2$ $L_3 = -G_4H_3$ $L_4 = -G_7H_4$ $L_5 = -G_8H_5$ $L_6 = -G_9H_6$	$L_1L_3 = G_2G_4H_1H_3$ $L_1L_4 = G_2G_7H_1H_4$ $L_1L_5 = G_2G_8H_1H_5$ $L_1L_6 = G_2G_9H_1H_6$ $L_2L_4 = G_3G_7H_2H_4$ $L_2L_5 = G_3G_8H_2H_5$ $L_2L_6 = G_3G_9H_2H_9$ $L_3L_4 = G_4G_7H_3H_4$ $L_3L_5 = G_4G_8H_3H_5$ $L_3L_6 = G_4G_9H_3H_6$ $L_4L_6 = G_7G_9H_4H_6$	$L_1L_3L_4 = -G_2G_4G_7H_1H_3H_4$ $L_1L_3L_5 = -G_2G_4G_8H_1H_3H_5$ $L_1L_3L_6 = -G_2G_4G_9H_1H_3H_6$ $L_1L_4L_6 = -G_2G_7G_9H_1H_4H_6$ $L_2L_4L_6 = -G_3G_7G_9H_2H_4H_6$ $L_3L_4L_6 = -G_4G_7G_9H_3H_4H_6$ **Four non-touching Loops** $L_1L_3L_4L_6 = G_2G_4G_7G_9H_1H_3H_4H_6$

$$\Delta = 1 - \{L_1 + L_2 + L_3 + L_4 + L_5 + L_6\} + \{L_1L_3 + L_1L_4 + L_1L_5 + L_1L_6 + +L_2L_4 + L_2L_5 + L_2L_6 + L_3L_4 + L_3L_5 + L_3L_6 + L_4L_6\} - \{L_1L_3L_4 + L_1L_3L_5 + L_1L_3L_6 + L_1L_4L_6 + L_2L_4L_6 + L_3L_4L_6\} + \{L_1L_3L_4L_6\}$$

Example 9

Consider the control system whose signal flow graph is shown in Fig. (2). Determine the system transfer function using Mason's formula.



* There are **TWO** Forward Paths:

P1= G1G2G3G4G5

P2=G6

* There are **EIGHT** feedback loops:

L1= - G2H1

L2= - G3H2

L3= - G4H3

L4= - G2H4

L5= - G3H5

L6= - G4H6

L7= - G6H7

L8= - G1G2G3G4G5H7

* There are **TEN** two-non-touching feedback loops:

L1L3 = G2G4H1H3

L1L6 = G2G4H1H6

L1L7 = G2G6H1H7

L2L7 = G3G6H2H7

L3L4 = G2G4H3H4

L3L7 = G4G6H3H7

L4L6 = G2G4H4H6

L4L7 = G2G6H4H7

L5L7 = G3G6H5H7

L6L7 = G4G6H6H7

* There are **FOUR** three-non-touching feedback loops:

L1L3L7 = - G2G4G6 H1H3H7

L1L6L7 = - G2G4G6H1H6H7

L3L4L7 = - G2G4G6H3H4H7

L4L6L7 = - G2G4G6H4H6H7

Δ = 1 + {G2H1+G3H2+G4H3+G2H4+G3H5+G4H6+G6H7+G1G2G3G4G5H7}+{ G2G4H1H3+G2G4H1H6+G2G6H1H7+G3G6H2H7+G2G4H3H4+G4G6H3H7+ G2G4H4H6+G2G6H4H7+G3G6H5H7+G4G6H6H7}+{G2G4G6H1H3H7+ G2G4G6H1H6H7+G2G4G6H3H4H7+G2G4G6H4H6H7}

$\Delta 1 = 1$

$\Delta 2$ = 1 + {G2H1+G3H2+G4H3+G2H4+G3H5+G4H6}+{ G2G4H1H3+G2G4H1H6 +G2G4H3H4+ G2G4H4H6}

Using Mason's Formula, the system Transfer Function is:

$$\frac{Y(S)}{R(S)} = \frac{G1G2G3G4G5 + G6\{1 + \{G2H1 + G3H2 + G4H3 + G2H4 + G3H5 + G4H6\} + \{G2G4H1H3 + G2G4H1H6 + G2G4H3H4 + G2G4H4H6\}\}}{1 + \{G2H1 + G3H2 + G4H3 + G2H4 + G3H5 + G4H6 + G6H7 + G1G2G3G4G5H7\} + \{G2G4H1H3 + G2G4H1H6 + G2G6H1H7 + G3G6H2H7 + G2G4H3H4 + G4G6H3H7 + G2G4H4H6 + G2G6H4H7 + G3G6H5H7 + G4G6H6H7\} + \{G2G4G6H1H3H7 + G2G4G6H1H6H7 + G2G4G6H3H4H7 + G2G4G6H4H6H7\}}$$

Example 10

For the control system whose signal flow graph is shown below, using Mason's formula, find the system transfer function Y(s)/R(s).

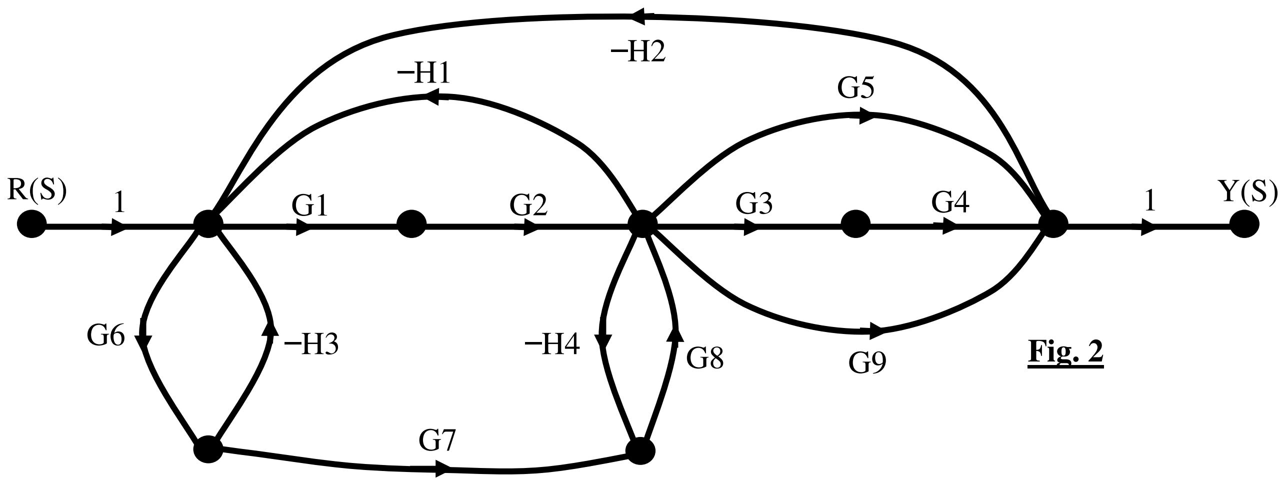


Fig. 2

Forward paths

$P_1 = G_1G_2G_3G_4$

$P_2 = G_1G_2G_5$

$P_3 = G_1G_2G_9$

$P_4 = G_6G_7G_8G_3G_4$

$P_5 = G_6G_7G_8G_5$

$P_6 = G_6G_7G_8G_9$

Feedback Loops:

$L_1 = -G_6H_3$

$L_2 = -G_8H_4$

$L_3 = -G_1G_2H_1$

$L_4 = -G_6G_7G_8H_1$

$L_5 = -G_1G_2G_3G_4H_2$

$L_6 = -G_1G_2G_5H_2$

$L_7 = -G_1G_2G_9H_2$

$L_8 = -G_6G_7G_8G_3G_4H_2$

$L_9 = -G_6G_7G_8G_5H_2$

$L_{10} = -G_6G_7G_8G_9H_2$

Two non-touching Feedback Loops:

$L_1L_2 = G_6G_8H_3H_4$

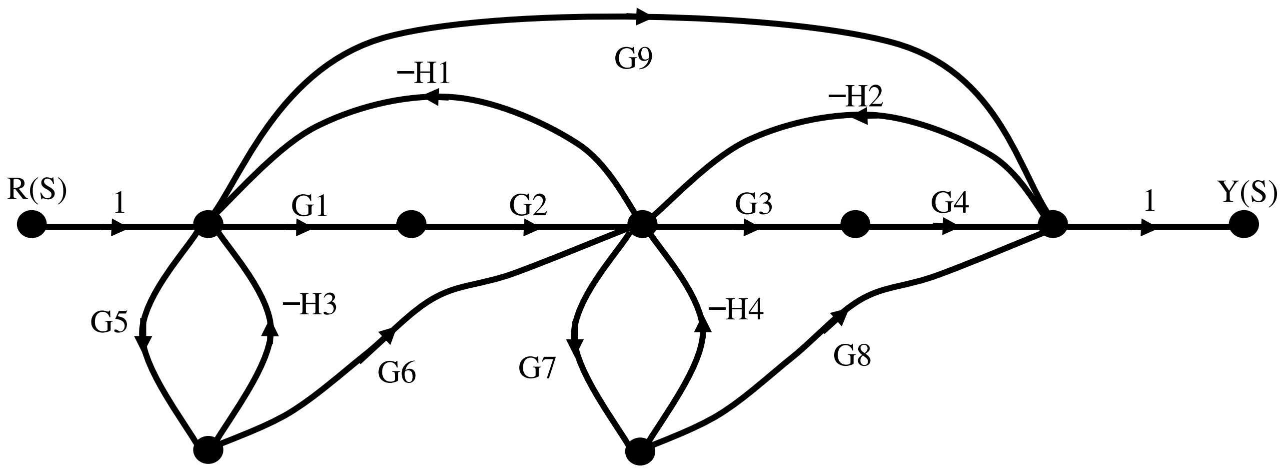
$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$

$\Delta = 1 + \{G_6H_3 + G_8H_4 + G_1G_2H_1 + G_6G_7G_8H_1 + G_1G_2G_3G_4H_2 + G_1G_2G_5H_2 + G_1G_2G_9H_2 + G_6G_7G_8G_3G_4H_2 + G_6G_7G_8G_5H_2 + G_6G_7G_8G_9H_2\} + G_6G_8H_3H_4$

$$\frac{Y(S)}{R(S)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6}{\Delta}$$

Example 11:

For the control system whose signal flow graph is shown below, using Mason's formula, find the system transfer function Y(s)/R(s).



Forward Paths:

P1 = G1 G2 G3 G4
P2 = G5 G6 G3 G4
P3 = G1 G2 G7 G8
P4 = G5 G6 G7 G8
P5 = G9

Feedback Loops

L1 = - G5 H3
L2 = - G7 H4
L3 = - G1 G2 H1
L4 = - G3 G4 H2

L5 = - G5 G6 H1
L6 = - G7 G8 H2
L7 =   G9 H2 H1

Two non-touching Loops
L1 L2 = G5 H3 G7 H4
L1 L4 = G5 H3 G3 G4 H2
L1 L6 = G5 H3 G7 G8 H2

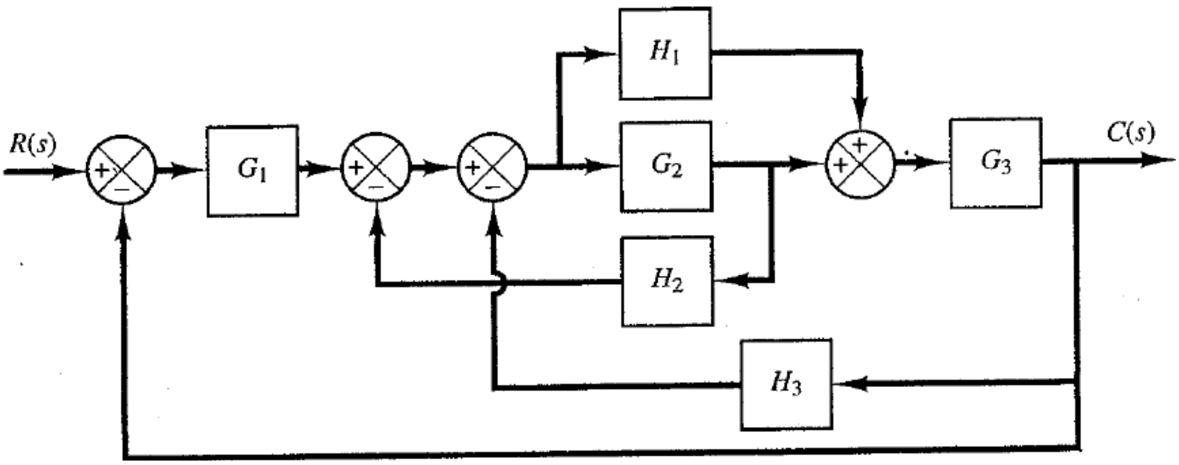
Δ1 = Δ2 = Δ3 = Δ4 = 1
Δ5 = 1 + G7 H4
Δ = 1 – {L1+L2+L3+L4+L5+L6+L7} + {L1 L2 + L1 L4 + L1 L6}

Using Mason's formula

$$\frac{C(S)}{R(S)} = \frac{P1\Delta1 + P2\Delta2 + P3\Delta3 + P4\Delta4 + P5\Delta5}{\Delta}$$

Report:

For the block diagram shown below,
a) Draw the corresponding signal flow graph
b) Using Mason's formula, obtain the system T.F. C(s)/R(s).



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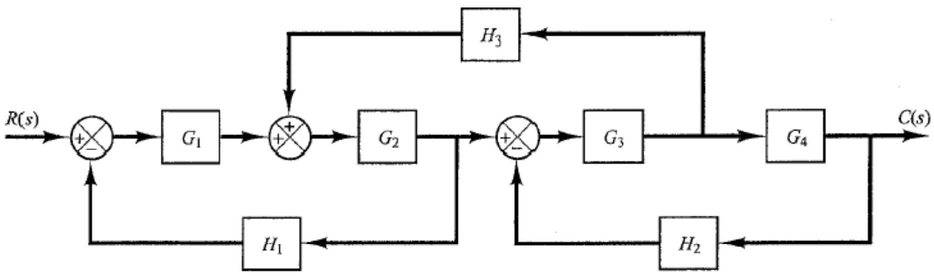
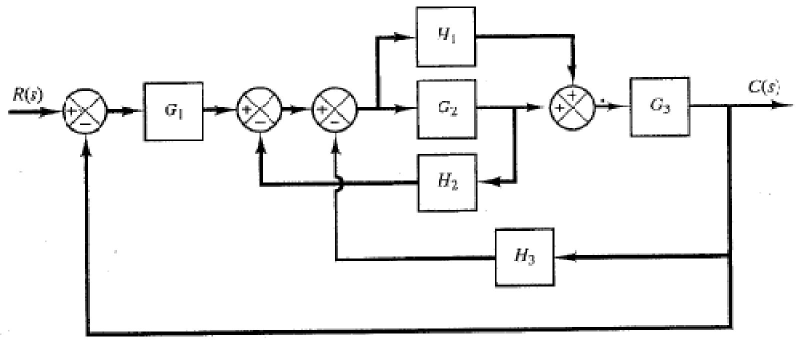
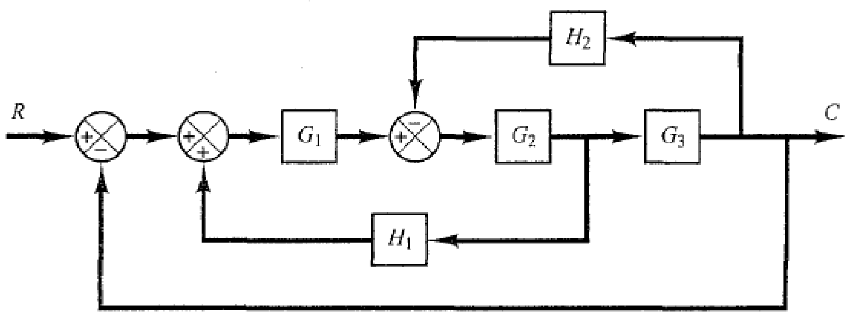
Sheet 3 (Signal Flow Graph)

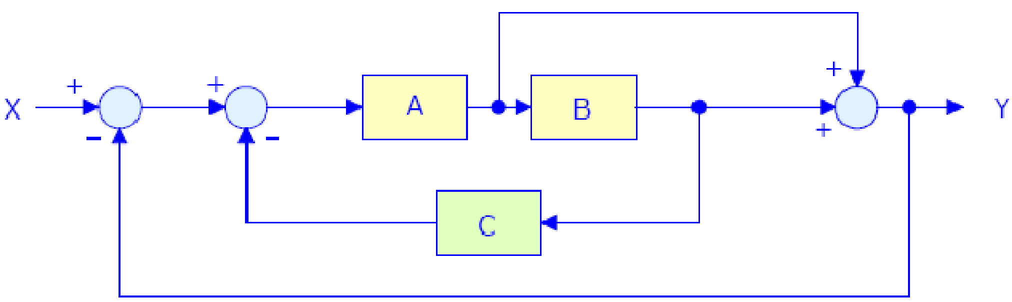
Problem #1

For the control systems represented by block diagrams shown in figure below, Draw the corresponding signal flow graph (SFG), then using Mason's rule to obtain the system transfer function.



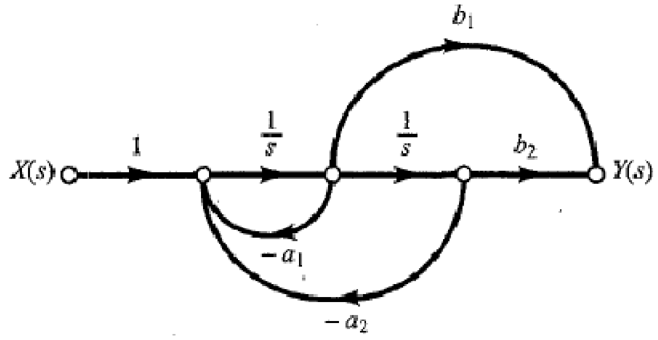


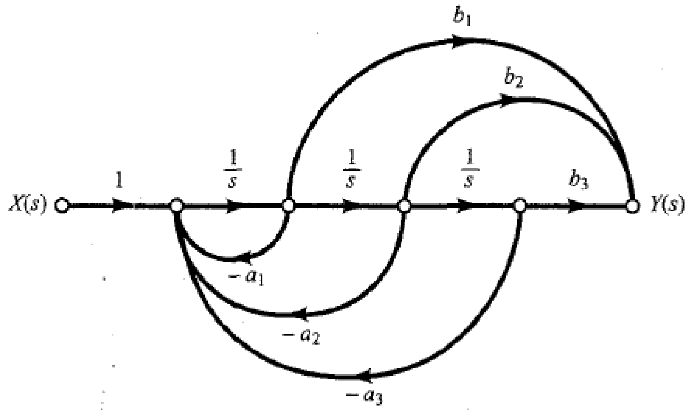


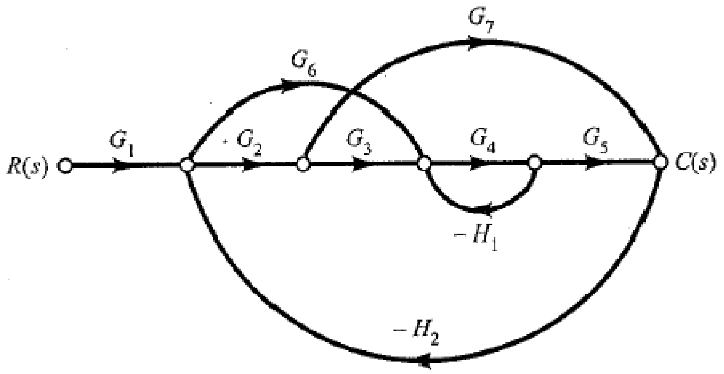


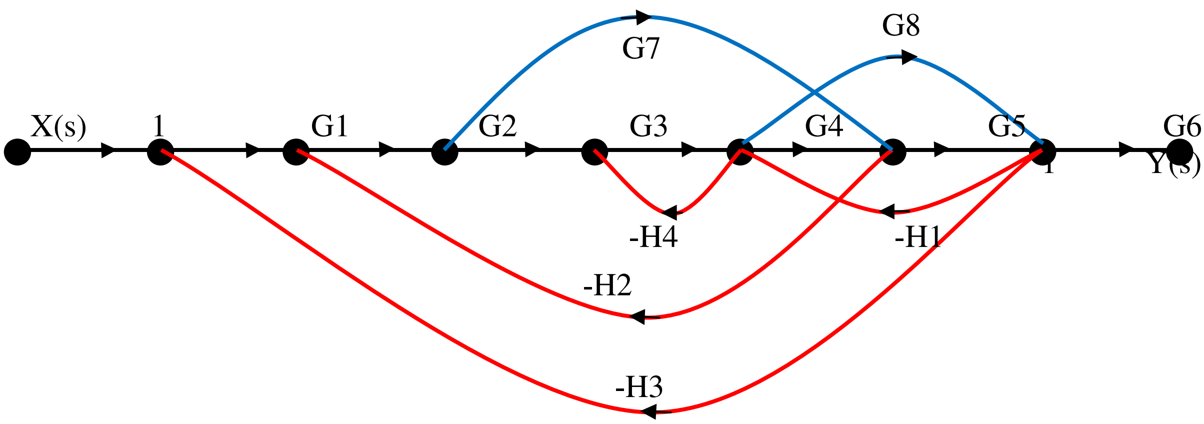
Problem #2

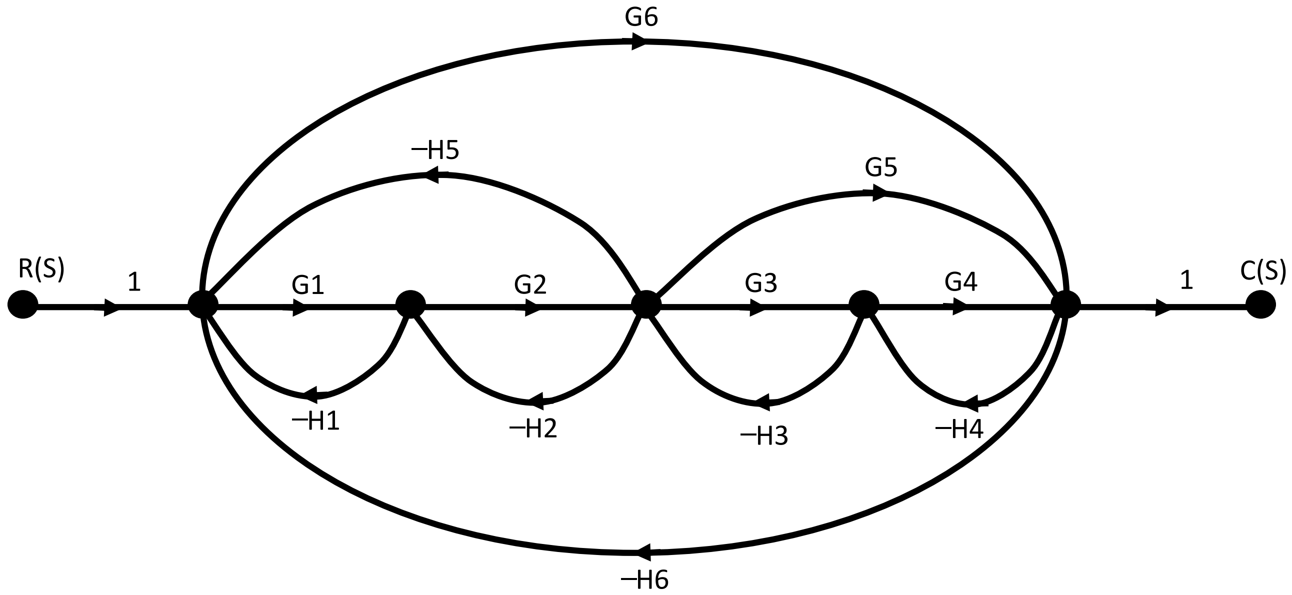
Using Mason's Rule, find the transfer function for the following SFG's

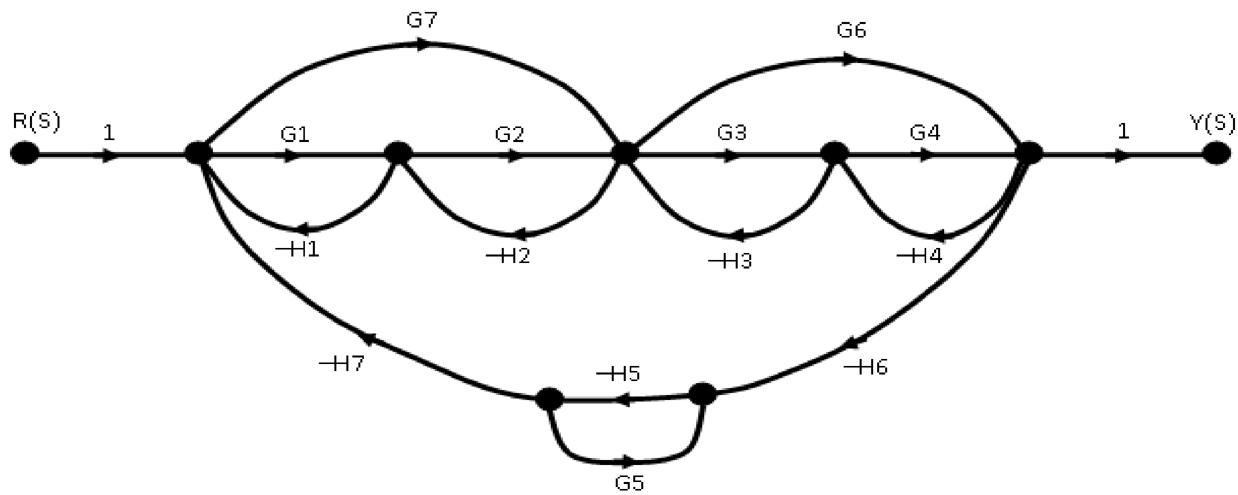


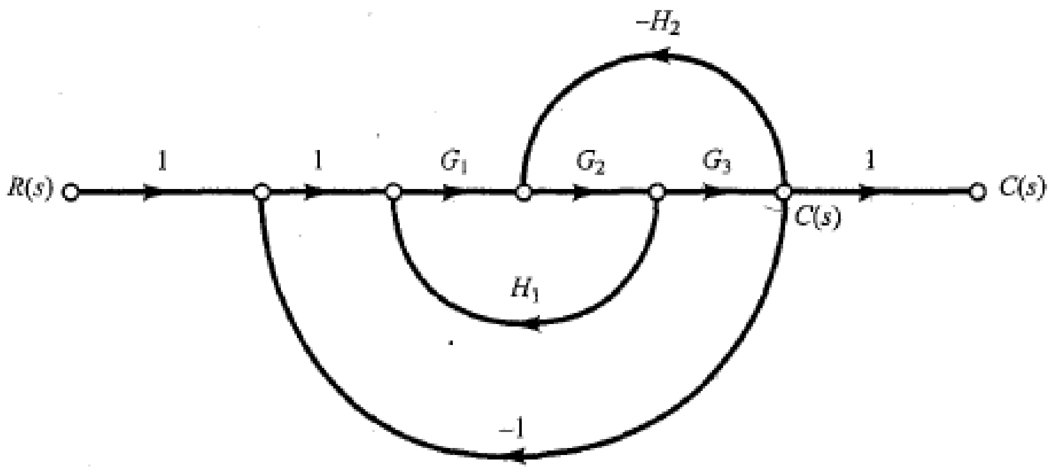


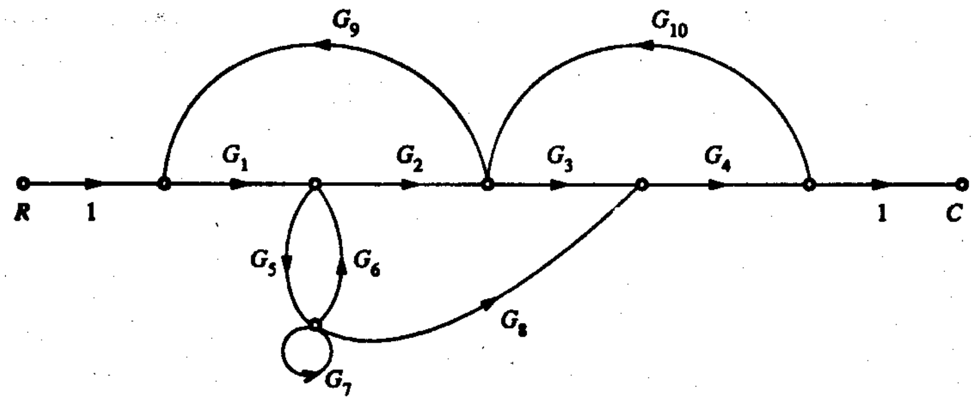




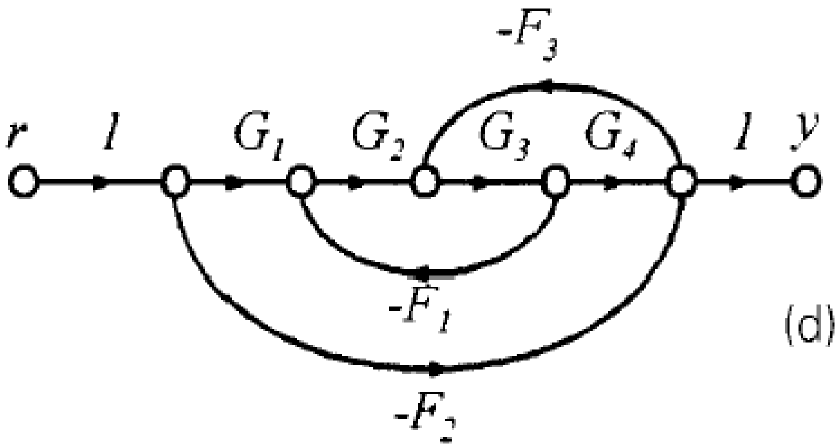

G6
−H5
G5
R(S)
1
G1
G2
G3
G4
1
C(S)
−H1
−H2
−H3
−H4
−H6


G7
G6
R(S)
1
G1
G2
G3
G4
1
Y(S)
−H1
−H2
−H3
−H4
−H7
−H5
−H6
G5

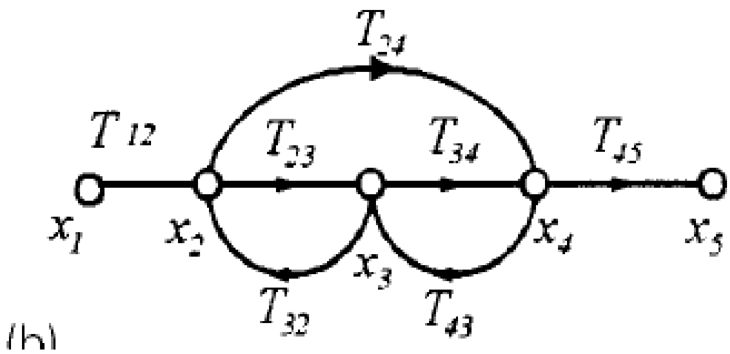

$-H_2$
$R(s)$
1
1
G_1
G_2
G_3
1
$C(s)$
$C(s)$
H_1
−1



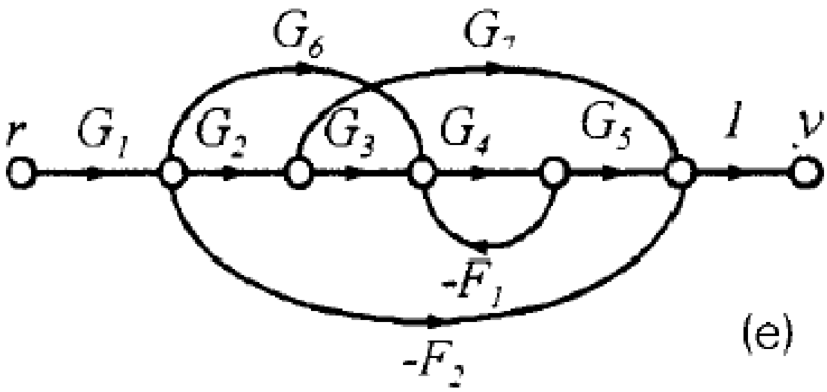
G9
G10
G1
G2
G3
G4
R
1
1
C
G5
G6
G8
G7



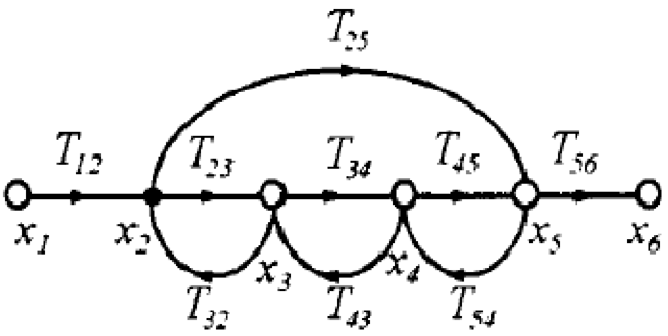
-F3
r
1
G1
G2
G3
G4
1
y
-F1
(d)
-F2

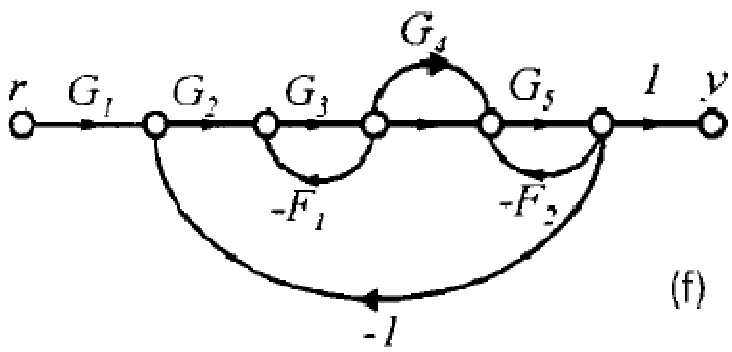


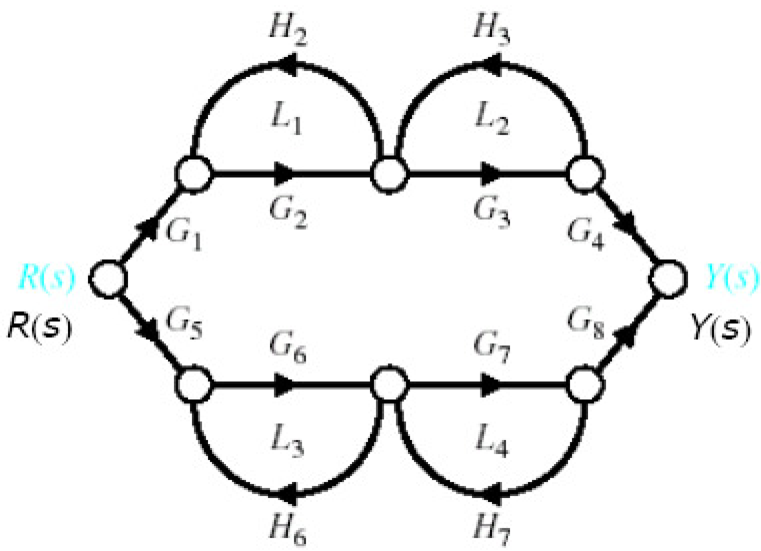
T24
T 12
T23
T34
T45
x1
x2
x3
x4
x5
T32
T43
(h)



G6
G7
r
G1
G2
G3
G4
G5
1
y
-F1
(e)
-F2


T_{25}
T_{12}
T_{23}
T_{34}
T_{45}
T_{56}
x_1
x_2
x_3
x_4
x_5
x_6
T_{32}
T_{43}
T_{54}


G_4
r
G_1
G_2
G_3
G_5
1
y
$-F_1$
$-F_2$
(f)
-1


H_2
H_3
L_1
L_2
G_2
G_3
G_1
G_4
R(s)
Y(s)
G_5
G_8
G_6
G_7
L_3
L_4
H_6
H_7