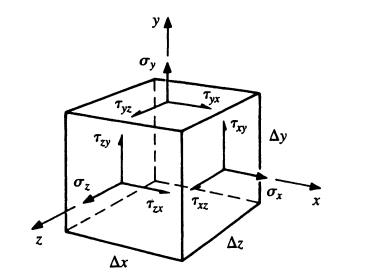
**Lecture No. 2**

**-Three dimensional stresses and Strains-**

**2-1 General.**

Consider a cube of infinitesimal dimensions shown in figure (1), All stresses acting on this cube are identified on the diagram. The subscripts (τ) are the shear stress, associate the stress with a plane perpendicular to a given axis, the second designate the direction of the stress, i.e.



τxy

Face Direction

The stress symbols in figure (1),

shows that three normal stresses: -

σx = τxx, σy = τyy , σz = τzz

and six shearing stresses, τxy , τxz ,

τyx , τyz , τzx , τzy. **Figure (1)**

The force vector (P) has only three components Px , Py and Pz.

And stress vector: -

= ..….2.1

This is a matrix representation of the stress tensor. It is a second –rank tensor requiring two indices to identify its elements or components. A vector is first- rank tensor, and scalar is a zero tensor.

Sometimes, for brevity, a stress tensor is written in identical natation as τij , where i, j and k designations x, y and z.

The stress tensor is symmetric, i.e. τij= τji  or

τxy= τyx

τxz= τzx …2.2

τyz= τzy

**2-2 Two dimensional stress (Biaxial stress): -**

For a two dimensional case of plane stress where (σ3=0).

Where:-

σy

= Normal stress

τxy

τ = Shear stress

σx

=

**2-3 Three dimensional stress (Triaxial stress): -**

There are nine components of stress. Moment equilibrium can be used to reduce the number of stress components to six.

y

τxy= τyx

x

σy

τxz= τzx

z

τyz= τzy

τyz

τyx

τxy

Stresses in 3D is represented by vector

τxz

σx

σx

tensor: -

σijk =

σy

A homogenous linear equation has a solution only if the determinant of the coefficient matrix is equal to zero this called Eigenvalue problem. Such as

= …2.3

In case that the equation is in eigenvalue problem such as: -

= 0 ...2.4

The determinant can be expanded to yield the equation

. ..2.5

Where I1, I2 and I3 are the first, second and third invariants of the Cauchy stress tensor.

..2.6

...2.7

...2.8

There are three roots the characteristic equation 2.5, σ1, σ2 and σ3. Each root is one of the principal stresses. The direction cosines can be found by substituting the principal stresses into the homogenous equation 2.3 and solving. The direction cosines define the principal direction or planes.

σ max ≥ σin ≥ σmin

σin

σ1≥ σ2 ≥ σ3

σmin

σ max

……2.9

..….2.10

=τmax ..….2.11

**2-4 Poisson’s Ratio (υ): -**

Biaxial and Triaxial deformation another type of elastic deformation is the change in transverse dimensions accompanying axial tension or compression.

Experiments show that if a bar is lengthened by axial tension, there is a reduction in the transverse dimensions. Simeon D. Poisson showed in 1811 that the ratio of the unit dimensions or strain in these directions is constant for stresses within the proportional limit,

**Poisson’s Ratio (υ) =** …2.12

**=**

**Where: -**

.

.

.

Note: - Minus sign indicates a decrease in transverse dimension position as in the case of tensile elongation.

δd

d

δL

**2-5 Three Dimensional Strains: -**

For Tri-axial stress state,

If

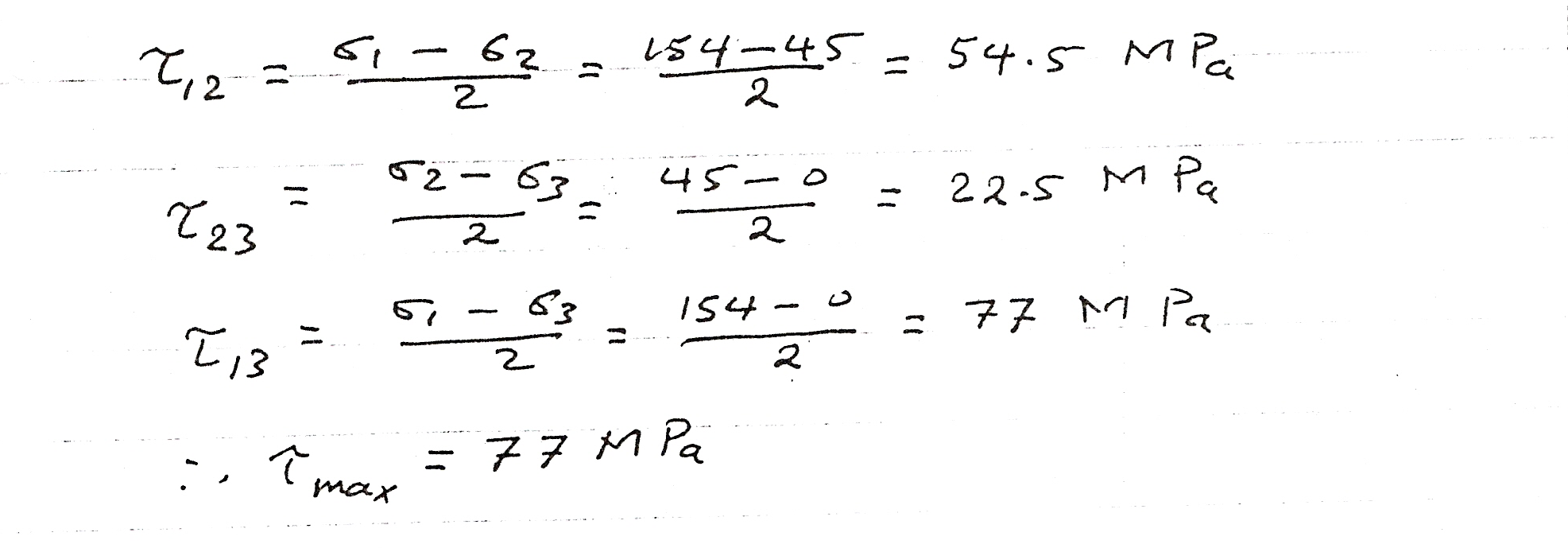
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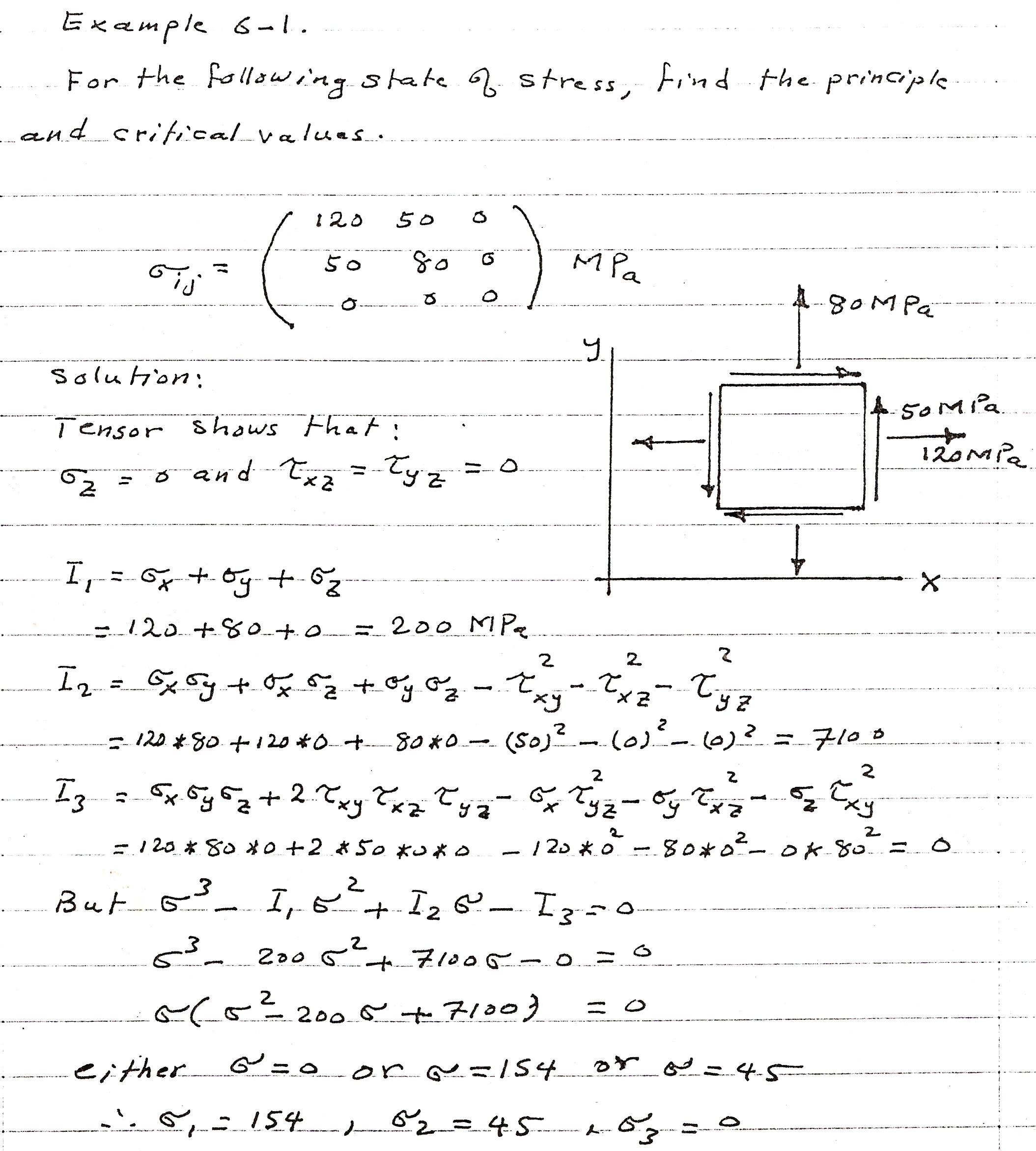
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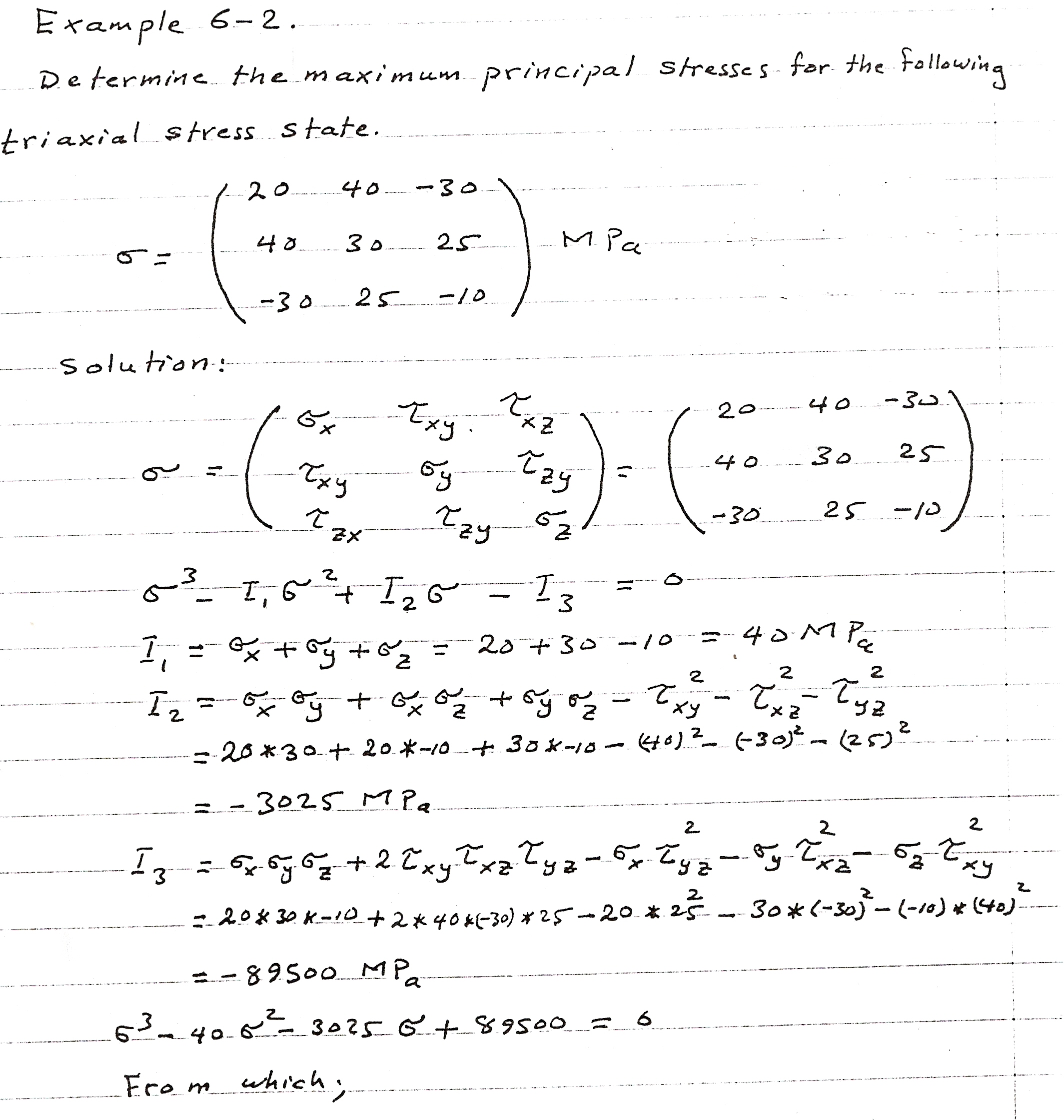
…2.13

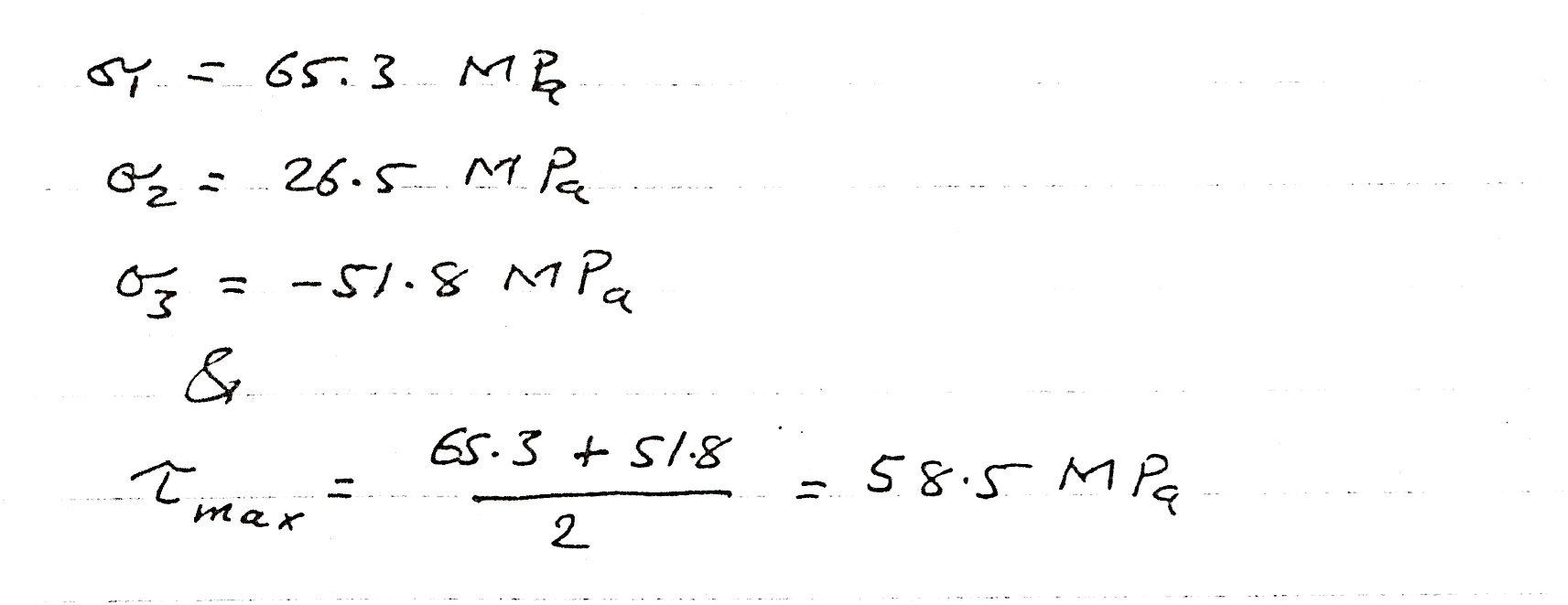
So, the principal strains in terms of principal stresses,

…2.14









…………………………….End…………………………