

Q1/ Determine whether the following series converge or diverge:

$$\sum_{n=1}^{\infty} \frac{n^2(-2)^n}{(n+2)!}$$

$$a_n = \frac{n^2(-2)^n}{(n+2)!}$$

$$a_{n+1} = \frac{(n+1)^2(-2)^{n+1}}{(n+3)!} = \frac{(n+1)^2(-2)^n \cdot (-2)}{(n+3)(n+2)!}$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)^2(-2)^n(-2)}{(n+3)(n+2)!} \cdot \frac{(n+2)!}{n^2(-2)^n} \right|$$

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$$= \lim_{n \rightarrow \infty} \left| \frac{(-2)(n+1)^2}{n^3 + 3n^2} \right| = \lim_{n \rightarrow \infty} \frac{2n^2 + 4n + 2}{n^3 + 3n^2} = 0$$

$0 < 1 \implies$ The series converges absolutely by ratio test

Q2/ Determine the interval and center of convergence for power series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{\sqrt{n+1}}$$

$$a_n (x-c)^n = \frac{(-1)^n (x-2)^n}{\sqrt{n+1}}$$

$$a_{n+1} (x-c)^{n+1} = \frac{(-1)^{n+1} (x-2)^{n+1}}{\sqrt{n+2}} = \frac{(-1)^n (-1) (x-2)^n (x-2)}{\sqrt{n+2}}$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_n (x-c)^n}{a_{n+1} (x-c)^{n+1}} \right| = \lim_{n \rightarrow \infty} \left| \frac{(-1)^n (-1) (x-2)^n (x-2)}{\sqrt{n+2}} \cdot \frac{\sqrt{n+1}}{(-1)^n (x-2)^n} \right|$$

$$= \lim_{n \rightarrow \infty} \left| (-1)(x-2) \sqrt{\frac{1+\frac{1}{n}}{1+\frac{2}{n}}} \right|$$

$$= |x-2| \left(\sqrt{\frac{1+\frac{1}{\infty}}{1+\frac{2}{\infty}}} - 1 \right)$$

$$-1 < x-2 < 1$$

$$1 < x < 3$$

if $x=3$

$$\sum_{n=1}^{\infty} \frac{(-1)^n (3-2)^n}{\sqrt{n+1}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$$

alternative series

- ① all a_n 's positive
② $|a_n| \geq |a_{n+1}|$
③ $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1}} = 0$
- } ∞ Converges

if $x = 1$

$$\sum_{n=1}^{\infty} \frac{(-1)^n (-1)^n}{\sqrt{n+1}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1}}$$

let $b_n = \frac{1}{\sqrt{n}} = \frac{1}{n^{\frac{1}{2}}}$ p-series
with $p = \frac{1}{2} < 1$
 ∞ Diverges

∞ $b_n > a_n$

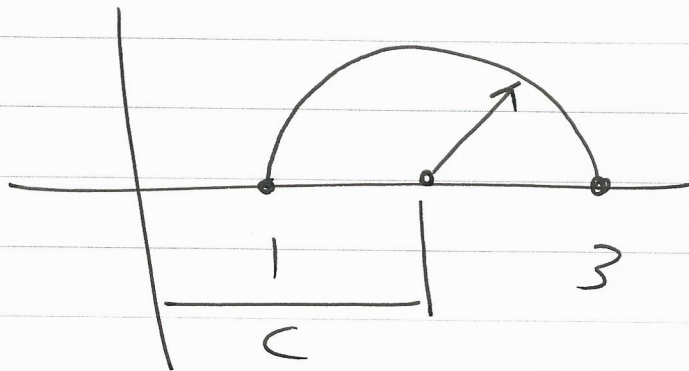
b_n Diverges we must use LCT

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{\frac{1}{\sqrt{n+1}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \sqrt{\frac{n}{n+1}} = 1$$

∞ both converges or both
Diverges

∞ a_n Diverges

* we can use integral test also to prove
 a_n Diverges.

$(1, 3]$ 

$$r = 1$$

$$c = 2$$