Evaporation التبخر

Evaporation: is the process in which a liquid changes to the gaseous state (vapor), below the boiling point through the transfere of heat energy.

هو العملية التي يتحول فيها السائل الى الحالة الغازية (بخار) عند السطح الحر له قبل نقطة الغليان من خلال انتقال الطاقة الحرارية .

The heat absorbed by a unit mass of Liquid required to change it to vapor state, is called "Latent heat of evaporation"

الطاقة الحرارية الممتصة من سائل بغية تحويله الى بخار تدعى «الحرارة الكامنة للتبخر» ويعبر عنها بالمعادلة

$$L_h = 597.3 - 0.564\, T_{c} \quad \text{-------------} (1)$$

حيث T بدرجات الحرارة السيليزية ، و L_h مقاسة (cal/gr).

لاحظ انه بإمكانك تحويلها الى (KJ/gr):

$$L_h = 2.50 - 0.00236\, T_{c}$$

حيث ان كل سعرة (cal) تعادل 4.186 جول / أو :

$$\boxed{Cal = 4.186\ J}$$

(4.186) هو مكافئ جول الميكانيكي

The rate of evaporation (معدل التبخر) is dependent on:

(i) The vapor pressure at the water and air above. يعتمد على الفرق بينهما
(ii) Air & water temperatures. زيادة درجة الحرارة ترفع من الضغط البخاري المشبع
(iii) Wind speed. يزداد مع زيادة الريح
(iv) atmospheric pressure. انخفاضه لصالحه زيادة في الضغط البخاري
(v) water quality (dissolved salt). اذابة الاملاح تقلل الضغط البخاري ويقل التبخر تبعاً لذلك.
(vi) Heat storage in water body. الخزين الحراري في الشتاء والصيف يؤثر على التبخر

Evaporation Estimation تقدير التبخر

The estimation of evaporation is of utmost important in many Hydrologic problems associated with planning and operation of reservoirs and irrigation systems.

يعد تقدير التبخر غاية في الأهمية لمسائل الهيدرولوجية المتعلقة بتخطيط وتشغيل السدود وأنظمة الري .

The estimation of evaporation can be carried out by :

(1) measurement by evaporimeter . القياس بأجهزة القياس

(2) Empirical evaporation equations . التخمين باستخدام معادلات تجريبية

(3) Analytical methods . الطرق التحليلية

(1) Evaporimeters القياس باستخدام مقاييس التبخر

All evaporimeters are of type of Pan evaporation;

1) Class A Pan evaporation ($\phi = 121\,cm$, $d = 25.5\,cm$) A الإناء صنف

2) Colorado Sunken Pan evaporation إناء كلورادو الغاطس

3) U.S.G Pan (Floating Pan evaporation) الوعاء الطوّاف

Evaporation pans are not exact models for large reservoirs, because:

(i) They differ in the heat-storing capacity and heat-transfer from the side and bottom.

(ii) The hight of the rim in the pan affects the wind action.

(iii) The heat tranfer is different in Pan & reservoir due to Pan material - and thus;

$$Evap)_{Lake} = Evap)_{Pan} * C_p \quad \text{—— (2)}$$

C_p = معامل إناء التبخر

C_p = coefficient of Pan ranged from 0.6 – 0.8 for class A.

The evaporation pans are usually installed in such location according to World Meteorological Organization (WMO) recommendation:

1. Arid Zones - on every 30 000 km^2 — المناطق الجافة
2. Humid temperate climate - one every 50 000 km^2 — المناطق المعتدلة الرطبة
3. Cold regions - One every 100 000 km^2 - المناطق الباردة

(2) Empirical equations for evaporation — معادلات التبخر التجريبية

A Large number of empirical equations are available, but most of these equations are based on Dalton-type equation.

John Dalton 1802 stated that

$$\boxed{E_L = C\,(e_s - e_a)}$$

E_L = Lake evaporation in (mm/day)
C = Coefficient
e_a = actual vapor pressure of air above water surface at specified height, in (mm Hg).
e_s = saturated vapor pressure at the water surface temperature, in (mm/day).

This basic equation was modified many times and finally takes the following form;

$$\underset{mm/day}{E_L} = K\,f(u)\,(\underset{mm\,Hg}{e_s} - \underset{mm\,Hg}{e_a}) \quad \text{------------} (3)$$

K = Coefficient
$f(u)$ = Wind speed correction function (Dimensionless)

Meyer Formula (1915)

$$\underset{mm/day}{E_L} = K_M \, (e_s - e_a)\left(1 + \frac{u_9}{16}\right) \text{ ------ } (4)$$

E_L, e_s & e_a as defined previously. $K_M = 0.36$ for deep lake; 0.5 for shallow water.
u_9 = wind velocity (monthly mean) in (Km/hr) measured at height 9 m above ground surface.
If the wind velocity was known at z_o, one can estimates a wind velocity at a specified height z, by using.

$$\frac{u}{u_o} = \left(\frac{z}{z_o}\right)^K \text{ ---------- } (5)$$

K; Power Law coefficient varied from $0.1 \rightarrow 0.6$.
Use $K = 1/7$, for natural ground roughess

Rohwer Formula (1931)

$K = 0.771(1.465 - 7.3\times10^{-4} P_a) \approx 0.70$ for $P_a = 760$ mm Hg
$f(u) = (0.44 + 0.0733\, u_o)$

$$E_L = K\, f(u)\, (e_s - e_a) \text{ ---------- } (6)$$

P_a = mean barometric pressure in (mm Hg).
u_o = mean wind velocity in (Km/hr) at ground level which can be taken at 0.6 m above ground surface.

These empirical equations are simple to use and permit the use of standard meteorological data. However, the various limitation of these formulae, they can, at best, give an approximate values of evaporation.

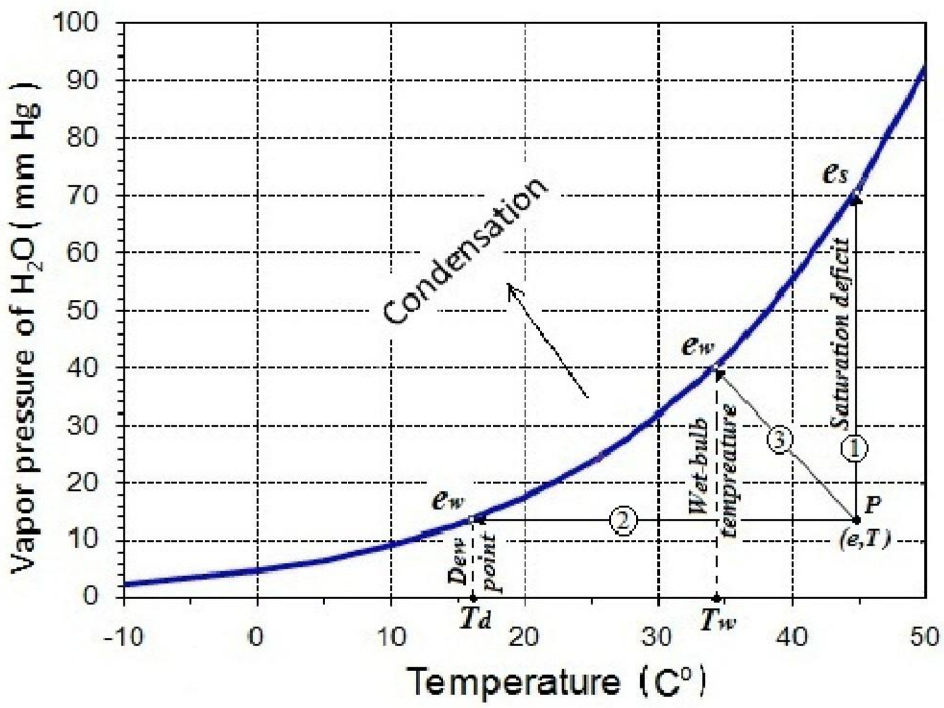
هذه المعادلات التجريبية بسيطة الاستخدام وتسمح باستخدام المعلومات
المناخية القياسية، لكن المحددات المتعددة لهذه المعادلات جعلها تعطي
نتائج تقريبية مقبولة في أحسن الأحوال .

لغرض تطبيق المعادلات آنفة الذكر يجب معرفة قيم e_s عند أية درجة حرارة

In using empirical equations, the saturated vapor pressure at a given temperature should be known. (e_s),

<u>Relation between $e_s - T$</u> <u>علاقة الضغط البخاري المشبع ودرجة الحرارة</u>

suppose an evaporating surface of water in a closed system and enveloped in air.



evaporating of water into air will take place untill a state of equilibrium is reached when $e_a = e_s$ and no more absorbtion occurs.
حالة التوازن حين يكون الهواء مشبعاً ببخار الماء ويسلط ضغطاً ثابتاً يدعى ضغط البخار المشبع ويتغير بتغير درجة الحرارة كما موضح بالشكل

1. since P lies below saturation vapor pressure curve, the air can absorb more water vapor, and if it did so, the position P move vertically (path 1) until e_s curve intersection, with no change in temperature.
2. If no change in a humidity of air, while it was cooled, the P move horizontally along path 2 until saturated line was intersected. Now, e_s will takes place at new temprature td called dew-point. Cooling beyond this temperature resulted condensation.
3. if water allowed to evaporate freely, P will move diagonally along path 3 until saturated vapor pressure curve is reached at point defined by (e_w, tw). tw called wet-bulb temp, which is an original air temperature that can be cooled by evaporating water into it. It can be found by bulb thermometer.

Relative humiedity (hr) الرطوبة النسبية

وتمثل كمية الرطوبة في الهواء الى كمية الرطوبة في حالة الاشباع ويعبر عنها :-

$$\% hr = \frac{e_a}{e_s} \times 100 \quad \text{------- (7)}$$

يمكن قياس e_a باستخدام المحرار ذو البصلة الرطبة ويسمى جهاز قياس الرطوبة "المرطاب" (Psycrometer) حيث يبين الأول درجة حرارة للبصلة t_w (wet bulb temp.) وهي الدرجة التي يصل اليها المحرار عند تبريده بواسطة تبخر الماء.

$$\frac{(\overset{mmHg}{e_w} - \overset{mmHg}{e_a})}{(\underset{C^\circ}{t_w} - \underset{C^\circ}{t_a})} = \gamma \quad \text{---------- (8)}$$

e_w = saturated vapor pressure corresponding to wet-bulb temperature

t_w = wet-bulb temperature

t_a = dry-bulb temprature

e_a = Corresponding air vapor pressure to "t_a".

γ = Psycrometer Constant ثابت المرطاب

$$\gamma = 6.5 \times 10^{-4}\ Pa / c^\circ \quad \text{------- (9)}$$

$\gamma = 0.49\ mmHg / c^\circ$

$\gamma = 0.66\ mb / c^\circ$

$\gamma = 0.066\ KPa / c^\circ$

Pa = atmospheric pressure.

ملاحظة

1. e_w يحسب من معادلات المنحني الذي سيتم لاحقاً
2. t_w تقاس من المرطاب
3. t حرارة الهواء الآني من المحرار
4. e_a يحسب بمعرفة قيمة γ

Note that:

$mb = \frac{1}{10}\ KPa \Rightarrow mb = 100\ Pa \Rightarrow Pa = 10^{-5}\ bar$

$mb = \frac{3}{4}\ mmHg \Rightarrow mmHg = 4/3\ mb$

$mmHg = \frac{2}{15}\ Kpa \Rightarrow Kpa = 7.5\ mmHg$

How to calculate e_s as a function of T !

After reviewing the variation of e_s with T, an attempt was made to introduce equation that can be calculated e_s and its gradient as a function of T.

$$e_s = 4.58\, e^{\left(\frac{17.27\,T}{237.3+T}\right)} \qquad (10)$$

$$\Delta = \frac{de_s}{dT} = \frac{4098\, e_w}{(237.3+T)^2} \qquad (11)$$

e_s in mm Hg , Δ in mm Hg/ C°.

If e_s required to be in Pa instead of mm Hg, then the factor 4.58 should be replaced by 611, whereas Δ remains as it is.

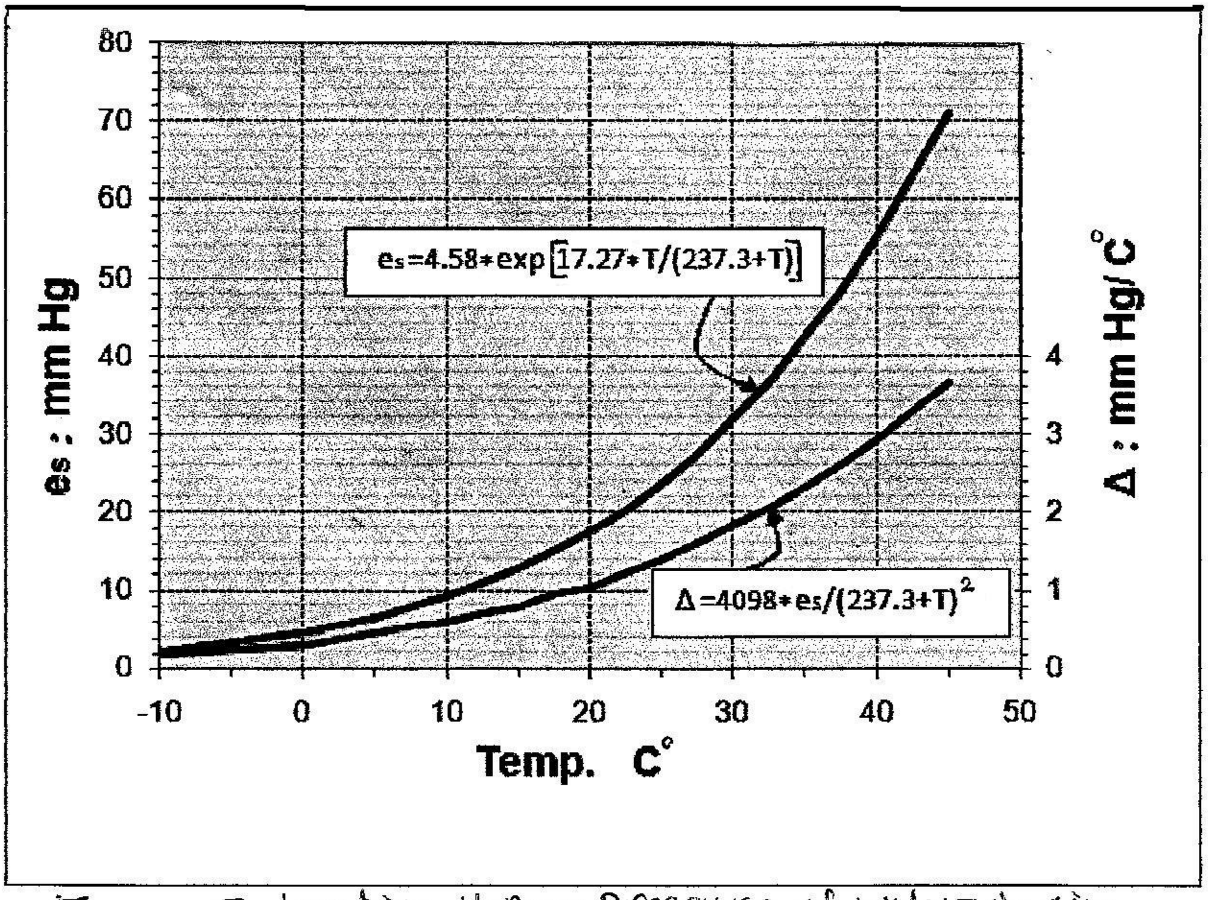


Fig : Saturation Vapor pressure of water in air as a function of temperature

Example

A Lake of 250 hectares surface area, had the following average values of climate parameters during a week of measurements,

- relative humidity = 40%
- wind velocity at 1.0 m above ground surface = 16 km/hr.
- Pan evaporation = 72 mm in the week.
- Pan coefficient = 0.8
- Temperature = 20 c°

ⓐ estimate average dialy evaporation from Lake using Meyer formula.
ⓑ estimate accuracy of this method compared with pan evaporation.
ⓒ estimate the volume of water evaporated from lake in that week.

Solution:

$$\boxed{hec = 10\,000\ m^2}$$

ⓐ from eq (10); $e_s = 4.58\, e^{\left(\frac{17.27(20)}{237.3+20}\right)} = 17.53$ mmHg.

From eq. (7);

$e_a = hr * e_s = 0.4\,(17.53) = 7.01$ mm Hg.

From eq. (5);

$$\frac{u_9}{u_1} = \left(\frac{9}{1}\right)^{1/7} \Rightarrow u_9 = 16 * 9^{1/7} = 21.9 \text{ km/hr}$$

From eq (4); $K_M = 0.36$ since the Lake is large.

$$E_L = 0.36\,(17.53 - 7.01)\left(1 + \frac{21.9}{16}\right) = \underline{8.97} \text{ mm/day}$$

ⓑ Daily evaporation from Pan evaporimeter $= \frac{72}{7} = 10.28$ mm/day

From eq (2)

$$E_{actual} = 10.28 * 0.8 = 8.23 \text{ mm/day}$$

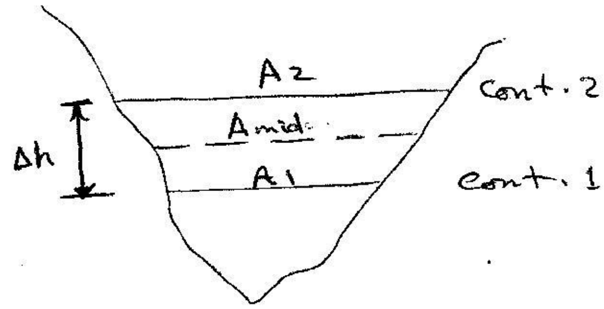
$$\%E = \frac{|actual - Calculated|}{actual} \times 100 = \frac{|8.23 - 8.97|}{8.23} \times 100 = \underline{9\%}.$$

ⓒ Volume of loss by evaporation $= \frac{8.23}{1000} * 7 \times 250 \times 10\,000 = 144{,}025\ m^3$

or $= 0.8 * \frac{72}{1000} * 250 * 10\,000 = 144{,}000\ m^3$

Why the two answers are different?

Note that the surface area of the Lake is assumed to be constant. Is it real??



$\Delta S =$ Volume change $= \Delta \forall$

$$\Delta \forall = \Delta h * \bar{A}$$

There are more than rule to calculate $\bar{A} = f(A_1, A_2)$

① for uniform shape;

$$\bar{A} = \frac{A_1 + A_2}{2}$$

② $\bar{A} = \dfrac{A_1 + A_2 + \sqrt{A_1 A_2}}{3}$ Cone formula

③ $\bar{A} = \dfrac{A_1 + A_2 + 4A_{mid}}{6}$ Prismoidal formula

A_m: is midway area between two successive Contours.

The practical formula for use is the Cone formula

$$\Delta S = \frac{h}{3}\left(A_1 + A_2 + \sqrt{A_1 A_2}\right)$$

(3) Analytical methods of estimation الطرق التحليلية لتقدير التبخر

The Lake evaporation can be estimated using analytical methods which may be classifred into three categories;

(i) water-budget الموازنة المائية
(ii) Energy-balance method موازنة الطاقة
(iii) Mass-transfer method انتقال الكتلة

(i) Water Budget method

It is the simplest method of the three analytical methods, but it has a least reliable. It involves writing hydrological continuity equation for lake and determining evaporation from knowledge of other variables.

أقل اعتمادا

$$\forall_P + \forall_{Qi} - \forall_{Qo} + \forall_{gi} - \forall_{go} - \forall_E - \forall_T = \Delta S \quad \text{-----} (12)$$

$\forall_P$ = Volume of daily precipitation over the Lake.
$\forall_{Qi}$ = Volume of daily inflow into lake.
$\forall_{Qo}$ = Volume of daily outflow from the lake.
$\forall_{gi}$ = Volume of daily groundwater inflow.
$\forall_{go}$ = Volume of daily groundwater outflow (seepage).
$\forall_E$ = Volume of daily Lake evaporation
$\forall_T$ = Volume of daily transpiration.
ΔS = increase in volume of storage in day.
All units are in m^3 or (m) depth over surface area of Lake.

تبخر مطر جريان مياه جوفية نتح زيادة خزين

$$\Rightarrow E = P + (Q_i - Q_o) + (G_i - G_o) - T_L - \Delta S \quad \text{---} (13)$$

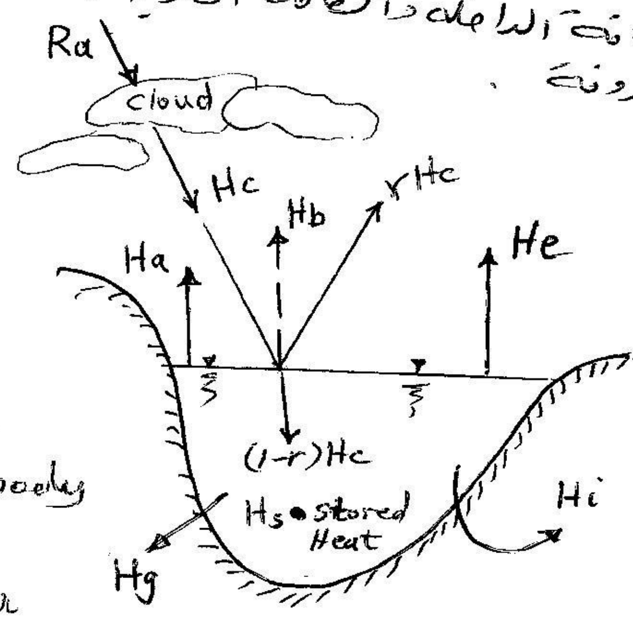
All parameters must be had same unit, if not any different unit of such parameter should be converted.

Note that, P, Q_i, Q_o, Δs can be measured while G_i, G_o, T_L were estimated, T_L can be neglected in some Lakes.

E_L can be well-estimated from eq(13), if the unit of time is kept large, say weeks, months. يمكن الحصول على تقدير جيد للتبخر اذا كانت فترة الزمن كبيرة كأن تكون أسابيع أو أشهر عدا ذلك في الفهم فإن هذه الطريقة غير مرجحة بسبب أن البيانات أحياناً قد تعطي نتائج مقبولة.

(ii) Energy Budget method طريقة موازنة الطاقة

It is application of Law of conservation of energy. The energy available for evaporation is determined by considering incoming energy, outcoming energy and energy stored in the water body over known time interval. الطريقة هي تطبيق لقانون حفظ الطاقة، فالطاقة المتوفرة للتبخر تحدد بموجب الطاقة الداخلة والطاقة الخارجة والطاقة المخزونة في الكتلة المائية خلال فترة زمنية معروفة.



Fig; Energy Balance in water Body.

R_a = incident solar radiation outside atmosphere, it is a function of the latitude and period of the year, see table in next pages.

H_c = incident solar radiation reached the water surface.

H_b = back radiation from water body (long wave)

H_a = sensible heat transfer from water surface to air (المحسوسة)

H_e = heat energy used up in evaporation.
$= \underset{gr/cm^3}{\rho} \; \underset{cal/gr}{L_h} \; \underset{cm \text{ or } mm}{E_L} \Rightarrow cal/cm^2$

H_g = Heat flux into ground

H_i = net heat conducted <u>out</u> of the system (advected energy)
الطاقة المنتقلة أفقياً بالجريان تمثل صافي حرارة التوصيل الخارجي .

H_s = heat stored in the system (water body)

and thus ;

H_n = Net heat energy received by water surface

$$\left.\begin{array}{l} H_n = H_s + H_i + H_g + H_a + H_e \\ \text{also } H_n = (1-r)\,H_c - H_b \end{array}\right\} \text{———} \dots (13)$$

where r = Coeficient of reflection (albedo)
$r = 0.05$ ~~from~~ water surface.

All terms of energy-Balance method in ($Cal/cm^2 . day$). If time interval is short, (H_s, H_i) can be neglected, since they are small.
All terms except H_a can be measured or evaluated.
كل الحدود عدا Ha يمكن قياسها أو حسابها بشكل مباشر ، ولذا يتم التعويض عن Ha بالمصطلح الآتي ;

$\beta = \dfrac{H_a}{H_e}$; β = Bowen's ration ——— (14)

where $$\beta = \gamma \, \frac{T_o - T_a}{e_s - e_a} \text{ ———} (15)$$

T_o = temperature of water surface in C°.
T_a = temperature of air in c°
e_s = saturated vapor pressure in mmHg at T_o
e_a = actual vapor pressure in mmHg at T_a
γ = constant estimated by eq (9) ≈ 0.49 mmHg/c°

Hence ;
by using: $\beta = \dfrac{H_a}{\rho L_h E_L}$ and From eq (13);

$$E_L = \frac{H_n - H_g - H_s - H_i}{\rho L_h (1+\beta)} \text{ ———} (16)$$

It was found that the evaporation in a lake estimated by energy-Budget gives satisfactory result with errors of order 5% if the budget was applied to period less than week.

(iii) Mass transfer method طريقة انتقال الكتلة

This method is based on the concept of turbulent transfer of water vapor mass from an evaporating surface to the atmosphere. It asumes that the diffusion of vapor pressure is analoge to temperature distribution. The equaton similar to Dalton form can be adopted as;

$$E_L = \alpha\,(e_s - e_a) \quad \text{-----} (17)$$

where α is vapor transfer coefficient, and

$$\alpha = \frac{0.102\,U_a}{\left[\ln\left(\frac{Z_a}{Z_0}\right)\right]^2}$$

; Z_o (cm) = roughness height of surface (0.01 – 0.06) cm

Z_a (cm) = elevation corresponding to velocity U_a (m/s)

Any other details is out of the current study.

Methods of Reduce evaporation LOSSES: تقليل خسائر التبخر

(1) reduce surface area. تقليل المساحة السطحية

(2) use mechanical covers. سقوف عائمة خفيفة الوزن

(3) use chemical films. الاغشية الكيمياوية

hexadecanol
octadecanol } use these (سيتايل الكحول وستريل الكحول)

a) strong and flexible قوي ومرن

b) If punctured due to impact, the film closes back soon after. يلتئم بسرعة اذا ثقب

c) colourless, odourless, and nontoxic غير سام عديم اللون والرائحة

Evapotranspiration التبخر الكلي

النتح

Transpiration: is the process by which water leaves the body of a living plant and reach the atmospher as a water vapor. هو العملية التي يغادر فيها الماء جسم النبات الحي كبخار ماء الى الجو

factors affecting the transpiration are; e_a, T, u, light intensity and type of the plant (root and leaf system).
الريح

العوامل التي تؤثر على التبخر الحر هي نفسها تؤثر على النبات ما .

The transpiration is restricted to daylight hours and its rate depends on the growth period of the plant. while the vaporation occurs during day and night with differed rate.

Evapotranspiration takes place from moist land and water body عندما يحصل النتح فان الاراضي التي تحتوي على النباتات تفقد رطوبتها أيضاً فضلاً عن التبخر من الاجسام المائية ويسمى هنا التبخر الكلي . ويسمى في هندسة الري "المقنن المائي" Consumptive use.

Potential evapotranspiration (PET) التبخر الكلي الكامن : occure when soil moisture at field capacity

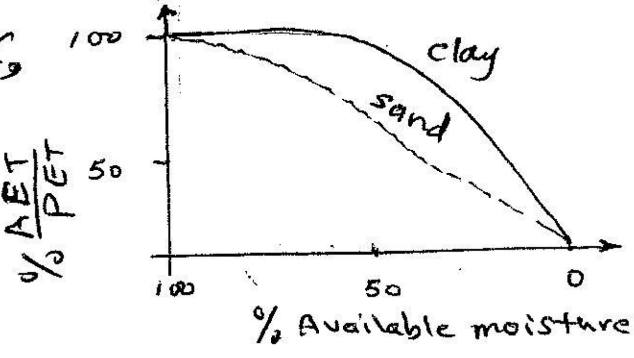
Actual evapotranspiration (AET): occure when soil moisture betwee field capacity and wilting point

$$\frac{AET}{PET} \leq 1$$

ويكون حساب AET في الزراعة الحالية كمية حقيقية

This ratio depends upon the available soil moisture and the type of soil.

as moisture $\rightarrow 0$ thus AET $\rightarrow 0$



Variation of AET

Measurement of Evapotranspiration

يمكن استخدام PET على الاقرب جداً للتبخر من البحيرات EL ،

It can be measured by Lysimeters مقاييس التسرب

Evapotranspiration is estimated as amount of water requred to maintain constant moisture. حيث التبخر الكلي بدلالة مقدار الماء اللازم للمحافظة على رطوبة ثابتة ، ويجب ان يصمم اللايسميترات بدقة تمثل المحتوى المائي ونوع النبات في التربة المجاورة .

Evapotranspiration Equations

معادلات التبخر الكلي

نقتصر في دراستنا على معادلة بنمان حيث سيتم التطرق

٢) معادلات بلاني-كريدل وثورنثويت في موضوع حسابات الري

Penman Equation:

معادلة بنمان

Penman's equation is based on combination of energy-balance and mass-transfer approach. This equation is modified many times and still modified by investigators.

$$PET = \frac{\Delta H_n + \gamma E_a}{\Delta + \gamma} \quad \text{--------} (18)$$

PET = dialy evapotranspiration (mm/day)

Δ = slope of e_s–T curve (see eq (11) Page 7) $\frac{mmHg}{C^\circ}$

γ = Psychrometric constant (see equation (9) Page 6) = 0.49 $\frac{mm\,Hg}{C^\circ}$

H_n = net radiation in mm of evaporable water/day. It is the same as used in energy-budget (see eq. (13) page 11).

E_a = Parameter quoted from mass-transfer, including wind velocity and saturation deficit. معيار مستنبط من مبدأ انتقال الكتلة ويشمل تأثير سرعة الرياح وحد التشبع البخاري

$$E_a = 0.35\left(1 + \frac{u_2}{160}\right)(e_s - e_a) \quad \text{------} (19)$$

u_2 = mean wind velocity at 2 m above ground surface in (Km/day)

H_n can be calculated as follow:

R_a = incident solar radiation outside atmosphere. It is a funtion of period of a year and Latitude. Table of R_a is given for North part of earth whereas it is same for South part of earth. (mm/day)

$$H_c = R_a\left(a + b\,\frac{n}{N}\right) \quad \text{--------} (20) \qquad \text{mm/day of water}$$

H_c = as defined previously in Energy-budget (page 10), incident radiation on water surface.

$$\text{but; } H_n = (1-r)H_c - H_b \quad \text{------------} (13)$$

$$\text{Thus; } H_b = \sigma T_a^4\left(0.56 - 2.54\sqrt{\frac{e_a}{P_o}}\right)\left(0.1 + 0.9\,\frac{n}{N}\right) \quad \text{---} (21)$$

a = Constant depends on latitude $\phi \Rightarrow a = 0.29 \cos\phi$.

b = constant, average value is of 0.52.

n = actual duration of actual sunshine, hrs فترة سطوع الشمس الحقيقي

N = max. possible of sunshine as a function of latitude and period of year, hrs (see table below).

في حالة عدم ورود القيمة في الجدول صراحةً يتم عمل استكمال ما بين القيمتين.

Table 1 ; Mean Monthly Hours of Possible Sunshine, **N**

North latitude	*Jan*	*Feb*	*Mar*	*Apr*	*May*	*Jun*	*July*	*Aug*	*Sept*	*Oct*	*Nov*	*Dec*
0°	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1
10°	11.6	11.8	12.1	12.4	12.6	12.7	12.6	12.4	12.9	11.9	11.7	11.5
20°	11.1	11.5	12.0	12.6	13.1	13.3	13.2	12.8	12.3	11.7	11.2	10.9
30°	10.4	11.1	12.0	12.9	13.7	14.1	13.9	13.2	12.4	11.5	10.6	10.2
40°	9.6	10.7	11.9	13.2	14.4	15.0	14.7	13.8	12.5	11.2	10.0	9.4
50°	8.6	10.1	11.8	13.8	15.4	16.4	16.0	14.5	12.7	10.8	9.1	8.1

Table 2 ; Mean Monthly Solar Radiation(in mm/day) at top of atmosphere, **Ra**

North latitude	*Jan*	*Feb*	*Mar*	*Apr*	*May*	*Jun*	*July*	*Aug*	*Sept*	*Oct*	*Nov*	*Dec*
0°	14.5	15.0	15.2	14.7	13.9	13.4	13.5	14.2	14.9	15.0	14.6	14.3
10°	12.8	13.9	14.8	15.2	15.0	14.8	14.8	15.0	14.9	14.1	13.1	12.4
20°	10.8	12.3	13.9	15.2	15.7	15.8	15.7	15.3	14.4	12.9	11.2	10.3
30°	8.5	10.5	12.7	14.8	16.0	16.5	16.2	15.3	13.5	11.3	9.1	7.9
40°	6.0	8.3	11.0	13.9	15.9	16.7	16.3	14.8	12.2	9.3	6.7	5.4
50°	3.6	5.9	9.1	12.7	15.4	16.7	16.1	13.9	10.5	7.1	4.3	3.0

σ = Stefan-Boltzman Constant $= 2.0 \times 10^{-9}$ mm/day / K°^{4}

one can convert mm of water/day to cal/cm².day ;

أي طاقة لكل وحدة مساحة ساقطة عليها خلال اليوم الواحد حسب التحويلات الآتية

$$\boxed{58.5 \text{ mm/day of } H_2O = \frac{\text{cal}}{\text{cm}^2.\text{day}}}$$

$$\Rightarrow \sigma = 11.703 \times 10^{-8} \frac{\text{cal}}{\text{cm}^2.\text{day}}$$

and thus;

$$41.86 \frac{\text{cal}}{\text{cm}^2.\text{day}} = \frac{\text{KJ}}{\text{day}.\text{m}^2}$$

$$= 4.89888 \times 10^{-6} \frac{\text{KJ}}{\text{day}.\text{m}^2}$$

$$28.35 \text{ mm/day} = \frac{\text{Watt}}{\text{m}^2}$$

$$= 5.67 \times 10^{-8} \frac{\text{J}}{\text{sec m}^2} = \frac{\text{Watt}}{\text{m}^2}$$

$$2449.44 \text{ mm/day} = \frac{\text{KJ}}{\text{day}.\text{m}^2}$$

هذه القيم المرادفة تفيد اذا أعطي الجدول بغير الوحدات المعتمدة (mm/day).

Ta = mean air temperature in degree kelvin, k°.
$K^\circ = 273 + c^\circ$.
r = reflection coefficient (albedo). The values of r the surface of incident, and given as;

Tale 3; Reflection coefficient (albedo)

Surface	Range of *r* values
Close ground corps	*0.15-0.25*
Bare lands	*0.05-0.45*
Water surface	*0.05*
Snow	*0.45-0.95*

تعتمد على الانعكاس على السطح المتلقي لمعاني الاشعاع الحراري .

Thus eq (13) can be rewritten as;

$$H_n = R_a\left(a + b\frac{n}{N}\right)(1-r) - \sigma T_a^4\left(0.56 - 2.536\sqrt{\frac{e_a}{P_o}}\right)\left(0.1 + 0.9\frac{n}{N}\right) \ldots (18)$$

Note that P_o = standard barametric pressure = 760 mmHg

Penman equation (18) can be used for evaporation estimation from Water surface by setting $r = 0.05$.

<u>Example:</u> given data as;

mean monthly temperature $= 19C^\circ$ (Nov.), Latitude $= 28^\circ 4' N$
mean relative humidity $= \% 75$, Elevation = 230 m amsL.
observed sunshine hours = 9 hr.
wind velocity at 2 m height = 85 km/day
Nature of surface cover = close-ground green cover.

(a) Calculate the vapotranspiration by Penman's equation
(b) Calculate the dialy evaporation from water body (Lake)

From eq(10) & (11) $\Rightarrow e_s = 16.47$ mmHg
$\Delta = 1.03$ mmHg/c°

$e_a = 0.75 \times 16.47 = 12.35$ mmHg
$a = 0.29\cos(28^\circ 4') = 0.29\cos(28.067) = 0.2559$
$b = 0.52$
$\sigma = 2.0 \times 10^{-9}$ mm/day·k°
$T_a = 273 + 19 = 292\ k^\circ$
$\sigma T_a^4 = 14.54$, From table (3) $r = 0.25$ (used as max.)
$R_a = 9.506$ mm/day 11.2 by interpolation! $\frac{11.2 - 9.1}{20 - 30} = \frac{11.2 - R_a}{20 - 28.067}$ (from Table
From table (1); $N = -\left(\frac{11.2 - 10.6}{10}\right) 8.067 + 10.6 = 10.716$ hrs.

From eq. (22);

$$H_n = (1-0.25)(9.506)\left(0.2559+0.52\frac{9.0}{10.716}\right) - (14.54)\left(0.56-2.536\sqrt{\frac{12.35}{760}}\right) * \left(0.1+0.9\frac{9.0}{10.716}\right)$$

$$H_n = 4.938 - 2.946$$

$H_n = 1.992$ mm of water/day.

From eq. (19) evaporating parameter E_a is:

$$E_a = 0.35\left(1+\frac{85}{160}\right)(16.47-12.35) = \underline{2.208} \text{ mm/day}$$

(a) From eq. (18); $PET = \dfrac{1.03(1.992)+0.49(2.208)}{1.03+0.49} = \underline{\underline{2.061}}$ mm/d

(b) $H_n = 4.938\dfrac{1-0.05}{1-0.25} - 2.946 = 3.308$ mm/day ← نفس قيمة السابق ل الأوراق

thus; $PET = \dfrac{1.03(3.308)+0.49(2.208)}{1.52} = \underline{\underline{2.953}}$ mm/day

Q: why PET from Lake evaporation is greater than PET from green-crop area?

اشتقاق معادلة بنمان

Derivation of Penman Equation

Penman assumed that the water vapor transferred in a way similar to temperature dispersion (eddy-turbulence) , i.e;

$$(e_s' - e_a) \propto (t_a' - t_a)$$

and thus; $(e_s' - e_a) = const.(\,t_a' - t_a)$

Recalling Eq.(15);

$$\beta = \gamma \frac{t_a' - t_a}{e_s' - e_a} \quad ; \quad here: \qquad const. = \frac{\gamma}{\beta}$$

While from Eq. (14) ; $\beta = \frac{H_a}{H_e}$

Thus; $H_n = H_e + H_a = (1+\beta)H_e$; $Assuming\ H_i\,,\ H_g\ \&\ H_s\ are\ neglected.$

$$\rightarrow H_n = \frac{H_n}{(1+\beta)} = \frac{H_n}{1+\gamma\,(\frac{t_a' - t_a}{e_s' - e_a})}$$

Note that;

e_s'=is a saturated vapor pressure at t_a' (which is the temperature of air close to water surface. t_a' is difficult to measure , and thus it was replaced by t_a and consequently e_s' by e_s .

According to Dalton; *Eo*= *C f(u)* $(e_s' - e_a)$ which can be easily convert to the ;

Ea= *C f(u)* $(e_s - e_a)$, where *Eo* is potential evaporation equivalent to H_e , whereas *Ea* is evaporation from water body.

How to substitute t_a ,t_a' , e_s and e_s'

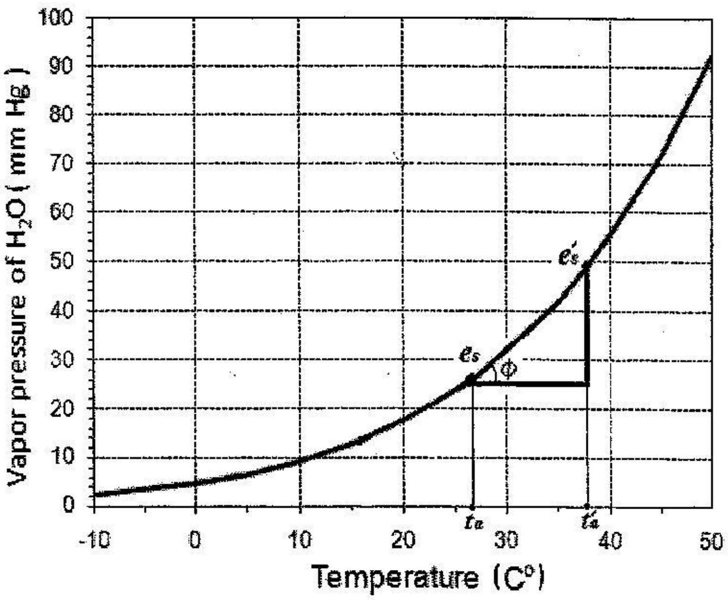
Δ= tan Φ , slope of the vapor pressure -temp. curve at t_a;

$$\Delta = \frac{t_a' - t_a}{e_s' - e_a}$$

And thus;

$$t_a' - t_a = \frac{e_s' - e_a}{\Delta}$$

$$H_e = \frac{H_n}{1 + \frac{\gamma}{\Delta}\,(\frac{e_s' - e_s}{e_s' - e_a})}$$



$$H_e = \frac{H_n}{1 + \frac{\gamma}{\Delta}\left(\frac{(e_s' - e_a) - (e_s - e_a)}{e_s' - e_a}\right)} = \frac{H_n}{1 + \frac{\gamma}{\Delta}\left(1 - \frac{e_s - e_a}{e_s' - e_a}\right)}$$

Since ;

$$\frac{E_a}{E_o} = \frac{E_a}{H_e} = \frac{e_s - e_a}{e_s' - e_a}$$

$$H_e = \frac{H_n}{1 + \frac{\gamma}{\Delta}\left(1 - \frac{E_a}{H_e}\right)}$$

$$\Delta H_n + \gamma E_a = (\gamma + \Delta) H_e$$

$$H_e = \frac{\Delta H_n + \gamma E_a}{\gamma + \Delta}$$

Universal Law of gases:

From Boyle & Charle laws, one can get the universe Law of gases as;

$$\boxed{P = \rho R T} \quad \text{——— (1)}$$

P = pressure Pa (N/m^2)

ρ = mass density Kg/m^3

R = universal gas constant, $R = 286.8 \frac{J}{Kg \cdot K^\circ}$ for air.

T = temperature in $K^\circ = C^\circ + 273$

expansion or compression take place according to the law of thermodynamics

(1) isothermal (constant temperature) يتعرض تبادل حراري لكتلة الغاز مع المحيط لتبقى الحرارة ثابتة

from eq (1); T = constat $\Rightarrow \frac{P}{\rho}$ = constant or;

$$\frac{P}{\gamma} = \text{constant}$$

(2) where is no heat exchange (لا يسمح بالتبادل الحراري (أديباتيك))

isentropic

$$\frac{P}{\rho^n} = \text{constat} \quad \text{or} \quad \frac{P}{\gamma^n} = \text{constant}$$

where n = is adiabatic exponent ≈ 1.4 for air.

Temperature Lapse rate

It is a change of temperature versus elevation, i.e $\frac{dT}{dz}$?

since $P = \rho R T = \gamma/g \, R T$ ——— (2)

set in general $\frac{P}{\gamma^n}$ = constant ——— (3)

i.e.; $\frac{\gamma R T}{g \gamma^n}$ = constant or $T = Const. \frac{g\gamma^{n-1}}{R}$

$$\frac{dT}{d\gamma} = Const. \frac{g}{R}(n-1)\gamma^{n-2}$$

$$\frac{dT}{d\gamma} = \frac{\gamma R T}{g\gamma^{n}} \cdot \frac{g}{R}(n-1)\gamma^{n-2}$$

$$\Rightarrow \frac{dT}{d\gamma} = (n-1)\frac{T}{\gamma} \quad \text{——} \quad (4)$$

and; $\frac{T}{\gamma}\frac{d\gamma}{dT} = \frac{1}{n-1}$

From eq(2) $dP = d\left(\frac{\gamma}{g}RT\right)$

but from principle of fluid mechanic ; $\frac{dp}{dz} = -\gamma$

$$\Rightarrow d\left(\frac{\gamma}{g}RT\right) = -\gamma dz$$

$$\{RT\,d\gamma + \gamma R\,dT = -\gamma g\,dz\,\} * \frac{1}{\gamma g\,dT}$$

$$\frac{dz}{dT} = -\frac{R}{g}\left(\frac{T}{\gamma}\frac{d\gamma}{dT} + 1\right)$$

subs eq(4) in above expression ; yields

$$\frac{dz}{dT} = -\frac{R}{g}\left(\frac{n}{n-1}\right)$$

or $$\frac{dT}{dz} = -\frac{g}{nR}(n-1) \quad \text{——} \quad (5)$$

For $n > 1$, which normal case for lower portion of atmosphere (Tropospher) $\Rightarrow \frac{dT}{dz} < 0$ (negative)

This occurred from elevation 0_m – 11019 m.

For standard atmosphere ($P = 101.3 \times 10^{3}$ Pa , Sea level, $T = 15 C^{\circ}$) and take $R = 286.8 \frac{J}{kg.K^{\circ}}$ $\Rightarrow n = 1.235$, Thus $\boxed{\frac{dT}{dz} = -0.0065 \ C^{\circ}/m}$

For adiabatic case where $n = 1.4$ $\frac{dT}{dz} = -0.0098 \ C^{\circ}/m$

For 11019 m to 20 000 m (stratosphere) الغلاف زمهريري

the temperature has been observed to be essentially constant at $(-56.5\,c^\circ) \Rightarrow n = 1.0$ and $\frac{dT}{dz} = 0.0$

وقد أشار القرآن الكريم الى حالة الانجماد في الطبقات العليا وذكر أن جبال من ثلج (برد).

سُورَةُ النُّور

أَلَمْ تَرَ أَنَّ اللَّهَ يُزْجِي سَحَابًا ثُمَّ يُؤَلِّفُ بَيْنَهُ ثُمَّ يَجْعَلُهُ رُكَامًا فَتَرَى الْوَدْقَ يَخْرُجُ مِنْ خِلَالِهِ وَيُنَزِّلُ مِنَ السَّمَاءِ
مِن جِبَالٍ فِيهَا مِن بَرَدٍ فَيُصِيبُ بِهِ مَن يَشَاءُ وَيَصْرِفُهُ عَن مَّن يَشَاءُ يَكَادُ سَنَا بَرْقِهِ يَذْهَبُ بِالْأَبْصَارِ ﴿٤٣﴾

example 1: Calculate the pressure and weight density of air in the U.S. atmosphere at altitude 10 km above sea level.

solution U.S. standard atmosphere are:

$P_1 = 101.3 \times 10^3$ Pa , $\gamma_1 = 12.01$ N/m^3 , $T_1 = 15 + 273 = 288$ K°

Since $\frac{P}{\gamma^n}$ = constant , for first 11 km $n = 1.235$

$$\Rightarrow \frac{P}{\gamma^{1.235}} = C \Rightarrow C = \frac{101.3 \times 10^3}{(12.01)^{1.235}} = 4702.987$$

Note that $dP = -\gamma\, dz$ (from fluid principle)

$$\frac{dP}{\gamma} = -dz$$

Thus ; $\left(\frac{P}{\gamma^{1.235}}\right)^{\frac{1}{1.235}} = (4702.987)^{\frac{1}{1.235}}$

$$\left[\frac{P^{0.81}}{\gamma} = 941\right] * \frac{dP}{P^{0.81}}$$ ضرب الطرفين *

$$\frac{dP}{\gamma} = \frac{941}{P^{0.81}}\, dP = -dz$$

$$\Rightarrow \int_{P_1}^{P_2} \frac{941}{P^{0.81}}\, dP = -\int_{z_1}^{z_2} dz$$

$$\Rightarrow z_2 - \cancel{z_1}^{=0} = \int_{P_1 = 101.3 \times 10^3} \frac{941}{P^{0.81}}\, dP$$

$$10 \times 10^3 = \frac{941}{0.19}\left[(101.3 \times 10^3)^{0.19} - P_2^{0.19}\right]$$

$\Rightarrow P_2 = 26.305$ KPa

هنا لنا نستنتج

$$z_2 - z_1 = \frac{n}{n-1} C^{\frac{1}{n}} \left[P_1^{\frac{n-1}{n}} - P_2^{\frac{n-1}{n}}\right]$$

يمكن حساب C من المعطيات

ولدينا : $\frac{P}{\gamma^n} = C$

$T_1 = 15 C^\circ = 288 K^\circ$

$\frac{dT}{dz} = -\alpha = -0.0065 \qquad (\alpha = 0.0065)$

$\frac{\Delta T}{\Delta z} = -\alpha \Rightarrow \Delta T = -0.0065\, \Delta z$

$T_1 - T_2 = 0.0065\,(Z_2 - Z_1)$

$T_1 - T_2 = 10\,000\,(0.0065)$

$15 - T_2 = 65$

$\Rightarrow T_2 = 15 - 65 = -50\,\overset{\circ}{C} \quad (223\,\overset{\circ}{K})$

$P = \frac{\gamma}{g} R T \Rightarrow \gamma = \frac{g P}{R T} = \frac{9.81\,(26.805 \times 10^3)}{286.8 \times 223} = 4.03\ N/m^3$

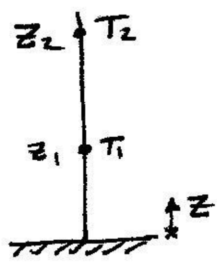
One can derive a direct relationship between P & T and solve it by applying the geven data.

$\frac{dP}{\gamma} = -dz \qquad$ and $\gamma = \frac{gP}{RT}$ from eq (2)

$\Rightarrow \frac{dP}{P} = -\frac{g}{RT}\, dz$

but $\frac{dz}{dT} = \frac{1}{-\alpha}$ where $\alpha = 0.0065$ [from eq.(5) after subst.]

$\frac{dP}{P} = \frac{g}{R\alpha}\left(\frac{dT}{T}\right)$



by integrating both sides yields;

$\ln\left(\frac{P_2}{P_1}\right) = \frac{g}{R\alpha} \ln\left(\frac{T_2}{T_1}\right) \quad ;$ or

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{g/R\alpha} \qquad (6)$$

ولكن عليك استخدام T بال $\overset{\circ}{K}$.

Now for example above;

$\frac{P_2}{P_1} = \frac{26\,805}{1013\,000}, \quad \left(\frac{T_2}{T_1}\right)^{g/R\alpha} = \left(\frac{T_2}{288}\right)^{5.2623}$

$T_2 = 223\,\overset{\circ}{K} \Rightarrow T_2 = -50\,\overset{\circ}{C}$.

example 2

Columns of air are heated by sun, above the standard values of atmosphere. So that the air is buoyant. If the air temperature is 17°C and it is buoyant rise is adiabatic, at what level will the rising air be the temperature and density in equilibrium [stop rising buoyantly

sol.

$T_{standard} = 15^\circ C - 0.0065\, Z$ (Standard atmosphere).

for adiabatic ($n = 1.4$)

$T_{thermal} = 17^\circ C - 0.0098\, Z$ (adiabatic condition).

$T_{standard} = T_{thermal}$ [for equilibrium T_e & Z_e]

$\Rightarrow 2 = 0.0033\, Z_e \Rightarrow Z_e = 607\,m$.

Density of vapor:

At same temperature and pressure, the specific gravity of water vapor to dry air is 0.622.

كثافة بخار الماء الى كثافة الهواء الجاف تساوي (0.622) لنفس درجة
الحرارة والضغط

$$\frac{\rho_v}{\rho_d} = 0.622 \qquad \text{——— (7)}$$

since ρ_d from eq.(2) is;

$\rho_d = \dfrac{P_d}{R_d T}$, therefor; for $e = P_d$

$$\rho_v = 0.622\, \frac{e}{R_d T} \qquad \text{——— (8)}$$

According to Dalton's Law "the pressure exerted by a gas (vapor pressure in this case) is independent on the pressure of other gases, hence;

$$\underset{\text{moist air}}{P_a} = \underset{\text{dry air}}{P_d} + \underset{\text{Vapor}}{e} \qquad \text{——— (9)}$$

or $P_d = P_a - e$

but $\rho_d = \rho_d + \rho_v$ $\left(\rho_a = \dfrac{m_e + m_d}{V}\right)$ m_e = vapor mass. m_d = dry air mass.

$$\Rightarrow \rho_a = \frac{P_d}{R_d T} + \frac{0.622\, e}{R_d T}$$

$$\rho_a = \frac{P_a - e}{R_d T} + \frac{0.622\, e}{R_d T}$$

equation (10) shows that moist air is actually lighter than dry air for same values of pressure and temperature. $\rho_a < \rho_d$ for $P_d = P_a$

or; $$\rho_a = \frac{P_a}{R_d T}\left(1 - 0.37\frac{e}{P_a}\right) \qquad (10)$$

The term $\left(1 - 0.37\frac{e}{P_a}\right)$ is less than one, since $\frac{e}{P_a} < 1$.

Thus; $(\rho_a < \rho_d)$ أي أن الهواء الرطب أخف من الهواء الجاف
وتبين ملاحظة المعادلة (10) بوجود البخار أو عدمه.

The R_d sometimes is slightly different with R_a in general one can assume $R_d = R_a$, if not

$$\rho_v = 0.622\frac{e}{R_d T} = \frac{e}{R_v T} \Rightarrow R_v = \frac{R_d}{0.622}.$$

Specific Humidity

The specific humidity, q_v is the ratio of density of vapor to a density of moist air. هي نسبة كثافة البخار إلى كثافة الهواء الرطب.

$$q_v = \frac{\rho_v}{\rho_a} \qquad (11)$$

تعتمد على درجة تشبع الهواء بالرطوبة وعلى درجة الحرارة وعلى كثافة الهواء الجاف.

since from eq (9); $P_a = P_d + e$

$$\Rightarrow P_a = \rho_d R_d T + \rho_v R_v T$$

$$P_a = \left[\rho_d + \frac{\rho_v}{0.622}\right] R_d T \qquad (12)$$

where $R_v = R_d / 0.622$

From eq (10) and eq (11);

$$q_v = 0.622\frac{e}{(P_a - 0.378\, e)} \qquad (13)$$

or $$\boxed{q_v \approx 0.622\frac{e}{P_a}}$$

since, $P_a = \rho_a R_a T$ with recalling eq.(12)

$$\Rightarrow \left(\rho_d + \frac{\rho_a}{0.622}\right) R_d = \rho_a R_a$$

$$\Rightarrow \left(\frac{\rho_d}{\rho_a} + \frac{\rho_v}{0.622\,\rho_a}\right) R_d = R_a$$

but $\rho_a = \rho_d + \rho_v \Rightarrow \frac{\rho_d}{\rho_a} = 1 - \frac{\rho_v}{\rho_a} = 1 - q_v$ —— (14)

$$\therefore \quad \frac{R_a}{R_d} = \left(1 - q_v + \frac{q_v}{0.622}\right)$$

$R_a \geq R_d$

or $R_a = R_d\,(1 + 0.608\, q_v)$ —— (15)

<u>example:</u>

If the moist air pressure ($P_a = 100\,kPa$), air temperature $20\,C°$; vapor pressure is $1871\ N/m^2$, calculate:

* saturated vapor pressure, relative humidity, specific humidity and air density & dry density.

$$e_s = K\, e^{\left(\frac{17.27\,T_{C°}}{237.3 + T_{C°}}\right)} \quad ; \; K = 611 \text{ for } P = N/m^2, \quad K = 4.58 \text{ for } P \text{ in mm Hg}$$

$$* \; e_s = 611\, e^{(17.27 * 20/(237.3+20))}$$

$$= 2339 \ N/m^2 \ (Pa)$$

$$* \; h_r = \frac{e}{e_s} = \frac{1871}{2339} = 0.8 = 80\%$$

$$* \; q_v = 0.622\,\frac{e}{P_a} = 0.622\,\frac{1871}{100\,000} = 0.0116 \ \text{Kg of vapor / Kg of moisture}$$

كتلي لكل كتلة رطب في الجو .

$P_a = \rho_a R_a T$; and From eq (15):

$$P_a = \rho_a \cdot R_d\,(1 + 0.608\, q_v)\, T$$

$$100000 = \rho_a \left[286.8\,(1 + 0.608 * 0.0116)\right](20 + 273)$$

$$10^5 = \rho_a\,(288.8)(293) \Rightarrow \rho_a = 1.180\ Kg/m^3$$

From eq (14) $\rho_d = \rho_a (1 - q_v)$

$$\rho_d = 1.18\,(1 - 0.0116) = 1.168\ kg/m^3$$

Precipitable water — الماء المطري

Precipitable water (M_P) is amount (mass) of moisture in an atmospheric column. يعرّف بأنه كمية الرطوبة (بخار الماء) في عمود الهواء الجوي والتي تتحول الى مطر وتسمى الماء المطري (القابل للأستمطار) ويرمز له M_P وتقاس بوحدات الكتلة خلال كتلة عمود الهواء أي (Kg in the column of atmosphere). إن كمية الماء المطري الى كتلة عمود الهواء الجوي تمثل معدل الرطوبة النوعية average specific humd. $\bar{q}_v$ ويمكن كتابة الآتي:-

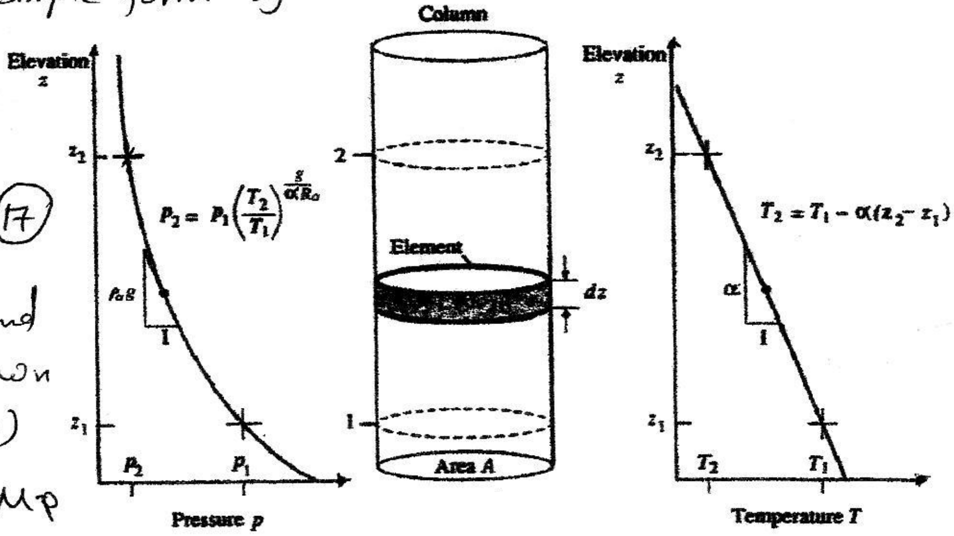
$$\bar{q}_v = \frac{M_P}{M_a} \Rightarrow M_P = \bar{q}_v M_a \quad \text{or;}$$

$$M_P = \int_{z_1}^{z_2} q_v \rho_a A \, dz \quad \text{——— (16)}$$

Eq(16) can be reduced to simple form by invoking ($dp = -\gamma dz$);

$$M_P = \int_{z_1}^{z_2} q_v \rho_a A \left(\frac{-dp}{\gamma_a}\right)$$

$$\text{or;} \quad M_P = -\frac{A}{g} \int_{z_1}^{z_2} q_v \, dp \quad \text{——— (17)}$$



Due to variation of q_v, T, ρ_a and γ with variation of z as shown in Figure, Eq(16) & Eq(17) was recommended to solve M_P incrementaly as;

$$M_P = \sum_{i=1}^{n} \Delta M_P = \sum_{i=1}^{n} \bar{q}_{v_i} \bar{\rho}_{a_i} \Delta z_i, \quad \text{where } \bar{q}_{vi} \text{ is an average value over } \Delta z_i$$

or;

$$M_P = -\frac{A}{g} \sum_{i=1}^{n} \bar{q}_{vi} \Delta P_i \quad \text{——— (18)}$$

Example: Calculate the precipitable water for saturated atmospheric column up to 10 km over 1 m² area (ground surface). The surface pressure is 101.3 kPa, and surface temperature is 30 C°, lapse rate is (0.0065) C°/m ($R_d = 286.8 \frac{KJ}{Kg \cdot K^\circ}$).

Solution: Divide the column in 5 increments to get small value of q_v over increment thus; $R_a = R_d$ since $R_a = R_d(1 + 0.608 q_v)$ [acceptable assumption].

$M_P = 76.905$ Kg in a column as shown in table below.

1	2	3	4	5	6	7	8	9	10
Z (km)	T (c°)	T (K°)	P (pa)	e (pa)	ρ_a (kg/m³)	q_v	$\bar{q}_v$	ΔP/g	$-A/g \cdot \bar{q}_v (\Delta P)$
0	30	303	101300	4244.5	1.147	0.0261			
2	17	290	80425	1938.4	0.958	0.0150	0.0205	-2127.9	43.679
4	4	277	63179	813.5	0.791	0.0080	0.0115	-1758.0	20.217
6	-9	264	49059	309.3	0.646	0.0039	0.0060	-1439.4	8.586
8	-22	251	37611	104.6	0.522	0.0017	0.0028	-1167.0	3.298
10	-35	238	28430	30.8	0.416	0.0007	0.0012	-935.9	1.125
								∑ Mp =	76.905

Kg, in column of atmosphere

$\Delta z = \frac{10}{5} = 2\,km$; $T_{i+1} = T_i - \alpha(z_{i+1})$; $T_K = T_{C^\circ} + 273$; $P_{i+1} = P_i\left(\frac{T_{i+1}\,K^\circ}{T_i\,K^\circ}\right)^{(g/R_d\alpha)}$

$e = 611\,EXP\left(\frac{17.27\,T_{C^\circ}}{237.3 + T_{C^\circ}}\right)$; $\rho_a = \frac{P_a}{R_d T_{K^\circ}}\left(1 - 0.378\,e/P_a\right)$; $q_v = \frac{\rho_v}{\rho_a} = 0.622\,e/P$

Col. 1: $\Delta Z = \frac{10\ \text{Km}}{5} = 2\ \text{Km}$

Col. 2: $T_2 = T_1 - \alpha(Z_2)$

Col. 3: $TK^\circ = Tc^\circ + 273$

Col. 4: $P_2 = P_1 \left(\frac{T_2 K^\circ}{T_1 K^\circ}\right)^{(g/R\alpha)}$

Col. 5: $e = 611\ \text{Exp}\left(\frac{17.27\, Tc^\circ}{237.3 + Tc^\circ}\right)$

Col. 6: $\rho_a = \frac{P_a}{R_d\, T_{K^\circ}}\left(1 - 0.378\frac{e}{P_a}\right)$

Col. 7: $q_v = \frac{\rho_v}{\rho_a} = 0.622\ e/P_a$

Col. 8: $\overline{q_v} = (q_{v\,i} + q_{v\,(i+1)})/2$

Col. 9: $\frac{P_{i+1} - P_i}{g}$

Col. 10: (Col. 8 * Col. 9)

which the sum of all cells of Col. 10 is equal to ΣM_p.

Note that $A = 1.0\ m^2$