Pavement Structural Design التصميم الانشائي للتبليط

Syllabus

1) Introduction (Pavement definition , Types, Wheel loads, Tire pressure, Types of pavement Distress, Serviceability , Design Process and Strategies).

2) Stresses in Flexible Pavements (Layered system concept , Material behavior &type of theory, Stresses & displacement in one layer or system, stresses &displacement in Burmister two layer system stresses & displacement in Burmister three layer system).

3) Stresses in Rigid Pavements (Bending thin plate (plate bending theories, the differential equation of the deflection surfaced , bouncing conditions simply supported rectangular plates under sinusoidal load, navier solution for simply supported rectangular plates, finite difference solution for simply supported rectangular plates).

4) Plates on the Elastic Foundations (foundation models, rectangular plates on Winkler foundation finite difference solution for plate in Winkler foundation)

5) Stresses in Rigid Pavements (causes of stresses, stresses due to wheel loads Westergaard solution, Pickett & Ray charts (influence charts) Reddy & Paranesh charts , stresses due to friction (stresses due to uniform temperature to drop or due to uniform temperature increase, temperature reinforcement , stresses due to warping , types of joints, stresses in dual bars, shear transfer , double group action).

6) Equivalent Single Wheel Load & AASHTO Load Equivalency Factors Equivalent single wheel load principle (ESWL) , ESWL criteria & concept , ESWL analysis , flexible pavement ESWL , rigid pavement ESWL, AASHTO equivalency factors, Traffic growth.

7) Design of Flexible Highway Pavement (General, Failure conditions of flexible highway pavement, Distress modes, Major methods for design of flexible highway pavement, AASHTO Guide 1993 method ,Applications).

8) Design of Highway Rigid Pavement (Pavements types, plain concrete pavement simply reinforcement concrete pavement , continuously reinforced concrete pavement , priestesses concrete pavements methods , design of highway rigid pavement , the Portland cement association method , AASHTO Guide 1993 method).

References

1) Yoder , E. J. & Witiczek , M. W. " Principles of Pavement Design " 2nd edition-1975.
2) AASHTO, “AASHTO Guide for Design of Pavement Structures”, 1993.
3) Huang, Y.H., “Pavement Analysis and Design”, Prentice Hall, New Jersey, 1993.

Periods

2 hr – theoretical.
1 hr – tutorial.

Lecture No. 1

Introduction

Pavement Design (Highway and Airport):- Involves a study of soils and paving materials their behavior under load and the design of pavement to carry that load under all climatic conditions.

التصميم الانشائي للتبليط (الطرق و المطارات) يتضمن دراسة سلوك التربة و مواد التبليط تحت الحمل المسلط وتصميمها لتحمل هذة الاحمال تحت الظروف البيئة المختلفة.(بمأن التربة تتاثر بالرطوبة الداخلية و بالتالي تقل مقاومتها لذلك تغطى بطبقة من المواد القوية).

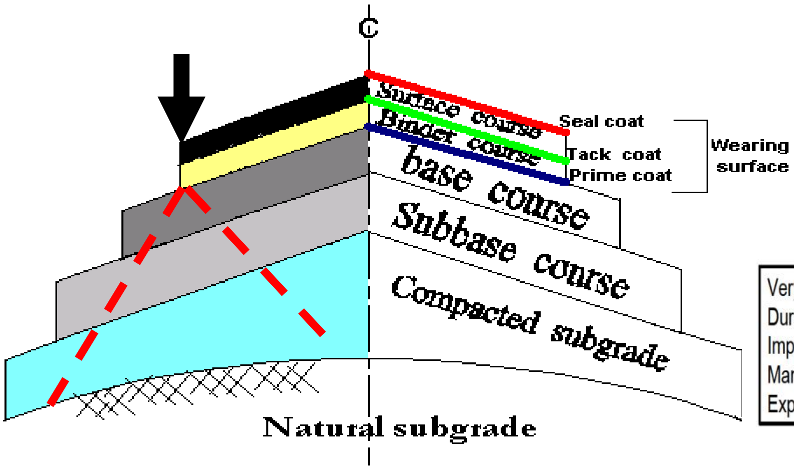
Pavement Structure = Subbase + Base + Surfacing

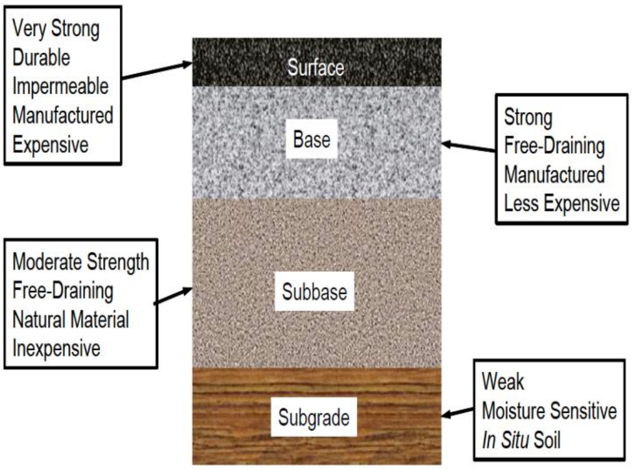
The purpose of the pavement system is to provide a smooth surface over which vehicles may safely pass under all climatic conditions for the specific performance period of the pavement

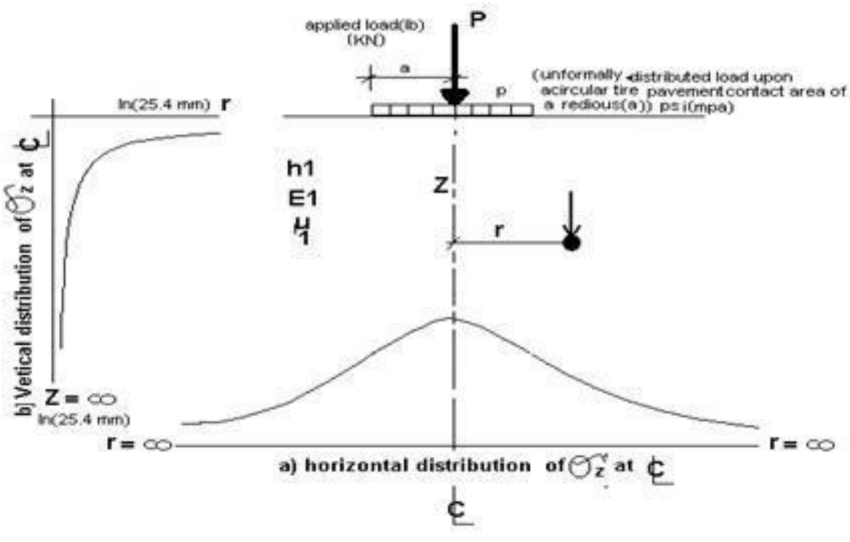
Pavement Types:- انواع التبليط

1) Flexible Pavement (Asphalt Pavement) **التبليط المرن او التبليط الاسفلتي**

the flexible pavement is a multi-layered system having different materials in different layer (better materials on the top and cannot be represented by a homogeneous mass). Multi-layer system consist of:-





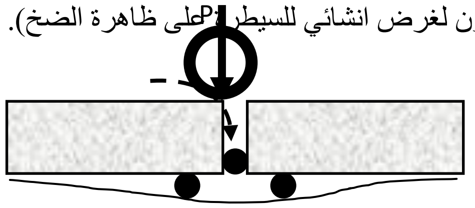


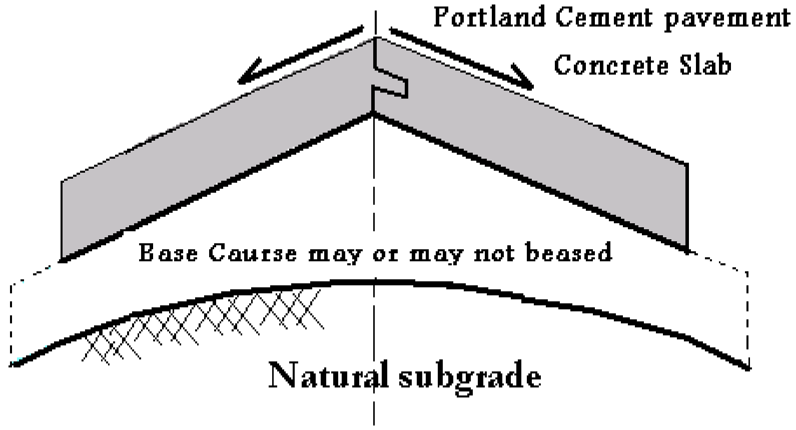
من خلال الشكل (1) نلاحظ كلمايزداد العمق تقل الاجهادات (اكبر اجهاد يكون عند السطح) لذلك يجب ان تكون مواد الطبقة السطحية قوية و عادة تكون مكلفة وكلما زاد العمق نستخدم مواد ضعيفة ورخيصة .

من خلال شكل (2) كلما يزداد العمق يزداد عرض الطبقات ؟ لغرض توزيع الاحمال على مساحة اكبر و تعمل كأسناد للطبقات العليا في حالة الفشل.

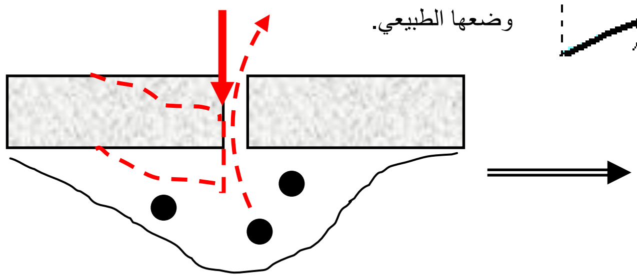
2) Rigid Pavement (Concrete Pavement- Slab) التبليط الصلد (التبليط الكونكريتي)

يسمى بالتبليط الصلد نسبة الى التبليط المرن (صلابة الكونكريت عالية وبالتالي فان مقاومة الاحمال سوف يتحملها هو فقط و لايحتاج الى اسناد (لايحتاج الى اساس) حتي لو وضع فتكون لغرض انشائي للسيطرة على ظاهرة الضخ).





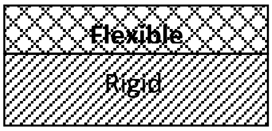
نتجة للضغط على جانبي البلاطة سوف تندفع الى داخل التربة ويدخل الماء من خلال المفاصل او الشقوق ونتجة الضغط الشديد والمستمر سوف تزداد الفجوة هذا الماء يعمل على ذوبان ذرات التربة الناعمة و تحت تأثير الضغط تخرج الى السطح وعند زوال الحمل المسلط فان البلاطة تعود الى وضعها الطبيعي.



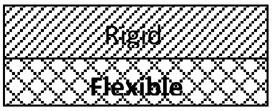
هذه الظاهرة تدعى (الضخ او النضح Pumping)

3- Composite Pavement التبليط المركب

a) Flexible over rigid.

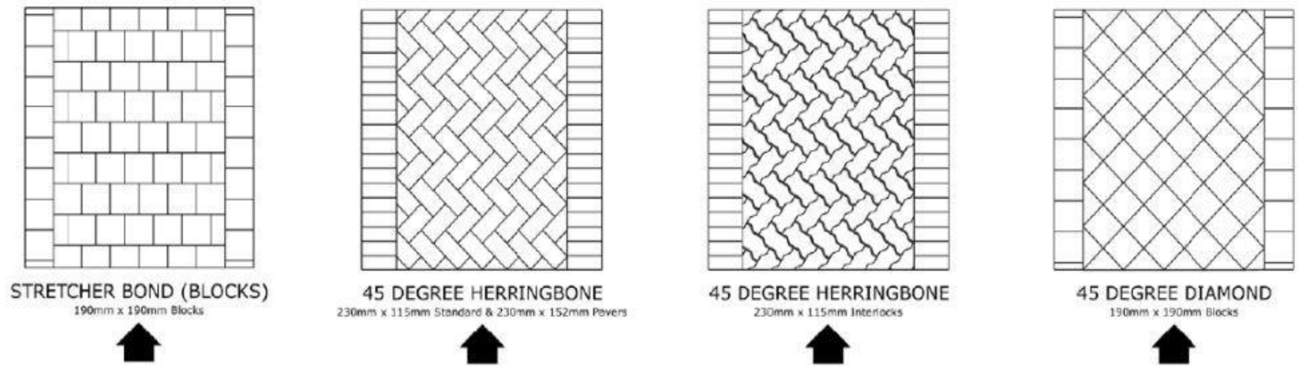


b) Rigid over flexible.



3- Block Pavement بلاطات التبليط

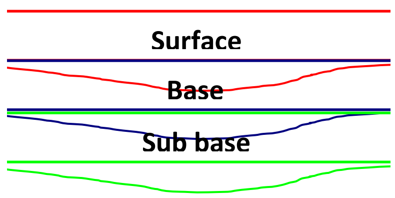
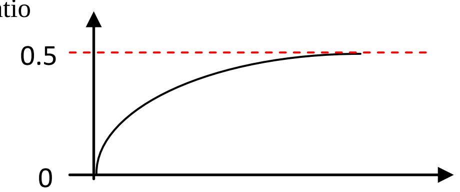
يوفر سهولة رفعه واعادته (الصيانة)



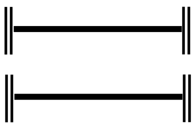
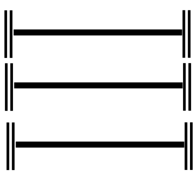
NOTES

1. PAVERS MUST ALWAYS BE LAID ACROSS THE TRAFFIC FLOW.
2. PAVERS ARE NOT TO BE LAID IN THE "STACL BOND" PATTERN BECAUSE THE PAVERS DON'T LOCK OR BOND TOGETHER.
3. IT IS RECOMMENDED THAT PAVERS BE LAID IN A HERRINGBONE PATTERN.

Comparison between مقارنة بين

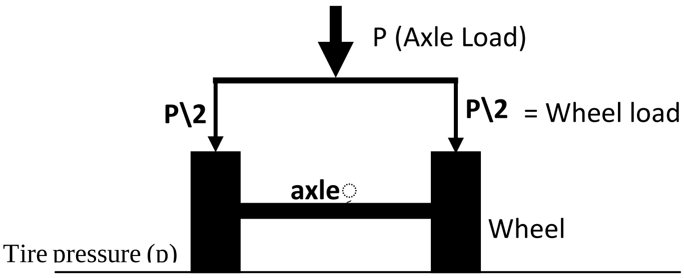
Flexible Pavement	Rigid Pavement
A pavement structure which a maintains intimate a contact with distributes load to subgrade depends on aggregate interlock particles fraction and cohesion for stability. منظومة متصلة بالتربة تقوم بتوزيع الحمل الى طبقة ماتحت الاساس وتعتمد على 1) التداخل بين الركام 2) الاحتكاك بين الجزئيات 3) قوة التماسك (اذا حدث تشوة بالتربة ينقل الى الطبقات اعلاه).  Flexible pavement consists of a series of layers with the highest quality at or near the surface. التبليط المرن يتكون من سلسلة من الطبقات والطبقة الاقوى تكون قرب السطح	A pavement structure which distributes load to subgrade has a one course of a Portland cement concrete slab of relatively high bending resistance (bridge action). التشوة بالتربة لا ينقل الى الطبقات العليا الاخرى حيث ان التبليط يعمل كجسر.  Rigid pavement consists of a Portland cement concrete slabs resting either directly on subgrade or on base course. التبليط الصلد من طبقة من الكونكريت تجلس مباشرة على طبقة ماتحت الاساس
The pavement possesses an asphalt surface. الطبقة السطحية من الاسفلتية	The pavement possesses an Portland cement concrete slabs surface.(الطبقة السطحية مصنوعة من الكونكريت (السمنت البورتلاني
The load carrying capacity of a truly flexible pavement is brought about by the load distributing characteristics of the layered system.	The strength of a rigid pavement is the result of the bending action of slab.
The thickness design of flexible pavement is influenced by the strength of subgrade.	The rigid pavement distributes the load over a relatively wide area of soils and minor variations in subgrade.(soil strength have little significance upon the structured capacity of the pavement.
The fundamental purpose of base course and subbase course it to provided a stresses distributing medium so that shear and place in the subgrade	Bases under rigid highway pavement are used mainly for controlled pumping.
Failure mode [rutting, cracking and raveling] with outs joints.	Failure mode [cracking and pumping] using with joints.
Temperature affects properties. Poisson's ratio  Modulus of elasticity (E)= 10^5 PSI ------ 10^6 PSI (Summer ------ Winter)	Temperatures are unaffected in Poisson's ratio and modulus of elasticity. Poisson's ratio ( )= 0.15 Modulus of elasticity (E)= $4*10^6$ PSI

Wheel loads and Axle Loads: (الاحمال المحورية و أحمال العجلات)

	**Front axle**	Single axle with single tire at each ends (أحادية المحور مع أطار منفرد في كل نهاية)
		Single axle with dual tire at each ends (أحادية المحور مع أطار مزدوج في كل نهاية)
	**Rear axle**	Tandem axle with dual tire tire at each ends (ثنائية المحور مع أطار مزدوج في كل نهاية)
		Triple axle with dual tire at each ends (ثلاثية المحور مع اطار مزدوج في كل نهاية)

Wheel configration and distrubted axle load

طريقة توزيع أحمال المركبات على الإطارات



According to Iraqi Specification

Maximum Observed axle load in Iraq		Maximum Available axle load in Iraq
Max. Front axle load	12 Ton >	6 Ton
Max. Rear single axle load	25.4- 30 Ton >	12 Ton
Max. tandem axle load	49 Ton >	18 Ton
Max. Floating axle load	21 Ton	--------
Max. Triple axle load	49.5 Ton >	25.5 Ton

Code for representing axle Configuration

Each axle represented by a digit namely (1 – single axle with single tire at each ends , 2- single axle with dual tire at each ends) depending on how many wheels on each end of the axle.

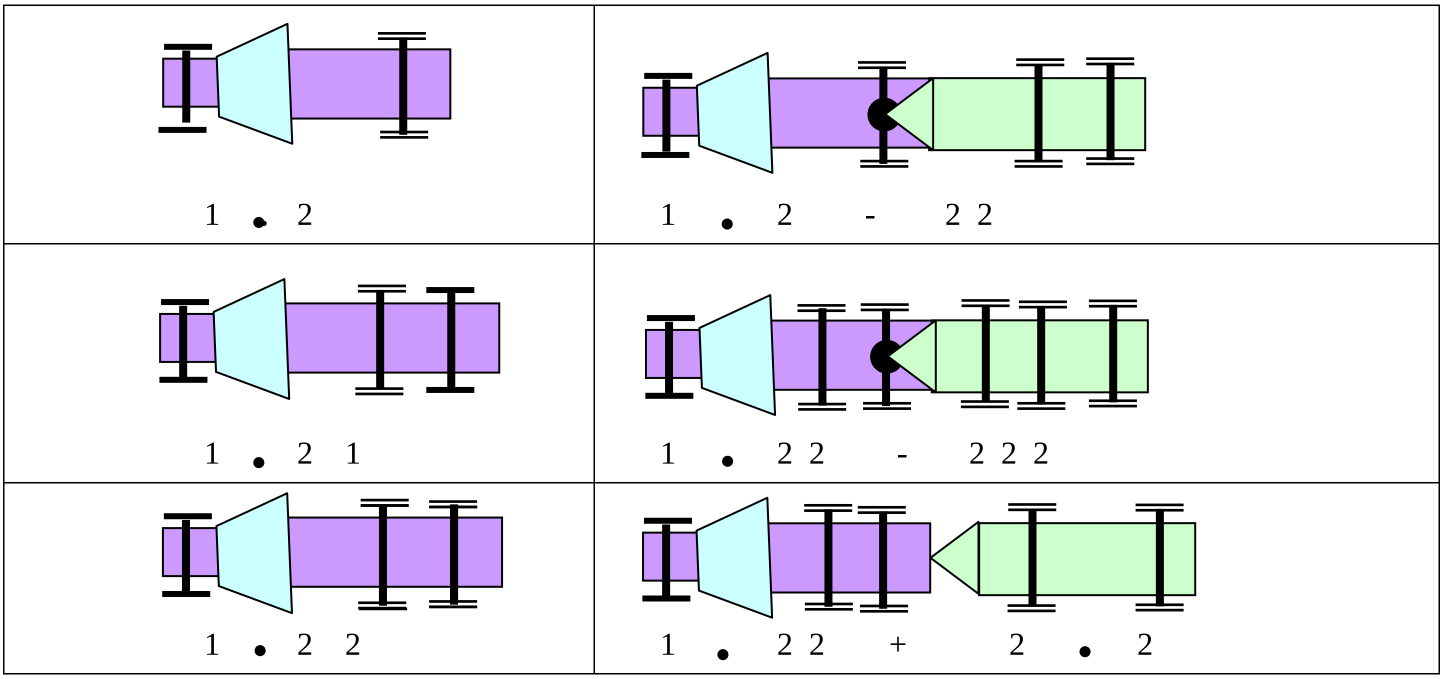
كل محور يمثل برقم (1- أحادي المحور مع أطار واحد في كل نهاية 2- أحادي المحور مع إطارين في كل نهاية)

Tandem axles are indicated by recording the digits directly after each other and decimal point is place between the code for vehicle's front wheels and its load axles.

في حالة المركبات ثنائية المحور يعبر عنها بتسجيل (عدد إطارات المحور الخلفي . نوع المحور الأمامي)

The code for trailers is recorded in same way as for trucks and is separated from the trucks code by a (plus sign +) for semi-trailers or (minus – sign) for articulated.

الشاحنات بنفس الطريقة السابقة ويفصل بينهما أشارة جمع او طرح وكما مبين بالجدول أدناه:-

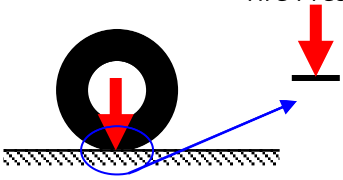


Tire Pressure, contact Pressure and the Imprint:-

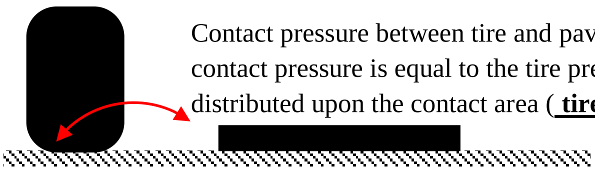
ضغط الاطار و ضغط التماس و اثر الاطار

ضغط الاطار :- الضغط الناتج من الهواء المحصور داخل الاطار .

ضغط التماس :- الضغط الناتج من تسليط ضغط الاطار على مساحة التماس.



P (Tire pressure) = allowable tire pressure in Iraq 95 PSI



Contact pressure between tire and pavement surface it's usually assumed that contact pressure is equal to the tire pressure and it's assumed uniformly distributed upon the contact area (**tire imprint**)

ضغط التماس الناتج من ضغط الاطار يفترض بانة يتوزع بصورة منتظمة فوق مساحة تدعى (مساحة التماس)

كلما يزداد الضغط في الاطار تزداد مساحة التماس ويزداد التشوة (الهبوط).

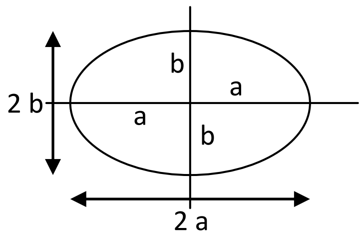
Contact Pressure (P) = Load on wheel \ Contact area

Calculate Contact Area

حساب مساحة التماس وتعتمد على شكل المساحة المفروضة

1) **Elliptical Tire Contact Area**

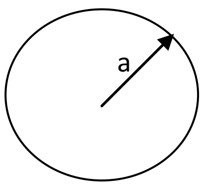
a- Major axis
b- Minor axis
$b/a \approx 1.5$



Contact area = area of ellipse
A= π*a*b

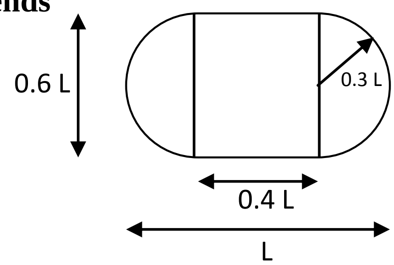
2) **Circular Shape:** most conmen assumptions for contact area (b=a)

Contact Area = πa^2



3) **Rectangular Contact area with semi-circular ends**

$A= 0.4 L * 0.6 L + \pi (0.3 L)^2$
$A= 0.52274 L^2$



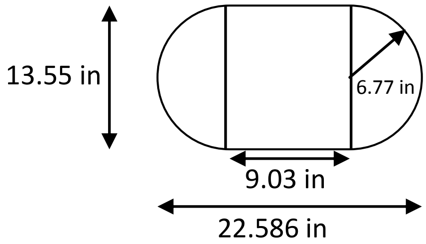
PSI = Pound per Square inch
Kips = 100 lb (Pound)

Example :- Wheel load 40 kips and tire pressure = 150 psi. Calculate contact area.

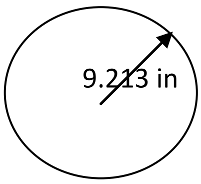
Solution

Contact area = wheel load / tire pressure = 40 *1000 lb / 150 lb/ in^2 = 266.67 in^2

1) If assumed contact area is rectangular contact with semi-circler load ($A= 0.52274\ L^2$)
266.67 in^2 = 0.52274 * L^2 ==== L = 22.586 in



2) If using circular pavement contact area = πa^2
$266.67 = \pi a^2$ == a= 9.213 in



Pavement Distresses (Failure) **عيوب التبليط**

1) Structural Distress (Structural Failure):- الفشل الانشائي

A collapse of pavement structure or a breakdown of one or more pavement components of such magnitude to make pavement incapable sustaining the loads imposed upon it's surface. == Complete Rebuilding

فشل او انهيار واحدة او اكثر من مركبات التبليط لدرجة انة لا يتحمل الحمل المسلط علية وهذا الفشل يحتاج الى اعادة أ نشاء.

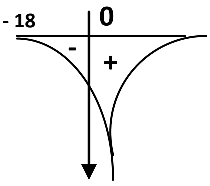
2) Functional Distress (Functional Failure):- الفشل االوظيفي

Is a distress such that the pavement will not carry out it intended function without causing discomfort to passenger or without causing high stresses in the plane or vehicle passenger it due to it's roughness.

الطريق بسنيطع انشائيا" تحمل الاحمال المسلطة علية ولكنه غير مريح للمستخدم نتيجة للخشونة (الخشونة تولد اهتزازت)

Causes of Pavement Distresses

1) Over Load
   a) Excessive grass loads (excessive axle load). الاجهادات الناتجة من زيادة الاحمال المسلطة
   b) High number of reputations of axle loads. تكرار الاحمال يؤدي الى حدوث تشوة الكلال
   c) High tire pressure ضغط الاطار العالي
2) Climatic and environmental conditions _ الضروف المناخية والبئية



   a) Frost heaving (frost action)
   الماء عندما ينجمد يزداد حجمه وبمان التربة قريبة من منسوب المياة الجوفية فعندما تنخفض درجة الحرارة ينجمد الماء داخل التربة ويزداد حجمه ويعمل على رفع التبليط .
   b) Volume change of soil due wetting and drying breakup resulting from freezing and thawing or improper drainage.
   التغيرات الحجمية الناتجة من الزيادة والنقصان بالرطوبة يحدث تقلص و تمدد
   c) Dissolution of gypsum due to rain fall
   الذوبان الجبس في الماء يعمل على تقليل قوة التماسك بين ذرات التربة وتقليل مقاومتها للقص.
3) Disintegration the paving materials due to freezing and thawing and or wetting, drying. Sealing of concrete pavement due to non durable aggregate. Base coarse material may break down.
تفتت مواد التبليط نتجة عملية الانجماد والذوبان المتكرر مما يؤدي الى تقشر الكونكريت وانعزاله عن الركام و الفشل.
4) Constriction practice (rutting of subgrade during instruction which permits the accumulation of water and sequent softeners of the subgrade after contraction is completed).
5) Use dirty aggregate a inadequate inspection during constriction. استخدام ركام غير نظيف او كفؤ
6) Lack of maintenance (sealing joint, sealing of concrete and joints at proper internals , sealing of flexible pavements surface). فقدان الصيانة نتجة عدم ختم المفاصل او
7) Inadequate structured design التصميم غير كفؤاو كافي

Layer Function

- *The* ***subgrade*** is the top surface of a roadbed upon which the pavement structure and shoulders are constructed. The purpose of the subgrade is to provide a platform for construction of the pavement and to support the pavement without undue deflection that would impact the pavement's performance. For pavements constructed on-grade or in cuts, the subgrade is the natural in-situ soil at the site. The upper layer of this natural soil may be compacted or stabilized to increase its strength, stiffness, and/or stability.
- *The* ***subbase*** is a layer or layers of specified or selected materials of designed thickness placed on a subgrade to support a base course. The subbase layer is usually of somewhat lower quality than the base layer. In some cases, the subbase may be treated with Portland cement, asphalt, lime, flyash, or combinations of these admixtures to increase its strength and stiffness. A subbase layer is not always included, especially with rigid pavements. A subbase layer is typically included when the subgrade soils are of very poor quality and/or suitable material for the base layer is not available locally, and is, therefore, expensive. Inclusion of a subbase layer is primarily an economic issue and alternative pavement sections with and without a subbase layer should be evaluated during the design process.

  In addition to contributing to the structural capacity of flexible pavement systems, subbase layers have additional secondary functions:
  - Preventing the intrusion of fine-grained subgrade soils into the base layer.
  - Minimizing the damaging effects of frost action.
  - Providing drainage for free water that may enter the pavement system.
  - Providing a working platform for construction operations in cases where the subgrade soil is very weak and cannot provide the necessary support.
- *The* ***base*** is a layer or layers of specified or select material of designed thickness placed on a subbase or subgrade (if a subbase is not used) to provide a uniform and stable support for binder and surface courses. The base layer typically provides a significant portion of the structural capacity in a flexible pavement system and improves the foundation stiffness for rigid pavements, as defined later in this section. The base layer also serves the same secondary functions as the subbase layer, including a graduation requirement that prevents subgrade migration into the base layer in the absence of a subbase layer. It usually consists of high quality aggregates, such as crushed stone, crushed slag, gravel and sand, or Combinations of these materials. The specifications for base materials are usually more stringent than those for the lower-quality subbase materials.
- *The* ***surface course*** is one or more layers of a pavement structure designed to accommodate the traffic load, the top layer of which resists skidding, traffic abrasion, and the disintegrating effects of climate. The surface layer may consist of asphalt (also called bituminous) concrete, resulting in "flexible" pavement, or Portland cement concrete (PCC), resulting in "rigid" pavement

Lecture No. 2

Serviceability مستوى الخدمة او الاداء للتبليط

Ability at time of observation of pavement to serve high speed, high volume automobile and track traffic.

قابلية التبليط لخدمة السرعة العالية و الحجم المروي العالي لاطول فترة ممكنة.لغرض تحديد مستوى الخدمة للطريق يتم اجراء فحوص حقلية وذلك باختيار مجموعة من الناس وتقسم الى :-

Serviceability

1) Present serviceability Index (PSI) – دليل مستوى الخدمة
2) Present Serviceability Rating (PSR)- دليل تقيم مستوى الخدمة

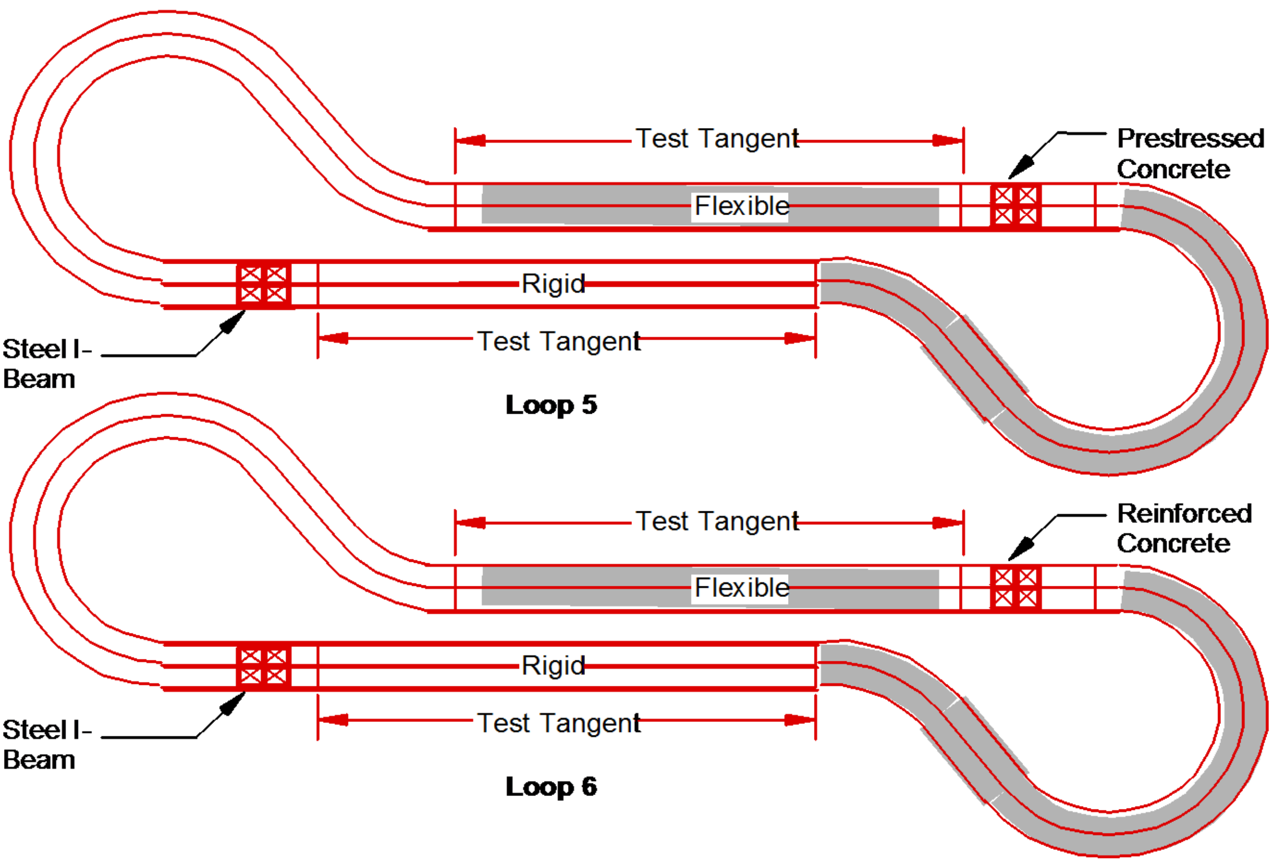
By AASHO Road Test (1959-1961, Illinois, USA)

The **AASHO Road Test** was a series of experiments carried out by the American Association of State Highway and Transportation Officials (AASHTO)to determine how traffic contributed to the deterioration of highway pavements. Officially, the Road Test was "...**to study the performance of pavement structures of known thickness under moving loads of known magnitude and frequency**." This study, carried out in the late 1950s in Ottawa, Illinois, is frequently quoted as a primary source of experimental data when vehicle damage to highways is considered, for the purposes of road design, vehicle taxation and costing.

المنظمة الامريكية للطرق و النقل قامت بهذا الفحص عام 1950 في ولاية اوناوا لدراسة الكفاءة الانشائية للطريق المعلوم السمك تحت الاحمال المتحركة المعلومة التردد وطول الموجة.

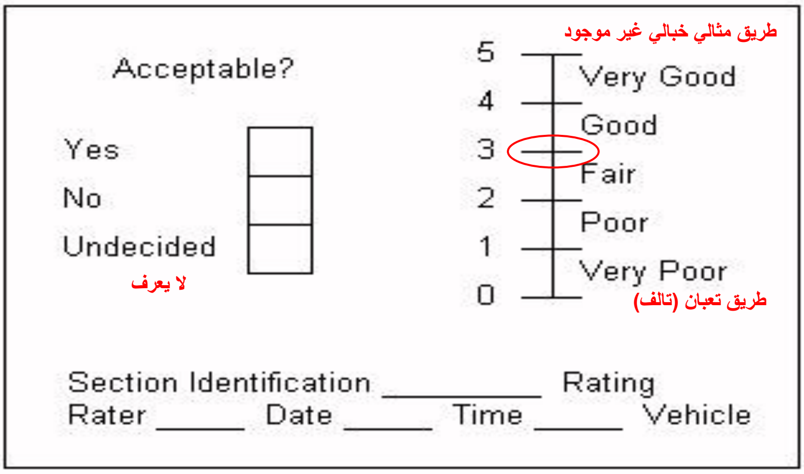
The Road test consisted of six two-lane loops along the future alignment of Interstate 80. Each lane was subjected to repeated loading by a specific vehicle type and weight. The pavement structure within each loop was varied so that the interaction of vehicle loads and pavement structure could be investigated. "Satellite studies" were planned in other parts of the country so that climate and subgrade effects could be investigated, but were never carried out

فحص الطريق يتكون من 6 دورات ثنائية الممر كل ممر يتعرض الى احمال مرورية متكررة من مركبة قياسية ومعلومة الوزن في كل دورة التيبليط يتغير انشائيا وكافة العوامل المؤثرة على التبليط تم دراستها من خلال الدراسات الفضائية



Percent Serviceability Rating (PSR)

يستخدم هذا الدليل لايجاد العلاقة بين اراء مستخدمي الطريق و القياسات (خشونة الطريق – الشقوق – الاخاديد – الترقيع) وتتم عملية جمع المعلومات (اراء مستخدمي الطريق) من خلال بطاقة

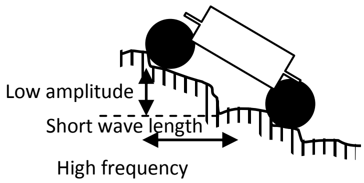


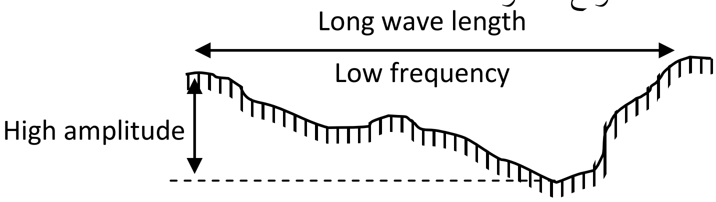
تتضمن البطاقة العوامل المؤثرة وهي :-

1) Longitudinal Distortions التشوة بالاتجاه الطولي للطريق

التبليط يجب ان تكون له القدرة الكافية لحمل الاحمال المرورية وذو سطح املس (خشونة بمقدار كافي) ولكن هذا صعب الحصول عليه.

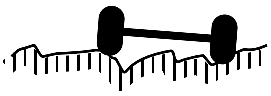
Types of Roughness انواع الخشونة





السطح الخشن (ينتج من الركام الخشن في الطبقة السطحية) هذه الخشونة تولد ضوضاء وتسبب اهتزازات بطول موجي وترددو وسعة هذة العوامل تعتمد على طول الحفرة حيث انة كلما يزداد طول الحفرة يزداد طول الموجة المتكونة ويقل ترددها وتزداد سعة الموجة.

2) Transverse Disturbance التشوة بالاتجاة العرضي للطريق



Rut:- surface distortion can be caused by consolidation of one or more of the paving layers

Ruts or settlement cause side way

3) Cracking. الشقوق
4) Faulting.
5) Surface deformation. التشوة السطحي

Number of Raters required estimating:-

Permissible error	Probability level	
	0.05	0.1
0.3	31	21
0.4	17	12
0.5	11	8
0.6	8	5
0.7	6	5
0.8	4	3
0.9	3	2
1.0	3	2

عدد الاشخاص يعتمد على الدقة المطلوبة فكلما تزداد الدقة يزداد العدد بصورة عامة تؤخذ دقة مقدارها 0.5 فالعدد المطلوب يكون من 8 الى 11 وهولاء الناس يجب ان يكونوا بمستتيات مختلفة (**مهندسين** – تكون توقعاتهم اسفل المخمنة و عادين – تكون توقعاتهم فوق المخمنة)

Engineer --- under estimation
General ---- Over estimation

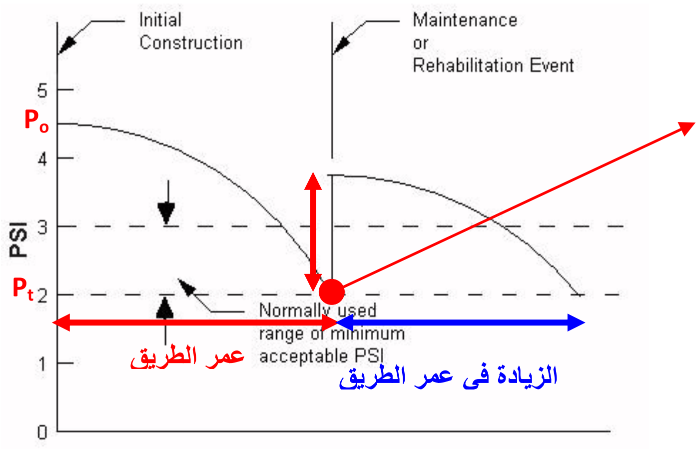
Failure Point (P_t):- Terminal level of serviceability

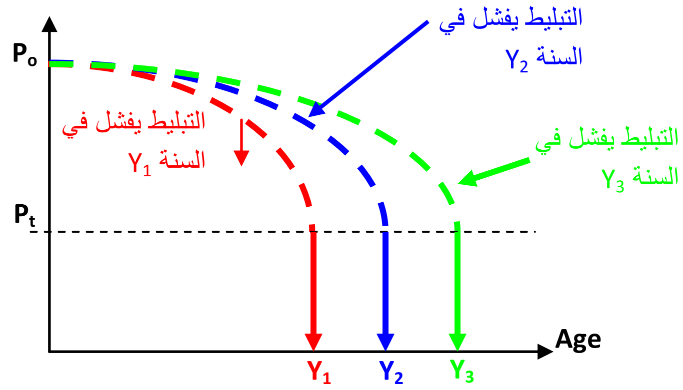
وهي النقطة التي يكون فيها مستوى الاداء او الخدمة خارج المستوى المطلوب (الطريق يحتاج الى صيانة) وتكون محددة حسب نوعية الطريق :

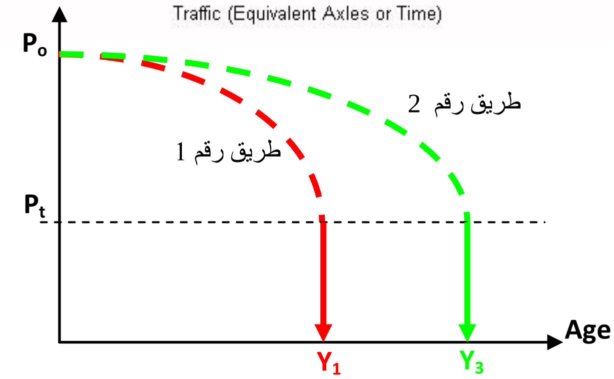
1) Express ways , Major highways $P_t = 3.0$
2) Primary Roads $P_t = 2.5$
3) Secondary Roads $P_t = 2.0$

Initial Level of Serviceability (P_o) مستوى الاداء او الخدمة في بداية الانشاء

يتم عمل تقيم بصورة منتظمة من بداية انشاء الطريق وبفترات زمنية مختلفة لمستوى الاداء ويتم رسم العلاقة بين تقيم الاداء لمعرفة او تحديد الوقت المناسب للصيانة (معرفة الوقت الذي يكون فيه مستوى الاداء او الخدمة اقل مايمكن تحتاج الى صيانه)







ملاحظة:-

اذا كان لدينا عدة طرق (فايهما يحتاج الى الصيانة قبل الاخر) ؟

نعمل تقييم الاداء لكل طريق ولفترات زمنية متفاوته ونرسم العلاقة بين العمر و مستوى الخدمة ونوصل النقاط بخط واحد ونمده الى P_t ونرسم من نقطة التقاطع الى محور العمر فنحصل على السنة التي تحتاج الى الصيانة).

* اذن الطريق رقم 1 يحتاج الى الصيانة قبل

Percent serviceability Index (PSI)

- PSI يربط العلاقة بين نتائج التقييم و الخواص الفيزيائية للتبليط

$$PSI = A_o + A_1(R) + A_2(F_1) + A_3(F_3)$$

Ao = constant
R= roughness
F1 – F3 = physical measurements for cracking, rutting and patch.

For Flexible Pavement

$$PSI = 5.03 - 1.91 \log_{10}(1 + SV) - 1.38 * RD^2 - 0.01\,(C+P)^{0.5} + error$$

Where:-

SV :- Slope variance مقدار التباين في قياس الميل

SV = Sum (yi) -1/n(ÿ)^2/ (n-1)

where Si = slope

ÿ = mean slope

RD :- Rut depth (inch) measured with a 4 ft straight edge عمق الاخدود

C+P :- Cracking & Patching area ft^2 / 1000 ft^2 of pavement area (class 2 (>1/8") & class 3 (> ¼ ") cracking)

مساحة الشقوق بالفوت المربع لكل مساحة 1000 فوت من التبليط

For Rigid Pavement

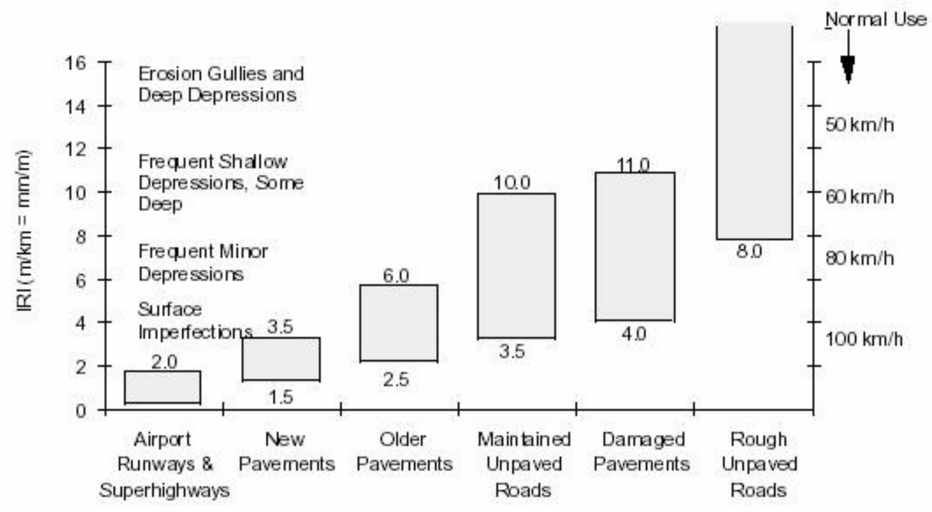
$$PSI = 5.41 - 1.80 \log_{10}(1+SV) - 0.09\,(C+P)^{0.5} + error$$

For rigid pavement major trouble are crack, scaling and roughness (RD=0)

AASHO Road Test	New flexible pavement at the road	PSI = 4.2
	New rigid pavement at the road	PSI = 4.5

International Roughness Index (IRI)

The international roughness index (IRI) was developed by the World Bank in the 1980s (UMTRI, 1998). IRI is used to define **a characteristic of the longitudinal profile of a traveled wheel track and constitutes a standardized roughness measurement**. The commonly recommended units are meters per kilometer (m/km) or millimeters per meter (mm/m). The IRI is based on the average rectified slope (ARS), which is a filtered ratio of a standard vehicle's accumulated suspension motion (in mm, inches, etc.) divided by the distance traveled by the vehicle during the measurement (km, mi, etc.). IRI is then equal to ARS multiplied by 1,000. The open-ended IRI scale is shown in following figure



Correlations between PSR and IRI

Various correlations have been developed between PSR and IRI. Two are presented here. One was reported in 1986 by Paterson:

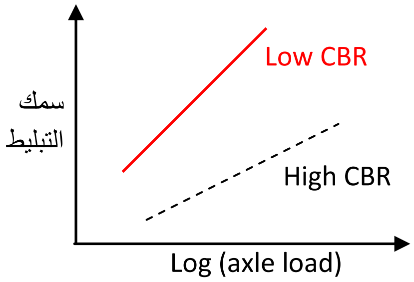
$$PSR = 5e^{-0.18(IRI)}$$

Another correlation was reported in a 1992 Illinois funded study performed by Al-Omari and Darter (1992):

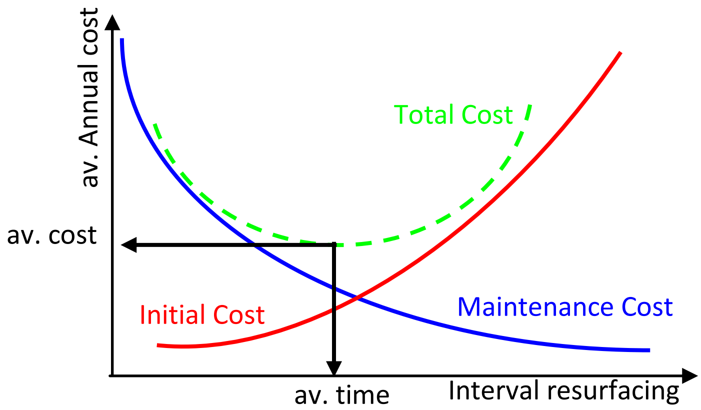
$$PSR = 5e^{-0.26(IRI)}$$

This study used data from the states of Indiana, Louisiana, Michigan, New Mexico, and Ohio for both flexible and rigid pavements. The associated regression statistics are R^2 = 0.73, SEE = 0.39, and n = 332 sections. Correlations are highly dependent upon the data that are used.

The design process, Design Strategies



اذا كانت طبقة التربة ضعيفة فنحتاج الى سمك اكبر
كلما تزداد الفترة التصميمية نحتاج الى زيادة السمك
كلما يزداد **CBR** للتربة فيقل السمك.



اذا كان السمك كبير من البداية فانة سوف يخدم اطول فترة ولكنه على حساب الكلفة.

السمك القليل (كلفة اقل – فترة خدمة اقل – كلفة صيانة اكبر.

السمك الاكبر (كلفة كبيرة – فترة خدمة اطول – كلفة صيانة اقل)

Total Cost

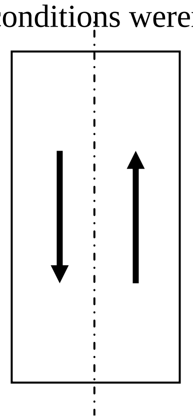
1) Construction cost.
2) Maintenance Cost.
3) Added road user cost due to short downed the facility.

Traffic

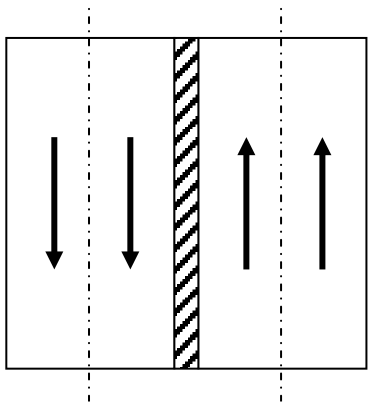
1) The predication of traffic for design purpose must rely on or information from the past traffic, by growth factors or other expected ranges.
2) Predications the traffic is made for some convenient period of times, referred to as the traffic analysis period.
   The traffic analysis is period often used is 20 years which is also a period used in traffic predictions for geometric design neither the traffic analysis period nor the time a pavements reaches it's terminal serviceability Index(P_t) should be confused with pavement life pavement life may be extended by provider renewal of the surface.

Traffic Distribution by Direction and Lanes

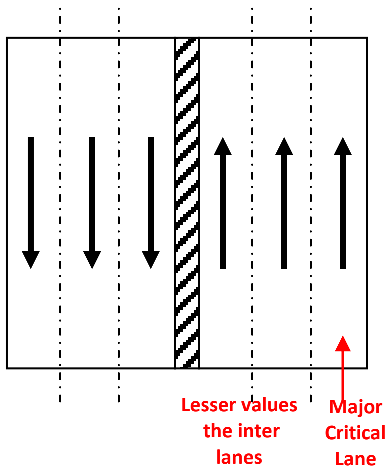
Direction distribution is usually made by assuming 50 % of the traffic and each direction unless special conditions weren't same other.



2- Lane
Single carriageway
Design based an
100 % on one direction



4- Lanes divided highway
2-laness dual carriageway
Design based an
80- 100 % on one direction

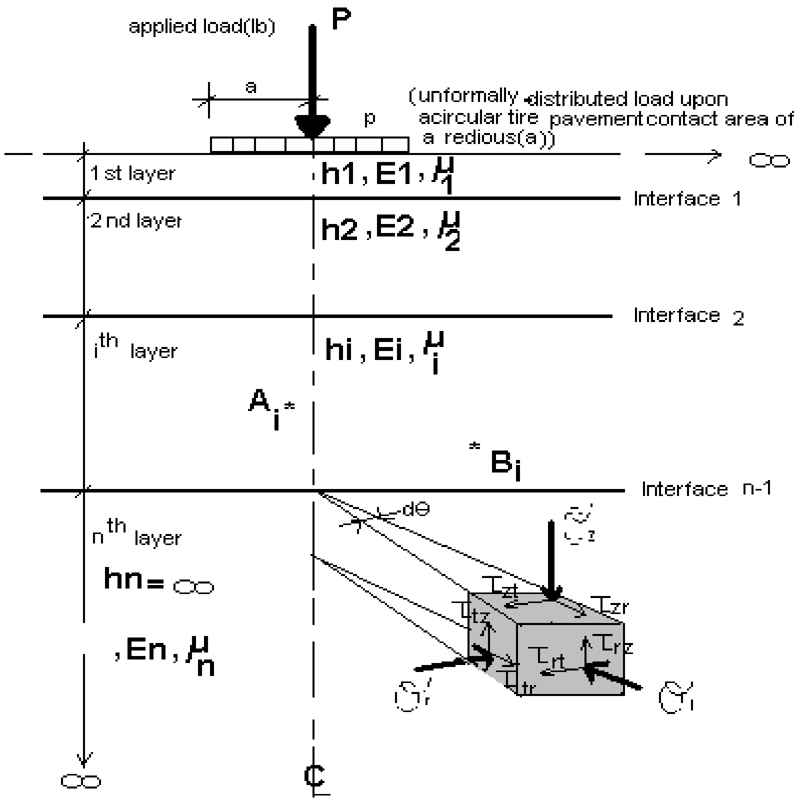


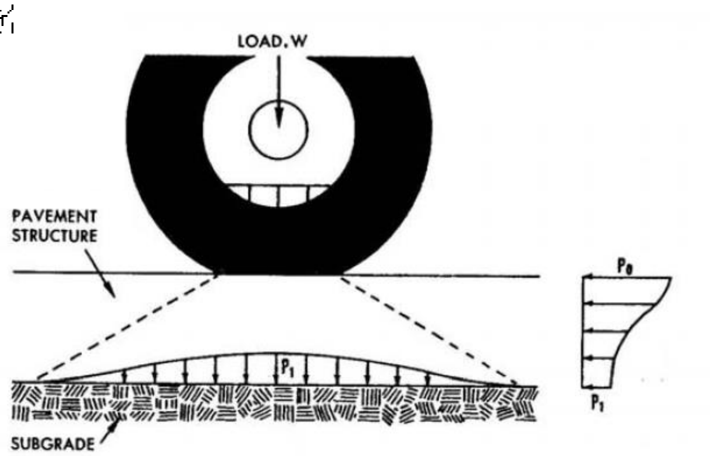
3- Lanes or more, Design based an
68- 80 % on one direction

Lecture No. 3

Stresses in Flexible Pavement

Typical cross section for the multi-layer elastic flexible pavement systems shown in the following figure:-





Generalized multi-layer elastic system and component of stresses under Axisymmetric Loading

Where:

H_i: -Thickness of ith layer.

E_i: - Modulus of elasticity of the ith layer.

μ_i: - Poisson's ratio of the ith layer.

σ_z, σ_r, σ_t :- Normal stresses (Vertical, Radial and Tangential respectively) at element surface.

τ_{zr}, τ_{zt}, τ_{rt}; Shearing stresses produced from vertical normal stress (σ_z) in the Radial and Tangential direction.

τ_{rz}, τ_{rt}: Shearing stresses produced from Radial normal stress (σ_r) in the Vertical and Tangential direction.

τ_{tz}, τ_{zr}: Shearing stresses produced from Tangential normal stress (σ_t) in the Vertical and Radial direction.

In this theory, the following assumptions were made:

1) The material properties of each layer are homogenous (the property at point A_i is the same at point B_i).
خصائص المواد المستخدمة بالطبقة تفرض بانة متجانسة (اي ان خصائص المواد في النقطتين متشابه).

2) Each layer is Isotropic that is the property at a specific point (point A_i is the same in every direction or orientation).
الاجهادات المسلطة (العمودية والافقية) على كل نقطة تكون متساوية

3) Each layer has finite thickness except the lower layer and all are infinite in the lateral direction.
لكل طبقة سمك محدد عدا الطبقة الطبقة الاخيرة وكل الطبقات غير محددة بالاتجاة الجانبي.

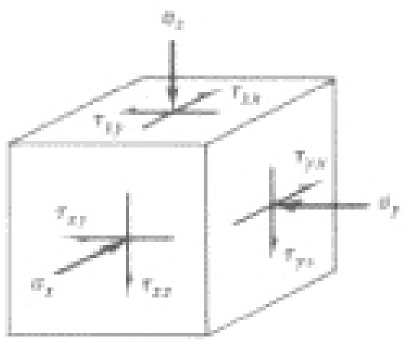
4) Full friction is developed between the layers at each interface.
وجود احتكاك بين الطبقات و لايسمح بالحركة الافقية (لا يجوز ان يحدث فيها زحف).

5) Surface shearing forces are not present at the surface.
تهمل قوى القص على سطح التربة والاطار (القص بين الاطار وسطح التبليط)

6) The stress solutions are characterized by two material properties for each layer they are :
   a- Poisson's ratio. (μ)
   b- Modulus of elasticity, (E).

7) A circular load acts at the surface of the top layer so that the problem can be regarded as being axisymmetric in properties as well as in load.

الحمل المسلط يفرض بانة دائري ويتوزع بصورة منتظمة.
اذا اخذنا عنصر من التربة فالاجهادات المسلطة علية تكون بالشكل التالي (9 اجهادات).



3- Stresses (normal stresses or perpendicular on the surface of element

σ_z :- Vertical Normal stresses.
σ_r :- Radial Normal stresses.
σ_t :- Tangential Normal stresses.
6- Stress (shearing stresses acting parallel to the face)

τ_{zr}, اجهاد القص الناتج من الضغط العمودي و باتجاة محور r
τ_{zt}, اجهاد القص الناتج من الضغط العمودي و باتجاة محور y
τ_{rt} اجهاد القص الناتج من الضغط العمودي و باتجاة محور t

Under static equilibrium conditions **$\tau_{zr}=\tau_{rz}$, $\tau_{zt}=\tau_{tz}$, $\tau_{rt}=\tau_{rt}$**

Bulk stress **$\sigma = \sigma_x + \sigma_y + \sigma_z$** (principle stress).

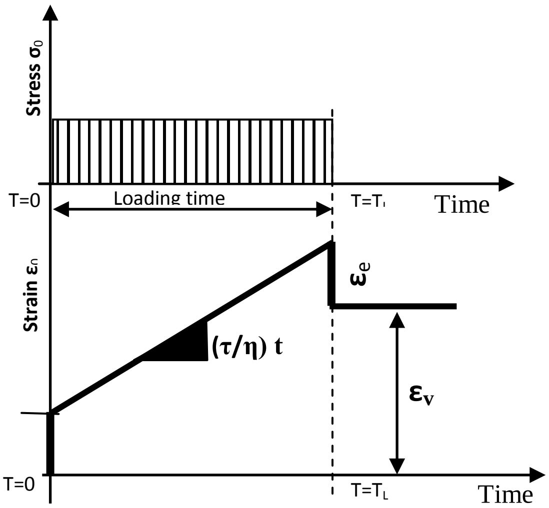
Theories depend on the properties of the materials:-

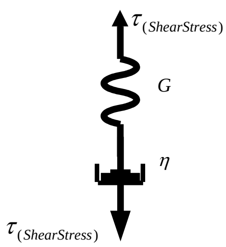
1) Stress, strain relationship (linear or non-linear).
2) Time dependency of strain under a constant stress level (viscous or non- viscous).

هنالك عدة نظريات لتفسير سلوك المادة منها:-

<u>a- Maxwell Model</u>: spring and dashpot series (heavy liquid)

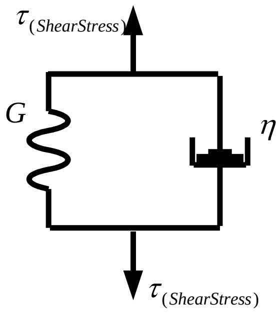
نموذج ماكسويل:- يفترض ماكسويل بان التبليط المرن يتكون من حبيبات مرنة و لدنة مربوطة على التوالي. و يستخدم لتمثيل السوائل الثقيلة.

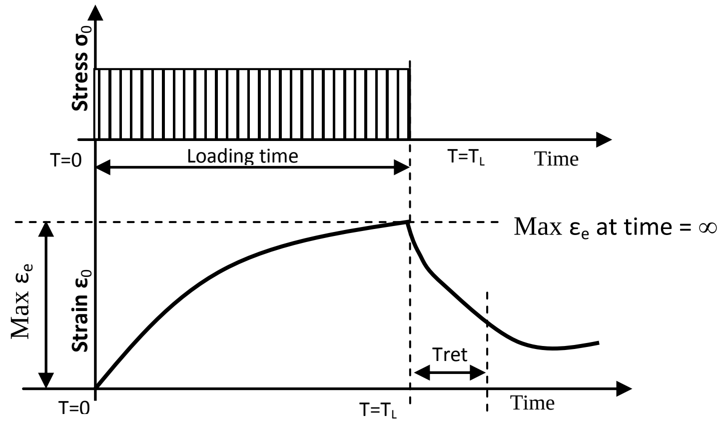




b- Kelvin (Voigt) Model (Spring and dish pot in parallel): Solid Material

نموذج كلفن :- يفترض كلفن بان التبليط المرن يتكون من حبيبات مرنة و لدنة مربوطة على التوازي. و يستخدم لتمثيل المواد الصلبة .

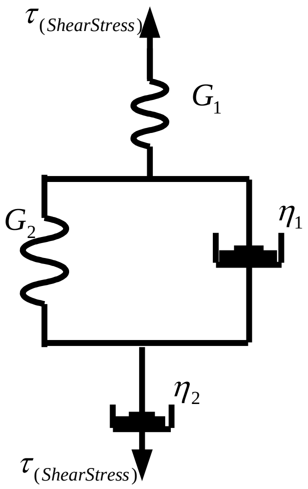


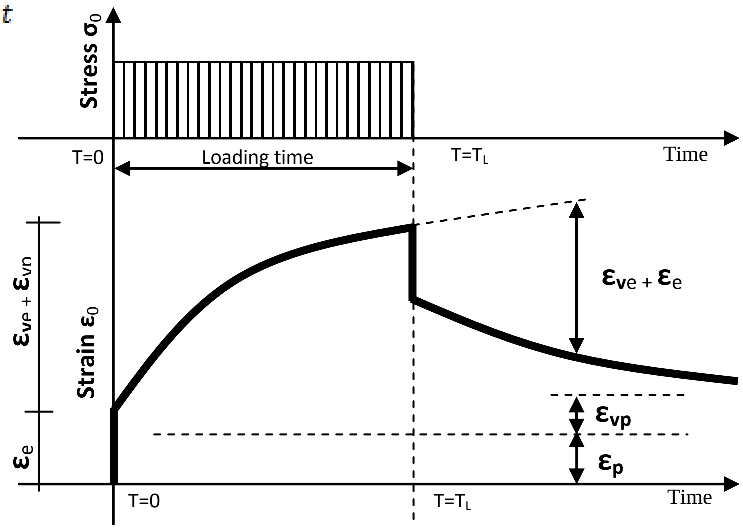


c- Burger 's Model (4-element model)- Viscoelastic Materials

نموذج بيركير:- يفترض بيركير بان التبليط المرن يتكون من حبيبات مرنة و لزجة مربوطة على التوالي و التوازي . و يستخدم لتمثيل المواد اللزجة-مرنة .

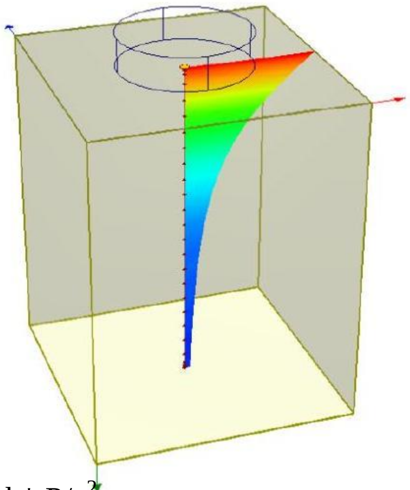
$$\gamma_T = \gamma_e + \gamma_d + \gamma_v = \frac{\tau}{G_1} + \frac{\tau}{G_2}\left[1 - e^{-\frac{G_2}{\mu_2}t}\right] + \frac{\tau}{\eta_1}t$$

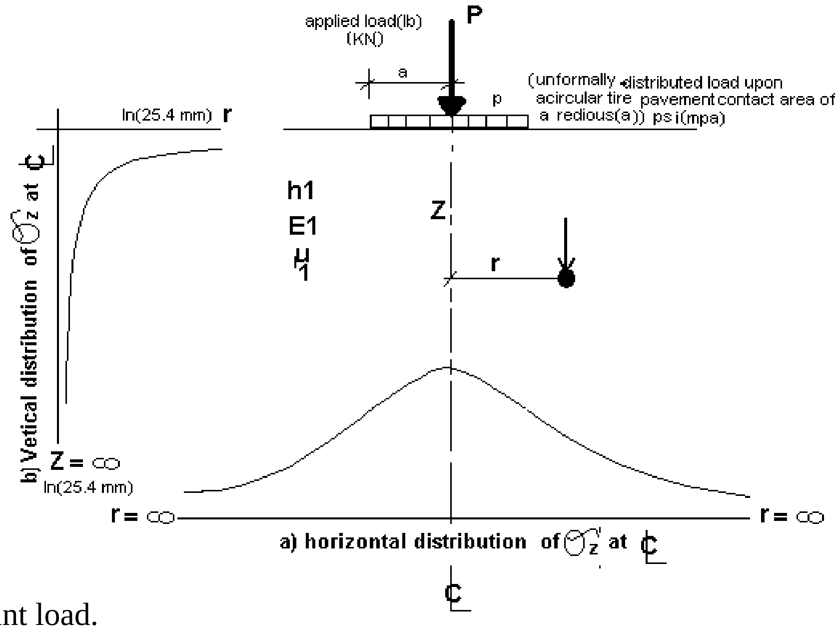




Boussinesq's half space (One-layer system).

Boussinesq* ,** assumed the pavement consists only one layer and it's a part of the soil (the same properties of soil (Poisson's ratio and Modulus of elasticity)) so that can be neglecting the effect of type materials using. According to ***Boussinesq's formula, the vertical stress at any depth below the earth's surface due to a point load at the Surface is as follows.





$\sigma_z = k * P/z^2$

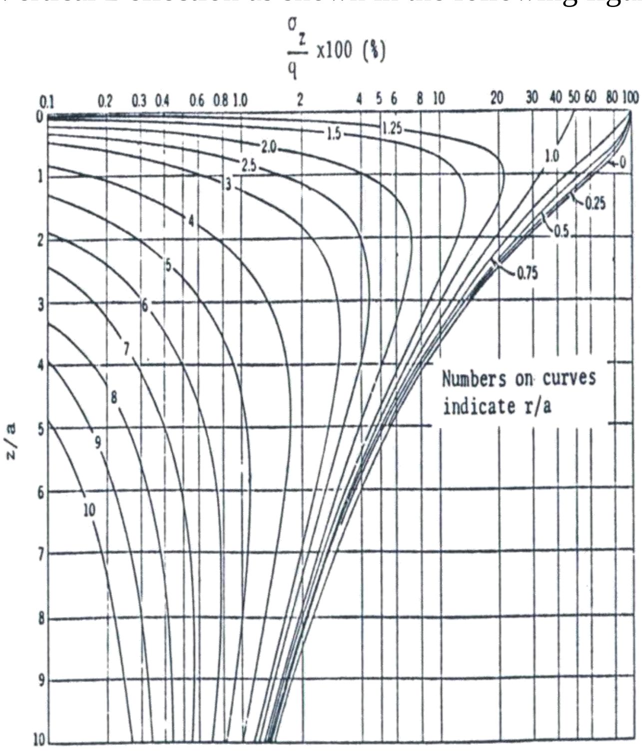
$K = 3/2\ \pi * 1/[1+(r/z)^2]^{5/2}$

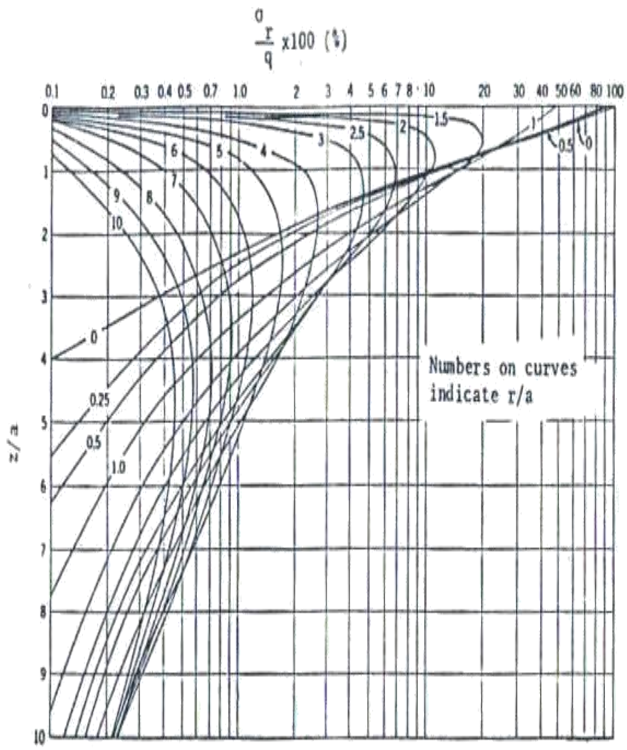
Where r = Redial distance from point load.

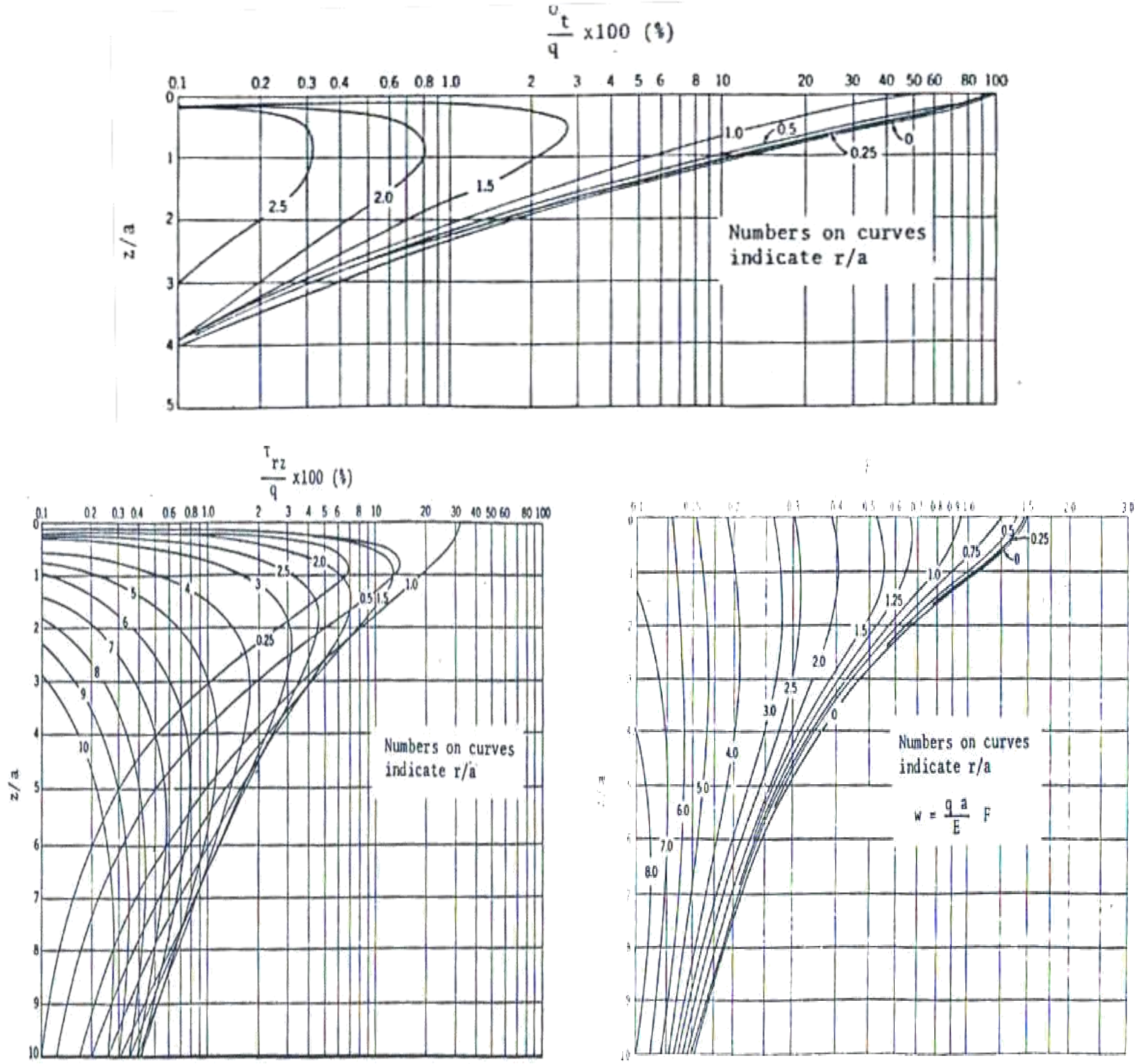
Z= Depth below point load.

It can be noticed that the vertical stress distribution is dependent on the depth and radial distance and is independent of properties of the transmitting medium.

Foster and Ahlvin , presented charts for determining Vertical, Radial, Tangential and Shear Stress and Vertical Deflection as shown in the following figure;







Foster and Alvin Chart (1954)
(Vertical, Radial, Tangential, Shear Stresses and Vertical Deflection due to Circular Loading)

$\Delta T = \Delta P + \Delta S$

ΔT = Total surface deflection

ΔP = deflection within the pavement layer. (ΔP = 0) (لان التبليط يتكون من طبقة واحدة فقط فنهمل الهبوط)

ΔS= deflection within the subgrade soil.

Foster and Ahlvin, assumed the Poisson's ratio of (μ=0.5) because Poisson's ratio has a relatively small effect on stresses and deflections.

Ahlvin and Ulery , who presented a series of equations and tables so that the stresses, strains, and deflections for any given Poisson's ratio can be, computed these equations as shown in following table Kenlayer computer program.

Summary of one-layer elastic Equations

Parameter	General Case	Special Case (μ =
Vertical stress	$\sigma_z = p[A + B]$	(same)
Radial horizontal stress	$\sigma_r = p[2\mu A + C + (1 - 2\mu)F]$	$\sigma_r = p[A + C]$
Tangential horizontal stress	$\sigma_t = p[2\mu A - D + (1 - 2\mu)E]$	$\sigma_t = p[A - D]$
Vertical radial shear stress	$\tau_{rz} = \tau_{zr} = pG$	(same)
Vertical strain	$\epsilon_z = \frac{p(1+\mu)}{E_1}[(1-2\mu)A + B]$	$\epsilon_z \frac{1.5p}{E_1} B$
Radial horizontal strain	$\epsilon_r = \frac{p(1+\mu)}{E_1}[(1-2\mu)F + C]$	$\epsilon_r = \frac{1.5p}{E_1} C$
Tangential horizontal strain	$\epsilon_t = \frac{p(1+\mu)}{E_1}[(1-2\mu)E - D)]$	$\epsilon_t = -\frac{1.5p}{E_1} D$
Vertical deflection	$\Delta_z = \frac{p(1+\mu)a}{E_1}\left[\frac{z}{a}A + (1-\mu)H\right]$	$\Delta_z = \frac{1.5pa}{E_1}\left(\frac{z}{a}A +\right.$
Bulk stress	$\theta = \sigma_z + \sigma_r + \sigma_t$	
Bulk strain	$\epsilon_\theta = \epsilon_z + \epsilon_r + \epsilon_t$	
Vertical tangential shear stress	$\tau_{zt} = \tau_{tz} = 0 \therefore [\sigma_t(\epsilon_t)$ is principal stress (strain)]	
Principal stresses	$\sigma_{1,2,3} = \frac{(\sigma_z + \sigma_r) \pm \sqrt{(\sigma_z - \sigma_r)^2 + (2\tau_{rz})^2}}{2}$	
Maximum shear stress	$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$	

Function A

Depth (z) in Radii	Offset (r) in Radii $\frac{r}{a}$																
	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	1.0	1.0	1.0	1.0	1.0	.5	0	0	0	0	0	0	0	0	0	0	0
0.1	.90050	.89748	.88679	.86126	.78797	.43015	.09645	.02787	.00856	.00211	.00084	.00042					
0.2	.80388	.79824	.77884	.73483	.63014	.38269	.15433	.05251	.01680	.00419	.00167	.00083	.00048	.00020			
0.3	.71265	.70518	.68316	.62690	.52081	.34375	.17964	.07199	.02440	.00622	.00250						
0.4	.62861	.62015	.59241	.53767	.44329	.31048	.18709	.08593	.03118								
0.5	.55279	.54403	.51622	.46448	.38390	.28156	.18556	.09499	.03701	.01013	.00407	.00209	.00118	.00053	.00025	.00014	.00009
0.6	.48550	.47691	.45078	.40427	.33676	.25588	.17952	.10010									
0.7	.42654	.41874	.39491	.35428	.29833	.21727	.17124	.10228	.04558								
0.8	.37531	.36832	.34729	.31243	.26581	.21297	.16206	.10236									
0.9	.33104	.32492	.30669	.27707	.23832	.19488	.15253	.10094									
1	.29289	.28763	.27005	.24697	.21468	.17868	.14329	.09849	.05185	.01742	.00761	.00393	.00226	.00097	.00050	.00029	.00017
1.2	.23178	.22795	.21662	.19890	.17626	.15101	.12570	.09192	.05260	.01935	.00871	.00459	.00269	.00115			
1.5	.16795	.16552	.15877	.14804	.13436	.11892	.10296	.08048	.05116	.02142	.01013	.00548	.00325	.00141	.00073	.00043	.00027
2	.10557	.10453	.10140	.09647	.09011	.08269	.07471	.06275	.04496	.02221	.01160	.00659	.00399	.00180	.00094	.00056	.0003
2.5	.07152	.07098	.06947	.06698	.06373	.05974	.05555	.04880	.03787	.02143	.01221	.00732	.00463	.00214	.00115	.00068	.00042
3	.05132	.05101	.05022	.04886	.04707	.04487	.04241	.03839	.03150	.01980	.01220	.00770	.00505	.00242	.00132	.00079	.00051
4	.02986	.02976	.02907	.02802	.02832	.02749	.02651	.02490	.02193	.01592	.01109	.00768	.00536	.00282	.00160	.00099	.0006
5	.01942	.01938				.01835			.01573	.01249	.00949	.00708	.00527	.00298	.00179	.00113	.0007
6	.01361					.01307			.01168	.00983	.00795	.00628	.00492	.00299	.00188	.00124	.0008
7	.01005					.00976			.00894	.00784	.00661	.00548	.00445	.00291	.00193	.00130	.0009
8	.00772					.00755			.00703	.00635	.00554	.00472	.00398	.00276	.00189	.00134	.0009
9	.00612					.00600			.00566	.00520	.00466	.00409	.00353	.00256	.00184	.00133	.0009
10								.00477	.00465	.00438	.00397	.00352	.00326	.00241			

Function *B*

Depth (z) in Radii	Offset (r) in Radii: 0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	.09852	.10140	.11138	.13424	.18796	.05388	−.07899	−.02672	−.00845	−.00210	−.00084	−.00042					
0.2	.18857	.19306	.20772	.23524	.25983	.08513	−.07759	−.04448	−.01593	−.00412	−.00166	−.00083	−.00024	−.00010			
0.3	.28362	.26787	.28018	.29483	.27257	.10757	−.04316	−.04999	−.02166	−.00599	−.00245						
0.4	.32016	.32259	.32748	.32273	.26925	.12404	−.00766	−.04535	−.02522								
0.5	.35777	.35752	.35323	.33106	.26236	.13591	.02165	−.03455	−.02651	−.00991	−.00388	−.00199	−.00116	−.00049	−.00025	−.00014	[illegible]
0.6	.37831	.37531	.36308	.32822	.25411	.14440	.04457	−.02101									
0.7	.38487	.37962	.36072	.31929	.24638	.14986	.06209	−.00702	−.02329								
0.8	.38091	.37408	.35133	.30699	.23779	.15292	.07530	.00614									
0.9	.36962	.36275	.33734	.29299	.22891	.15404	.08507	.01795									
1	.35355	.34553	.32075	.27819	.21978	.15355	.09210	.02814	−.01005	−.01115	−.00608	−.00344	−.00210	−.00092	−.00048	[illegible]	[illegible]
1.2	.31485	.30730	.28481	.24836	.20113	.14915	.10002	.04378	.00023	−.00995	−.00632	−.00378	−.00236	−.00107			
1.5	.25602	.25025	.23338	.20694	.17368	.13732	.10193	.05745	.01385	−.00669	−.00600	−.00401	−.00265	−.00126	−.00068	−.00040	[illegible]
2	.17889	.18144	.16644	.15198	.13375	.11331	.09254	.06371	.02836	.00028	−.00410	−.00371	−.00278	−.00148	−.00084	−.00050	[illegible]
2.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.09130	.07869	.06022	.03429	.00661	−.00130	−.00271	−.00250	−.00156	−.00094	−.00059	[illegible]
3	.09487	.09394	.09099	.08635	.08033	.07325	.06551	.05354	.03511	.01112	.00157	−.00134	−.00192	[illegible]	[illegible]	[illegible]	[illegible]
4	.05707	.05666	.05562	.05383	.05145	.04773	.04532	.03995	.03066	.01515	.00595	.00155	−.00029	−.00109	−.00094	−.00068	[illegible]
5	.03772	.03760				.03384			.02474	.01522	.00810	.00371	.00132	−.00043	−.00070	−.00068	[illegible]
6	.02666					.02468			.01968	.01380	.00867	.00496	.00254	.00028	−.00037	−.00047	[illegible]
7	.01980					.01868			.01577	.01204	.00842	.00547	.00332	.00093	−.00002	[illegible]	[illegible]
8	.01526					.01459			.01279	.01034	.00779	.00554	.00372	.00141	.00035	[illegible]	[illegible]
9	.01212					.01170			.01054	.00888	.00705	.00533	.00386	.00178	.00066	[illegible]	[illegible]
10	[illegible]							.00924	.00879	.00764	.00631	.00501	.00382	.00199			

Function *C*

Depth (z) in Radii	Offset (r) in Radii: 0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	−.04926	−.05142	−.05903	−.07708	−.12108	.02247	.12007	.04475	.01536	.00403	.00164	.00082					
0.2	−.09429	−.09755	−.10872	−.12977	−.14552	.02419	.14896	.07892	.02951	.00796	.00325	.00164	.00094	.00039			
0.3	−.13181	−.13484	−.14415	−.15023	−.12990	.01988	.13394	.09816	.04148	.01169	.00483						
0.4	−.16008	−.16188	−.16519	−.15985	−.11168	.01292	.11014	.10422	.05067								
0.5	−.17889	−.17835	−.17497	−.15625	−.09833	.00483	.08730	.10125	.05690	.01824	.00778	.00399	.00231	.00098	.00050	.00029	.00018
0.6	−.18915	−.18663	−.17336	−.14934	−.08967	−.00304	.06731	.09313									
0.7	−.19244	−.18831	−.17393	−.14147	−.08409	−.01061	.05028	.08253	.06129								
0.8	−.19046	−.18481	−.16784	−.13393	−.08066	−.01744	.03582	.07114									
0.9	−.18481	−.17841	−.16024	−.12664	−.07828	−.02337	.02359	.05993									
1	−.17678	−.17050	−.15188	−.11995	−.07634	−.02843	.01331	.04939	.05429	.02726	.01333	.00726	.00433	.00188	.00098	.00057	.00036
1.2	[illegible]	[illegible]	−.13467	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			
1.5	−.12801	−.12277	−.11101	−.09145	−.06711	−.04124	−.01702	.01088	.03154	.02652	.01570	.00933	.00585	.00266	.00141	.00083	.00039
2	−.08944	−.08491	−.07976	−.06925	−.05560	−.04144	−.02687	−.00782	.01267	.02070	.01527	.01013	.00321	.00327	.00179	.00107	.00069
2.5	−.06403	−.06068	−.05839	−.05259	−.04522	−.03605	−.02800	−.01536	.00103	.01384	.01314	.00987	.00707	.00569	.00209	.00128	.00083
3	−.04744	−.04560	−.04339	−.04089	−.03642	−.03130	−.02587	−.01748	−.00528	.00792	.01030	.00888	.00689	.00392	.00232	.00145	.00096
4	−.02854	−.02737	−.02562	−.02585	−.02421	−.02112	−.01964	−.01586	−.00956	.00038	.00492	.00602	.00561	.00389	.00254	.00168	.00115
5	−.01886	−.01810				−.01568			−.00939	−.00293	−.00128	.00329	.00391	.00341	.00250	.00177	.00127
6	−.01333					−.01118			−.00819	−.00405	−.00079	.00129	.00234	.00272	.00227	.00173	.00130
7	−.00990					−.00902			−.00678	−.00417	−.00180	−.00004	.00113	.00200	.00193	.00161	.00128
8	−.00763					−.00699			−.00552	−.00393	−.00225	−.00077	.00029	.00134	.00157	.00143	.00120
9	−.00607					−.00423			−.00452	−.00353	−.00235	−.00118	−.00027	.00082	.00124	.00122	.00110
10								−.00381	−.00375	−.00314	.00233	−.00137	−.00063	.00040			

Depth (z) in Radii	Offset (r) in Radii 0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	.04926	.04998	.05235	.05716	.06687	.07635	.04108	.01803	.00691	.00193	.00080	.00041					
0.2	.09429	.09552	.09900	.10546	.11431	.10932	.07139	.03444	.01359	.00384	.00159	.00081	.00047	.00020			
0.3	.13181	.13305	.14051	.14062	.14267	.12745	.09078	.04817	.01982	.00927	.00238						
0.4	.16008	.16070	.16229	.16288	.15756	.13696	.10248	.05887	.02545								
0.5	.17889	.17917	.17826	.17481	.16403	.14074	.10894	.06670	.03039	.00921	.00390	.00200	.00116	.00049	.00025	.00015	.0000[illegible]
0.6	.18915	.18867	.18573	.17887	.16489	.14137	.11186	.07212									
0.7	.19244	.19132	.18679	.17782	.16229	.13926	.11237	.07551	.03801								
0.8	.19046	.18927	.18348	.17306	.15714	.13548	.11115	.07728									
0.9	.18481	.18349	.17709	.16635	.15063	.13067	.10866	.07788									
1	.17678	.17503	.16886	.15824	.14344	.12513	.10540	.07753	.04456	.01611	.00725	.00382	.00224	.00096	.00050	.00029	.0001[illegible]
1.2	.15742	.15618	.15014	.14073	.12823	.11340	.09757	.07484	.04575	.01796	.00835	.00446	.00264	.00114			
1.5	.12801	.12754	.12237	.11549	.10657	.09608	.08491	.06833	.04539	.01983	.00970	.00532	.00320	.00140	.00073	.00043	.0002[illegible]
2	.08944	.09080	.08668	.08273	.07814	.07187	.06566	.05589	.04103	.02098	.01117	.00643	.00398	.00179	.00095	.00056	.0003[illegible]
2.5	.06403	.06565	.06284	.06068	.05777	.05525	.05069	.04486	.03532	.02045	.01183	.00717	.00457	.00213	.00115	.00068	.0004[illegible]
3	.04744	.04834	.04760	.04548	.04391	.04195	.03963	.03606	.02983	.01904	.01187	.00755	.00497	.00242	.00133	.00080	.0005[illegible]
4	.02854	.02928	.02996	.02798	.02724	.02661	.02568	.02408	.02110	.01552	.01087	.00757	.00533	.00280	.00160	.00100	.0006[illegible]
5	.01886	.01950				.01816			.01535	.01230	.00939	.00700	.00523	.00299	.00180	.00114	.0007[illegible]
6	.01333					.01351			.01149	.00976	.00788	.00625	.00488	.00301	.00190	.00124	.0008[illegible]
7	.00990					.00966			.00899	.00787	.00662	.00542	.00445	.00292	.00192	.00130	.0009[illegible]
8	.00763					.00759			.00727	.00641	.00554	.00477	.00402	.00275	.00192	.00131	.0009[illegible]
9	.00607					.00746			.00601	.00533	.00470	.00415	.00358	.00260	.00187	.00133	.0009[illegible]
10								.00542	.00506	.00450	.00398	.00364	.00319	.00239			

Function *E*

Depth (z) in Radii	Offset (r) in Radii 0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	.5	.5	.5	.5	.5	.5	.34722	.22222	.12500	.05556	.03125	.02000	.01389	.00781	.00500	.00347	.00255
0.1	.45025	.44949	.44698	.44173	.43008	.39198	.30445	.20399	.11806	.05362	.03045	.01959					
0.2	.40194	.40043	.39591	.38660	.36798	.32802	.26598	.18633	.11121	.05170	.02965	.01919	.01342	.00762			
0.3	.35633	.35428	.33809	.33674	.31578	.28003	.23311	.16967	.10450	.04979	.02886						
0.4	.31431	.31214	.30541	.29298	.27243	.24200	.20526	.15428	.09801								
0.5	.27639	.27407	.26732	.25511	.23639	.21119	.18168	.14028	.09180	.04608	.02727	.01800	.01272	.00734	.00475	.00332	.00246
0.6	.24275	.24247	.23411	.22289	.20634	.18520	.16155	.12759									
0.7	.21327	.21112	.20535	.19525	.18093	.16356	.14421	.11620	.08027								
0.8	.18765	.18550	.18049	.17190	.15977	.14523	.12928	.10602									
0.9	.16552	.16337	.159[illegible]1	.15179	.14168	.12954	.11634	09686									
1	.14645	.14483	.14610	.13472	.12618	.11611	.10510	.08865	.06552	.03736	.02352	.01602	.01157	.00683	.00450	.00318	.00237
1.2	.11589	.11435	.11201	.10741	.10140	.09431	.08657	.07476	.05728	.03425	.02208	.01527	.01113	.00664			
1.5	.08398	.08356	.08159	.07885	.07517	.07088	.06611	.05871	.04703	.03003	.02008	.01419	.01049	.00636	.00425	.00304	.00228
2	.05279	.05105	.05146	.05034	.04850	.04675	.04442	.04078	.03454	.02410	.01706	.01248	.00943	.00590	.00401	.00290	.00219
2.5	.03576	.03426	.03489	.03435	.03360	.03211	.03150	.02953	.02599	.01945	.01447	.01096	.00850	.00546	.00378	.00276	.00210
3	.02566	.02519	.02470	.02491	.02444	.02389	.02330	.02216	.02007	.01585	.01230	.00962	.00763	.00505	.00355	.00263	.00201
4	.01493	.01452	.01495	.01526	.01446	.01418	.01395	.01356	.01281	.01084	.00900	.00742	.00612	.00431	.00313	.00237	.00185
5	.00971	.00927				.00929			.00873	.00774	.00673	.00579	.00495	.00364	.00275	.00213	.00168
6	.00680					.00632			.00629	.00574	.00517	.00457	.00404	.00309	.00241	.00192	.00154
7	.00503					.00493			.00466	.00438	.00404	.00370	.00330	.00264	.00213	.00172	.00140
8	.00386					.00377			.00354	.00344	.00325	.00297	.00273	.00228	.00185	.00155	.00127
9	.00306					.00227			.00275	.00273	.00264	.00246	.00229	.00194	.00163	.00139	.00116
10								.00210	.00220	.00225	.00221	.00203	.00200	.00171			

Function *F*

Depth (z) in Radii	Offset (r) in Radii																
	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	.5	.5	.5	.5	.5	0	-.34722	-.22222	-.12500	-.05556	-.03125	-.02000	-.01389	-.00781	-.00500	-.00347	-.00255
0.1	.45025	.44794	.43981	.41954	.35789	.03817	-.20800	-.17612	-.10950	-.05151	-.02961	-.01917					
0.2	.40194	.39781	.38294	.34823	.26215	.05466	-.11165	-.13381	-.09441	-.04750	-.02798	-.01835	-.01295	-.00742			
0.3	.35633	.35094	.34508	.29016	.20503	.06372	-.05346	-.09768	-.08010	-.04356	-.02536						
0.4	.31431	.30801	.28681	.24469	.17086	.06848	-.01818	-.06835	-.06684								
0.5	.27639	.26997	.24890	.20937	.14752	.07037	.00388	-.04529	-.05479	-.03595	-.02320	-.01590	-.01154	-.00681	-.00450	-.00318	-.00237
0.6	.24275	.23444	.21667	.18138	.13042	.07068	.01797	-.02749									
0.7	.21327	.20762	.18956	.15903	.11740	.06963	.02704	-.01392	-.03469								
0.8	.18765	.18287	.16679	.14053	.10604	.06774	.03277	-.00365									
0.9	.16552	.16158	.14747	.12528	.09664	.06533	.03619	.00408									
1	.14645	.14280	.12395	.11225	.08850	.06256	.03819	.00984	-.01367	-.01994	-.01591	-.01209	-.00931	-.00587	-.00400	-.00289	-.00219
1.2	.11589	.11360	.10460	.09449	.07486	.05670	.03913	.01716	-.00452	-.01491	-.01337	-.01068	-.00844	-.00550			
1.5	.08398	.08196	.07719	.06918	.05919	.04804	.03686	.02177	.00413	-.00879	-.00995	-.00870	-.00723	-.00495	-.00353	-.00261	-.00201
2	.05279	.05348	.04994	.04614	.04162	.03593	.03029	.02197	.01043	-.00189	-.00546	-.00589	-.00544	-.00410	-.00307	-.00233	-.00183
2.5	.03576	.03673	.03459	.03263	.03014	.02762	.02406	.01927	.01188	.00198	-.00226	-.00364	-.00386	-.00332	-.00263	-.00208	-.00166
3	.02566	.02586	.02255	.02395	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4	.01493	.01536	.01412	.01259	.01386	.01331	.01256	.01134	.00912	.00508	.00209	.00026	-.00076	-.00148	-.00153	-.00137	-.00120
5	.00971	.01011				.00905			.00700	.00475	.00277	.00129	.00031	-.00066	-.00096	-.00099	-.00093
6	.00680					.00675			.00538	.00409	.00278	.00170	.00088	-.00010	-.00053	-.00066	-.00070
7	.00503					.00483			.00428	.00346	.00258	.00178	.00114	.00027	-.00020	-.00041	-.00049
8	.00386					.00380			.00350	.00291	.00229	.00174	.00125	.00048	.00003	-.00020	-.00033
9	.00306					.00374			.00291	.00247	.00203	.00163	.00124	.00062	.00020	-.00005	-.00019
10								.00267	.00246	.00213	.00176	.00149	.00126	.00070			

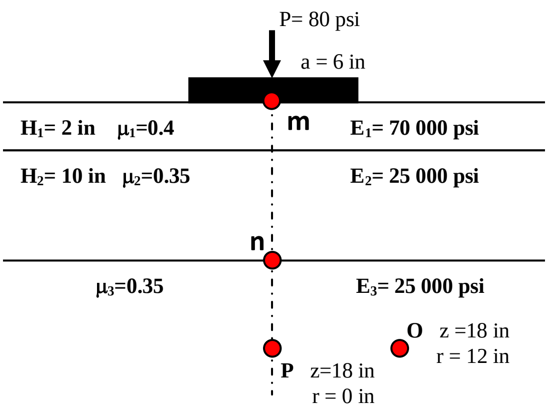
Function *G*

Depth (z) in Radii	Offset (r) in Radii																
	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	0	0	0	0	0	.31831	0	0	0	0	0	0	0	0	0	0	0
0.1	0	.00315	.00802	.01951	.06682	.31405	.05555	.00865	.00159	.00023	.00007	.00003					
0.2	0	.01163	.02877	.06441	.16214	.30474	.13592	.03060	.00614	.00091	.00026	.00010	.00005	.00002			
0.3	0	.02301	.05475	.11072	.21465	.29228	.18216	.05747	.01302	.00201	.00059						
0.4	0	.03460	.07883	.14477	.23442	.27779	.20195	.08233	.02138								
0.5	0	.04429	.09618	.16426	.23652	.26216	.20731	.10185	.03033	.00528	.00158	.00063	.00030	.00009	.00004	.00002	.00001
0.6	0	.04966	.10729	.17192	.22949	.24574	.20496	.11541									
0.7	0	.05484	.11256	.17126	.21772	.22924	.19840	.12373	.04718								
0.8	0	.05590	.11225	.16534	.20381	.21295	.18953	.12855									
0.9	0	.05496	.10856	.15628	.18904	.19712	.17945	.28881									
1	0	.05266	.10274	.14566	.17419	.18198	.16884	..2745	.06434	.01646	.00555	.00233	.00113	.00036	.00015	.00007	.00004
1.2	0	.04585	.08831	.12323	.14615	.15408	.14755	.12038	.06967	.02077	.00743	.00320	.00159	.00051			
1.5	0	.03483	.06688	.09293	.11071	.11904	.11830	.10477	.07075	.02599	.01021	.00460	.00233	.00078	.00033	.00016	.00009
2	0	.02102	.04069	.05721	.06948	.07738	.08067	.07804	.06275	.03062	.01409	.00692	.00369	.00129	.00055	.00027	.00015
2.5	0	.01293	.02534	.03611	.04484	.05119	.05509	.05668	.05117	.03099	.01650	.00886	.00499	.00185	.00082	.00041	.00023
3	0	.00840	.01638	.02376	.02994	.03485	.03843	.04124	.04039	.02886	.01745	.01022	.00610	.00241	.00110	.00057	.00032
4	0	.00382	.00772	.01149	.01480	.01764	.02004	.02271	.02475	.02215	.01639	.01118	.00745	.00340	.00167	.00090	.00052
5	0	.00214				.00992		.01343	.01551	.01601	.01364	.01105	.00782	.00404	.00216	.00122	.00073
6	0					.00602		.00845	.01014	.01148	.01082	.00917	.00733	.00432	.00243	.00150	.00092
7	0					.00396			.00687	.00830	.00842	.00770	.00656	.00432	.00272	.00171	.00110
8	0					.00270			.00481	.00612	.00656	.00631	.00568	.00413	.00278	.00185	.00124
9	0					.00177			.00347	.00459	.00513	.00515	.00485	.00381	.00274	.00192	.00133
10	0							.00199	.00258	.00351	.00407	.00420	.00411	.00346			

Function H

Depth (z) in Radii	Offset (r) in Radii 0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	2.0	1.97987	1.91751	1.80575	1.62553	1.27319	.93676	.71185	.51671	.33815	.25200	.20045	.16626	.12576	.09918	.08346	.07023
0.1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]					
0.2	1.63961	1.62068	1.56242	1.46001	1.30614	1.09996	.90098	.70074	.51382	.33726	.25162	.20072	.16683	.12512			
0.3	1.48806	1.47044	1.40979	1.32442	1.19210	1.02740	.86726	.68823	.50966	.33638	.25124						
0.4	1.35407	1.33802	1.28963	1.20822	1.09555	.96202	.83042	.67238	.50412								
0.5	1.23607	1.22176	1.17894	1.10830	1.01312	.90298	.79308	.65429	.49728	.33293	.24996	.19982	.16668	.12493	.09996	.08295	.07123
0.6	1.13238	1.11998	1.08350	1.02154	.94120	.84917	.75653	.63469									
0.7	1.04131	1.03037	.99794	.91049	.87742	.80030	.72143	.61442	.48061								
0.8	.96125	.95175	.92386	.87928	.82136	.75571	.68809	.59398									
0.9	.89072	.88251	.85856	.82616	.77950	.71495	.65677	.57361									
1	.82843	.85005	.80465	.76809	.72587	.67769	.62701	.55364	.45122	.31877	.24386	.19673	.16516	.12394	.09952	.08292	.07104
1.2	.72410	.71882	.70370	.67937	.64814	.61187	.57329	.51552	.43013	.31162	.24070	.19520	.16369	.12350			
1.5	.60555	.60233	.57246	.57633	.55559	.53138	.50496	.46379	.39872	.29945	.23495	.19053	.16199	.12281	.09876	.08270	.07064
2	.47214	.47022	.44512	.45656	.44502	.43202	.41702	.39242	.35054	.27740	.22418	.18618	.15846	.12124	.09792	.08196	.07026
2.5	.38518	.38403	.38098	.37608	.36940	.36155	.35243	.33698	.30913	.25550	.21208	.17898	.15395	.11928	.09700	.08115	.06980
3	.32457	.32403	.32184	.31887	.31464	.30969	.30381	.29364	.27453	.23487	.19977	.17154	.14919	.11694	.09558	.08061	.06897
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	.19805	.19785				.19455			.18450	.17080	.15575	.14130	.12785	.10585	.08915	.07675	.06695
6	.16554					.16326			.15750	.14868	.13842	.12792	.11778	.09990	.08562	.07452	.06522
7	.14217					.14077			.13699	.13097	.12404	.11620	.10843	.09387	.08197	.07210	.06377
8	.12448					.12352			.12112	.11680	.11176	.10600	.09976	.08848	.07800	.06928	.06200
9	.11079					.10989			.10854	.10548	.10161	.09702	.09234	.08298	.07407	.06678	.05976
0								.09900	.09820	.09510	.09290	.08980	.08300	.07710			

Example:- for the flexible pavement shown in following figure calculate the following values



a) Δ at point (m).
b) σ_z and ε_t at point o.
c) $\sigma_{1,2,3}$ and τ_{max} at point p .

Solution:-

$$\Delta T = \Delta P + \Delta S \qquad \Delta P = 0$$

$$\Delta m = \Delta n$$

For p=80 psi , $E_{subgrade}$ =16000 psi , μ = 0.40 , a=6 in , z=12 in

$$\frac{z}{a} = \frac{12}{6} = 2 \qquad \frac{r}{a} = \frac{0}{6} = 0$$

From above table $\Delta m = \Delta n = \frac{p(1+\mu)a}{E1}[\frac{z}{a}A + (1-\mu)H]$

From functions table A= 0.10557 H=0.47214

Δ m =0.021 in / in

For z =18 in z / a= 3 and r=12 in r / a = 2

From equations table $\sigma_z = p.[A+B]$ $\varepsilon_r = \frac{p(1+\mu)}{E_1}[(1-2\mu)F + C]$

From table A= 0.0315 C=-0.00523 B=0.03511 F=0.01144

$$\sigma_z = 5.3 psi \qquad \varepsilon_r = -20.9 * 10^{-6} in / in$$

Because of symmetry about the plate centerline ($\sigma_t = \sigma_r - -\tau_{rz} = 0$

$$\sigma_t = \sigma_r = p[2\mu A - D + (1-2\mu)E]$$

$$\sigma_z = p[A+B]$$

$$\tau_{rz} = pG$$

$$\sigma_{1,2,3} = \frac{(\sigma_z + \sigma_r) \pm \sqrt{(\sigma_z - \sigma_r)^2 + (2\tau_{rz})^2}}{2}$$

$$\tau_{max} = \frac{\sigma_1 + \sigma_2}{2}$$

From table: - A=0.05132, D=0.04744, G=0, B=0.09487, E= 0.02566

$$\sigma_t = \sigma_r = -0.1 psi$$
$$\sigma_z = 11.7 psi$$
$$\tau_{rz} = 0$$
$$\sigma_{1,2,3} = 11.7 psi$$
$$\tau_{\max} = 5.9 psi$$

Burmister Theory (Two-Layer System)

Burmister, gives effect of material properties used in pavement much attention on the stress, strain and deflection distribution.

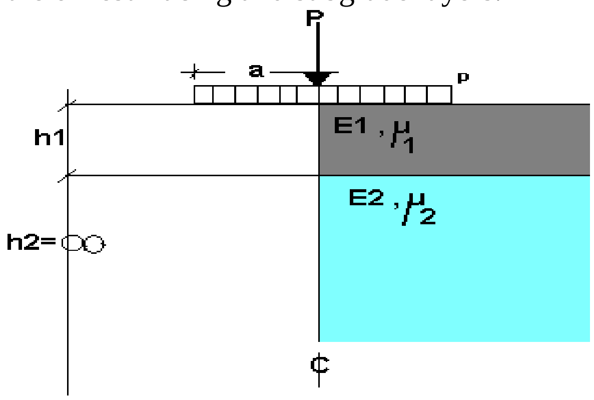
The materials in the layers are assumed (homogeneous, isotropic, linearity elastic and the surface layer is assumed to be infinite in extent in lateral direction but of finite depth.

المواد في كل طبقة تفرض بانة متجانس ومرنة والاجهادات المسلطة عليها تتوزع بصورة متساوية وسطح الطبقة بفرض انة غير محدد على امتداد الاتجاة الجانبي و محدد العمق.

Full frication is developed at the interface (نفرض حدوث تتداخل تام بين الطبقات)

Burmister, represented that stresses and deflection values are dependent upon the strength ratio (E_1 / E_2) of layers or Modular ratio where the E_1, E_2 are the module of resurfacing and subgrade layers.

في معادلة بوسنك لا يعطي اهمية لقوة المواد المستخدمة وهذا غير اقتصادي اما برمستر فقد اعطى اهمية كبيرة لقوة المواد المستخدمة حيث يقل السمك كلما ازدادت قوة المواد المستخدمة و بالتالي تقليل الاجهادات المسلطة على سطح التربة.

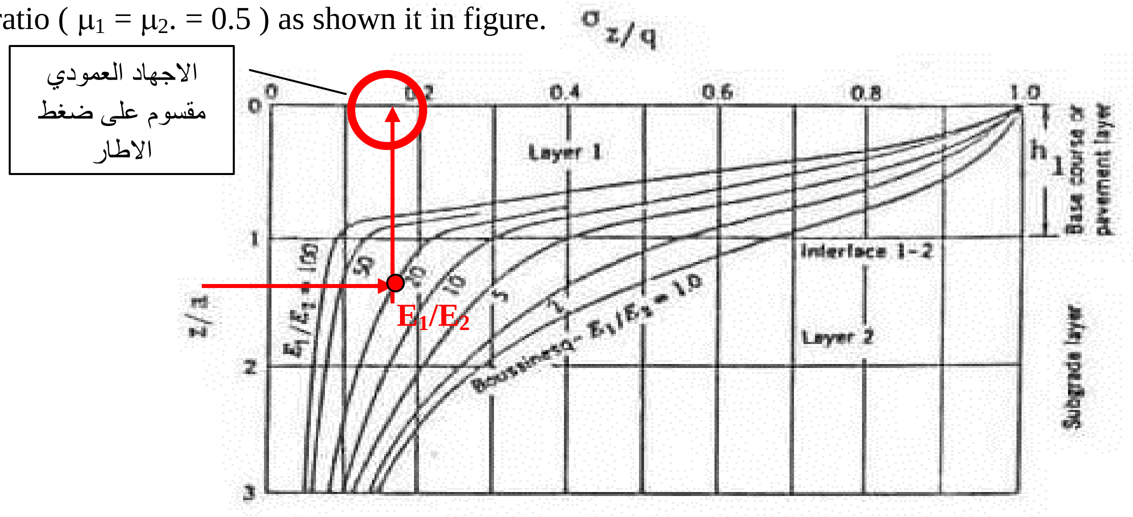


1. Vertical stress:

The vertical stress on the top of subgrade is an important factor in pavement design. The function of a pavement is to reduce the vertical stress on the subgrade so that detrimental pavement deformations will not occur. This stress depends on the strength or modulus of the subgrade.

الاجهاد العمودي فوق طبقة ماتحت الاساس عامل مهم في تصميم التبليط حيث ان وظيفة التبليط هي تقليل هذا الاجهاد وبالتالي تقليل التشوة و هذا الاجهاد يعتمد على معامل مرونة طبقة ماتحت الاساس. قيمة الاجهات في نظام الطبقتين يعتمد على نسبة معامل المرونة و نسبة السمك الى نصف قطر الحمل المسلط مع ثبات نسبة بوسان.

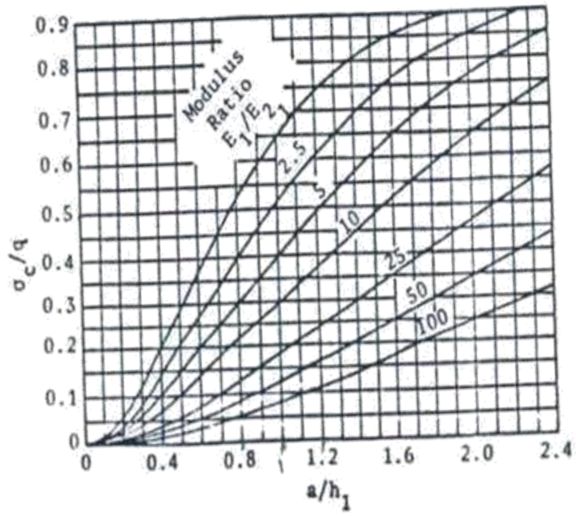
The stresses in two-layer system depend on the modulus ratio (E_1/E_2) and thickness-ratio (h_1/a) and Poisson's ratio ($\mu_1 = \mu_2 = 0.5$) as shown it in figure.



Vertical stress distribution in two-layer system (after Burmister)

Huang , show the effect of pavement thickness and modulus on the vertical stress (σ_z) at the pavement subgrade interface under the center of a circular loaded area for a given applied pressure. The vertical stress increases with the increase in contact radius and decreases with the increase in thickness as shown in figure.

هونك بين تاثير سمك التبليط و معامل المرونة على الاجهادات الشاقولية (حيث نلاحظ ان الاجهادات الشاقولية تزداد بزيادة مساحة التماس ونصف قطر الحمل المسلط ويقل بزيادة السمك



Vertical interface stresses for two-layer system (after Huang)

2. Vertical Surface Deflection : الهبوط السطحي

عند تسليط حمل سوف يحدث تشوة بالتبليط او التربة او يسبب هبوط وهذا الهبوط يكون على نوعين :-

a) Flexible loading (the load applied from tire to pavement is similar to a flexible plate with a radius (a) and a uniform pressure (p) p الحمل المسلط من الاطار على التبليط يكون مشابة للوحة مرنة بنصف a وضغط منتظم قطر same load on all point but different deflection

Deflection at center of circular tire pavement contact area $\Delta_T = 1.5 * F_2 * \frac{p*a}{E_2}$

where :- p = unit load on circular plate.

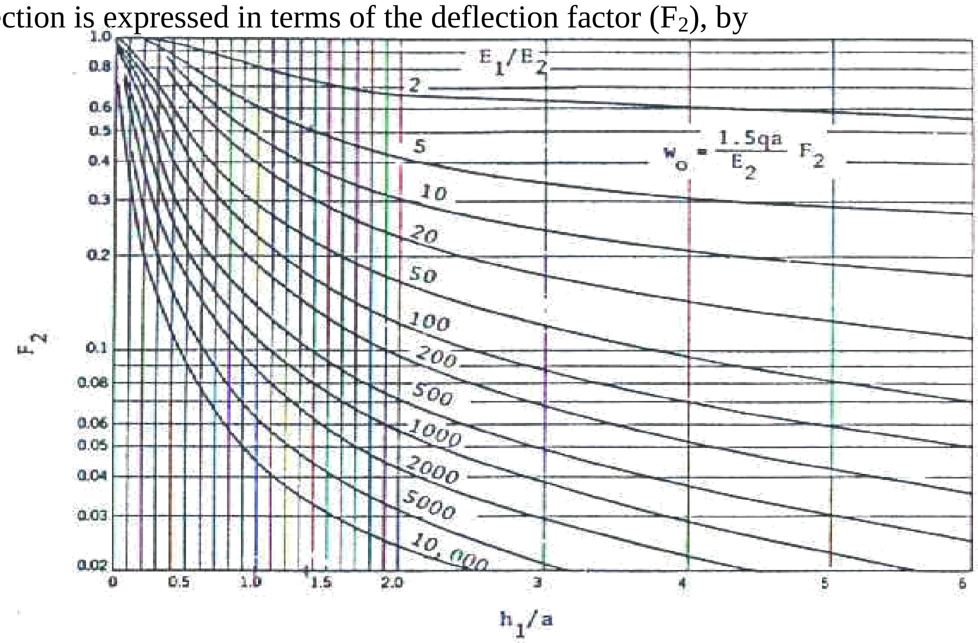
a = radius of circular plate.

E_2 = Modules of elasticity of lower layer.

F_2 = Dimensionless surface deflection factor at center of applied load dependences on E_1/E_2 and h/a

Burmister, presented graphical solution for determination of vertical surface deflection (as shown in figure) for two-layer system. The deflection is expressed in terms of the deflection factor (F_2), by

برمستر بين كيفية حساب الهبوط في الطبقة السطحية في نظام الطبقتين من الشكل التالي الذي يمثل بمعامل الهبوط السطحي F_2



Vertical Surface Deflection for two-layer systems. (After Burmister 1958)

The deflection factor is a function of (E_1/E_2 and (h_1/a), can be calculated

$$F2 = \frac{E2}{E1}\int_0^{\infty} J0\left(\frac{mr}{h}\right)J1\left(\frac{ma}{h}\right)\frac{\left[1+4Nme^{-2m} * N^2e^{-4m}\right] * dm/m}{1-2N\left(1+2m^2\right)e^{-2m}+N^2e^{-4m}}$$

Where m= a parameter $N = \frac{E_1 - E_2}{E_1 + E_2} = \frac{E_1/E_2 - 1}{E_1/E_2 + 1}$

Where E_1= average Modulus of elasticity of first pavement layer.
E_1 / E_2= Modular ratio.

Jo = Bessel function of the 1st kind, order zero.

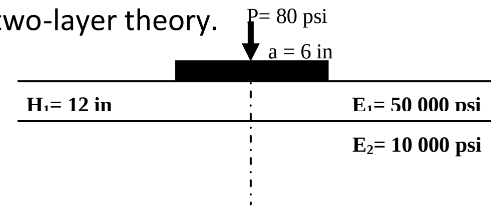
$$J_0 = \left(\frac{mr}{h}\right)\sum_{K=0}^{\infty}(-1)^K\left(\frac{mr}{2h}\right)^{2K} \Big/ (K!)^2$$

J_1= Bessel function of the 1st kind order one

$$J_1 = \left(\frac{ma}{h}\right)\sum_{K=0}^{\infty}(-1)^K\left(\frac{ma}{h}\right)^{(2K+1)} \Big/ \left(K!(K+1)!\right) \quad و \quad Jn(x) = X^N * \sum_{m=0}^{\infty}\frac{(-1)^m * X^{2m}}{2^{(2m+n)} * m!(n+m)!}$$

For a homogeneous half-space with $\frac{h_1}{a}$ = 0 the value of F_2=1 , this equation used when μ=0.5.

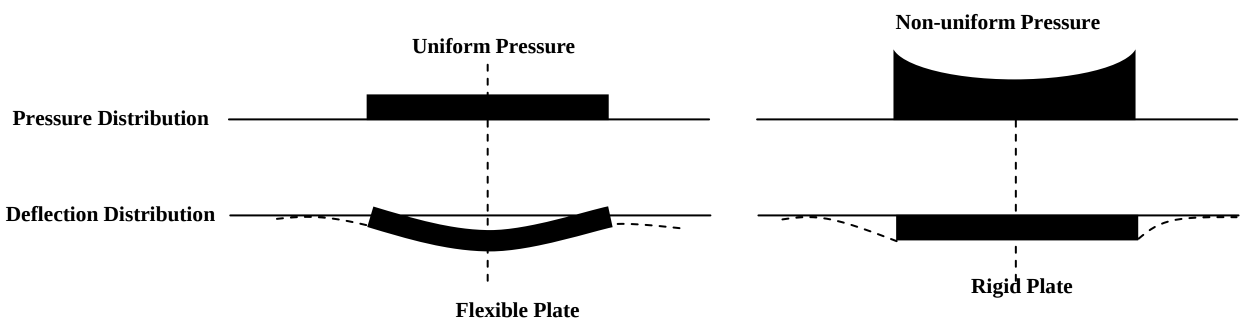
<u>Example :-</u> calculate the surface deflection under the center of a tire (a=6 in, p=80 psi) for a 12 in pavement having a 50000 psi modulus and subgrade modulus of 10000 psi from two-layer theory.



<u>Solution:-</u> from above figure F_2=0.42

$$\Delta_T = 1.5 * 0.42 * \tfrac{80*6}{10000} = 0.03in$$

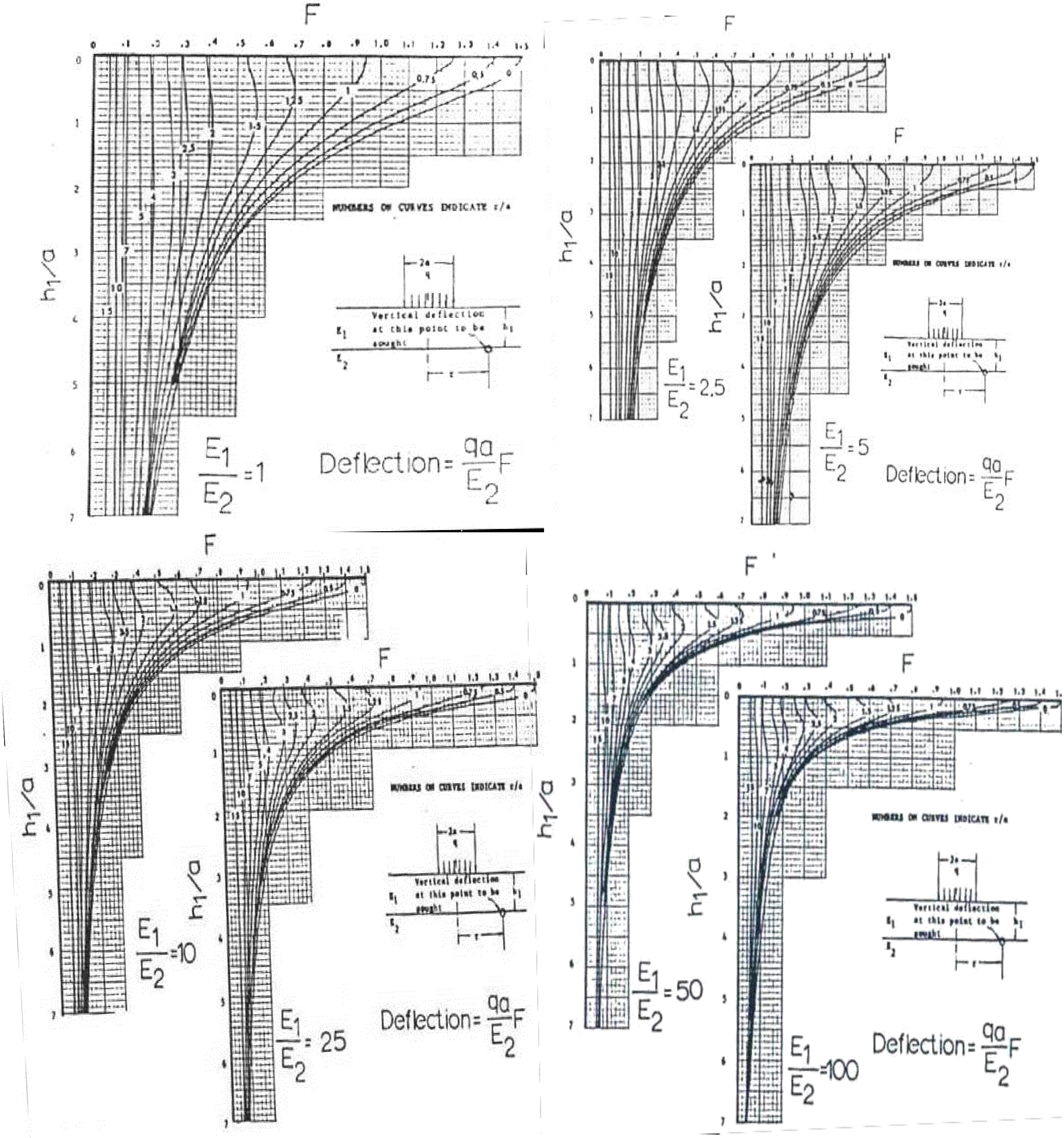
<u>b) Rigid loading</u> (all the above analyses are based on the assumption that the load is applied on a flexible plate such as a rubber tire. if the load is applied on a rigid plate such as that used in plate loading test, the deflection is the same at all points on the plate, but the pressure distribution under the plate is not uniform. The differences between a flexible and a rigid plate are shown in the following figure;



3. Vertical Interface Deflection:

Huang, presented a graphical solution for the determination of the vertical interface deflection as shown in following figure, the deflection is expressed in terms of the interface deflection factor (F) by

$\Delta_s = F * \frac{p*a}{E_2}$ (F) -- factor is function of (h/a , E_1/E_2, r/a)



Vertical Interface deflection for two layer Systems (after Huang 1969)

Example: - Calculate the interface deflection, and the deflection that takes place within the pavement layer for the information noted previous example.

Solution:- $E_1/E_2=5$, $E_2=10000$ psi , r=0 , z=12 in , p=80 psi , a= 6 in)
z/a = 12/6 = 2 the F = 0.48

$$\Delta_s = 0.48 * \frac{80*6}{10000} = 0.023 in$$

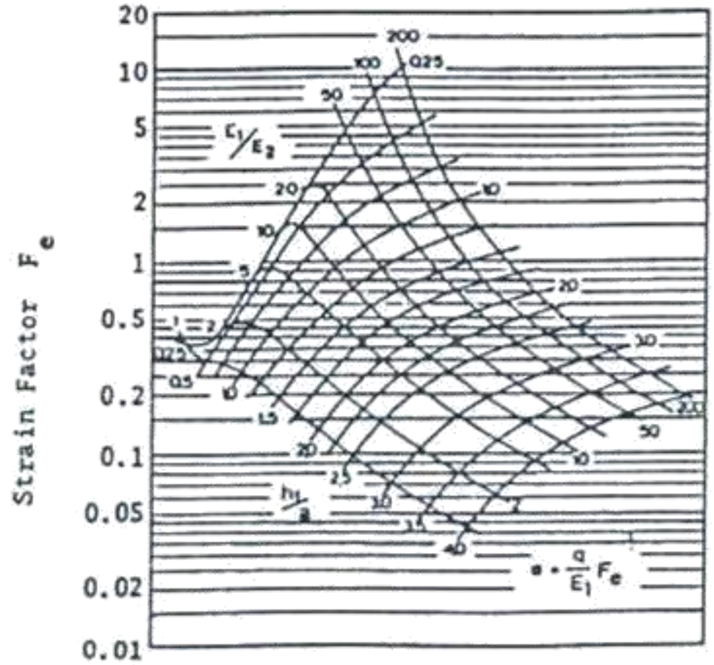
$$\Delta_T = \Delta_p + \Delta_s \Rightarrow \Delta_T - \Delta_s = \Delta_p \Rightarrow \Delta p = 0.007 in$$

4. **Critical Tensile Strain:** تشوة الشد الحرج

Huang, developed charts for determining the critical tensile strain at the bottom of layer one for a two – layer system as shown in following figure.

$$e = F_e * \frac{p}{E_1}$$

Where e = Critical tensile strain
F_e = Tensile strain factor.

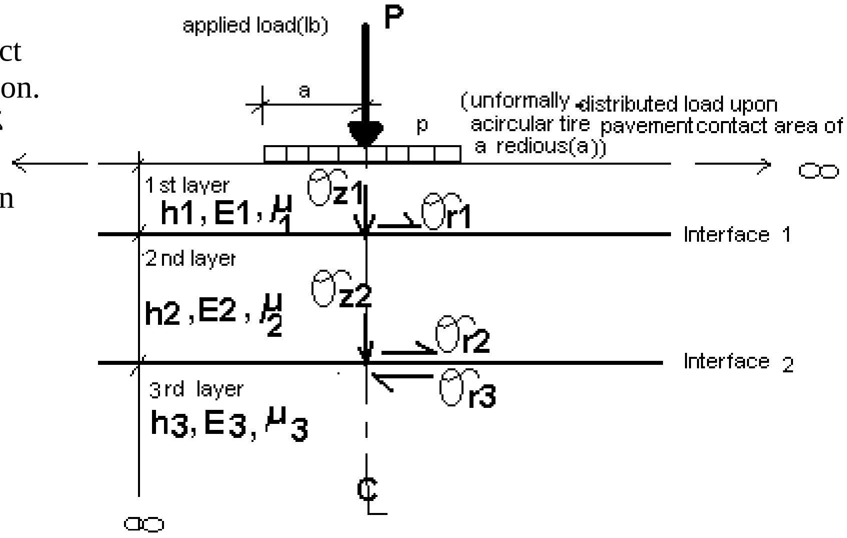


Multi-Layer System (Burmister's 3-layers system).

Following Figure. Shows a three-layer system and stresses at the interfaces on the axis of symmetry.

Assumptions

1) each layer has infinite in depth expect lower layer & finite in lateral direction.
كل طبقة محددة السمك عدا الطبقة الاخيرة فانة غير محددة السمك ∞
2) full frication is development between layer at each interface.
نفرض حدوث تتداخل كامل بين الطبقات اي يوجد احتكاك بينهما



Three –layer system.

Where:

σ_{z1} = Vertical stress at center of load at interface (1).
σ_{z2} = Vertical stress at center of load at interface (2).
σ_{r1} = Horizontal stress at center of load at the bottom of 1st Layer.
σ_{r2} = Horizontal stress at center of load at the bottom of 2nd layer.
σ_{r3} = Horizontal stress at center of load at the top of 3rd layer.

Fox* and *Acum, produced the first excusive tabular summary of normal and radial stresses in three-layer systems at the intersection of the plate axis with the layer interfaces.

Jones* and *Peattie, subsequently expanded these solutions to a much wider range of solution parameters. The details are presented in the following figure and tables.

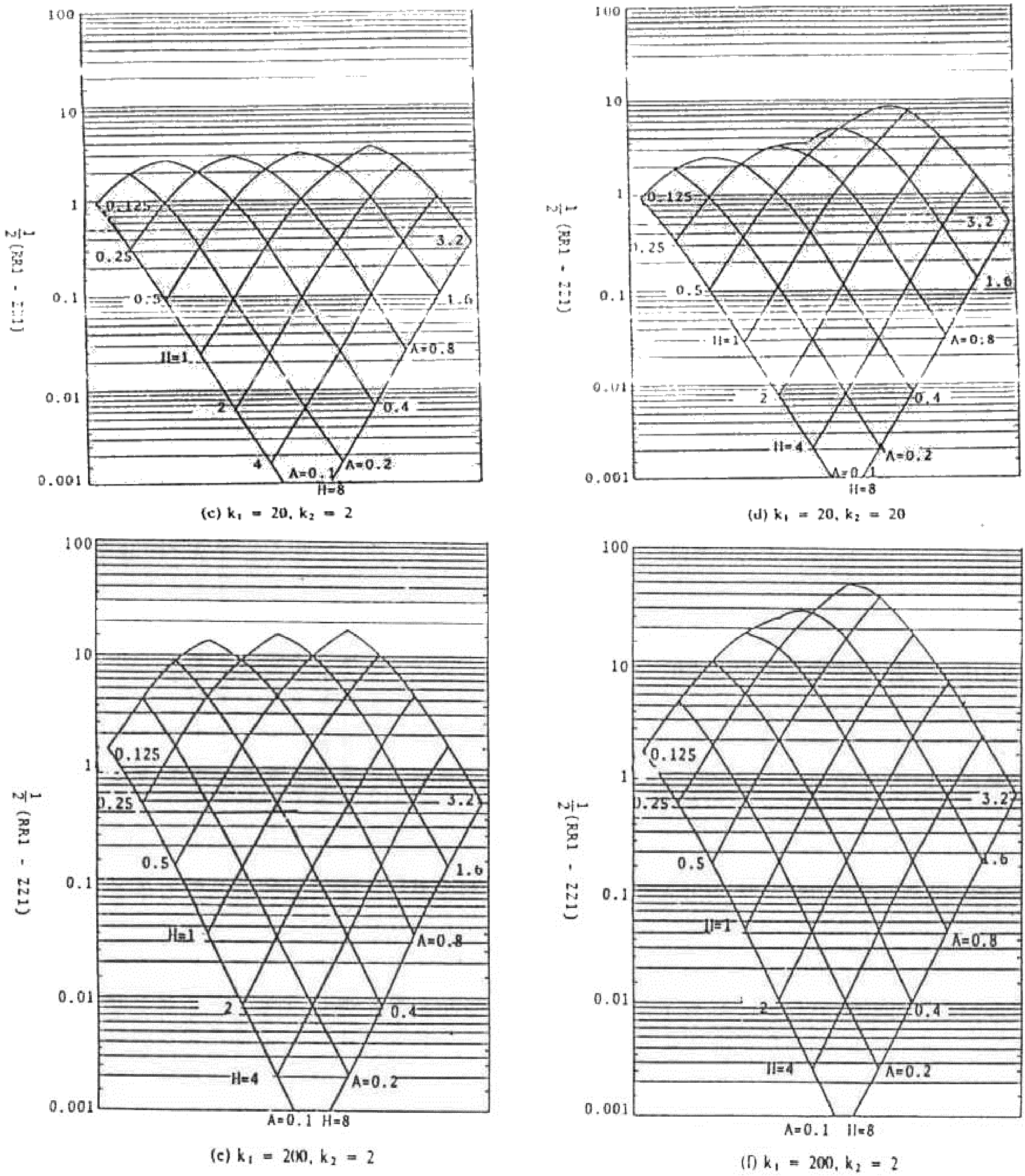
Parameters

1) $\mu_1 = \mu_2 = \mu_3 = 0.5$.
2) Geometry condition and Modular ratio :-
   a) Modular ratio $k_1=K_1= E_1/E_2$ $k_2=K_2=E_2/E_3$
   b) Geometry ratio $a_1=A=a/h_2$ $H=h_1/h_2$
3) Values of the parameters :- $k_1=K_1=0.2, 2, 20, 200$
   $k_2=K_2=0.2, 2, 20, 200$
   $a_1=A=0.1, 0.2, 0.4, 0.8, 1.6, 3.2$
   $H=0.125, 0.25, 0.5, 1.0, 2.0, 4.0, 8.0$
4) Stresses
   a) Vertical stress $\sigma_{z1}= p\,(ZZ1)$
   $\sigma_{z2} = p\,(ZZ2)$
   ZZ1 :- vertical stress factor σ_{z1}
   ZZ2 :- vertical stress factor σ_{z2}
   b) Horizontal stress $\sigma_{z1}- \sigma_{r1}= p\,(ZZ1-RR1)$
   $\sigma_{z2}- \sigma_{r2}= p\,(ZZ2- RR2)$
   $\sigma_{z3} - \sigma_{r3} = p\,(ZZ1-RR3)$ ($\sigma_{z2}= \sigma_{z3}$, ZZ2=ZZ3)
5) Strains

$$\varepsilon_{r1} = \frac{1}{E_1}[\sigma_{r1} - \mu_1(\sigma_{t1} + \sigma_{z1})] \quad \text{at center line } (\sigma_{t1} = \sigma_{r1})$$

$$\varepsilon_{r1\,\text{at}\,\not\in} = \frac{1}{E_1}[\sigma_{r1} - \mu_1\sigma_{t1} - \mu_1\,\sigma_{z1})] \quad for\ \mu = 0.5$$

$$\varepsilon_{r1\,\text{at}\,\not\in} = \frac{1}{E_1}[\sigma_{r1} - 0.5\,\sigma_{t1} - 0.5\,\sigma_{z1})] = \frac{1}{2E_1}[\sigma_{r1} - \sigma_{z1})]$$



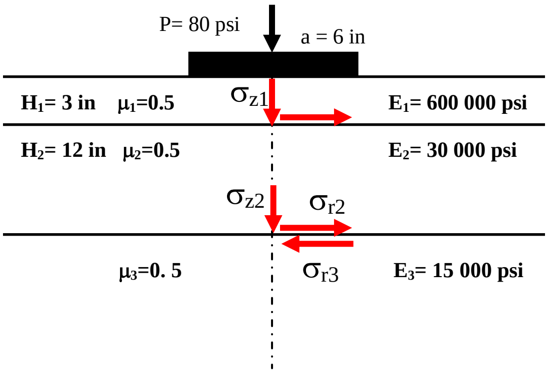
Three-Layer Stress Factors. (After Peattie 1962)

H	k_2	A	$k_1 = 2$ ZZ1	$k_1 = 2$ ZZ2	$k_1 = 2$ (ZZ1 − RR1)	$k_1 = 2$ (ZZ2 − RR2)	$k_1 = 20$ ZZ1	$k_1 = 20$ ZZ2	$k_1 = 20$ (ZZ1 − RR1)	$k_1 = 20$ (ZZ2 − RR2)	$k_1 = 200$ ZZ1	$k_1 = 200$ ZZ2	$k_1 = 200$ (ZZ1 − RR1)	$k_1 = 200$ (ZZ2 − RR2)
0.125	2	0.1	0.42950	0.00896	0.70622	0.01716	0.14529	0.00810	1.81178	0.01542	0.03481	0.00549	3.02259	0.00969
		0.2	0.78424	0.03493	0.97956	0.06647	0.38799	0.03170	3.76886	0.06003	0.11491	0.02167	8.02452	0.03812
		0.4	0.98044	0.12667	0.70970	0.23531	0.78651	0.11650	5.16717	0.21640	0.33218	0.08229	17.64175	0.14286
		0.8	0.99434	0.36932	0.22319	0.63003	1.02218	0.34941	3.43631	0.60493	0.72695	0.27307	27.27701	0.45208
		[illegible]	[illegible]	[illegible]	−0.19982	0.97707	0.99860	0.69014	1.15211	0.97146	1.00203	0.63916	23.38638	0.90861
		3.2	0.99922	0.96148	−0.28916	0.84030	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	20	[illegible]	[illegible]	0.00228	0.69332	0.03467	0.14447	0.00182	1.80664	0.02985	0.03336	0.00128	3.17763	0.01980
		0.2	0.78414	0.00899	0.92086	0.13541	0.38469	0.00716	3.74573	0.11697	0.10928	0.00509	8.66097	0.07827
		0.4	[illegible]	0.03392	0.46583	0.49523	0.77394	0.02710	5.05489	0.43263	0.31094	0.01972	20.12259	0.29887
		0.8	[illegible]	0.11350	−0.66535	1.49612	0.98610	0.08861	2.92533	1.33736	0.65934	0.07045	36.29943	1.01694
		1.6	[illegible]	0.31263	−2.82859	3.28512	0.93712	0.24528	−1.27093	2.99215	0.87931	0.20963	49.40857	2.64313
		3.2	0.98591	0.68433	−5.27906	5.05952	0.96330	0.55490	−7.35384	5.06489	0.93309	0.49938	57.84369	4.89895
0.25	2	0.1	0.15524	0.00710	0.28362	0.01353	0.04381	0.00530	0.63215	0.00962	0.00909	0.00259	0.96553	0.00407
		0.2	0.42809	0.02783	0.70225	0.05278	0.14282	0.02091	1.83766	0.03781	0.03269	0.01027	3.10763	0.01611
		0.4	0.77939	0.10306	0.96634	0.19178	0.37882	0.07933	3.86779	0.14159	0.10684	0.04000	8.37852	0.06221
		0.8	0.96703	0.31771	0.66885	0.55211	0.75904	0.26278	5.50796	0.44710	0.30477	0.14513	18.95534	0.21860
		[illegible]	[illegible]	[illegible]	0.17331	0.95080	0.98743	0.61673	4.24281	0.90115	0.66786	0.42940	31.18909	0.58553
		3.2	0.99840	0.93798	−0.05691	0.89390	1.00064	0.91258	1.97494	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	20	0.1	0.15136	0.00179	0.25780	0.02728	0.04236	0.00123	0.65003	0.01930	0.00776	0.00065	1.08738	0.00861
		0.2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.07623	0.02741	0.00257	3.59448	0.03421
		0.4	0.76647	0.02697	0.84462	0.39919	0.35716	0.01888	4.13976	0.29072	0.08634	0.01014	[illegible]	[illegible]
		0.8	0.92757	0.09285	0.21951	1.26565	0.68947	0.06741	6.48948	0.98565	0.23137	0.03844	26.41442	0.49135
		1.6	0.91393	0.26454	−1.22411	2.94860	0.85490	0.20115	6.95639	2.55231	0.46835	0.13148	57.46409	1.53833
		3.2	0.95243	0.60754	−3.04320	4.89878	0.90325	0.48647	6.05854	4.76234	0.71083	0.37342	99.29034	3.60964
0.5	2	0.1	0.04330	0.00465	0.08250	0.00878	0.01122	0.00259	0.17997	0.00440	0.00215	0.00094	0.26620	0.00128
		0.2	0.15325	0.01836	0.28318	0.03454	0.04172	0.01028	0.64779	0.01744	0.00826	0.00373	0.98772	0.00509
		0.4	0.42077	0.06974	0.70119	0.12954	0.13480	0.03998	1.89817	0.06722	0.02946	0.01474	3.19580	0.01996
		0.8	0.75683	0.23256	0.96681	0.41187	0.35175	0.14419	4.09592	0.23476	0.09508	0.05622	8.71973	0.07434
		1.6	0.93447	0.56298	0.70726	0.85930	0.70221	0.42106	6.22002	0.62046	0.27135	0.19358	20.15765	0.23838
		3.2	0.98801	0.88655	0.33878	0.96353	0.97420	0.82256	5.41828	0.93831	0.62399	0.52912	34.25229	0.54931
	20	0.1	0.04193	0.00117	0.08044	0.01778	0.00990	0.00063	0.19872	0.00911	0.00149	0.00023	0.31847	0.00257
		0.2	0.14808	0.00464	0.27574	0.07027	0.03648	0.00251	0.72264	0.03620	0.00564	0.00094	1.19598	0.01025
		0.4	0.40086	0.01799	0.67174	0.26817	0.11448	0.00988	2.19520	0.14116	0.01911	0.00372	1.02732	0.04047
		0.8	0.69098	0.06476	0.86191	0.91168	0.27934	0.03731	5.24726	0.51585	0.05574	0.01453	12.00885	0.15452
		1.6	0.79338	0.19803	0.39588	2.38377	0.50790	0.12654	10.30212	1.59341	0.13946	0.05399	32.77028	0.53836
		3.2	0.85940	0.49238	−0.41078	4.47022	0.70903	0.35807	16.38520	3.69109	0.30247	0.18091	77.62943	1.56409
1	2	0.1	0.01083	0.00241	0.02179	0.00453	0.00263	0.00100	0.04751	0.00160	0.00049	0.00029	0.06883	0.00035
		0.2	0.04176	0.00958	0.08337	0.01797	0.01029	0.00347	0.18481	0.00637	0.00195	0.00116	0.26966	0.00138
		0.4	0.14665	0.03724	0.28491	0.06934	0.03810	0.01565	0.66727	0.02498	0.00746	0.00460	1.00131	0.00545
		0.8	0.39942	0.13401	0.71341	0.24250	0.12173	0.05938	1.97428	0.09268	0.02647	0.01797	3.24971	0.02092
		1.6	0.71032	0.38690	1.02680	0.63631	0.31575	0.20098	4.37407	0.29253	0.08556	0.06671	8.92442	0.07335
		3.2	0.92112	0.75805	0.90482	0.97509	0.66041	0.53398	6.97695	0.65446	0.25186	0.22047	20.83387	0.21288
	20	0.1	0.00963	0.00061	0.02249	0.00920	0.00193	0.00024	0.05737	0.00322	0.00027	0.00007	0.08469	0.00062
		0.2	0.03697	0.00241	0.08618	0.03654	0.00751	0.00098	0.22418	0.01283	0.00104	0.00028	0.33312	0.00248
		0.4	0.12805	0.00950	0.29640	0.14241	0.02713	0.00387	0.82430	0.05063	0.00384	0.00110	1.25495	0.00985
		0.8	0.33263	0.03578	0.76292	0.51815	0.08027	0.01507	2.59672	0.19267	0.01236	0.00436	4.26100	0.03825
		1.6	0.52721	0.12007	1.25168	1.56503	0.17961	0.05549	6.77014	0.66326	0.03379	0.01683	12.91809	0.13989
		3.2	0.65530	0.33669	1.70723	3.51128	0.34355	0.18344	15.23252	1.88634	0.08859	0.06167	36.04291	0.45544
2	2	0.1	0.00250	0.00100	0.00555	0.00188	0.00059	0.00033	0.01219	0.00051	0.00011	0.00008	0.01737	0.00009
		0.2	0.00991	0.00397	0.02199	0.00750	0.00235	0.00130	0.04843	0.00203	0.00045	0.00033	0.06913	0.00036
		0.4	0.03832	0.01569	0.08465	0.02950	0.00922	0.00518	0.18857	0.00803	0.00179	0.00131	0.27103	0.00142
		0.8	0.13516	0.05974	0.29365	0.11080	0.03412	0.02023	0.68382	0.03093	0.00685	0.00520	1.00808	0.00553
		1.6	0.36644	0.20145	0.75087	0.35515	0.10918	0.07444	2.04134	0.10864	0.02441	0.02003	3.27590	0.02043
		3.2	0.67384	0.51156	1.17294	0.77434	0.29183	0.23852	4.60426	0.30709	0.08061	0.07248	9.02195	0.06638
	20	0.1	0.00181	0.00025	0.00652	0.00378	0.00033	0.00008	0.01568	0.00094	0.00005	0.00002	0.02160	0.00014
		0.2	0.00716	0.00099	0.02586	0.01507	0.00130	0.00031	0.06236	0.00374	0.00018	0.00007	0.08604	0.00058
		0.4	0.02746	0.00394	0.10017	0.05958	0.00503	0.00123	0.24425	0.01486	0.00071	0.00030	0.33866	0.00229
		0.8	0.09396	0.01535	0.35641	0.22795	0.01782	0.00485	0.90594	0.05789	0.00261	0.00119	1.27835	0.00901
		1.6	0.23065	0.05599	1.00785	0.78347	0.05012	0.01862	2.91994	0.21190	0.00819	0.00467	4.35311	0.03390
		3.2	0.37001	0.17843	2.16033	2.13215	0.11331	0.06728	7.95104	0.67732	0.02341	0.01784	13.26873	0.11666
4	2	0.1	0.00057	0.00034	0.00147	0.00065	0.00013	0.00010	0.00312	0.00015	0.00003	0.00002	0.00437	0.00002
		0.2	0.00228	0.00137	0.00587	0.00260	0.00054	0.00039	0.01245	0.00029	0.00011	0.00009	0.01746	0.00009
		0.4	0.00905	0.00544	0.02324	0.01032	0.00214	0.00154	0.04944	0.00235	0.00042	0.00036	0.06947	0.00036
		0.8	0.03500	0.02135	0.08957	0.04031	0.00837	0.00610	0.19247	0.00924	0.00168	0.00142	0.27221	0.00144
		1.6	0.12354	0.07972	0.31215	0.14735	0.03109	0.02358	0.69749	0.03488	0.00646	0.00560	1.01140	0.00553
		3.2	0.34121	0.25441	0.81908	0.43632	0.10140	0.08444	2.09049	0.11553	0.02332	0.02126	3.28913	0.01951
	20	0.1	0.00030	0.00008	0.00201	0.00128	0.00005	0.00002	0.00413	0.00025	0.00001	0.00000	0.00545	0.00003
		0.2	0.00119	0.00034	0.00803	0.00510	0.00021	0.00009	0.01651	0.00099	0.00003	0.00002	0.02178	0.00014
		0.4	0.00469	0.00134	0.03191	0.02032	0.00083	0.00035	0.06569	0.00396	0.00013	0.00008	0.08673	0.00054
		0.8	0.01790	0.00532	0.12427	0.07991	0.00321	0.00138	0.25739	0.01565	0.00050	0.00031	0.34131	0.00215
		1.6	0.06045	0.02049	0.45100	0.29991	0.01130	0.00542	0.95622	0.05993	0.00186	0.00124	1.28773	0.00833
		3.2	0.14979	0.07294	1.36427	0.97701	0.03258	0.02061	3.10980	0.20906	0.00612	0.00483	4.38974	0.03010

Stress Factor for Three layer system (After Jones 1962)

Example:- for the 3-layer system shown in figure below; calculate (σ_{z1} , σ_{z2} , $\sigma_{r\,1,2,3}$, ε_{r1} , ε_{r3})

solution:-



1) Calculate parameters

Geometry condition and Modular ratio:-

a) Modular ratio

$k_1=K_1=600000/30000=20$ $k_2=K_2=30000/15000=2$

b) Geometry ratio

$a_1=A=6/12=0.5$ $H=3/12=0.25$

2) stresses

a) Vertical stress $\sigma_{z1}=p\,(ZZ1)$, $ZZ1 = 0.47$ then σ_{z1}

$\sigma_{z2} = p\,(ZZ2)$, $ZZ2= 0.10$ then σ_{z2} 8.0 psi

b) Horizontal stress $\sigma_{z1}-\sigma_{r1}=p\,(ZZ1-RR1)$,

$\sigma_{z2}-\sigma_{r2}=p\,(ZZ2-RR2)$

$\sigma_{z3}-\sigma_{r3}=p\,(ZZ1-RR3)$ ($\sigma_{z2}=\sigma_{z3}$, $ZZ2=ZZ3$)

a_1=A	ZZ1-RR1	σ_{z1} - σ_{r1}	σ_{r1} psi
0.1	0.63215	50.57	-13.0 psi
0.2	1.83766	147.01	-109.4 psi
0.4	3.86779	309.42	-271.8 psi
0.8	5.50796	440.64	-403.0 psi
1.6	4.24281	339.42	-301.8 psi
3.2	1.97494	157.00	-120.4 psi

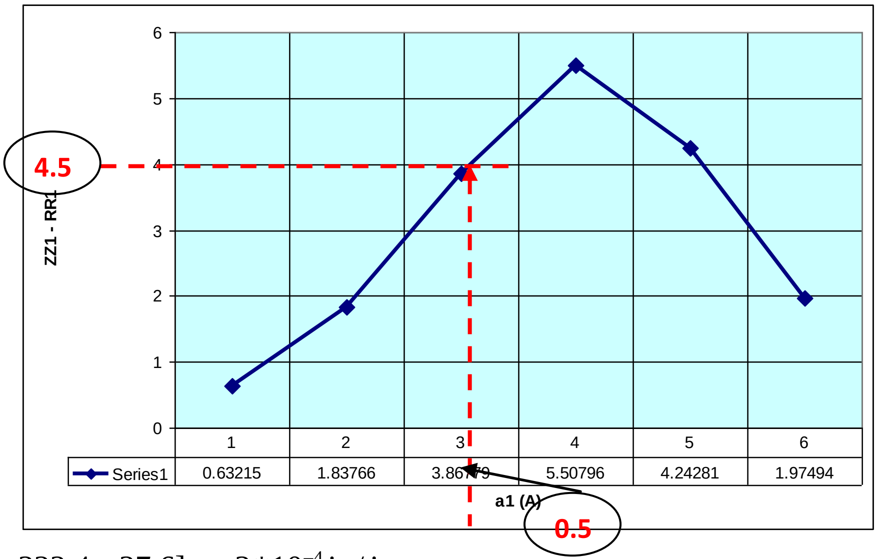
نلاحظ عدم وجود قيم خاصة ب a1=0.5 لذلك نعمل Interpolation للعلاقة بين قيم معاملات الاجهادات العمودية و الافقية

Relationship between vertical and horizontal stresses;

at a1=0.5 the ZZ1-RR1= 4.5

$\sigma_{z1}-\sigma_{r1}=80 * 4.5 = 360$ psi

$\sigma_{r1}=360 - 37.6 = -322.4$ psi



3) Strains

$$\varepsilon_{r1}=\frac{1}{2E_1}[\sigma_{r1}-\sigma_{z1}]=\frac{1}{2*600000}[-322.4-37.6]=-3*10^{-4}\,in/in$$

$$\varepsilon_{z3}=\frac{1}{E_3}[\sigma_{z2}-\sigma_{r3}]=\frac{1}{15000}[8-(-1)]=6*10^{-4}\,in/in$$

$$\varepsilon_{r2}=\frac{1}{2E_2}[\sigma_{r2}-\sigma_{z2}]$$

$$\varepsilon_{r3}=\frac{1}{2E_3}[\sigma_{r3}-\sigma_{z2}]$$

Odemark's Method (Method of Equivalent Thickness (MET))

To determine pavement responses in multi-layered system by using the elastic theory is quite complicated. Alternatively, Odemark (1949) suggested the approximate solution to determine the stresses, strains and deflections of multi-layered system. Odemark's procedure is based on the assumption that the stresses and strains below a layer depend on the stiffness of that layer only. If the thickness, modulus and Poisson's ratio of a layer are changed, but the stiffness remains unchanged, the stresses and strains below the layer should also remain unchanged. This stiffness of a layer is proportional to:

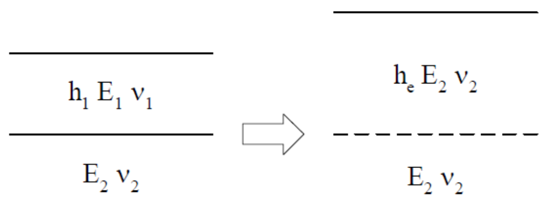
$$\frac{h^3 E}{1-\mu}$$

Where h is the thickness of the layer.
The transformation shown in following Figure; should not affect the stresses or strains in layer 2 if:

$$\frac{h_1^3 E_1}{1-\nu_1^2} = \frac{h_e^3 E_2}{1-\nu_2^2} \qquad \text{or} \qquad h_e = h_1 \sqrt[3]{\frac{E_1}{E_2} \times \frac{1-\nu_2^2}{1-\nu_1^2}}$$

where h_e is the *equivalent thickness*

The system after transformation in Figure is then a half-space material which Boussinesq's equations can be used, but only for stresses, strains and displacement *below* the interface.



Odemark's method is not mathematically correct. In order to obtain results that close to the theory of elasticity, a correction factor, f, should be introduced. Moreover, if the Poisson's ratio is assumed to be the same for all layers,

$$h_e = f * h_1 \sqrt[3]{\frac{E_1}{E_2}}$$

Reasonably good results agreement with the theory of elasticity is obtained with a correction factor of 0.8, except for the first interface where a factor of 0.9 is used for two-layer system and 1.0 for a multi-layer system. If the thickness of layer one, h_1, is less than the radius of the loaded area, a, then a factor of $1.1(a/h_1)_{0.3}$ will bring the horizontal tensile strain at the bottom of layer one closer to that obtained from the theory of elasticity.
For a multi-layered system the equivalent thickness of the upper n – 1 layers with respect to the modulus of layer n, may be calculated from:

$$h_{e,n} = f \times \sum_{i=1}^{n-1} h_1 \sqrt[3]{\frac{E_i}{E_n}}, \text{ or}$$

$$h_{e,n} = f \times \left\{ \ldots \left[\left(h_1 \sqrt[3]{\frac{E_1}{E_2}} + h_2 \right) \times \sqrt[3]{\frac{E_2}{E_3}} + h_3 \right] \times \ldots + h_{n-1} \right\} \times \sqrt[3]{\frac{E_{n-1}}{E_n}}$$

Layers below layer n are assumed to have the modulus E_n in the transformed system.
Deflections are calculated as the sum of the compression of the layers plus the deflection of the subgrade. The compression of an individual layer is found as the difference between the deflection at the top and the bottom of the layer in the transformed system. For the top layer the transformed system is a half-space with modulus E_1. In addition to the correction factor given previously, the Odemark's method will give results close to the theory of elasticity provided that:
- Module are decreasing with depth ($E_i/E_{i+1} > 2$)
- The equivalent thickness of each layer is larger than the radius of the loaded area.

Computer Program.

Several computer programs are available for obtaining the solution of surface deflections for layered elastic media have given elastic material parameters and layer thickness .These programs have been adopted to develop computer programs for calculation of stresses, strains and deflection.

Computer Programs Employing Closed Form

Various organizations have developed computer programs such as

1. ***BISAR*** (*Bitumen- structures- Analysis – In Road*) developed at *koniklijke/shell* laboralorium, Amsterdam, Netherlands have the capability to analyze layered systems without interface friction mobilized and the presence of surface shearing forces.
2. ***CHEVRON*** (*Chevron-Research-Company*). Analysis of stresses and displacements in an N-layered elastic system under a load uniformly distributed on a circular area.
3. ***ELSYM5*** Is a linear elastic layer system composed of a maximum up to five layers. The pavement may be loaded with one or more identical uniform circular loads normal to the surface. The program superimpose the various loads and computes the stresses, strains and displacements in three dimensions, along with the three principle stresses and strains at locations specified by the user.
4. ***Kenlayer Computer program***

   Kenlayer computer program was developed at the *Kentucky University* by *Huang H. Yang* and was written in Fortran 77 and requires storage of 509 KB. In it's present dimensions, It can be applied to a maximum of 19 layers with output at 10 different radial coordinates and 19 different vertical coordinates, or a total of 190 points. For multiple wheels in addition to the 19 vertical coordinates, solutions can be obtained at a total of 25 points by specifying the X and Y coordinates of each point creep compliance's can be specified at a maximum of 15 time duration's. Damage analysis can be made by dividing each year into a maximum of 24 periods each with a maximum of 24-load group's.

   In this theses based on the Kenlayer program**.** This program can be applied to layered systems under single, dual, dual–tandem or dual-tridem wheels with each layer behaving different set of material properties. Each period can have a maximum of 24 load groups, either single or multiple. The damage caused by Fatigue cracking and Permanent deformation in each period over all load groups is summed up to evaluate the design life.

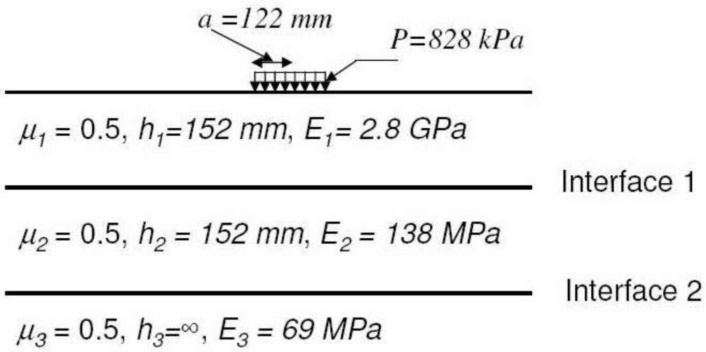
Problems:-

1) determine the complete state of stress (σ_z , σ_r , σ_t) using one layer theory under the center line of tire having a 50000 lb load, 100 psi pressure for the following depth tire radius ratios: 0, 0.2,0.5,1.0, 2.0, 4.0, 8.0. Assume the pavement is characterized by μ=0.5 and E=3000 psi.
2) If μ=0.2, repeat problem 1; determined effect of μ upon the computed stresses.
3) A plate bearing test using a 30 in diameter rigid plate was made on a subgrade as well as on 10 in of gravel base course. The unit load required to cause settlement of 0.2 in was 10 psi and 40 psi, respectively. Determine the required thickness of base course to sustain a 50 000 lb tire , 100 psi pressure and maintain a deflection of 0.2 in.
4) A pavement structure is comprised of the following layers; 5.75 in asphalt concrete surface, E= 400000 psi; 23 in of granular base E= 20000 psi; and a sugrade having an E=10000 psi . All layers are assumed to have μ=0.5 . Calculate the horizontal tensile strain at the bottom of the asphalt concrete layer and the vertical compressive strain at the top of the subgrade layer under the center line of a 40000 lb wheel load, 150 psi pressure.
5) Calculate the surface deflection under the center of a tire (a = 152 mm, p =552 kPa) for a 305 mm pavement having a 345 MPa modulus and subgrade modulus of 69 MPa from two-layer theory. Also calculate the interface deflection and the deflection that takes place within the pavement layer.
6) A circular load with a radius of 152 mm and a uniform pressure of 552 kPa is applied on a two-layer system. The subgrade has an elastic modulus of 35 kPa and can support a maximum vertical stress of 55 kPa. What is the required thickness of full depth AC pavement, if AC has an elastic modulus of

3.45 GPa. Instead of a full depth AC pavement, if a thin surface treatment is applied on a granular base (with elastic modulus of 173 MPa), what is the thickness of base course required?

7) A plate bearing test using 750 mm diameter rigid plate was made on a subgrade as well as on 254 mm of gravel base course. The unit load required to cause settlement of 5 mm was 69 kPa and 276 kPa, respectively. Determine the required thickness of base course to sustain a 222.5 kN tyre, 690 kPa pressure and maintain a deflection of 5 mm.

8)
- Given the three layer system shown in figure, determine all the stresses and strains at the two interfaces on the axis of symmetry.



9) A three layer system (subgrade is counted as a "layer") with the thicknesses and moduli shown in Fig A1 is loaded by a uniformly distributed load with radius 150 mm and contact stress 0.7 MPa. Poisson's ratio is assumed to be 0.35 for all materials.

1) Horizontal strain at the bottom of the asphalt

Equivalent thickness of asphalt corresponding to modulus of second layer,

$$h_{e,2} = 1.0 \times 150 \times \sqrt[3]{\frac{3000}{300}} = 323 \text{ mm}$$

When,

$z/a = 323/150 = 2.15$ and
$r/a = 0$

From Fig 5.2, $\frac{\sigma_z}{q} \times 100 = 26$

Thus, $\sigma_z = 0.26 \times 0.7 = 0.18$ MPa

Similarly, from Fig 5.3 and Fig 5.4, $\frac{\sigma_t}{q} \times 100 = \frac{\sigma_r}{q} \times 100 = 1.6$

Thus, $\sigma_r = \sigma_t = 1.6/100 \times 0.7 = 0.01$ MPa

Horizontal strain, $\varepsilon_r = \frac{1}{E}[\sigma_r - \nu(\sigma_t + \sigma_z)]$

$\varepsilon_r = \frac{1}{300}[0.01 - 0.35(0.01 + 0.18)]$

$= -0.0002 = -200\ \mu\varepsilon$ (minus sign indicates tension)

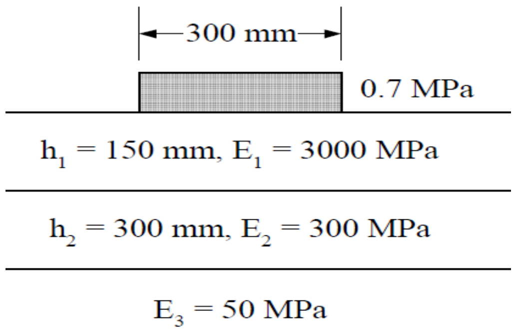


Figure A1

2) Vertical stress on subgrade

$$h_{e,3} = 0.8 \times (323 + 300) \times \sqrt[3]{\frac{300}{50}} = 906 \text{ mm}$$

When

$z/a = 906/150 = 6.04$ and
$r/a = 0$

From Fig 5.2, $\frac{\sigma_z}{q} \times 100 = 4$

Therefore, $\sigma_z = 4/100 \times 0.7 = 0.028$ MPa

Lecture No. 4

Stresses in Rigid Pavement (Bending of Thin Plate)

Theories of plate bending according to Timoshenko ((تومنشكو

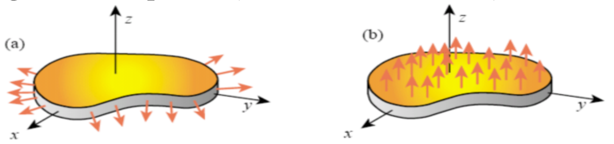
1) Bending of thin plate with small deflection (Plan stress)

وهي النظرية التي تناقش الطبقات القليلة السمك بالمقارنة مع بقية الأبعاد (الطول و العرض) ومقارنته مع الهبوط (الهبوط قليل بالمقارنة مع السمك).مقدار الهبوط المسموح بة هو 0.1 in من سمك الطبقة 12 in) هذه النظرية هي المطبقة في الطرق حيث لا يسمح بحدوث هبوط عالي).

2) Bending of thin plate with large deflection(Plan Strain)

وهي النظرية التي تناقش الطبقات القليلة السمك مقارنة مع اصغر بعد فيها ويكون مقدار التشوه تحت الطبقة متغير (غير متساوي).

3) Bending of Thick plates (Roof or Foundation) .وهي النظرية التي تناقش الطبقات السميكة (الأسس) في المطارات.



Plates theories

	moderately thick	thin	very thin
h/lx, h/ly	1/5 to 1/10	l/5 to l/50	<1/50
	with transverse shear deformation	without transverse shear deformation, mostly used for practical applications	geometrically nonlinear, with membrane deformation
theory	Reissnen Mindlin	Kirchhoff	von Karman
related beam theory	Timoshenko	Euler, Bernoulli	theory of second order

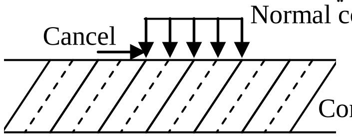
Bending of Thin Plate with Small Deflection (Plan stress)

Assumptions

تفرض هذه النظرية أن التبليط يتكون من طبقة ذات سمك قليل و مقدار التشوه تحتها قليل.

1) The load acting on the plate is normal to it surface.

تفرض هذه النظرية بان الحمل المسلط عمودي على الطبقة و تهمل القوى الأفقية على السطح ولكن في الحقيقة هذه القوى موجودة ولكنها قليلة المقدار).

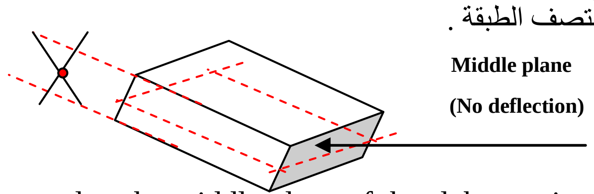


2) Deflections are small in comparison with thickness of plate (the thickness of plate assumed uniform).

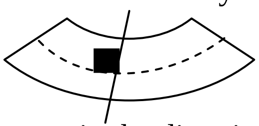
الهبوط قليل مقارنه مع سمك الطبقة الذي يفرض بأنة منتظم.

3) There is no deflection in the middle plane of the slab .this plane remains neutron during bending.

لا يوجد هبوط في منتصف الطبقة .



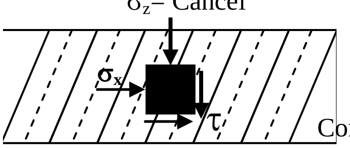
4) Those planes of the slab initially laying normal to the middle plane of the slab remain normal after bending.



المستوى العمودي على middle plane يبقى عمودي بعد التشوه

5) The normal stresses in the direction transverse to the plane of slab can be disagreed.

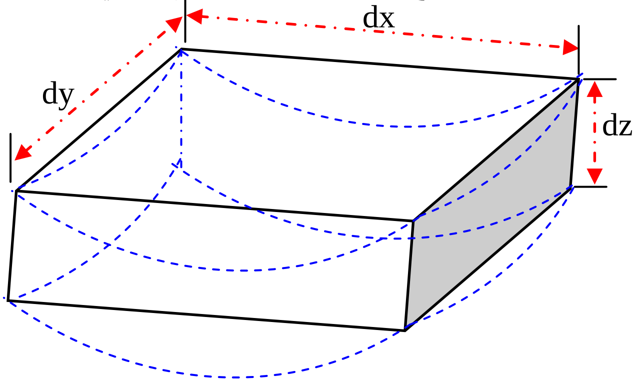
مقدار الإجهاد العمودي داخل الطبقة يمكن إهماله (اكبر أجهاد عمودي يمكن تسليطه بالطرق 90-95 psi

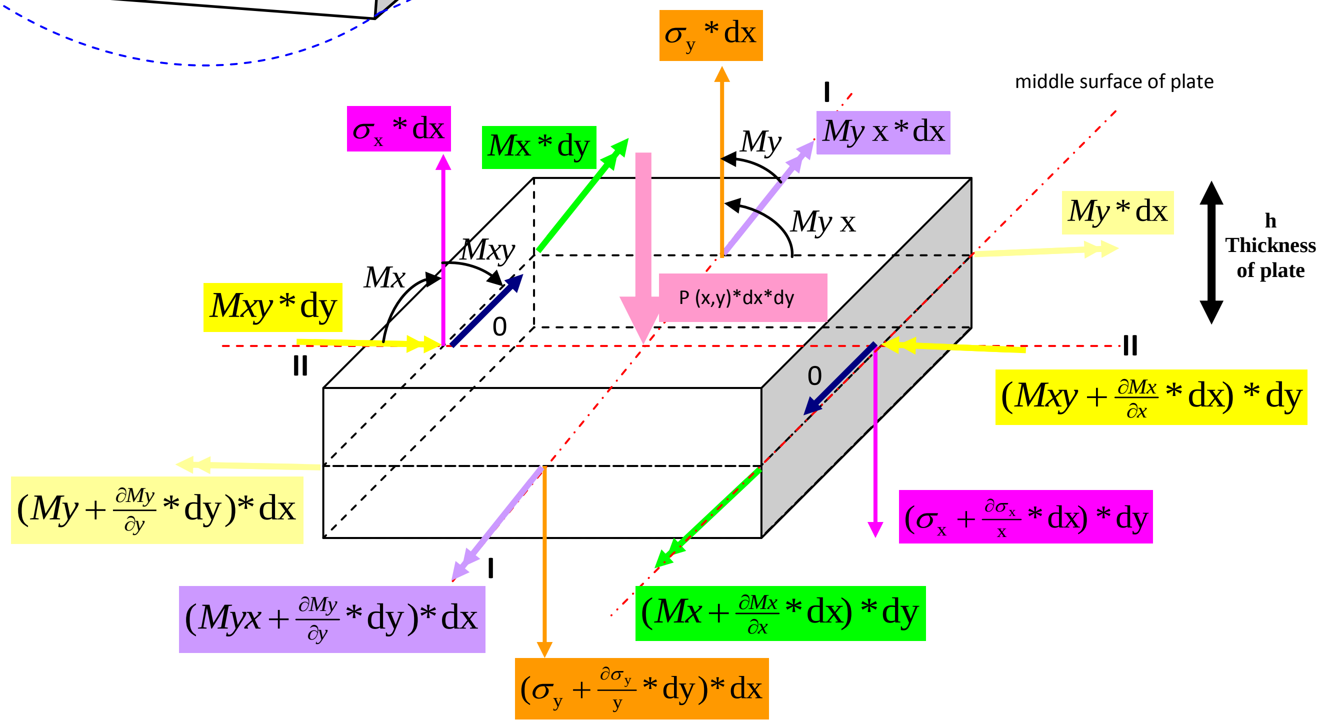


6) Friction with subgrade assumed = zero. .(الاحتكاك = صفر) نهمل الاحتكاك مع طبقة ما تحت الأساس

The Differential Equation of the Surface Deflection of the Plate

لغرض اشتقاق المعادلة التفاضلية التي تربط التشوه السطحي للطبقة فنأخذ عنصر من الطبقة ونوضح عليه كافة القوى والعزم كما في الشكل أدناه:-





σ_x , σ_y :- Shear forces KN/m

Mx , My :- Bending moment producing moment stresses in x-direction and y-direction (KN.m/m).

Mxy , Myx :- Twisting moment (KN.m/m)

1) المستوي الوسطي لا يحدث فيه إي تشوه وكذلك فان الحمل الموازي يهمل (الاجهادات الأفقية) .

2) اجهادات القص الموازية للسطح تهمل.

$$\sum Fy = 0 \qquad \downarrow \oplus$$

$$P * dx * dy + (\sigma_x + \frac{\partial \sigma_x}{\partial x} * dx) * dy + (\sigma_y + \frac{\partial \sigma_y}{\partial y} * dy) * dx - \sigma_x * dy - \sigma_y * dx = 0$$

$$P * dx * dy + \sigma_x * dy + \frac{\partial \sigma_x}{\partial x} * dx * dy + \sigma_y * dx + \frac{\partial \sigma_y}{\partial y} * dy * dx - \sigma_x * dy - \sigma_y * dx = 0$$

$$P * dx * dy + \frac{\partial \sigma_x}{\partial x} * dx * dy + \frac{\partial \sigma_y}{\partial y} * dy * dx = 0 \qquad / \, dx * dy$$

$$P(x, y) + \frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \qquad (1)$$

$\sum M_{I-I}=0$ ↲⊕

$$My*dx+(M_y+\frac{\partial M_y}{\partial y}*dy)*dx+(M_{xy}+\frac{\partial M_{xy}}{\partial x}*dx)*dy-\sigma_y*dx*\frac{dy}{2}-(\sigma_y+\frac{\partial \sigma_y}{\partial y}*dy)*dx*\frac{dy}{2}=0$$

$$\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_y}{\partial y}-\sigma_y-\frac{\partial \sigma_y}{\partial y}*\frac{dy}{2}=0$$ dy is very little, so that can be cancel

$$\frac{\partial M_y}{\partial y}+\frac{\partial M_{xy}}{\partial x}-\sigma_y=0 \quad \text{and} \quad \frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}-\sigma_x=0$$

$$\sigma_y=\frac{\partial M_y}{\partial y}+\frac{\partial M_{xy}}{\partial x} \qquad (2)$$

$$\sigma_x=\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y} \qquad (3)$$

in order to calculate change in shear stresses taken derivative the above equation;

$$\frac{\partial \sigma_y}{\partial y}=\frac{\partial^2 M_y}{\partial y^2}+\frac{\partial^2 M_{xy}}{\partial x*\partial y} \quad , \quad \frac{\partial \sigma_x}{\partial x}=\frac{\partial^2 M_x}{\partial x^2}+\frac{\partial^2 M_{xy}}{\partial y*\partial x}$$

Substitute into equation (1)

$$P+\frac{\partial^2 M_x}{\partial x^2}+\frac{\partial^2 M_{xy}}{\partial x*\partial y}+\frac{\partial^2 M_y}{\partial y^2}+\frac{\partial^2 M_{xy}}{\partial y*\partial x}=0 \Rightarrow P+\frac{\partial^2 M_x}{\partial x^2}+2*\frac{\partial^2 M_{xy}}{\partial x*\partial y}+\frac{\partial^2 M_y}{\partial y^2}=0$$

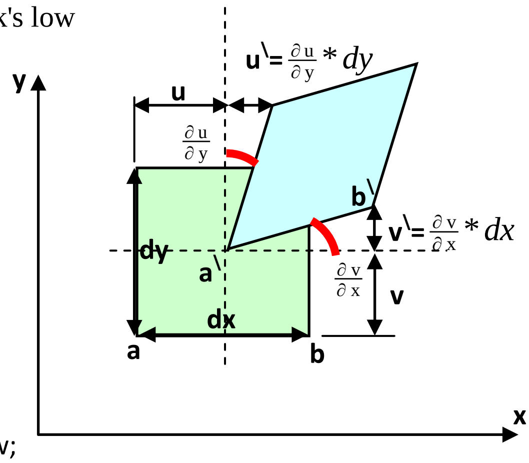
$$-P=\frac{\partial^2 M_x}{\partial x^2}+2*\frac{\partial^2 M_{xy}}{\partial x*\partial y}+\frac{\partial^2 M_y}{\partial y^2} \qquad (4)$$

Strains (نتجه لتسليط الأحمال تحت اجهادات وتشوهات) according to Hook's low

$$\varepsilon_x=\frac{1}{E}[\sigma_x-\mu(\sigma_z+\sigma_y)] \Rightarrow \varepsilon_x=\frac{1}{E}[\sigma_x-\mu\sigma_y]=\frac{\partial u}{\partial x} \qquad (5)$$

$$\varepsilon_y=\frac{1}{E}[\sigma_y-\mu(\sigma_z+\sigma_x)] \Rightarrow \varepsilon_y=\frac{1}{E}[\sigma_y-\mu\sigma_x]=\frac{\partial v}{\partial y} \qquad (6)$$

$$\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}=\frac{\tau_{xy}}{G}$$ shearing strain



u:- displacement in X-direction.
v:- displacement in Y-direction.
$\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}$:- shear strain
μ :- Passion's ratio.
G:- shear modulus = E/(1-μ)
E :- Modulus of elasticity
So, the relationship between Shear stresses and strain are shown below;

$$\varepsilon_x=\frac{1}{E}[\sigma_x-\mu\sigma_y]=\frac{\partial u}{\partial x} \quad *\mu \Rightarrow \varepsilon_x=\frac{1}{E}[\mu\sigma_x-\mu^2\sigma_y]=\mu\frac{\partial u}{\partial x} \qquad (7)$$

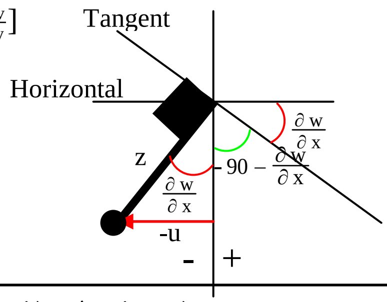
$$\varepsilon_y=\frac{1}{E}[-\mu\sigma_x+\sigma_y]=\frac{\partial v}{\partial y} \qquad (8)$$

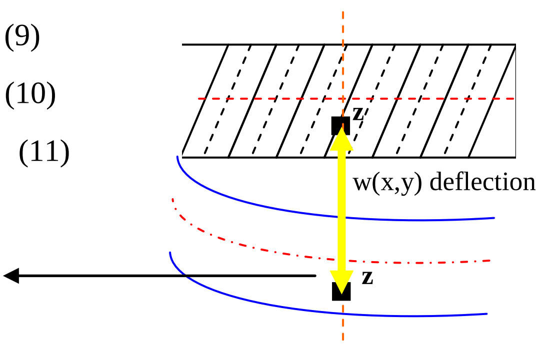
$$-\frac{\mu^2}{E}\sigma_y+\frac{\sigma_y}{E}=\mu\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} \Rightarrow \frac{(1-\mu)}{E}\sigma_y=\mu\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}$$

$$\therefore \sigma_y=\frac{E}{(1-\mu^2)}[\frac{\partial v}{\partial y}+\mu\frac{\partial u}{\partial x}] \qquad (9)$$

$$\therefore \sigma_x=\frac{E}{(1-\mu^2)}[\frac{\partial u}{\partial x}+\mu\frac{\partial v}{\partial y}] \qquad (10)$$

$$\therefore \tau_{xy}=G[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}] \qquad (11)$$





displacement and deflection   نتجه لحصول هذا التشوه سوف تحدث إزاحة

$$\frac{\partial w}{\partial x} = -\frac{u}{z} \qquad \Rightarrow u = -z\frac{\partial w}{\partial x} \qquad \Rightarrow \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} \qquad \frac{\partial u}{\partial y} = -z\frac{\partial^2 w}{\partial x \partial y} \qquad \frac{\partial v}{\partial x} = -z\frac{\partial^2 w}{\partial y.\partial x}$$

$$v = -z\frac{\partial w}{\partial y} \qquad \Rightarrow \frac{\partial v}{\partial y} = -z\frac{\partial^2 w}{\partial y^2}$$

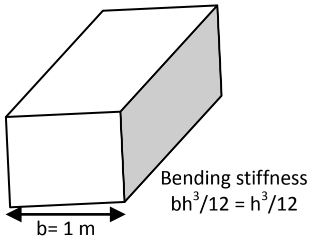
Substitute into equation (8,9,10)

$$\therefore \sigma_y = \frac{E}{(1-\mu^2)}[-z\frac{\partial^2 w}{\partial y^2} - z\,\mu\frac{\partial^2 w}{\partial x^2}] = -\frac{E\,z}{(1-\mu^2)}[\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}]$$

$$\therefore \sigma_x = \frac{E}{(1-\mu^2)}[-z\frac{\partial^2 w}{\partial x^2} - z\,\mu\frac{\partial^2 w}{\partial y^2}] = -\frac{E\,z}{(1-\mu^2)}[\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}]$$

$$\therefore \tau_{xy} = -2\,z\,G\,\frac{\partial^2 w}{\partial x \partial y} = -2\,\frac{E\,z}{1-\mu} * \frac{\partial^2 w}{\partial x \partial y}$$

Connect relation with deflection and stresses



$$M_x = \int_{-h/2}^{h/2} \sigma_x.z.dx = \int_{-h/2}^{h/2} -\frac{E\,z}{(1-\mu^2)}[\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}].z.dx \Rightarrow -\frac{E}{(1-\mu^2)}[\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}] * \left[\frac{z^3}{3}\right]_{-h/2}^{h/2} \Rightarrow$$

$$M_x = -\frac{E\,h^3}{12(1-\mu^2)}[\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}] \qquad \because D\,(\text{flexture rigidity of the plate}) = \frac{E\,h^3}{12(1-\mu^2)}$$

$$M_x = -D[\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}] \qquad M_y = -D[\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}] \qquad M_{x,y} = -2D(1-\mu)[\frac{\partial^2 w}{\partial x \partial y}]$$

$$\sigma_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} \qquad (2)$$

$$\sigma_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} = -D[\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}] - 2D(1-\mu)[\frac{\partial^2 w}{\partial x \partial y}] = -D\frac{\partial}{\partial x}[\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}] = -D\frac{\partial}{\partial x}[\Delta w]$$

$$\sigma_y == -D\frac{\partial}{\partial y}[\Delta w]$$

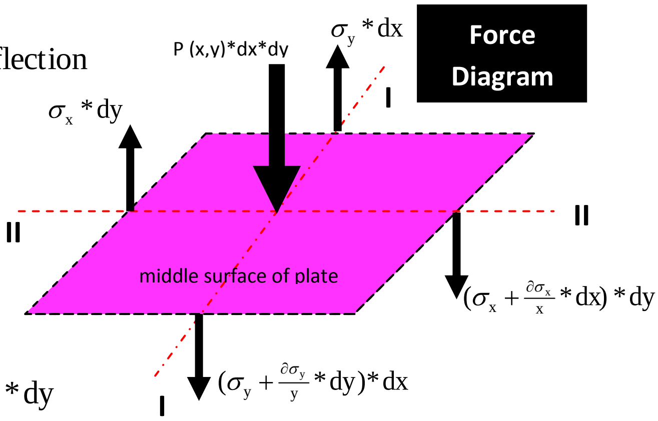
$$\Delta\,w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \quad \text{subtituteon the above eq. (4)}$$

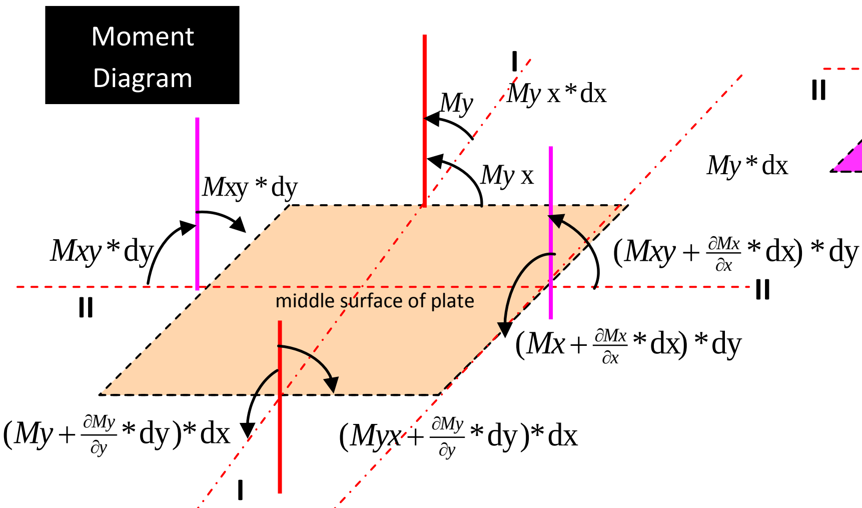
$$-D[\frac{\partial^4 w}{\partial x^4} + \mu\frac{\partial 4\,w}{\partial x^2 \partial y^2}] - 2D(1-\mu)[\frac{\partial^4 w}{\partial x^2 \partial y^2}] - D[\frac{\partial^4 w}{\partial y^4} + \mu\frac{\partial 4\,w}{\partial x^2 \partial y^2}] = -P$$

$$[\frac{\partial^4 w}{\partial x^4} + \mu\frac{\partial 4\,w}{\partial x^2 \partial y^2}] + 2(1-\mu)[\frac{\partial^4 w}{\partial x^2 \partial y^2}] + [\frac{\partial^4 w}{\partial y^4} + \mu\frac{\partial 4\,w}{\partial x^2 \partial y^2}] = \frac{P}{D}$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial 4\,w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{P}{D} \quad \text{the differential equation of surface deflection}$$

$$\Delta\Delta w = \frac{P}{D}$$





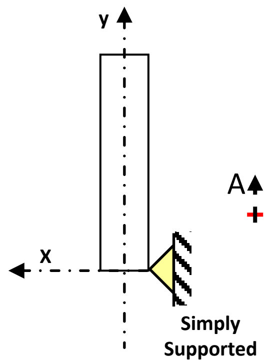
Differential equation of the surface deflection in plate

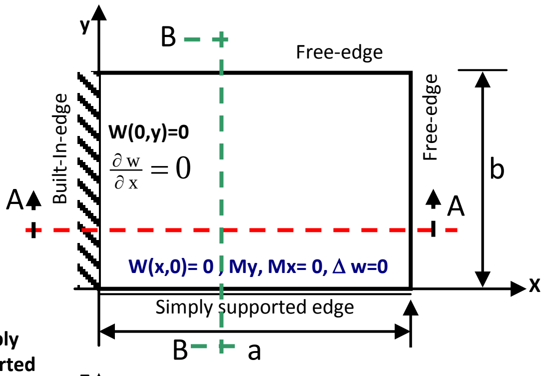
$$\sum M = \frac{M_x\,M_y}{1+\mu} = \frac{1}{1+\mu}[-D(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}) - D[\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}] \Rightarrow \; M = D\Delta w$$

$$\Delta M = D\Delta\Delta w$$

1) $\frac{\partial^2 w}{\partial x2} + \frac{\partial 2\, w}{\partial y^2} = -P$ (if P known)

2) $\frac{\partial^2 w}{\partial x2} + \frac{\partial 2\, w}{\partial y^2} = -\frac{M}{D}$ (if M known)





Boundary Condition: (simply, free and fixed)

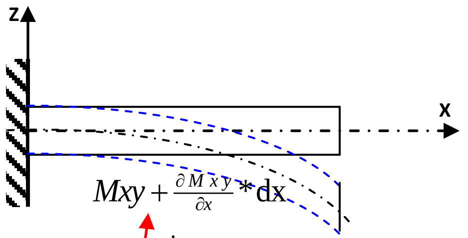
1) $\frac{\partial w}{\partial x}_{(0,y)} = 0$

2) Simply –supported edge

   a) w(x,0)=0 , My(x,0)=0

   b) w(x,0)=0 , $(\Delta w)_{x,0}$ =0

   c) w(x,0)=0 , $Mx_{(x,0)}$ = 0

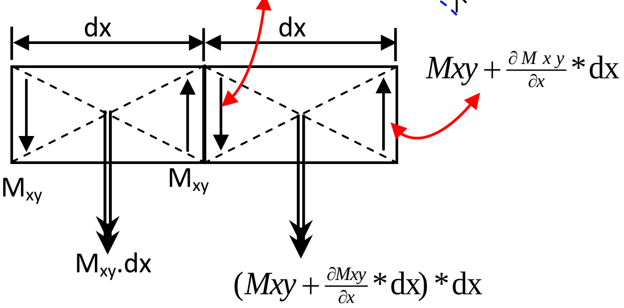


Simply Support Edge

$\sum Fy = 0 \quad \uparrow \oplus$

$M_{xy} - M_{xy} - \frac{\partial M_{xy}}{\partial x} dx - F = 0$

$F = -\frac{\partial M_{xy}}{\partial x} dx$



Resultant shear force $\sigma''_y \begin{cases} \sigma_y & \text{orignal shear force} \\ \frac{\partial M_{xy}}{\partial x} & \text{shear result from twice} \end{cases}$

$\sigma''_y = \sigma_y + \frac{\partial M_{xy}}{\partial x}$

$\sigma''_{y(x,0)} = \sigma_{y(x,0)} + (\frac{\partial M_{xy}}{\partial x})_{x,0} = -D(\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial x^2 \partial y}) - [(1-\mu)D^2 \frac{\partial^3 w}{\partial x^2 \partial y}]$

$\sigma''_{y(x,0)} = reaction\ \text{force} = -D[\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial x^2 \partial y}) + (1-\mu)^3 \frac{\partial^3 w}{\partial x^2 \partial y}] = -D[\frac{\partial^3 w}{\partial y^3} + (2-\mu)\frac{\partial^3 w}{\partial x^2 \partial y}]$

Free edge

- for the free edge y=b ⇒ 1) $M_{y(x,b)}$=0 ⇒ M
- For the free edge x=a

هذه الشروط تطبق فقط على الالواح المفصولة عن بعضها البعض والغير مترابطة بمفاصل او قضبان الحديد .

Simply Supported Rectangular Plate under Sinusoidal Load.

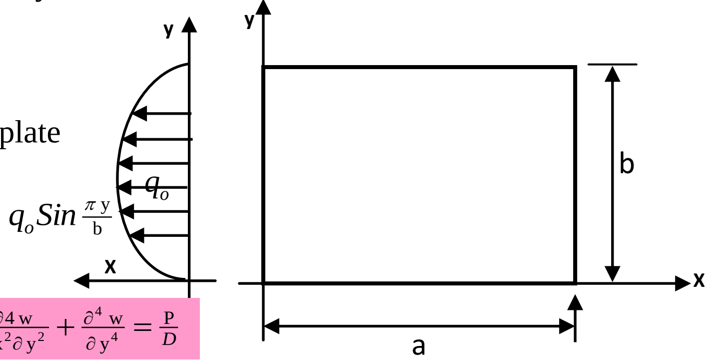
كيفية توزيع الاحمال على لوحة مستطيلة (على فرض بان احمل يتوزع على شكل موجة جيبية واللوح مستطيل).

The load distribution over the surface of the plate given by

$$q_{(x,y)} = q_o * Sin\frac{\pi x}{a} * Sin\frac{\pi y}{b}$$

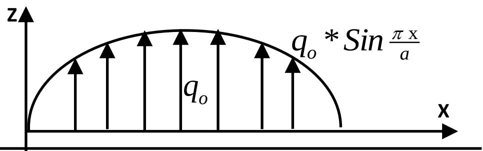
q_o= represents the intensity of the load at the center of plate

$at \begin{cases} y = \frac{b}{2} \Rightarrow q_o * Sin\frac{\pi x}{a} * Sin\frac{\pi b}{2b} = q_o * Sin\frac{\pi x}{a} \\ x = \frac{a}{2} \Rightarrow q_o * Sin\frac{\pi a}{2a} * Sin\frac{\pi y}{b} = q_o Sin\frac{\pi y}{b} \end{cases}$



the differential equation of surface deflection $\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial 4\, w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{P}{D}$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial 4\, w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q_o}{D} * Sin\frac{\pi x}{a} * Sin\frac{\pi y}{b}$$



Boundary conditions 4-edges simply supported are;

w(0,y)=0 and Mx(0,y)=0
w(x,0)=0 and My(x,0)=0
w(a,y)=0 and Mx(a,y)=0
w(x,b)=0 and My(x,b)=0

اذن يجب فرض معادلة تحقق الشروط المذكورة اعلاه وتوضح كيفية توزيع الاحمال على اللوح المستطيل. و هنالك طريقتين هما:-

1) Assumed solution by Timoshenko
   a) Max. deflection occur at x=a/2 and y=b/2 $w_{max} = w\left(\frac{a}{2}, \frac{b}{2}\right) = \frac{q_o}{D\pi^4\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2}$
   b) Max. M_x occurs at b/2 $$Max.\ M_x = \frac{q_o\left(\frac{1}{a^2}+\frac{\mu}{b^2}\right)*Sin\frac{\pi x}{a}}{\pi^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)}$$
   c) Max. M_y occurs at a/2 $$Max.\ M_y = \frac{q_o\left(\frac{\mu}{a^2}+\frac{1}{b^2}\right)*Sin\frac{\pi y}{b}}{\pi^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)}$$

For a square plate a=b Max. deflection $w_{Max}=q_o/D\pi^4(2/a^2)^2$ so that, $$w_{Max} = \frac{q_o * a^4}{4 D \pi^4}$$

2) Navier solution for simply supported rectangular plate.

q=f(x,y) general load — نفرض ان الحمل هو دالة يمكن فتحها و توزيعها بسلسلة

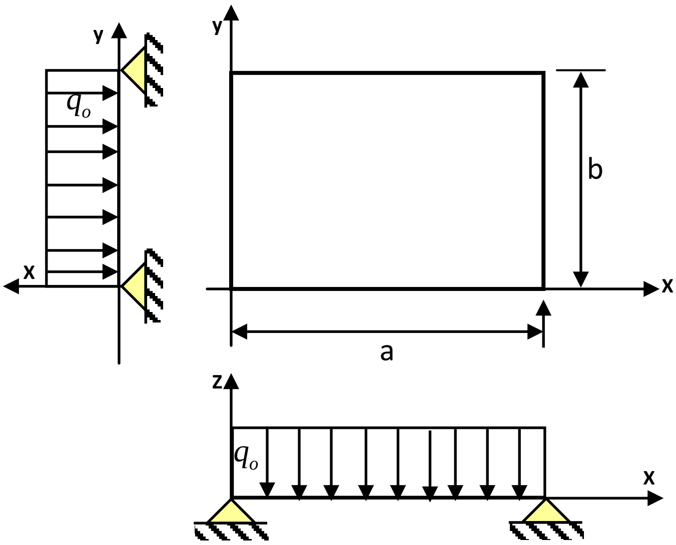
Can be expanded in both directions in terms of double Fourier Sine Series. (Double trigonometric series (متوالية تثليث مزدوجة)

$$q = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} * Sin\frac{m\pi x}{a} * Sin\frac{n\pi y}{b}$$

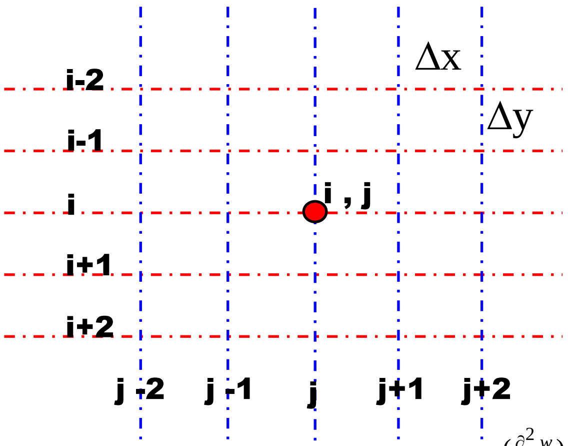
$$a_{mn} = \frac{4}{ab}\int_0^a\int_0^b f(x,y) * Sin\frac{m\pi x}{a} * Sin\frac{n\pi y}{b} * dx * dy$$

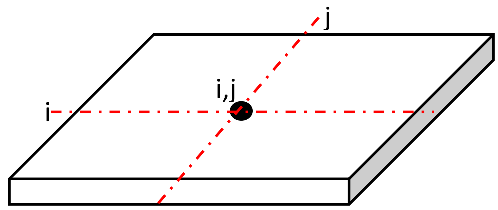
$$w_{(x,y)} = \frac{1}{\pi^4 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{a_{mn}}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)} * Sin\frac{m\pi x}{a} * Sin\frac{n\pi y}{b}$$

Example:- find surface deflection and Max. deflection for a rectangular plate subjected to a uniformly distributed load through out the whole plate.



Application Finite Difference to the Bending of Simply Supported Plate





$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial 4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q_o}{D}$$

$$(\frac{\partial^{4qq} w}{\partial y^4})_{i,j} = [\frac{\partial}{\partial y^2}(\frac{\partial^2 w}{\partial y^2})]_{i,j} = \frac{\partial}{\partial y^2}[\frac{(\frac{\partial^2 w}{\partial y^2})_{i-1,j} - 2(\frac{\partial^2 w}{\partial y^2})_{i,j} + (\frac{\partial^2 w}{\partial y^2})_{i+1,j}}{(\Delta y)^2}]$$

$$= \frac{\partial}{\partial y^2}[\frac{w_{i-2,j} - 2w_{i-1,j} + w_{i,j} - 2(w_{i-1,j-2} - 2w_{i,j} + w_{i+1,j}) + w_{i,j} - 2w_{i+1,j} + w_{i+2,j}}{(\Delta y)^2}]$$

$$= \frac{w_{i-2,j} - 4w_{i-1,j} + 6w_{i,j} - 4w_{i+1,j} + w_{i+2,j}}{(\Delta y)^4}$$

$$(\frac{\partial^4 w}{\partial x^4})_{i,j} = [\frac{\partial^2}{\partial x^2}(\frac{\partial^2 w}{\partial x^2})]_{i,j} = \frac{\partial}{\partial x^2}[\frac{(\frac{\partial^2 w}{\partial x^2})_{i,j+1} - 2(\frac{\partial^2 w}{\partial x^2})_{i,j} + (\frac{\partial^2 w}{\partial x^2})_{i,j-1}}{(\Delta x)^2}]$$

$$= \frac{\partial}{\partial x^2}[\frac{w_{i,j+2} - 2w_{1,j+1} + w_{i,j} - 2(w_{i,j+1} - 2w_{i,j} + w_{i,j-1}) + w_{i,j} - 2w_{i,j-1} + w_{i,j-2}}{(\Delta x)^2}]$$

$$= \frac{w_{i,j+2} - 4w_{i,j+1} + 6w_{i,j} - 4w_{i,j-1} + w_{i,j-2}}{(\Delta x)^4}$$

$$\frac{\partial 4 w}{\partial x^2 \partial y^2} = [\frac{\partial^2}{\partial x^2}(\frac{\partial^2 w}{\partial y^2})]_{i,j} = \frac{\partial}{\partial x^2}[\frac{(\frac{\partial^2 w}{\partial y^2})_{i,j+1} - 2(\frac{\partial^2 w}{\partial y^2})_{i,j} + (\frac{\partial^2 w}{\partial y^2})_{i,j-1}}{(\Delta y)^2}]$$

$$= \frac{w_{i-1,j+1} - 2w_{1,j+1} + w_{i+1,j+1} - 2(w_{i-1,j-2} - 2w_{i,j} + w_{i+1,j}) + w_{i-1,j-1} - 2w_{i,j-1} + w_{i+1,j-1}}{(\Delta x)^2 (\Delta y)^2}$$

substite in differential eq.

$$\frac{q_{i,j}}{D} = \frac{w_{i-2,j} - 4w_{i-1,j} + 6w_{i,j} - 4w_{i+1,j} + w_{i+2,j}}{(\Delta y)^4} + 2*[\frac{w_{i-1,j+1} - 2w_{1,j+1} + w_{i+1,j+1} - 2(w_{i-1,j-2} - 2w_{i,j} + w_{i+1,j}) + w_{i-1,j-1} - 2w_{i,j-1} + w_{i+1,j-1}}{(\Delta x)^2 (\Delta y)^2}]$$

$$+ \frac{w_{i,j+2} - 4w_{i,j+1} + 6w_{i,j} - 4w_{i,j-1} + w_{i,j-2}}{(\Delta x)^4}$$

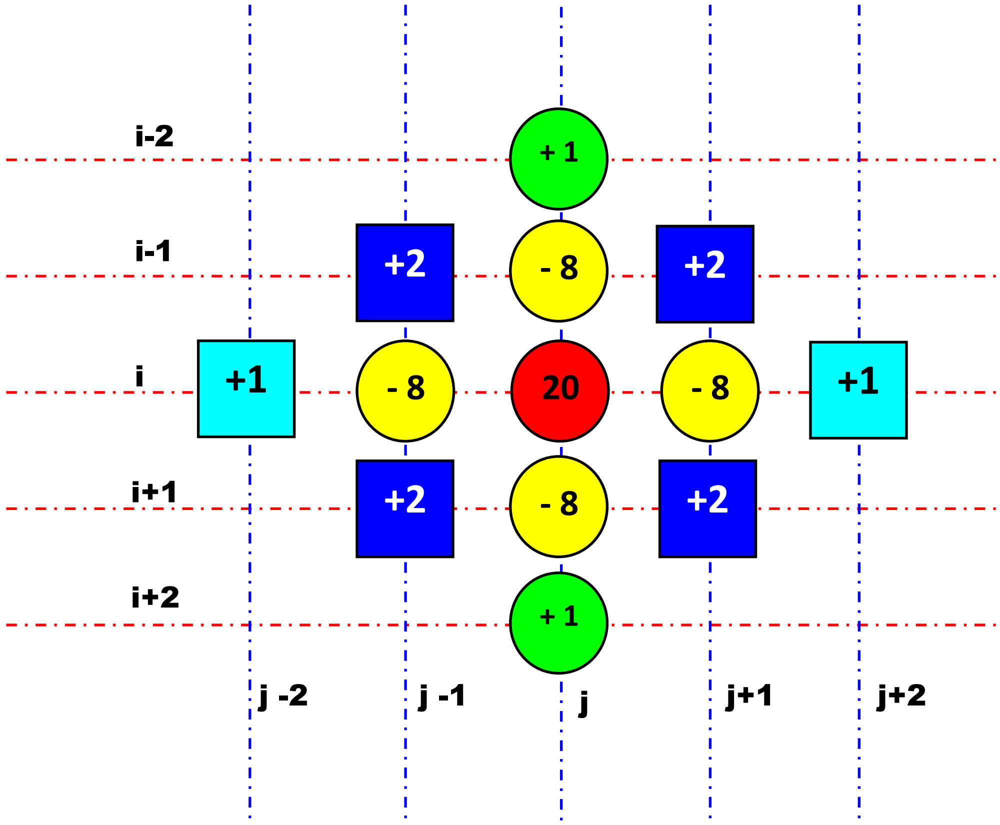
$$\frac{q_{i,j}}{D} * (\Delta x)^4 = w_{i-2,j} - 4w_{i-1,j} + 6w_{i,j} - 4w_{i+1,j} + w_{i+2,j} +$$

$$2(w_{i-1,j+1} - 2w_{i,j} + w_{i+1,j} - 2(w_{i-1,j-2} - 2w_{i,j} + w_{i+1,j}) + w_{i-1,j-1} - 2w_{i,j-1} + w_{i+1,j-1})$$

$$+ w_{i,j+2} - 4w_{i,j+1} + 6w_{i,j} - 4w_{i,j-1} + w_{i,j-2}$$

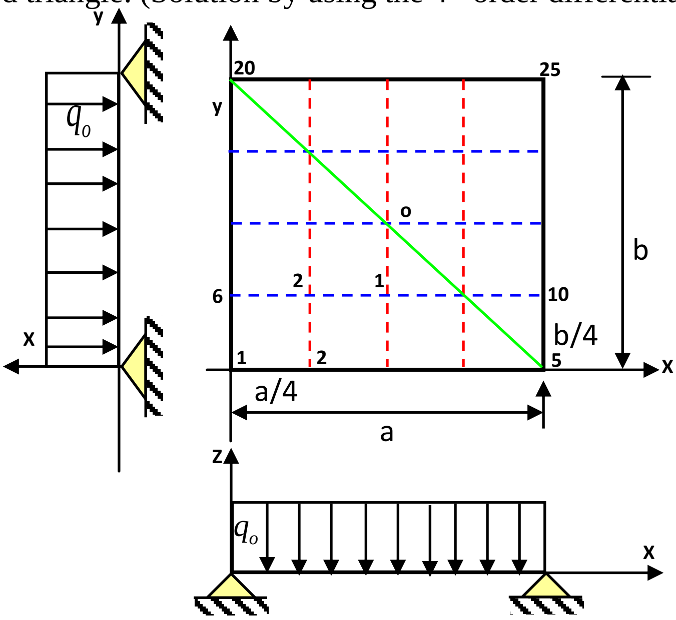
$$\frac{q_{i,j}}{D} * (\Delta x)^4 = 20w_{i,j} - 8w_{i,j-1} + w_{i,j+2} - 8w_{i,j-1} + w_{i,j-2} +$$

$$2w_{i+1,j+1} + 2w_{i-1,j+1} - 8w_{i+1,j} - 8w_{i-1,j+2} + 2w_{i+1,j-1} + 2w_{i-1,j-1} + w_{i+2,j} + w_{i-2,j}$$



Example:- calculate moments and deflections for the case of a uniformly loaded and simply supported square plate using 4 equal mesh intervals in each direction.
Solution it's evident from symmetry that the calculations need be extended over an area of one-eighth of the plate only as shown in following figure by the shaded triangle. (Solution by using the 4th order differential equation.

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial 4\, w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q_o}{D}$$



at Node Point (o) (نقطة كلها داخل اللوح)
$20w_o - 8w_1*4 + 2w_2*4 + 4*1*w_3 = q_o/D*(\Delta x)^4$ $w_3=0$
$20w_o - 32w_1 + 8w_2 = q_o/D(a/4)^4$
$N = q_o a^4 / 256\ D$
$20w_o - 32w_1 + 8w_2 = N$ (1)
at Node Point (1)
$-8w_o + 248w_1 - 16w_2 = q_o a^4 / 256\ D$
$-8w_o + 248w_1 - 16w_2 = N$ (2)
at Node Point (2)
$-20w_2 - 16w_1 + 2w_o = q_o a^4 / 256\ D$
$-20w_2 - 16w_1 + 2w_o = N$ (3)
In matrix form

$$\begin{bmatrix} 20 & -32 & 8 \\ -8 & 24 & -16 \\ 2 & -16 & 20 \end{bmatrix} \begin{bmatrix} w_o \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} N \\ N \\ N \end{bmatrix} \Rightarrow Aw = Q \Rightarrow w = \tfrac{Q}{A} \Rightarrow w = A^{-1}Q$$

$$A^{-1} = \tfrac{adj(\mathrm{A})}{|\mathrm{A}|}$$

$$adj(\mathrm{A}) = \mathrm{A}^{\mathrm{T}} = \begin{bmatrix} (-1)^{1+1}\begin{vmatrix} 24 & -16 \\ -16 & 20 \end{vmatrix} & (-1)^{1+2}\begin{vmatrix} -8 & -16 \\ 2 & 20 \end{vmatrix} & (-1)^{1+3}\begin{vmatrix} -8 & 20 \\ 2 & -16 \end{vmatrix} \\ (-1)^{2+1}\begin{vmatrix} -32 & 8 \\ -16 & 20 \end{vmatrix} & (-1)^{2+2}\begin{vmatrix} 20 & 8 \\ 2 & 20 \end{vmatrix} & (-1)^{2+3}\begin{vmatrix} 20 & -32 \\ 2 & -16 \end{vmatrix} \\ (-1)^{3+1}\begin{vmatrix} -32 & 8 \\ 24 & -16 \end{vmatrix} & (-1)^{3+2}\begin{vmatrix} 20 & 8 \\ -8 & -16 \end{vmatrix} & (-1)^{3+3}\begin{vmatrix} 20 & -32 \\ -8 & 24 \end{vmatrix} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 224 & 512 & 320 \\ 128 & 384 & 256 \\ 80 & 256 & 224 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 20 & -32 & 8 \\ -8 & 24 & -16 \\ 2 & -16 & 20 \end{vmatrix} \begin{matrix} 20 & -32 \\ -8 & 24 \\ 2 & -16 \end{matrix} \quad \begin{matrix} =10624 \\ \\ =11648 \end{matrix} \quad = 11648 - 10624 = 1024$$

$$\begin{bmatrix} w_o \\ w_1 \\ w_2 \end{bmatrix} = \tfrac{N}{1024} \begin{bmatrix} 224 & 512 & 320 \\ 128 & 384 & 256 \\ 80 & 256 & 224 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \tfrac{q*a^4}{256*1024D} \begin{bmatrix} 224+512+320 = 1056 \\ 128+384+256 = 768 \\ 80+256+224 = 560 \end{bmatrix} \Rightarrow \begin{bmatrix} w_o = 0.004028\,\frac{qa^4}{D} \\ w_1 = 0.00293\,\frac{qa^4}{D} \\ w_2 = 0.00213\,\frac{qa^4}{D} \end{bmatrix}$$

BM at node point (o) $M_{xo} = -D(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2})_o \Rightarrow M_{xo} = -D\left[\frac{2w_1 - 2w_o}{(\Delta x)^2} + \mu \frac{2w_1 - 2w_o}{(\Delta y)^2}\right]$

$\mu_{concrete(\Delta x = \Delta y) = 0.2}$

$$M_{xo} = -D\left[\tfrac{2w_1 - 2w_o}{(\Delta x)^2} + 0.2\,\tfrac{2w_1 - 2w_o}{(\Delta x)^2}\right] \Rightarrow \tfrac{-D}{(\Delta x)^2}\left[2w_1 - 2w_o + 0.4w_1 - 0.4w_o\right]$$

$$M_{xo} = \tfrac{-D}{(\frac{a}{4})^2}\left[2.4w_1 - 2.4w_o\right] = \tfrac{-D*16}{a^2}\left[2.4*0.00293\tfrac{qa^4}{D} - 2.4*0.004028\tfrac{qa^4}{D}\right]$$

$$M_{xo} = +0.04224*q*a^2$$

Lecture No. 3 Plates on the Elastic Foundations

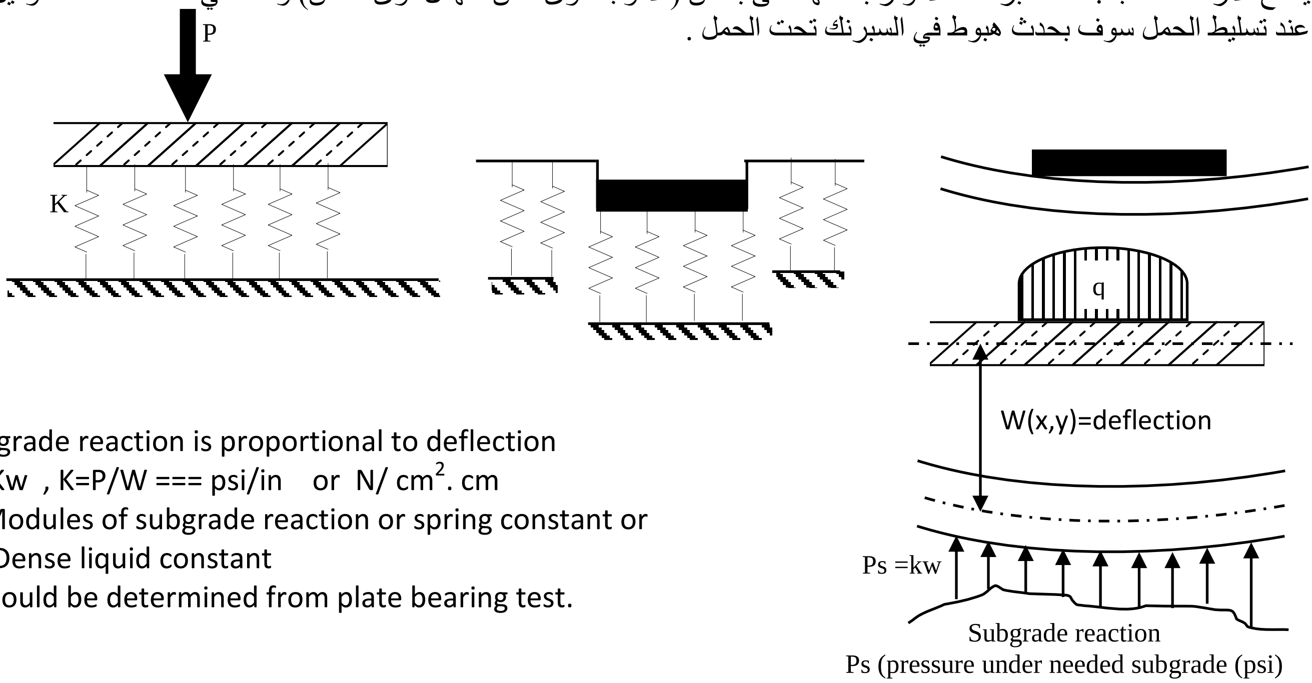
كيفية تمثيل الاجهادات الموجودة بالتربة تحت التبليط (هل يمكن تمثيل التربة رياضيا" بنموذجموديل بسيط).

Foundation Modulus

1) Winkler Foundation (Dense Liquid Model 1876).

Represented the subgrade soil by menus of indicial closely spaced discrete springs for this winkler model pressure at any point is proportional to deflection at that point.

ولنكر بفرض بان الكونكريت يجلس على التربة (طبقة ما تحت الاساس) ممثلة بسبرنك وقوى السبرنك تتناسب طرديا مع مقدار التشوة (الهبوط) اي انة يمثل التربة بسبرنكات مستقلة بعضها مع بعض ومتشابها في خواصها ومتقاربة ومقدار القوة الموجودة في السيرنك تتناسب طرديا مع الازاحة الناتجة.هذة السبرنكات لاتؤثر بعضها على بعض (لاتوجد قوى قص- نهمل قوى القص) وهذة هي نقطة ضعف الموديل لانه عند تسليط الحمل سوف يحدث هبوط في السبرنك تحت الحمل .



Subgrade reaction is proportional to deflection

P_s=Kw , K=P/W === psi/in or N/ cm^2. cm

K=Modules of subgrade reaction or spring constant or Dense liquid constant

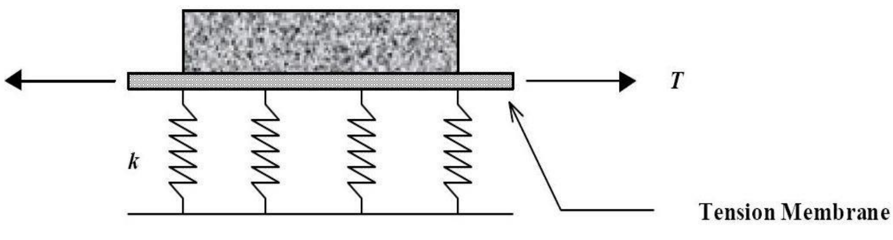
K should be determined from plate bearing test.

2) Filonenko-Borodish Model (1940) **نموذج فلنكو – بورداش**

This foundation model includes a stretched elastic membrane that connects to the top of the springs and is subjected to a constant tension field *T*. The tension membrane allows for interaction between adjacent spring elements. The relation between the subgrade surface stress field *q(x,y)* and the corresponding deflection is defined by

$$q(x, y) = kw - T\nabla^2 w$$

where ∇^2 is the Laplace operator in the *x* and *y* directions.

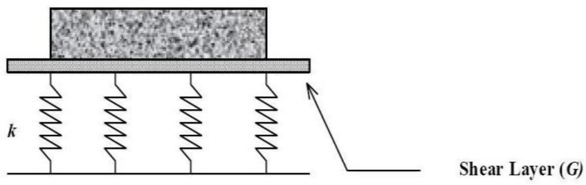


في هذا الموديل او النموذج يفرض بان السبرنكات مرتبطة بعضو شد في اعلى الطبقة مما يؤدي الى اجهاد سطح الطبقة

3) Pasternak foundation model , 1954

Pasternak (1954) allowed the transmission of shear stresses in the dense liquid foundation by inserting a thin shear layer between the spring elements and the bottom of the slab. On a microscopic level, the shear layer consisted of incompressible vertical elements that deform only in response to transverse shear stresses. In addition to the modulus of subgrade reaction (*k*-value), this model includes a shear characteristic parameter (*G*). Pasternak defined the relationship between subgrade reaction and deflection as

$$q = kw - G\nabla^2 w$$



4) Vlasov and Leont'ev

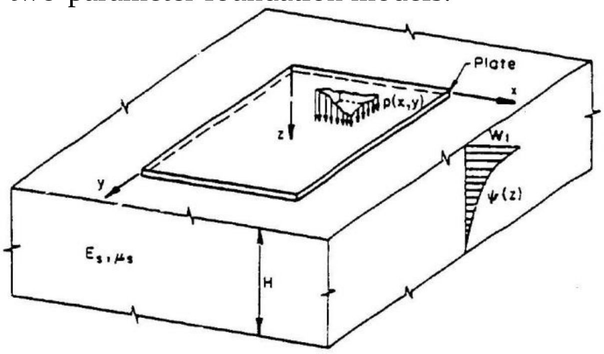
Vlasov and Leont`ev (1966) introduced a different approach to the problem of simulating the foundation of a pavement structure. The system was modeled as a plate supported by an elastic solid layer of thickness *H*, and subject to a vertical pressure *p(x,y)*, as illustrated in figure 2.11. Horizontal displacements (*u, v*) are assumed to be negligible in comparison with the vertical (*w*) displacement because there is no horizontal Loading. An unknown displacement of a point in the layer is determined through a summation of the form:

$$w(x, y, z) = \text{Sum } w_k(x, y)\varphi_k(z)$$

In this summation, $w_k(x,y)$ are unknown generalized displacement functions. These functions are calculated for a given section (i.e., z = constant) to determine the magnitude of the vertical displacement *w(x,y)* in this section. They have dimensions of length. On the other hand, φ_k are known functions that satisfy the boundary conditions, i.e., for $z = 0$ and $z = H$. These functions represent the distribution of displacements with depth and are dimensionless. After simplifying the problem to its two-dimensional case and applying the principle of virtual displacements, Vlasov and Leont`ev formulated the relationship between the subgrade reaction and deflection as

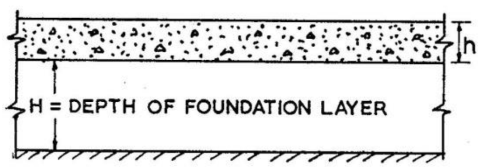
$$G\nabla^2 w - kw + q = 0$$

where *k* and *G* characterize the compressive and shear strain in the foundation, respectively. The form of this equation is essentially identical to those applying to other two-parameter foundation models.



5) **Reissner Model** (this model is more general than winker and retains the mathematical of winkler model).
Is based on continuum in approach with assumptions;

1) In plane stresses through out the continuum are negligibly small [$\varepsilon_x=\varepsilon_y=\tau_{ry}=0$]
2) Horizontal displacement at upper and lower surfaces of the foundation layers are zero.



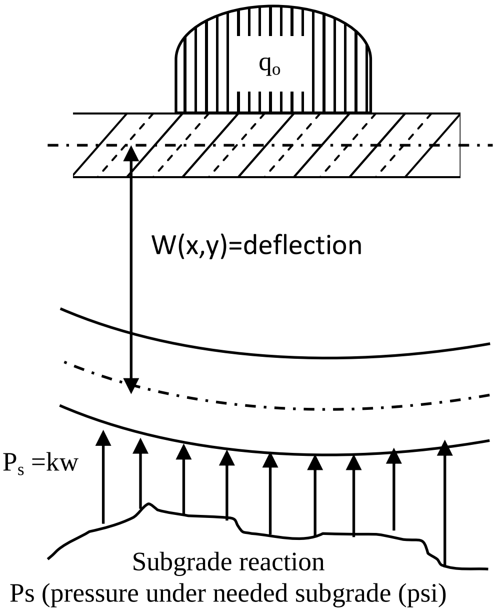
6) **Elastic Continuums (Boussinesq's half spa**ce)
7) **Layered Continuum متعدد الطبقات**

Plates on Elastic Foundation

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial 4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q_o}{D}$$

$$\Delta\Delta w = \frac{q_o - q_s}{D} = \frac{q_o\text{-Kw}}{D}$$

$$\Delta\Delta w + \frac{Kw}{D} = \frac{q_o}{D}$$



Finite difference solution for plates on winkler foundation

$$\Delta\Delta w = \frac{q_o}{D} - \frac{K}{D}w \Rightarrow \frac{D}{\lambda^4}[S]\{w\} = \{P\} - k\{w\}$$

$$\lambda = \Delta x = \Delta y$$

$$\frac{D}{\lambda^4}[S]\{w\} = \{P\} - kI\{w\}$$

$$\frac{D}{\lambda^4}\begin{bmatrix} S_{11} & S_{12} & - & - & S_{1n} \\ S_{21} & S_{22} & - & - & S_{2n} \\ - & & & & - \\ - & & & & - \\ S_{n1} & S_{m2} & - & - & S_{nn} \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \\ \\ \\ w_n \end{bmatrix} = \begin{bmatrix} P_1 \\ \\ \\ \\ p_n \end{bmatrix} - k\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ & 1 & 0 & 0 & 0 \\ & & 1 & 0 & 0 \\ & & & 1 & 0 \\ & & & & 1 \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \\ \\ \\ w_n \end{bmatrix}$$

$$\frac{D}{\lambda^4}\begin{bmatrix} S_{11} & S_{12} & - & - & S_{1n} \\ S_{21} & S_{22} & - & - & S_{2n} \\ - & & & & - \\ - & & & & - \\ S_{n1} & S_{m2} & - & - & S_{nn} \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \\ \\ \\ w_n \end{bmatrix} + k\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ & 1 & 0 & 0 & 0 \\ & & 1 & 0 & 0 \\ & & & 1 & 0 \\ & & & & 1 \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \\ \\ \\ w_n \end{bmatrix} = \begin{bmatrix} P_1 \\ \\ \\ \\ p_n \end{bmatrix}$$

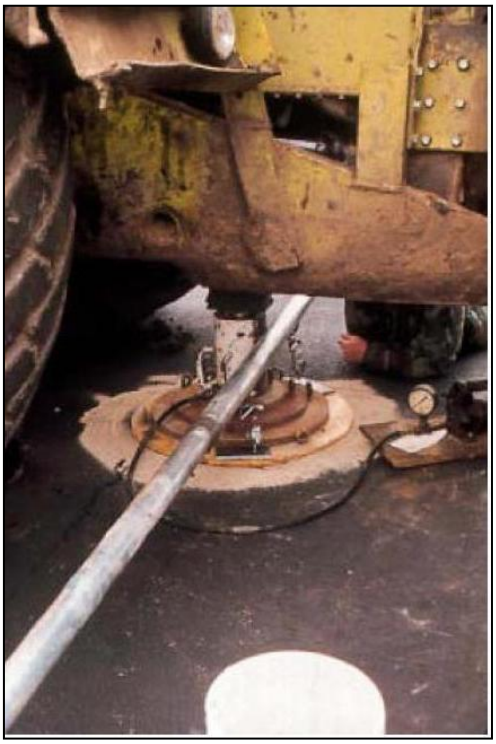
λ= length of square mesh grid.

[s]= Finite difference coefficient matrix

Equivalent stiffness matrix

I= unit matrix (identity Matrix)

D= $Eh^3/12(1-\mu^2)$



Lecture No. 5

Stresses in Rigid Pavement

Causes of stresses

1) Wheel load (Externally applied)
2) Cyclic change in temperature (warping , shrinkages , expansion)
التغيرات الدورية بدرجة الحرارة بين اعلى واسفل التبليط والذي تؤدي الى حركة (تقلص او تمدد) هذا الاختلاف بدرجة الحرارة يسبب اجهادات (شد (تمدد) او ضغط (تقلص)) فعند ارتفاع درجة حرارة السطح يحاول ان يتمدد و القعر يحاول ان يتقلص)
3) Changes in moisture
4) Volumetric changes in the subgrade or base due to (frost action , loss of support, plastic deformation)

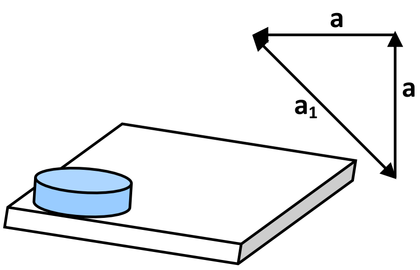
Stresses due to Wheel Loads (Axle loads)

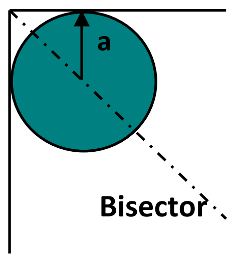
Stresses in rigid pavement due to a single wheel load (Westergaard Theory)

هذا النوع من التبليط بفشل ب Bending moment ويجب ان ناخذ الحالة الاسوا (Max. Bending Moment) لذلك يجب تحديد مواقع العزم الاسواء (واستر كارد يناقش او يحسب الاحهادات الناتجة من تسليط الحمل في المواقع المختلفة:-

1) Corner Load (wheel load distributed uniformly upon a circular tire pavement contact area)

واستركارد يفرض بان الحمل المسلط يتوزع بصورة منتظمة على مساحة تماس دائرية الشكل موضوعها بتماس في حافة التبليط بحيث ان منصف زاوية المركز يمر في مركز الحمل. وكذلك يفرض بان التبليط يجلس على اساس ولنكر و بالتالي يتصرف cantilever





a:- radious of circular tire pavement contact area.

$a_1^2 = 2a^2 \Rightarrow a_1 = a\sqrt{2}$

Tensile stress at corner

$$\sigma_c = \frac{3p}{h^2}\left[1 - \left(\frac{a_1}{\ell}\right)^{0.6}\right]$$

$\sigma_c \Rightarrow$ Max. tensile stress at the top of the slab in direction parallel to bisector of corner angle.
اكبر اجهاد اشد يحدث في سطح الطبقة وباتجاة موازي لمنصف زاوية المركز.

P $\Rightarrow$ effective wheel load = static wheel load*Impact factor
نتجة لحركة المركبة على الطريق تحدث اهتزازات بالاتجاةالعمودي فتزداد قيمة الحمل و هذة الزيادة تؤخذ بنظر الاعتبار.

$\ell \Rightarrow$ Radius of relative stiffness $\Rightarrow$ $\ell = \sqrt[4]{\frac{Eh^3}{12K(1-\mu^2)}}$

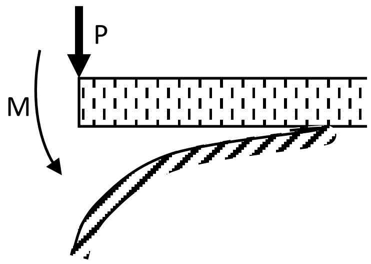
E:- Modulus of elasticity of a slab
h:- uniform thickness of the slab.
K:- Modulus of subgrade reaction (PCI, N/cm^3)
μ:- Poisson ratio of the slab.

Older formula

اولدر يفرض بان التبليط يكون cantilever وتحدث فيه تشقق بزاوية 45 درجة وبمسافة x

$$M = -Px \Rightarrow \sigma_{max} = \frac{MC}{I}$$

$$\sigma_{max} = \frac{-Px\ h/2}{2x*h^3/12} = \frac{-6P\ x\ h}{2\ x\ h^3} = -\frac{3P}{h^2}(tension = comp)$$



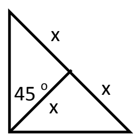


TABLE 3.1. Radius of Relative Stiffness
(μ = 0.15 E = 4,000,000 psi)

h (in.)	k = 50	k = 100	k = 200	k = 300	k = 400	k = 500
9.0	47.22	39.71	33.39	30.17	28.08	26.55
9.5	49.17	41.35	34.77	31.42	29.24	27.65
10.0	51.10	42.97	36.14	32.65	30.39	28.74
10.5	53.01	44.57	37.48	33.87	31.52	29.81
11.0	54.89	46.16	38.81	35.07	32.64	30.87
11.5	56.75	47.72	40.13	36.26	33.74	31.91
12.0	58.59	49.27	41.43	37.44	34.84	32.95
12.5	60.41	50.80	42.72	38.60	35.92	33.97
13.0	62.22	52.32	43.99	39.75	36.99	34.99
14.0	65.77	55.31	46.51	42.02	39.11	36.99
15.0	69.27	58.25	48.98	44.26	41.19	38.95
16.0	72.70	61.13	51.41	46.45	43.23	40.88
17.0	76.08	63.98	53.80	48.61	45.24	42.78
18.0	79.41	66.78	56.16	50.74	47.22	44.66
19.0	82.70	69.54	58.48	52.84	49.17	46.51
20.0	85.95	72.27	60.77	54.92	51.10	48.33
21.0	89.15	74.97	63.04	56.96	53.01	50.13
22.0	92.31	77.63	65.28	58.98	54.89	51.91
23.0	95.44	80.26	67.49	60.98	56.75	53.67
24.0	98.54	82.86	69.68	62.96	58.59	55.41

2) Interior Load (واستركارد يفرض بان الحمل يسلط مساحة تماس دائرية الشكل على لوح يمتد الى الملانهاية في كل الاتجاهات

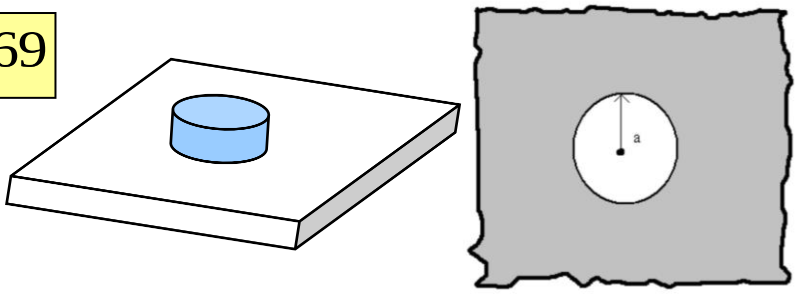
$$\sigma_i = \frac{0.316P}{h^2}[4\log_{10}(\tfrac{\ell}{b}) + 1.069$$

1) if $a < 1.724\,h$ special theory

$$b = \sqrt{1.6a^2 + h^2} - 0.675\,h$$

2) if $a > 1.724\,h$ ordinary theory

$$b = a$$

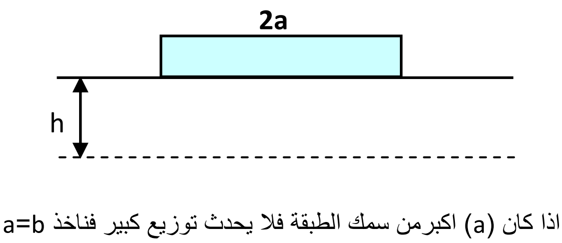


Where:-

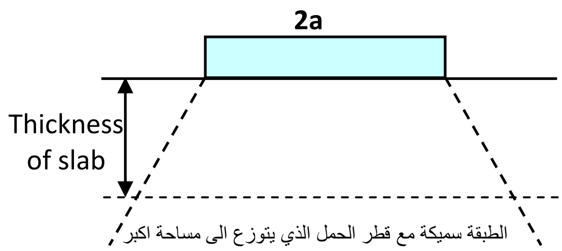
b:- radius of equivalent distribution of pressure at the bottom of the slab.

P:- wheel load (increased by the Impact factor = effective wheel load)

σ_i:- Max. load produced tensile stress in the interior of the slab (Occurs in the bottom of the slab directly under the load)



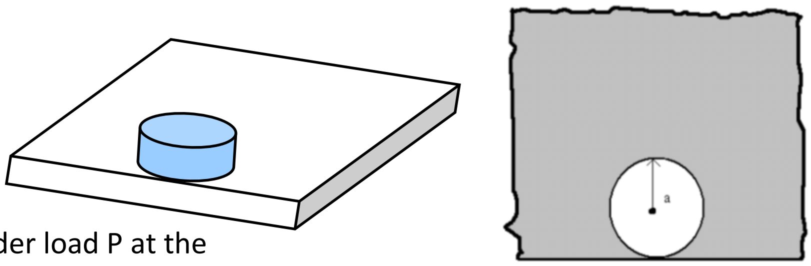
Ordinary theory



Special theory

3) Edge Load

$$\sigma_e = \frac{0.572P}{h^2}[4\log_{10}(\tfrac{\ell}{b}) + 0.359]$$



σ_e : Max. Load produced tensile stress
In the bottom of the slab directly under load P at the Edge and the direction parallel to the edge.

تحسب الاجهادات في الوسط و الحافة و المركز ونأخذ الاكبر.

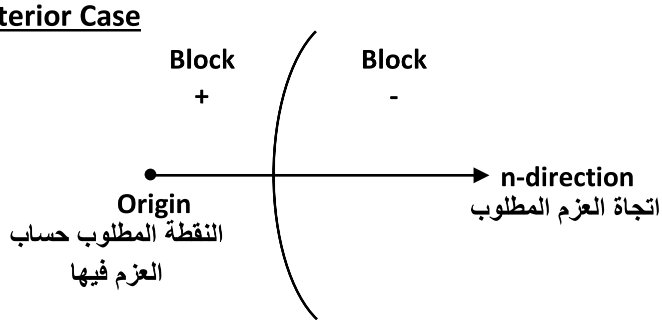
Multiple Wheel Loads (Influence Charts) (نتجة لتتطور المركبات والطائرات فتزداد حمولته(اين يحدث اكبر اجهاد وباي اتجاة

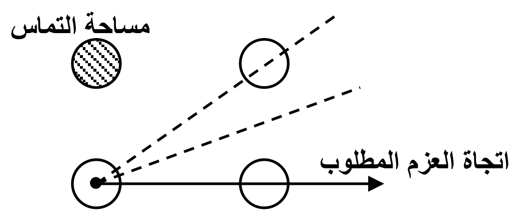
Pickett and Ray's Influence charts (the main assumptions are;

1) for interior load a) winkler springs (dense liquid) b) elastic continuums

2) for edge loading.

1) Interior Case





خطوات الحل

1) يتم تثبيت النقطة المطلوب حساب العزم فيها فوق نقطة الاصل. Origin point
2) يتم تثبيت اتجاه العزم للنقطة المطلوب حساب العزم فيها n-direction
3) كل عنصر element يمثل مساحة معينة من العزم (كل عنصر له قيمة معينة من عزم الانحناء bending Moment)
4) هنالك تشابه بالاتجاهين الافقي والعمودي.
5) هذة الخطوات اعلاه مرسومة بمقياس رسم معين بحيث يحول كل المخطط الى المقياس المطلوب.
6) المخططات مرسومة على اساس $\mu_{of\ concrete}=0.15$

Scale (radius of relative stiffness)

1) Find the radius of relative stiffness $\ell = \sqrt[4]{\frac{E\,h^3}{12K(1-\mu^2)}}$
2) assumed $\ell = 5\ cm$

نحول الرسم الى المقياس المطلوب على فرض ان ℓ تساوي 5 سم. اي نعيد رسم شكل الحمل بمقياس رسم اعلاه.

اين يتم دراسة العزم وباي اتجاه؟؟

1) بالاتجاه الافقي (نضع النقطة المطلوبة فوق o ونحسب عدد المربعات التي يغطيها ونستفيد من التشابة بالشكل)
2) اذا كان الاتجاة يميل بزاوية معينة مع n-direction
   a) اذا كان المطلوب حساب اكبر عزم في هذة الحالة ندور الشكل بزوايا مختلفة من ضمنها الزاوية اعلاه.
   b) نجد قيم العزم لكل زاوية ونرسم العلاقة بين الزوايا و العزم

حساب العزم

1) Uniform load $M = \frac{P\,\ell^2\,N}{10000}$
2) Non-uniform load $M_n = \sum_{i=1}^{m} \frac{P_i\,\ell^2\,N_i}{10000} = \frac{\ell^2}{10000}\sum_{i=1}^{m} P_i * N_i$

Where:-

m :- number of tires.
P_i :- Tire pressure for i^{th} tire.
N_i :- number of blocks with in the tire.

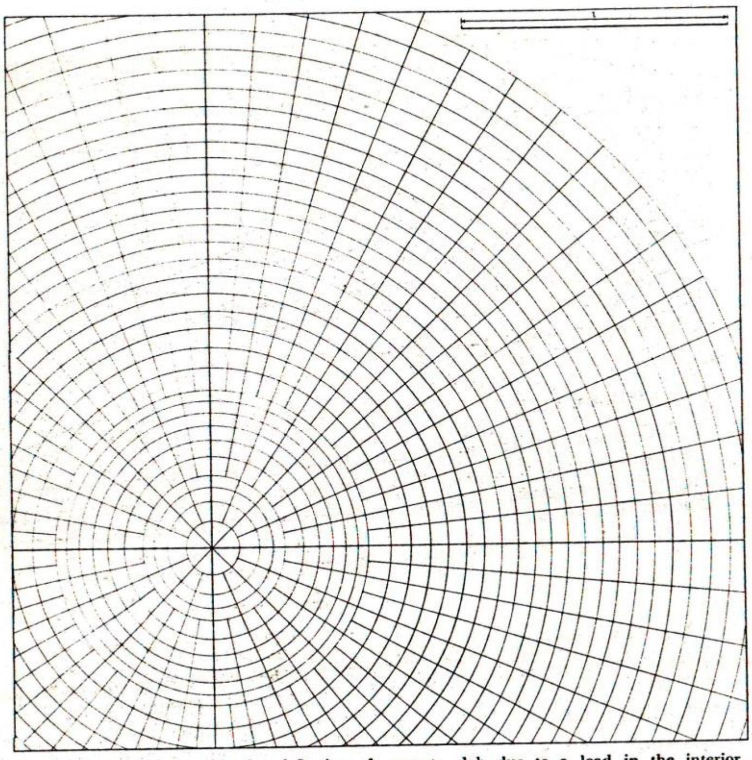


Figure 3.19. Influence chart for deflection of concrete slab due to a load in the interior. (Subgrade assumed to be a dense liquid. Poisson's ratio for the concrete = 0.15.) (From Figure 1 of Pickett and Ray, "Influence Charts for Concrete Pavements," *Transactions*, ASCE, 1951.)

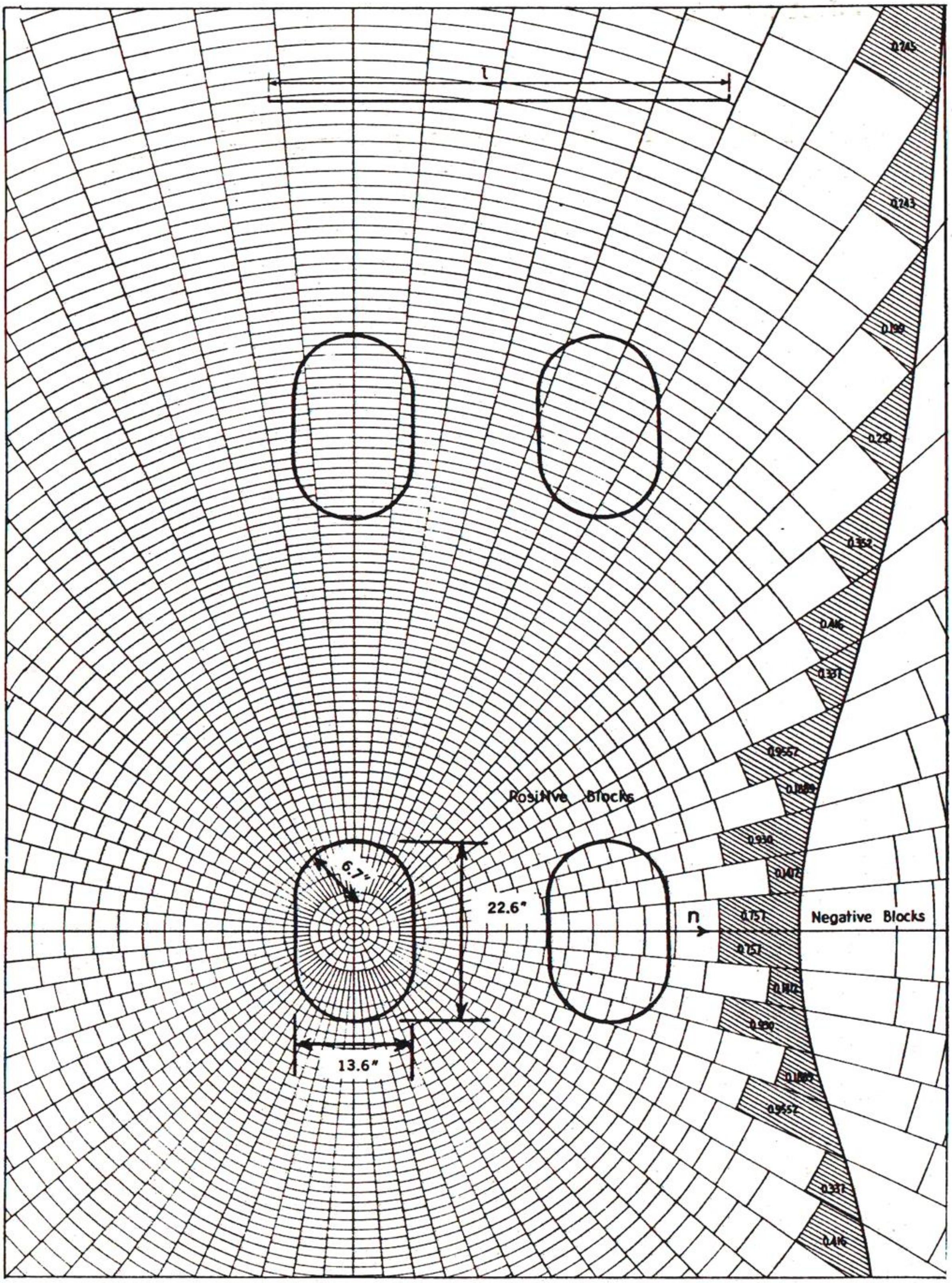


Figure 3.21. Influence chart for the moment M_n in a concrete pavement due to a load in the interior of the slab. (Subgrade assumed to be a dense liquid. Poisson's ratio for pavement assumed to be 0.15.) (From Figure 3 of Pickett and Ray, "Influence Charts for Concrete Pavements," *Transactions*, ASCE, 1951.)

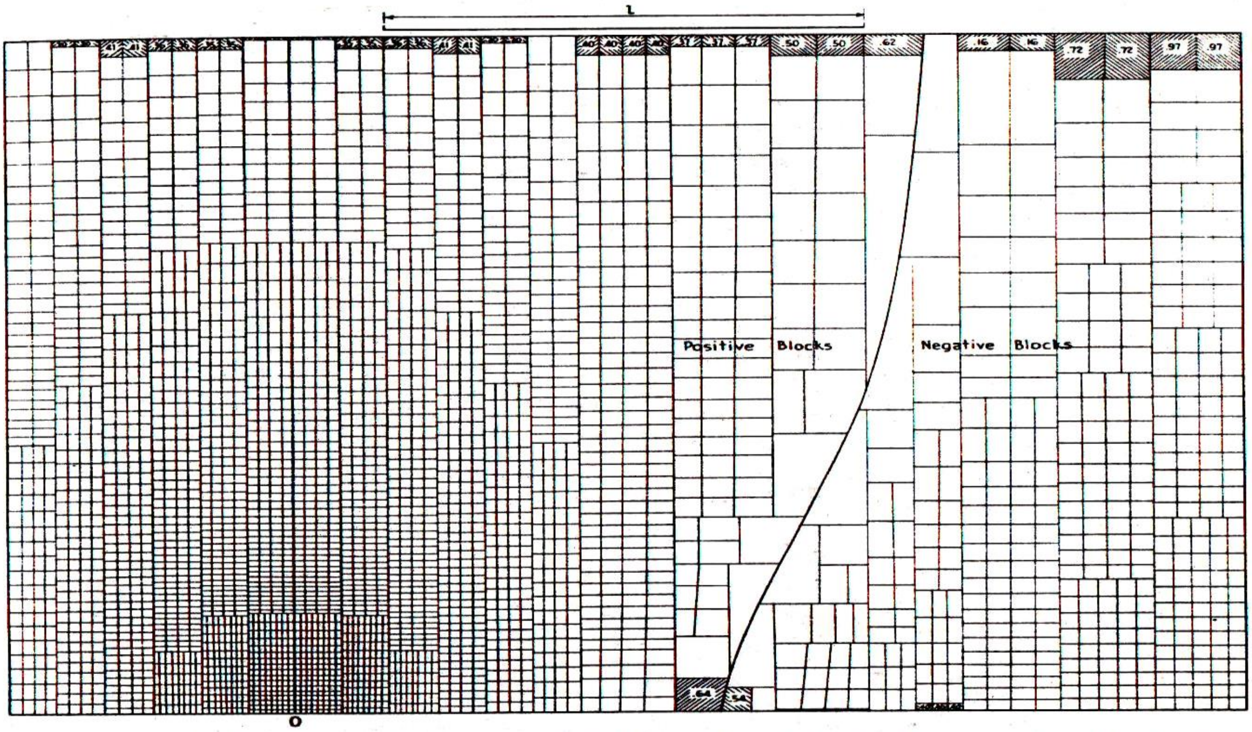


Figure 3.22. Influence chart for the moment *M* at edge (point 0) of a concrete slab due to a load in the vicinity of the edge. (Subgrade assumed to be a dense liquid. Poisson's ratio for pavement = 0.15.) (From Figure 4 of Pickett and Ray, "Influence Charts for Concrete Pavements," *Transactions,* ASCE, 1951.)

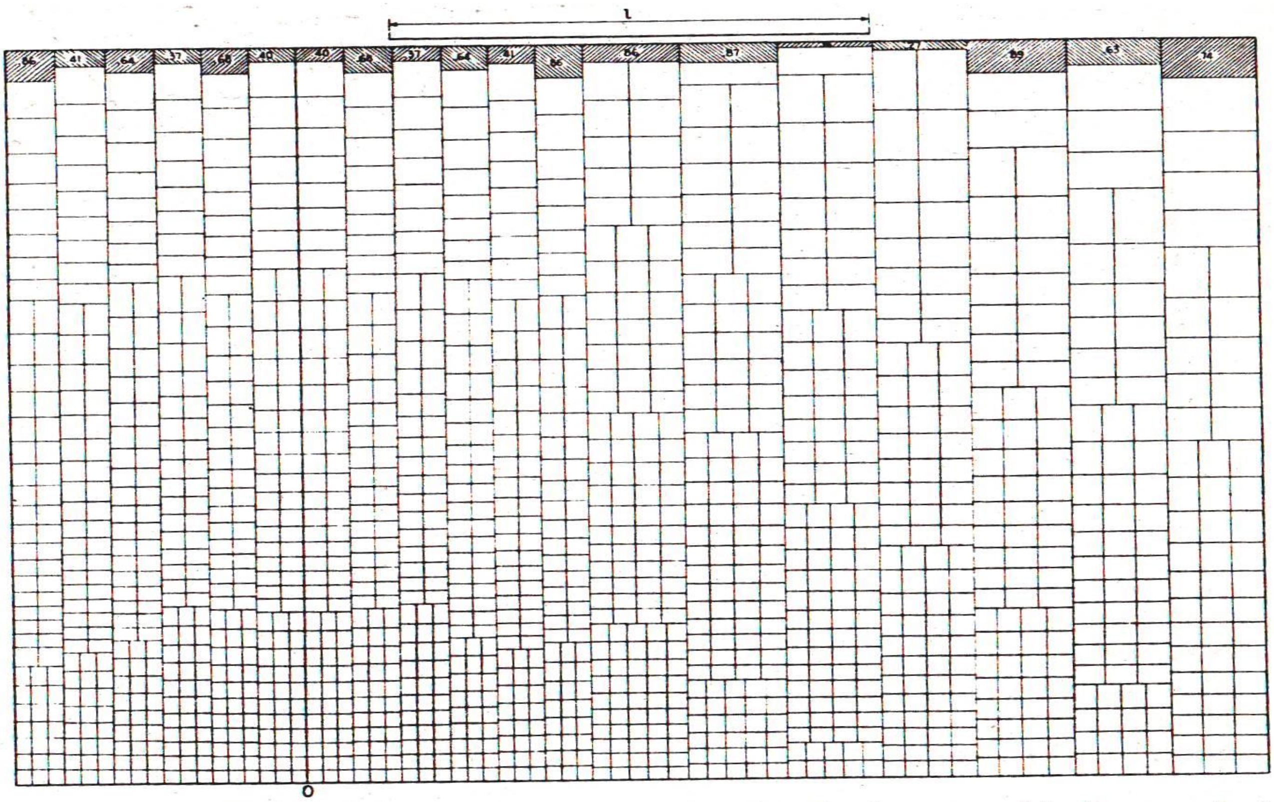
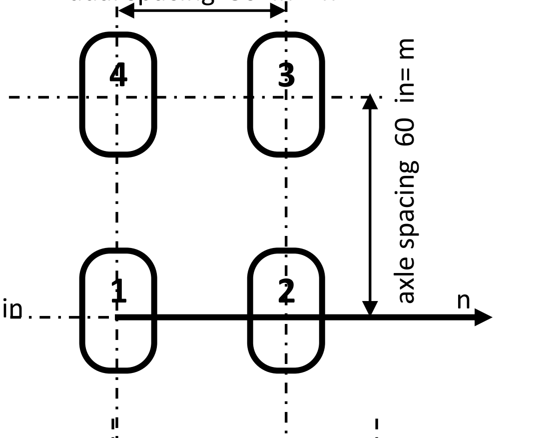


Figure 3.20. Influence chart for deflection at edge (point 0) of concrete slab due to a load in vicinity of the edge. (Subgrade assumed to be a dense liquid. Poisson's ratio for the concrete = 0.15.) (From Figure 2 of Pickett and Ray, "Influence Charts for Concrete Pavements," *Transactions,* ASCE, 1951.)

Example: - Determine the stress (Max. Stress) under the central of tire No.1 of the multiple wheel assemble shown in the following Figure. For indirect of position on the interior of 14 in thick concrete pavement knowing that the total load on the assembly is 160 kips, tire pressure 150 psi , K=100 pci , E=4*10^6 psi, μ=0.15. Assume the tire pavement contact area having the shape of rectangular with semi-circular at ends.



Solution

1) بمانة لم يذكر كيفية توزيع الحمل فنفرض انة يتوزع بصورة منتظمة.(قد يتوزع الحمل بصورة غير منتظمة اي كنسبة من الحمل الكلي).

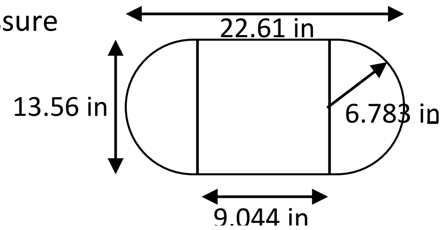
Load on each tire= 160/4=40 kips

2) نحسب مساحة التماس

Contact area = axle load/ tire pressure

A= 40 000/ 150 = 267 in^2

A=0.52274 L^2 $\Rightarrow$ L= 22.61 in



3) نحسب

$$\ell = \sqrt[4]{\frac{E\,h^3}{12K(1-\mu^2)}} = \sqrt[4]{\frac{4*10^6*14^3}{12*100(1-(0.15)^2)}} = 55.31 \text{ in}$$

4) نحسب مقياس الرسم ونرسم الاحمال اعلاه بهذا المقياس

55.31 = 5.5 cm

22.6 = L $\Rightarrow$ L= 2.247 cm

13.56 =0.6 L $\Rightarrow$ 0.6L= 1.348 cm

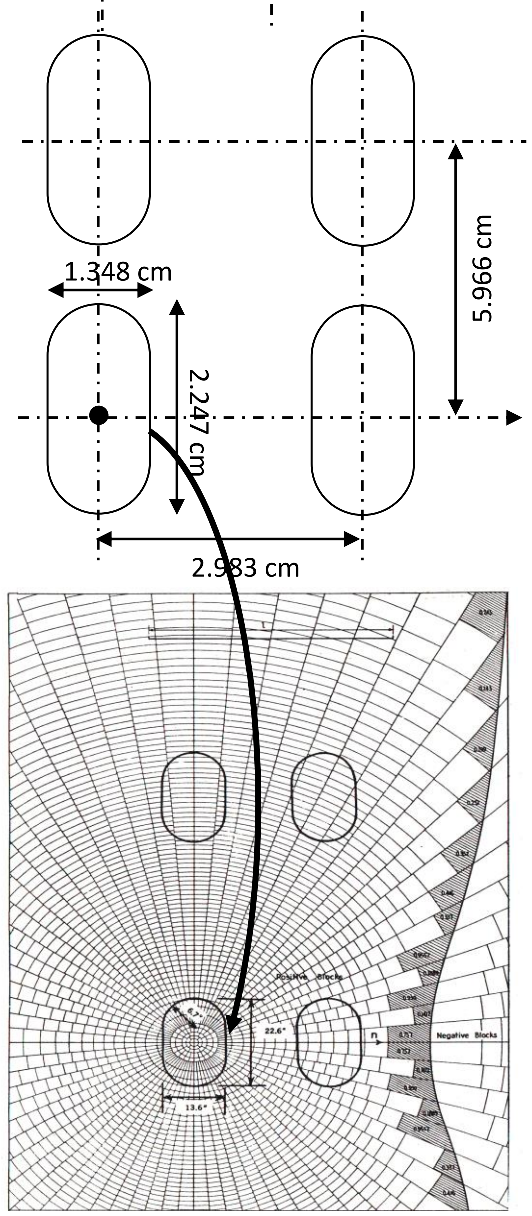
60 in = m $\Rightarrow$ m= 5.966 cm

30 in = n $\Rightarrow$ n=2.983 cm

5) نسقط هذا المخطط فوق المنحنيات اعلاه.

6) نحسب عدد block الذي يغطيها كل أطار ويمكن الاستفادة من التشابة:-

Tire No.	Ni (no. of enclosed block	Sum
1	4*49	196
2	2*17	34
3	1*24	24
4	20.5*2	41
	total	295 blocks



7) نحسب العزم

$$M_n == \frac{\ell^2}{10000}\sum_{i=1}^{m} P_i * N_i = \frac{55.31)^2}{10000} * 150 * 295 = 13537 \frac{lb.in}{in}$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{6M}{h^2} = \frac{6*13537}{14^2} = 414 \ psi$$

حيث ان عزم القصور الذاتي

$$\frac{h/2}{1*h^3/12} = \frac{12h}{2h^3} = \frac{6}{h^2}$$

ملاحظات:-

a. المساحات الغير كاملة تقرب.

b. قد تقع بعض المناطق في الجزء السالب فتطرح.

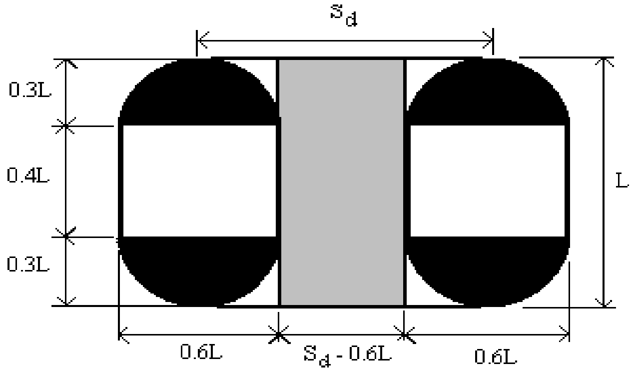
c. في المناطق الواقعة على الخط الفاصل بين + و – تؤخذ النسبة الذي تمثل نسبة المظلل من المساحة الكلية.

Dual Tires Simulation

Since all Westergaard's loading stress equations for a single tire print are circular loading area based. It is necessary to convert dual tires into a single circular loading area, following equation allows for the conversion from dual tires to a single tire (Huang, 1993).

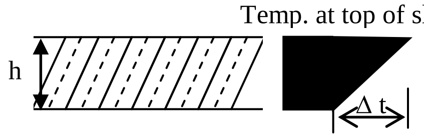
$$a = \sqrt{\frac{0.8521 \cdot P_d}{q \cdot \pi} + \frac{S_d}{\pi} \cdot \left(\frac{P_d}{0.5227 \cdot q}\right)^{\frac{1}{2}}}$$

Where P_d = load on one tire
q = contact pressure (one tire)
S_d = dual spacing (center to center)



Stresses due to Warping (Curing stresses) الاجهادات الناتجة من الالتواء او الاعوجاج

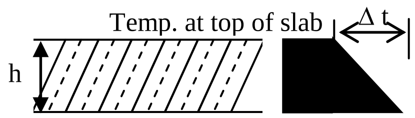
at day-time (summer)



درجة الحرارة في الاعلى تكون اكبرلذلك تميل الطبقة الى التمدد ولكننا نضغطها فيحدث فيها ضغط فترتفع الاركان الى الاعلى درجة الحرارة في الاسفل قليلة لذلك تميل الى التقلص ولكنها يسلط عليها ضغط فيحدث فيها شد .

Δt= temp. differential between top and bottom.

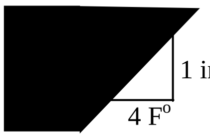
1) at night-time (winter)



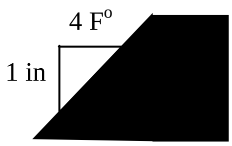
درجة الحرارة في الاعلى قليلة فيحاول اللوح للتقلص ولكن يسلط علية ضغط فيحدث فيها شد فترتفع في الوسط .وبالعكس فس اسفل الطبقة

ف

Temp. gradient from top to the bottom of slab observation of temp. different between top and bottom of slabs , temp. differentional in day time partially in summer after the sun has heated the upper surface ranged up to 4 F° per one inch of depth.



at day time بالنهار



at night time بالليل

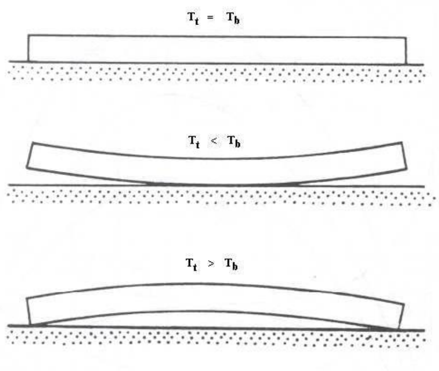
At night after cooling the temp. of the surface was below that at the bottom by as by as much as 1.5 F° per inch of depth.

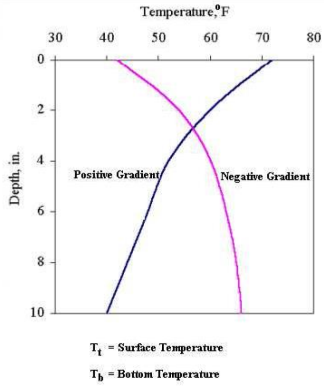
Differentional in temp. between day and night. (Westergaard – Bradbury , Edge stresses)

$$\sigma = \frac{C^* E^* \varepsilon_t {}^* \Delta_t}{2} \quad \Rightarrow \text{ at edge}$$

$$\sigma = \frac{E^* \varepsilon_t {}^* \Delta_t}{2}\left[\frac{C_1 + \mu^* C_2}{1-\mu^2}\right] \quad \Rightarrow \text{ at interior edge}$$

C_1 :- Coefficient in the desired direction (C_x).
C_2 :- Coefficient in the direction perpendicular to this direction (C_y)
ε_t :- coefficient of linear thermal expansion (10^{-5} per °C).
μ :- Poisson's ratio.
E :- Modulus of elasticity.

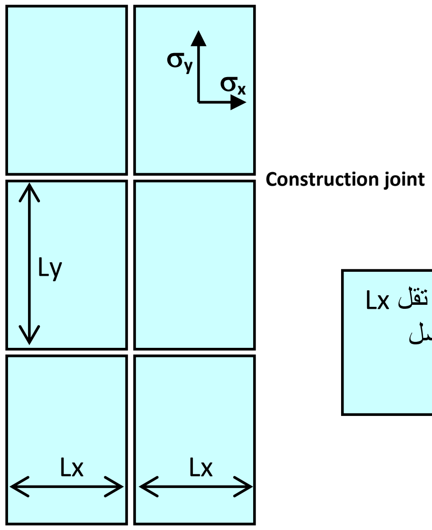




Warping Joint (اقصى مسافة بين مفصلين)

Lx :- free length of the slab اقصى مسافة بين مفصلين باتجاه x

Ly :- free width of the slab اقصى مسافة بين مفصلين باتجاه y



كلما تقل المسافة بين المفاصل تقل Lx
كل مفاصل التمدد تعامل كمفصل
حتي يكون العزم فيها صفر

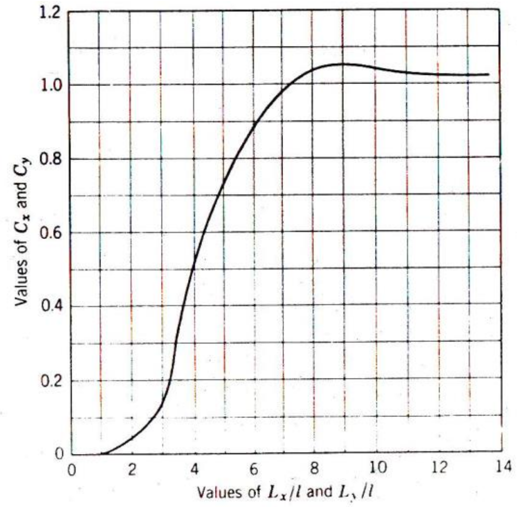


Figure 3.4. Warping stress coefficients. (From Bradbury.)

Max. Cx, Cy = 1.025 at Lx and Ly=9 at prestressing large length (airport)

اذن للتخلص من الاجهادات الحرارية نزيد ابعاد اللوح لتقليل الاجهادات.

Example:- for a concrete slab thickness=10 in , width=10 ft , length= 30 ft with the properties given below . Find Max. Combined flexural stresses in the interior slab;

- Static wheel load= 8000 lb.
- impact factor = 1.475
- Radius of circular tire pavement contact area (a)= 7.8 in.
- E of concrete = $5*10^6$ psi
- μ of concrete = 0.15.
- Coefficient of linear thermal expansion of concrete = $5*10^{-5}$ in/in/F^o.
- Temp. differentional between top and bottom of the slab at day time= -3 F^o per inches thickness.
- K=100 pci.

Solution:-

1) Stresses due to wheel load.

$$\ell = \sqrt[4]{\frac{E\,h^3}{12K(1-\mu^2)}} = \sqrt[4]{\frac{5*10^6\,(10)^3}{12*100*(1-(0.15)^2)}} = 45.43\,\text{in}$$

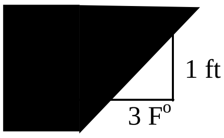
1.724h $\Rightarrow$ 1.724 *10= 17.24 in > a=7.8 in so that used special theory

Than $b = \sqrt{1.6a^2 + h^2} - 0.675\,h = 7.297$ in

$$\sigma_{xi} = \frac{0.316*11800}{10^2}[4\log_{10}(\frac{45.43}{7.297}) + 1.069] = 15.83 \ \text{psi}$$

$$\sigma_{xe} = \frac{0.572*11800}{10^2}[4\log_{10}(\frac{45.43}{7.297}) - 0.359] = 238.65 \ \text{psi}$$

2) <u>Stresses due to warping temperature.</u>



at day time بالنهار

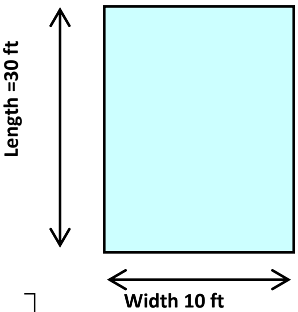
a) Stress at day time tangential stress.

مقدار التغير بدرجة الحرارة يتناسب طرديا مع العمق

1 in = 4 F^o

10 in = x so that x= 30 F^o differnational temp. Between

$\frac{L_x}{\ell} = \frac{30*12}{45.43} = 7.924 \text{ in}$ Using above figure to determined C_x= 1.05



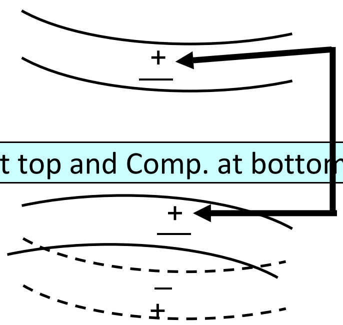
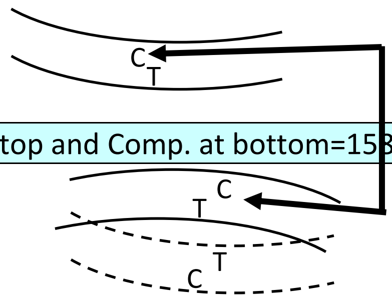
$$\sigma_{xe} = \frac{C* E* \varepsilon_t *\Delta_t}{2} = \frac{1.05*5*10^6*30*5*10^{-6}}{2} = 393.75 \text{ psi}$$

b) Interior stresses

$\frac{L_y}{\ell} = \frac{10*12}{45.43} = 2.641 \text{ in} \Rightarrow C_y = 0.07$

$$\sigma_{xi} = \frac{E* \varepsilon_t *\Delta_t}{2}\left[\frac{C_1 + \mu * C_2}{1-\mu^2}\right] = \frac{5*10^6*30*5*10^{-6}}{2}\left[\frac{1.05+0.17*0.07}{1-(0.15)^2}\right] = 406.841 \text{ psi}$$

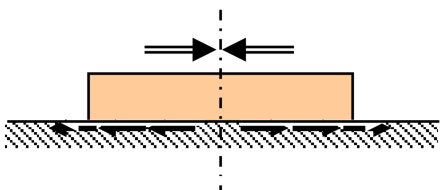
3) <u>Combined stresses</u>

	Edge stress	Interior stress
Load	Comp. at top and tension at bottom=393.75	Comp. at top and tension at bottom=406.841
		
Temp.	tension at top and Comp. at bottom=238.65	tension at top and Comp. at bottom=158.308
total	393.75 +238.65=632.4 psi	406.841+158.308= 565.149 psi

<u>Stress due to Friction(stresses due to uniform temperature)</u>

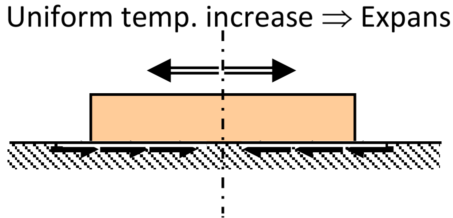
الاجهادات الناتجة من التغير المنتظم بدرجات الحرارة يسبب حركة للوحة و هذة الحركة تولد اجهادات نتجة لمقاومة الحركة الناتجة من الاحتكاك مع اسفل الطبقة .

1) Uniform temp. drop ⇒ Construction

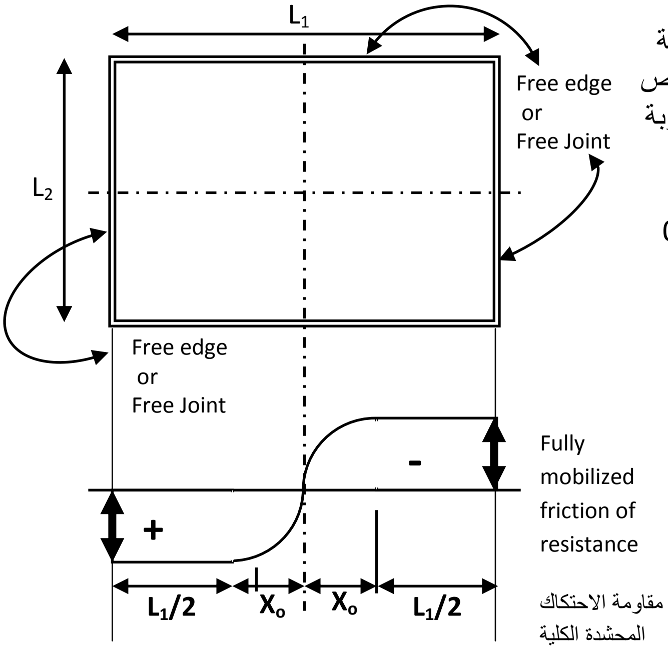


نتجة انخفاض درجة الحرارة تزداد قوى التلاصق فتحدث حركة انكماش تقاوم من قبل قوى الاحتكاك

2) Uniform temp. increase ⇒ Expansion



نتجة لزيادة درجة الحرارة تقل قوى التلاصق فتحدث حركة تمدد تقاوم من قبل قوى الاحتكاك



نتجة التقلص والتمدد تحدث حركة للوح حيث ان اللوح ليس حر لعمل هذة الحركة لانه جالس على الارض وبالتالي سوف تحدث قوى احتكاك معاكسه لاتجاه التقلص ويعتمد على قوة الحركة المسلطة على التربة (pressure earth resistance) (في المركز لا تحدث حركة وبالتالي فان الاحتكاك = صفر وكلما ابتعدنا عن المركز تزداد الحركة ويزداد الاحتكاك).

من خلال البحوث وجد ان المسافة التي يحدث فيها احتكاك كلي تساوي 0.06 in

Min. amount of displacement required to be fully friction at 0.06 in from center of plate

$X_o(ft) = 1000/\Delta t$

Δt= uniform temp. drop (F^o)

Two cases for X_o

مقدار هذة المسافة قد يكون باحتمالين هما:-

نلاحظ بان شكل توزيع اجهادات الاحتكاك يتكون من منطقتين

Case (1) if $X_o \le L/2$ (L in both direction L_1 and L_2)

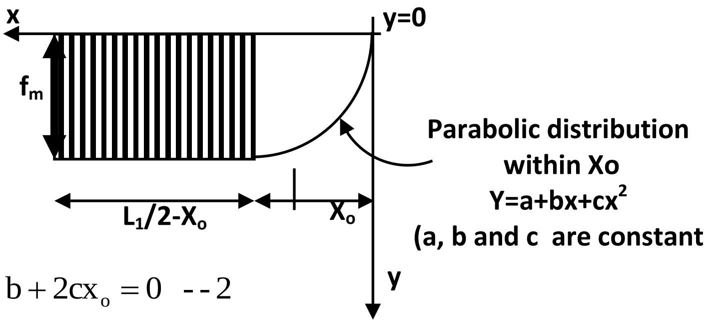
a) Normal distribution

1) X=0, y(0)=0 $\Rightarrow$ a=0

X=x_o , $\sigma(x_o)$=fm(Max. value for the coefficient of subgrade resistance)

(Max. mobizied friction resistance)

$fm = a + bx_o + cx_o^2$ ---- 1



2) $(\frac{dy}{dx})_{x=x_o} = 0 \quad \Rightarrow y' = b + 2cx \quad \Rightarrow y'(x_o) = 0 \Rightarrow b + 2cx_o = 0 \quad --2$

$fm = -2cx_o^2 + cx_o^2 \quad \Rightarrow fm = x_o^2 \quad \Rightarrow c = fm/x_o^2$

$y = 0 - 2 * \frac{fm}{x_a} x + \frac{fm}{x_b} x^2$

parabolic distribution equation $\quad y = \frac{2\,fm}{x_o} x - \frac{fm}{x_o^2} x^2 \qquad$ for $0 \le x \le x_o$

b) Equivalent uniform distribution of subgrade resistance.

1) نجد المساحة تحت القطع المكافئ

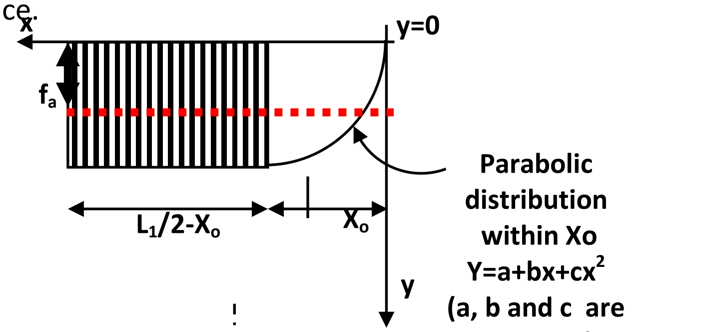
$$Area \text{ under the parabolic} = \int_{x=0} y.dx$$

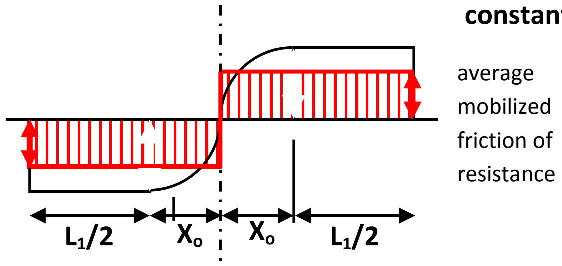
$$\int_{x=0}^{x=x_o} \left[\frac{2\,fm}{x_o} * x - \frac{fm}{x_o^2} x^2\right] dx \quad \Rightarrow \quad \text{area} = \frac{2}{3}\, fm * x_o$$

$$Total \text{ area of friction resistance for } 0 \le x \le \frac{L}{2}$$

$$\frac{2}{3}\, fm * x_o + fm * (\frac{L}{2} - x_o) = f_a * \frac{L}{2}$$

$$fa = fm\,(1 - \frac{2}{3} * \frac{x_o}{L}) \quad \text{for } x_o \le \frac{L}{2}$$

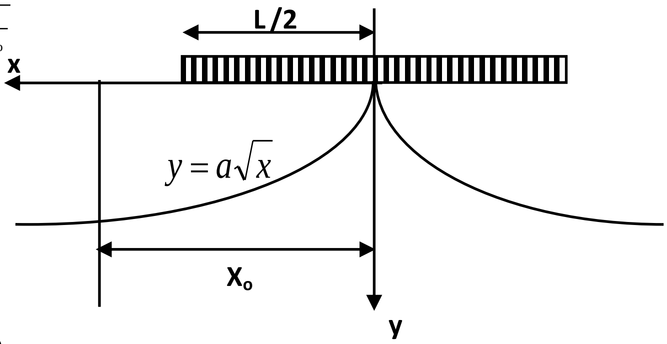




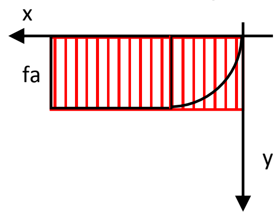
<u>Case (2) if $X_o > L/2$</u> [short slab] المسافة اللازمة للاحتكاك الكلي تكون اكبرمن $L/2$ اي x_o تكون خارج الاساس

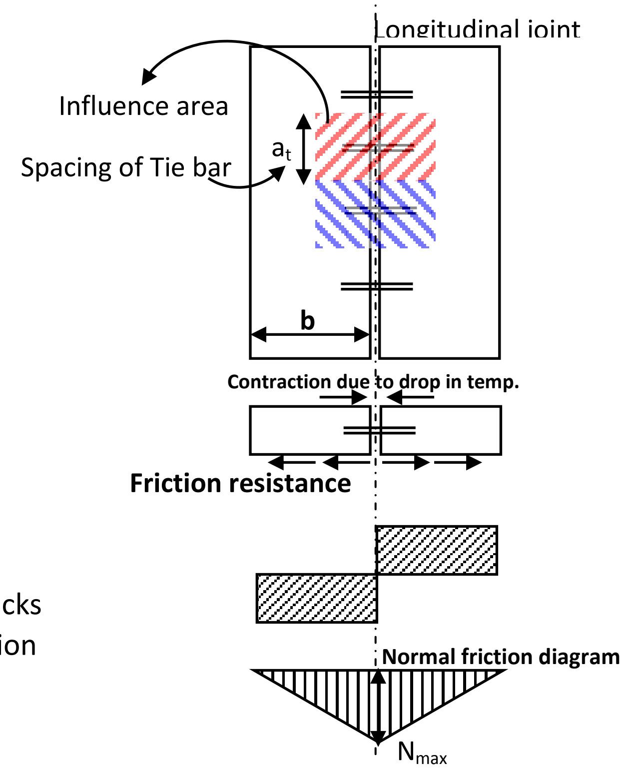
a) Normal distribution

$X = x_o$, $y = fm = c\sqrt{x_o}$ $\therefore c = \frac{fm}{\sqrt{x_o}} \Rightarrow y = fm\sqrt{\frac{x}{x_o}}$



c) Equivalent uniform distribution of subgrade resistance.





N $\Rightarrow$ frictional resistance =fa*w*1*X

w $\Rightarrow$ weight of slab per unit area

N α x $\Rightarrow$ Max. N at x=L/2

$$N_{max} = \frac{fa*w*L_1}{2}$$

σ_c ; unit tensile strength in concrete.

fa; average coefficient of subgrade resistance.

h; thickness of the slab

$$A_c * \sigma_c = \frac{fa*w*L_2}{2} \Rightarrow \sigma_c = N_{max} = \frac{fa*w*L_2}{2*A_c} = \frac{fa*w*L_1}{2*h*1}$$

$$\sigma_c = \frac{fa*w*L_1}{24*h}(psi) \quad h = in\,, w = psf\,, L = ft$$

av. fa=1.5 (passive earth pressure)

- Uniform temp. drop $\Rightarrow$ contraction $\Rightarrow$tension = cracks
- Uniform temp. increase $\Rightarrow$ expansion $\Rightarrow$ compression

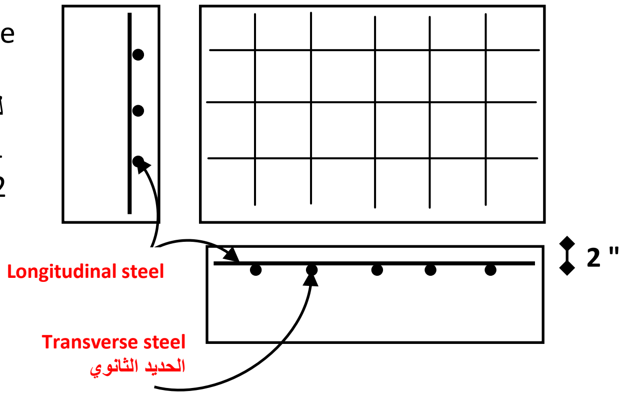
Temperature Reinforcement

Wire fabric or bar mat reinforcement may be used in rigid pavement slabs for control of temp. cracks .The amount steel needed to hold attack intact is calculate by balancing forces long the horizontal plane.

استخدام شبكة BRC او حديد التسليح في طبقات التبليط الصلد لغرض السيطرة على الشقوق الحرارية (الكونكريت له القابلية لتحمل الاجهادات الناتجة من تسليط الاحمال اما الحديد فيستخدم لامتصاص الاجهادات الحرارية.

1) Steel distribution توزيع الحديد

2 inch below top surface of slab help in preventing shrinkage Cracks.



لماذا حديد التسليح فوق؟

1) لغرض السيطرة على الشقوق الناتجة من الانكماش.

2) في مناطق الاركان تحدث مشكلة النضح ويتحول اللوح من بسيط الاسناد الى جسر سائب فيحدث فيه (شد فوق وضغط تحت) فنضع الحديد فوق

2) distribution equation.

a) Longitudinal reinforcement الحديد بالاتجاه الطولي

$$N_{max}\,(steel \text{ in longitudinal direction}) = \frac{fa*w*L_1}{2*\sigma_s}$$

$$A_L*\sigma_s = \frac{fa*w*L_1}{2} \Rightarrow A_L = \frac{fa*w*L_1}{2*\sigma_2} \quad in^2/ft \text{ width}$$

where:-

AL : required steel area per unit width (strip unit) [steel in longitudinal direction]

L_1 ; length of slab (ft).

σ_s ; Allowable stress in steel (psi).

w; weight of slab=γ*h=psf (lb/ft^2).

γ; unit weight of concreter , h= thickness of slab.

b) Transverse reinforcement

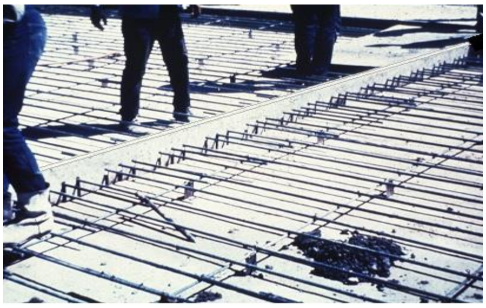
$$A_T*\sigma_s = \frac{fa*w*L_2}{2} \Rightarrow A_T = \frac{fa*w*L_2}{2*\sigma_2} \quad in^2/ft \text{ length}$$

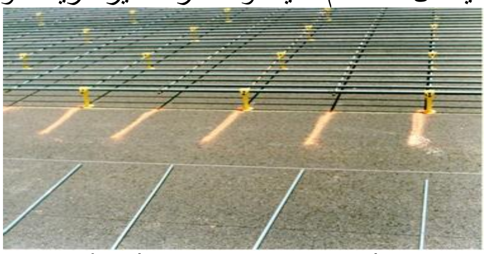
A_T= required steel area in transverse direction per unit length.

1) كلما يزداد طول وعرض السقف تزداد كمية الحديد (يمكن تقليل كمية الحديد بزيادة عدد المفاصل – سقوف قصيرة-

2) كلما يزداد وزن السقف عند زيادة سمك السقف سوف يزداد الحديد. كما في المطارات.

3) يفضل استخدام حديد ذو اقطار صغيرة لزيادة قوى التلاصق





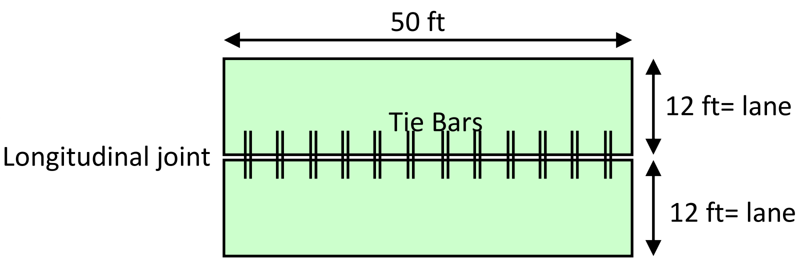
Example:- a 2 lane highway rigid pavement is 24 ft wide with a longitudinal warping joint in the center , transverse construction joints were placed at 50 ft intervals , calculate the amount of longitudinal and transverse reinforcement in the pavement if the slab thickness is 12 in , assume average coefficient of subgrade resistance (fa) =1.5, unit weight of concrete (γ) =150 pcf , working stress of steel =43 000 psi.

Solution

w=γ*h = 150 lb/ft^3* (12 in/12 in) 1 ft = 150 psf

$$A_L = \frac{fa*w*L_1}{2*\sigma_2} = \frac{1.5*150*50}{2*43000} = 0.131 \quad in^2/ft \text{ width}$$

$$A_T = \frac{1.5*150**24}{2*43000} = 0.0628 \quad in^2/ft \text{ length}$$



Wire Reinforcement Institute Guidelines:

- Minimum wires W4 or D4 (because wires are subjected to bending and tension)
- Minimum spacing 4in (allow for PCC placement and vibration) – Maximum 12x24
- Wire fabric should have end and side laps:
  - Longitudinal: 30*Diam. but no less than 12”
  - Transverse: 20*Diam. but no less than 6”
- Fabric should extend to about 2in but no more than 6in from the slab edges

=

TABLE 4.3 WEIGHTS AND DIMENSIONS OF WELDED WIRE FABRIC

Wire size no.		Diameter	Weight	Cross-sectional area (in.²/ft) center-to-center spacing (in.)						
Smooth	Deformed	(in.)	lb/ft	2	3	4	6	8	10	12
W31	D31	0.628	1.054	1.86	1.24	.93	.62	.465	.372	.31
W30	D30	0.618	1.020	1.80	1.20	.90	.60	.45	.36	.30
W28	D28	0.597	.952	1.68	1.12	.84	.56	.42	.336	.28
W26	D26	0.575	.934	1.56	1.04	.78	.52	.39	.312	.26
W24	D24	0.553	.816	1.44	.96	.72	.48	.36	.288	.24
W22	D22	0.529	.748	1.32	.88	.66	.44	.33	.264	.22
W20	D20	0.504	.680	1.20	.80	.60	.40	.30	.24	.20
W18	D18	0.478	.612	1.08	.72	.54	.36	.27	.216	.18
W16	D16	0.451	.544	.96	.64	.48	.32	.24	.192	.16
W14	D14	0.422	.476	.84	.56	.42	.28	.21	.168	.14
W12	D12	0.390	.408	.72	.48	.36	.24	.18	.144	.12
W11	D11	0.374	.374	.66	.44	.33	.22	.165	.132	.11
W10.5		0.366	.357	.63	.42	.315	.21	.157	.126	.105
W10	D10	0.356	.340	.60	.40	.30	.20	.15	.12	.10
W9.5		0.348	.323	.57	.38	.285	.19	.142	.114	.095
W9	D9	0.338	.306	.54	.36	.27	.18	.135	.108	.09
W8.5		0.329	.289	.51	.34	.255	.17	.127	.102	.085
W8	D8	0.319	.272	.48	.32	.24	.16	.12	.096	.08
W7.5		0.309	.255	.45	.30	.225	.15	.112	.09	.075
W7	D7	0.298	.238	.42	.28	.21	.14	.105	.084	.07
W6.5		0.288	.221	.39	.26	.195	.13	.097	.078	.065
W6	D6	0.276	.204	.36	.24	.18	.12	.09	.072	.06
W5.5		0.264	.187	.33	.22	.165	.11	.082	.066	.055
W5	D5	0.252	.170	.30	.20	.15	.10	.075	.06	.05
W4.5		0.240	.153	.27	.18	.135	.09	.067	.054	.045
W4	D4	0.225	.136	.24	.16	.12	.08	.06	.048	.04

<u>AASHTO Guide 1993 Method(</u>Maximum Recommended Tie bar Spacings)

Note : 48" maximum spacing recommended.

BAR SIZE		# 4 BAR										# 5 BAR									
GRADE STEEL		GRADE 40					GRADE 60					GRADE 40					GRADE 60				
DIST TO FREE EDGE (ft.)		10	12	16	22	24	10	12	16	22	24	10	12	16	22	24	10	12	16	22	24
PVMT THICKNESS	TYPE OF JOINT																				
9"	Warp	37	31	23	17	16	48	47	35	25	23	48	48	36	25	24	48	48	48	40	36
	Butt	26	22	16	12	11	40	34	25	18	16	42	35	26	19	17	48	48	39	29	26
10"	Warp	34	28	22	16	14	48	42	32	23	20	48	44	33	24	22	48	48	48	36	32
	Butt	24	20	16	11	10	36	30	23	16	14	38	31	24	17	16	48	47	35	26	23
11"	Warp	31	25	20	15	13	47	38	29	21	19	48	40	30	22	20	48	48	44	32	30
	Butt	22	18	14	11	9	34	27	21	15	14	34	29	21	16	14	48	43	31	23	21
12"	Warp	28	23	18	13	12	42	35	27	19	18	44	36	28	20	18	48	48	41	30	28
	Butt	20	16	13	9	9	30	25	19	14	13	31	26	20	14	13	47	39	29	21	20

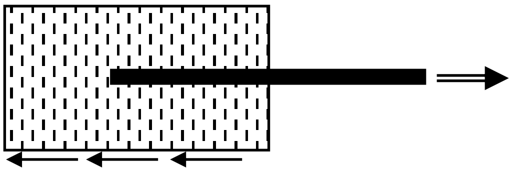
Warp joint: a sawed or construction joint with a keyway

Butt joint: a construction joint with no keyway

Design of Tie Bars (Load Transfer Device); lying 2 slabs together without transfer load

Load transfer" is a term used to describe the transfer (or distribution) load across discontinuities such as joints or cracks (AASHTO, 1993). When a wheel load is applied at a joint or crack, both the loaded slab and adjacent unloaded slab deflect. The amount the unloaded slab deflects is directly related to joint performance. If a joint is performing perfectly, both the loaded and unloaded slabs deflect equally. This efficiency depends on several factors, including temperature (which affects joint opening), joint spacing, number and magnitude of load applications, foundation support, aggregate particle angularity, and the presence of mechanical load transfer devices.

ناقل الحمل مصطلح يستخدم لوصف عملية نقل او توزيع الحمل خلال المفاصل او الشقوق فعند تسليط حمل العجلة يشترك كلا اللوحين في مقاومة الهبوط الناتج من تسليط الحملوحسب كفاءة المفصل.(وهي الاداة التي تستخدم لربط السقفين)



$N_{max} = \sigma_t * A_t$

$N_{max} = f * w * a_t * b$

$\sigma_t * A_t = f * w * a_t * b$

At $\Rightarrow$ cross sectional area of the bar (circle or square)

$\sigma_t \Rightarrow$ allowable working stresses in tie bar.

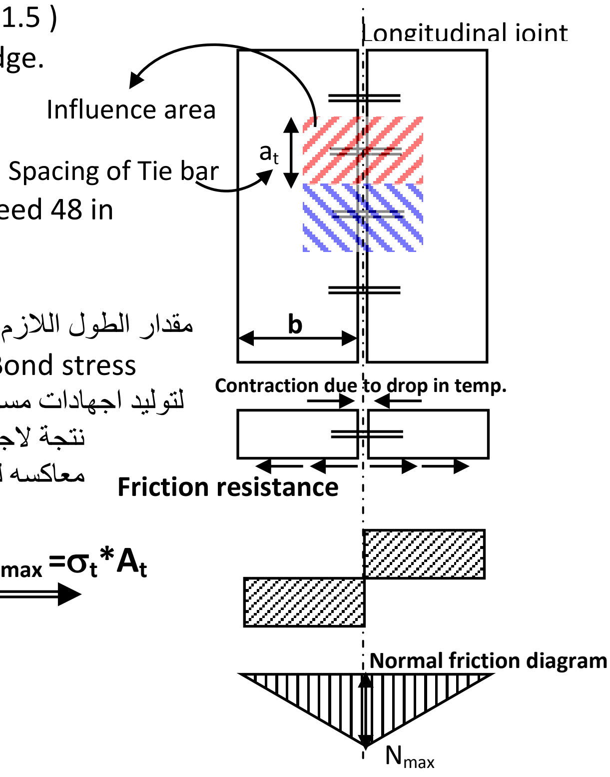
f $\Rightarrow$ frictional coefficient (coefficient of subgrade resistance $\approx$ 1.5)

b $\Rightarrow$ distance between tie joint and the nearest free joint or edge.

w $\Rightarrow$ weight of slab per unit area = γ*h

$$a_t = \frac{\sigma_t * A_t}{f * w * b}$$ required spacing for tie bar

AASHTO ; recommended that spacing of tie bar should not exceed 48 in (120 cm $\Rightarrow a_t \leq$ 120 cm).

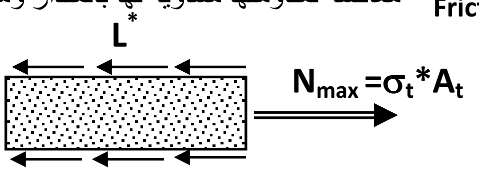


Length of Tie bar

مقدار الطول اللازم ادخاله باللوح الكونكريتي ويعتمد على مقدار الاجهادات المتولدة (اجهاد الالتصاق Bond stress قوى التلاصق الموجودة بين الحديد والكونكريت تولد اجهادات (المسافة اللازمة لتوليد اجهادات مساوية لقوى السحب)

نتجة لاجهادات الشد فيحدث تقلص (انكماش) بالحديد فيجعل الكونكريت يولد اجهادات معاكسه لمقاومتها مساوية لها بالمقدار ومعاكسها بالاتجاه تدعى اجهادات التلاصق .

For rounded bar



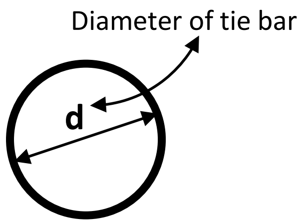
Total length (L)= 2*L* + 3 in

3 in ; allowance for centering

So $N_{max} = \sigma_t * A_t \Rightarrow \tau_b * (L^*) * \pi * d = \sigma_t * A_t$

The $$L^* = \frac{\sigma_t * A_t}{\pi * d * \tau_b}$$

Note; the bond stress assumed = 350 psf



Tie bar transfer tension stress

اضعف منطقة في التبليط هي منطقة المفصل

Joints in Concrete Pavement

Joints are vital to control cracking and horizontal movements of the pavement. Plain concrete pavements without joints would be riddled with cracks within one or two years after placement.

المفاصل وسيلة يمكن استخدامها للسيطرة على الشقوق و الحركة الافقية لطبقة التبليط فبدون المفاصل تحدث الشقوق بعد سنة او اثنين من صبها

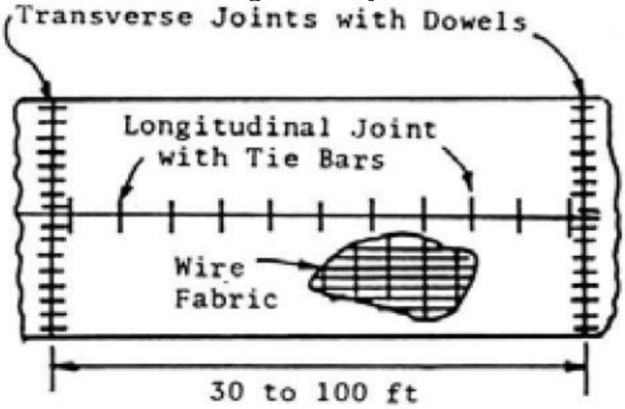
- prevents random cracking and other potential problems,
- allows concrete to crack at predetermined locations,
- prevents faulting, and
- extends pavement life.

- Classification of joint according to location;

1. **Longitudinal Joint** المفصل بالاتجاه الطولي للطريق

تعمل المفاصل الطولية اذا زاد العرض عن 4.5 m وتعمل لتقليل اجهادات warping الناتجة عن التغير بدرجات الحرارة بين السطح وقعر طبقة التبليط وتربط مع بعضها البعض باستخدام Tie bars (ربطات حديدية) يمكن تصورها كشقوق اصطناعية تعمل كمفصل hinge ينقل عزم الناتج من الضغط.

are necessary to control cracking in the longitudinal direction due to warping, expansion and shrinkage stresses caused by temperature variations when concrete is placed in great widths. They are constructed at lane lines, typically in multiples of 12 feet. Where there are no lanes, longitudinal joints should be spaced 12 feet apart, but no more than 14 feet apart. When widened slabs are used, longitudinal joints are omitted at the edge of traveled way.



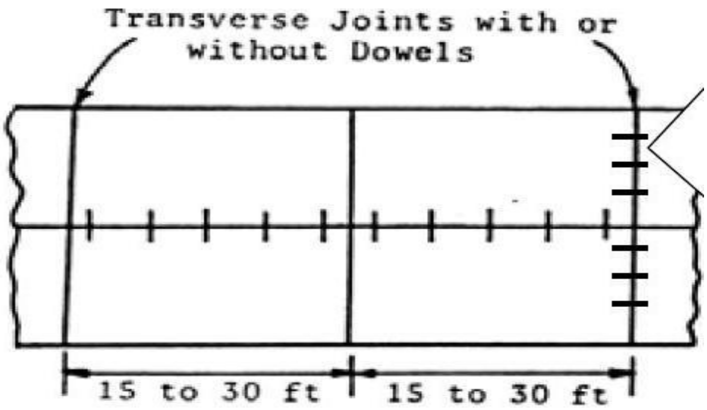
2. **Transverse Joint** المفصل بالاتجاة العرضي للطريق

تعمل بالاتجاة العرضي (مفاصل تمدد او تقلص او التواء او انشائية) وتكون عمودية على اتجاة الحركة.

Is a joint formed when?

(i) concrete is placed at different times, or
(ii) when paving is interrupted at the end of each day's paving operation, or
(iii) The paving is interrupted for more than 30 minutes.

An additional 42-inch long longitudinal reinforcement is placed in the transverse contact joint on the same plane and twice the distance as the longitudinal reinforcements3.

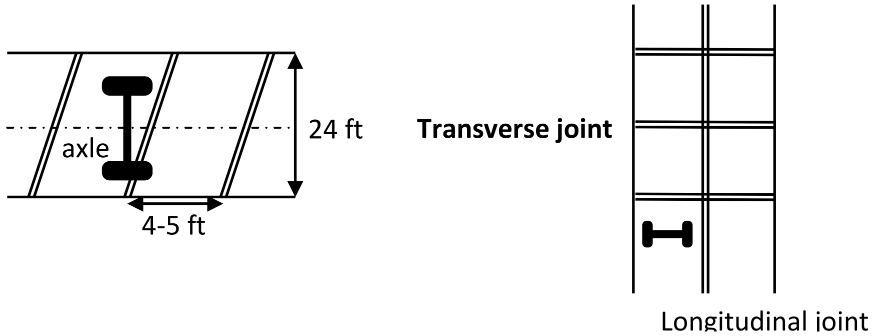


3. **Skewed Joint**

نتجة لحركة المركبات فان المحور قد يقع جزأ منه على مفصل وبالتالي فان مقدار الهبوط الحاصل يزداد (مفصل بالاتجاه العرضي ولكن ليس عموديا على اتجاه الحركة.

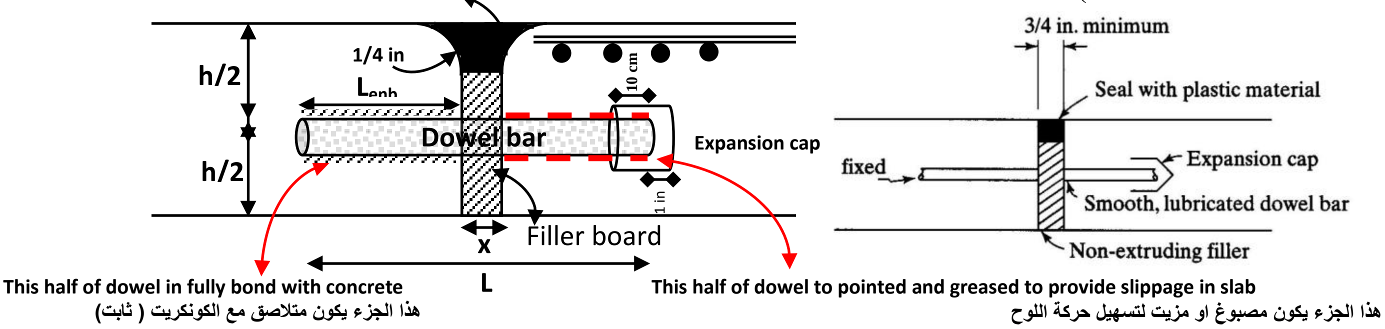
Note:- a 4 to 5 ft skew on a 24 ft width pavement will result in only;

1) one wheel crossing the joint at any one time.
2) This skew result in better load transfer and improved riding quality across the joint.



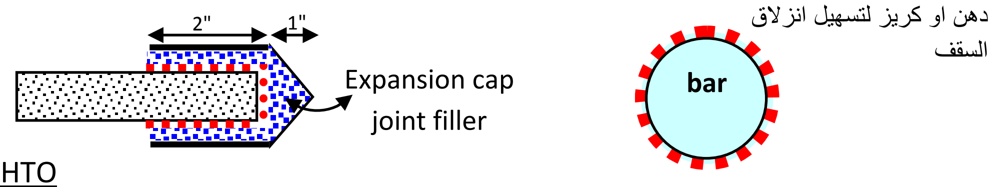
- Classification of joint according to function ; التصنيف حسب الوظيفية
    a. <u>Expansion joint (</u>Transverse joints for the relief of compressive stress)

عبارة عن انفصال تام بين القطع الخرسانية بكامل سمكها وعرضها بحيث تسمح بحدوث التمدد الناتج من التغير بدرجة الحرارة والرطوبة.
مواصفات المفصل يجب ان تكون 1) حرية الحركة.
2) يجب ان يملأ المفصل بمادة لا تسمح بمرور الماء.
3) نعمل لها ختم sealing لعدم السماح للماء بالدخول.
4) لها القدرة على التمدد مع الكونكريت.



X= Joint width or opening (3/16" to 1 " depending on the local and distance between joints)

<u>Expansion Cap</u>



<u>According to AASHTO</u>

- The paving function an expansion joint to prevent the development of damaging compressive stress due to volume changes in the pavement slab and to prevent excessive pressure being transmitted load adjacent structures.

الوظيفية الرئسية للمفصل هي تقليل اجهاد الضغط الناتج من التغيرات الحجمية وتقليل الاجهادات المنتقلة للسقف المجاور.

- In general it is considered that expansion joints are not necessary for rigid extensional adjacent to structure.

ليس من الضروري الاخذ بنظر الاعتبار تمدد السقف المجاور.

- Unused conditions, such as cold weather construction or the use of materials with a higher coefficient of thermal expansion may require special construction

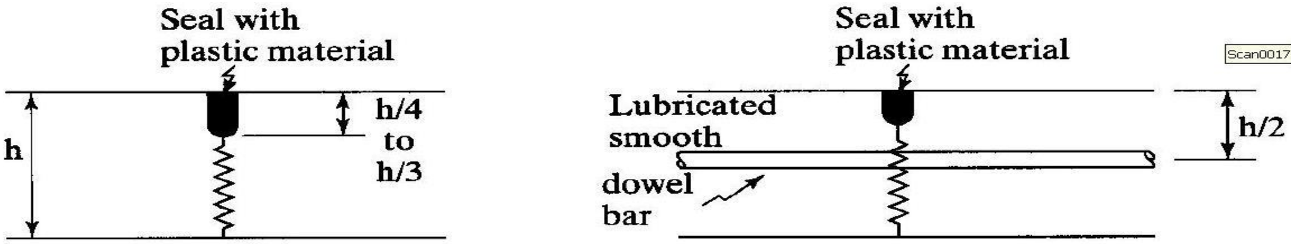
في ظروف الجو البارد او استخدام مواد ذات معامل تمدد حراري عالي

المواصفات البريطانية:- ممكن ان نلغي مفصل التمدد بشرط ان يكون انشاء المفصل في الاشهر الحارة ففي هذة الحالة لانحتاج الى المفصل ففي هذة الحالة يكون الكونكريت يكون قد تعرض الى اعلى درجة حرارة و تمدد حسب هذة الدرجة و بالتالي فيحدث فيه تمدد ولكن يكون اقل من تمدد عند الانشاء .

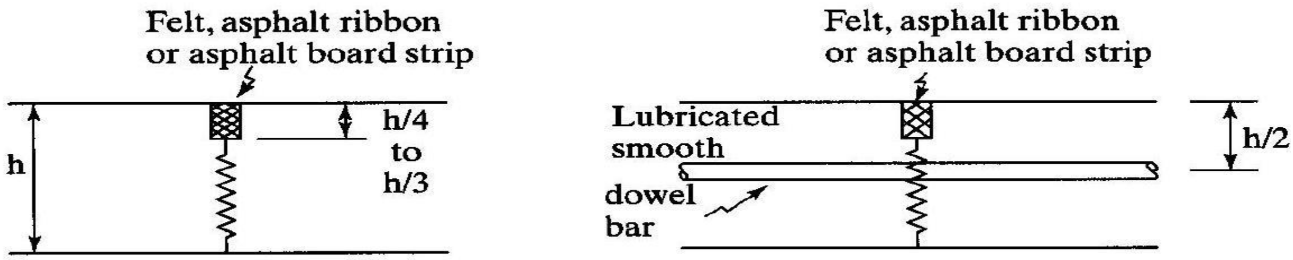
b. Contraction Joint (Transverse joints used to relieve tensile stresses from temperature drops and moisture variations in concrete)

يستخدم هذا المفصل للتخلص من اجهادات الشد المباشر الناتجة من التغير بدرجة الحرارة او الرطوبة.

The purpose of contraction joints is to provide for any ornately arrangement of cracking that occurs, if the joints are property designed and spaced a min. of cracking outside the joint would be expected. This joint may be sawed in hardened concrete or formed by plastic inserts



(a) Dummy Groove



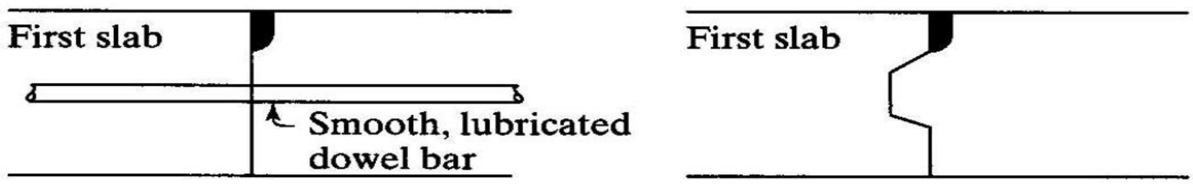
(b) Premolded Strip

1) نعمل على تقليل مساحة المقطع وذلك من بان نضع قطعة خشب بالاسفل ومن فوق نضع اسفلت فسوف تقل مساحة المقطع وتزداد الاجهادات نتجة لذلك يحدث اجهادات شد وتحدث شقوق في المنطقة المحصورة بينهما.

2) لغرض امتصاص هذة الاجهادات نستخدم حديد Dowel bar

c. Construction joint

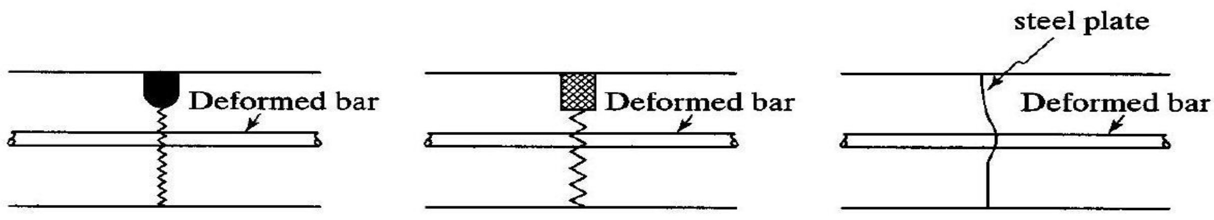
The transverse construction joint should be placed at the location of the contraction joint



(a) Butt Joint at Contraction Joint.

(b) Key Joint for Emergency.

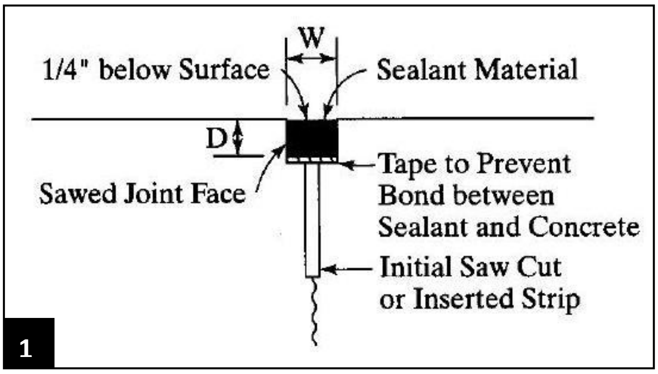
can be used to relieve curling and warping stresses , as shown in fig

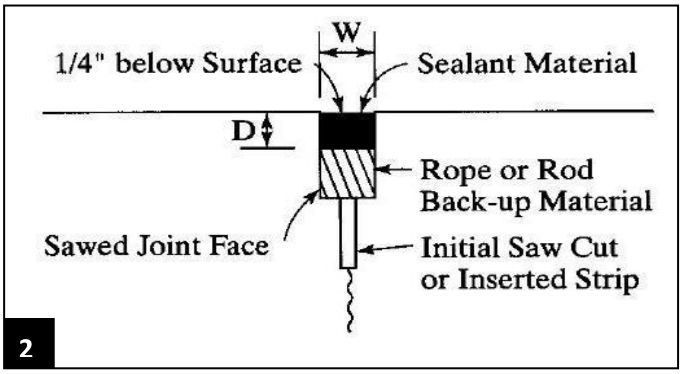


(a) Dummy Groove. (b) Ribbon or Premolded Strip. (c) Deformed Plate.

Joint Sealing Materials ; which provided

- Capable of withstanding repeated extension and compression as the temperature and moisture change
- Classified as field molded and preformed.
- The compression of seal ranges from 20 to 50%





There are 2 types of sealing presently for sealing joint

1) Liquid Sealants (these materials are placed in the joint in liquid form allowed to set.
   a) asphalt (Mastic asphalt).
   b) Hot –poured rubber (المطاط الساخن)
   c) Polymers.
2) Performed elastomeric seals (these are extruded neoprene seals having internal webs that exert an out word force against the joint face.

Reservoir Dimensions for Field Molded Sealants

Joint spacing (ft)	Reservoir width (in.)	Reservoir depth (in.)
15 or less	1/4	½ minimum
20	3/8	½ minimum
30	1/2	½ minimum
40	5/8	5/8

Joint and Sealant Width for Preformed Seals

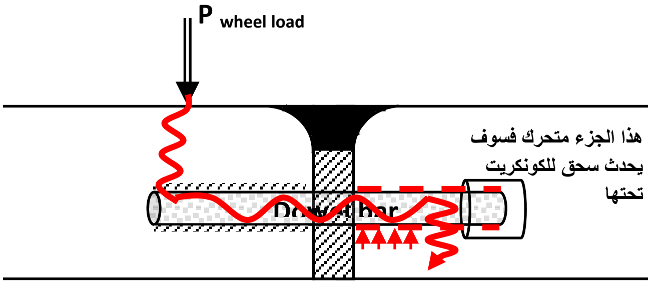
Joint spacing (ft)	Reservoir width (in.)	Reservoir depth (in.)
20 or less	1/4	7/16
30	3/8	5/8
40	7/16	3/4
50	1/2	7/8

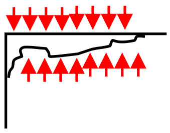
Load transfer device

Dowel bars:-

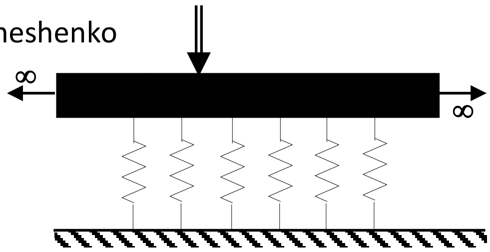
Dowel bar stresses (shear) result from bending and bearing .these stresses can be analyzed analytically to determine factors which affect load transfer characters.

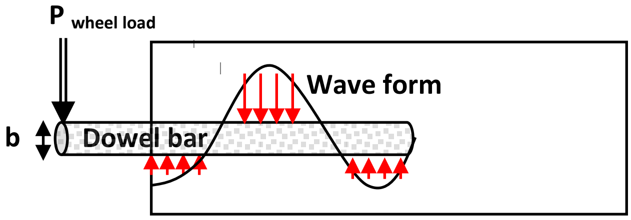
نتجة لتسليط الحمل سوف يولد اجهادات (قص) تنتقل الى اللوح وينتقل جزء منه الى اللوح المجاور من خلال dowel الذي يكون حر من جهة ومن الجهة الثانية يجلس بتماس مع الكونكريت فيحدث سحق للكونكريت تحته فاذا كان مربوط بشكل جيد فان الحديد ينقل بحدود 50 %.



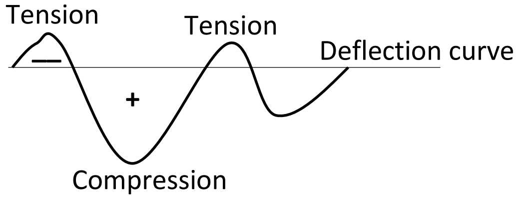


The stress analysis of dowels is based upon work presented by Timeshenko (infinite beam on elastic foundation- rail way trucks).





Bradbury and Friberg have presented mathematical analysis of dowel design which are all based upon the principle 1st presented by Timeshenko



1) relative stiffness of a bar embedded المطمور in concrete ;

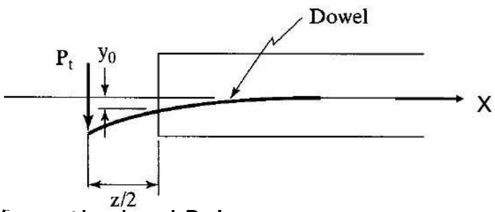
$$\beta = \sqrt[4]{\frac{K\,b}{4\,E\,I}}$$ where;

K; Modulus of dowel support (pci, N/cm ^{3})

b; diameter of the dowel bar.

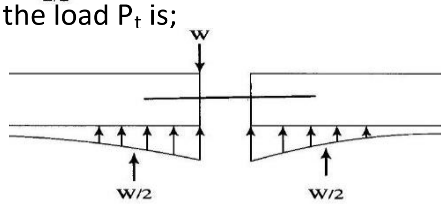
E; Modulus of elasticity of the dowel bar.

I; moment of inertia of the dowel bar.



According to Timeshenko theory; the deflection of the bar resulting from the load P_t is;

$$y = \frac{e^{-\beta x}}{2\,\beta^3\,\mathrm{EI}}\left[P_t * Cos(\beta x) - \beta * Mo\left[Cos(\beta x) - Sin(\beta x)\right]\right]$$



100% load transfer

e; natural logarithm base.

x; distance a long the dowel from face of concrete المسافة من حافة الكونكريت

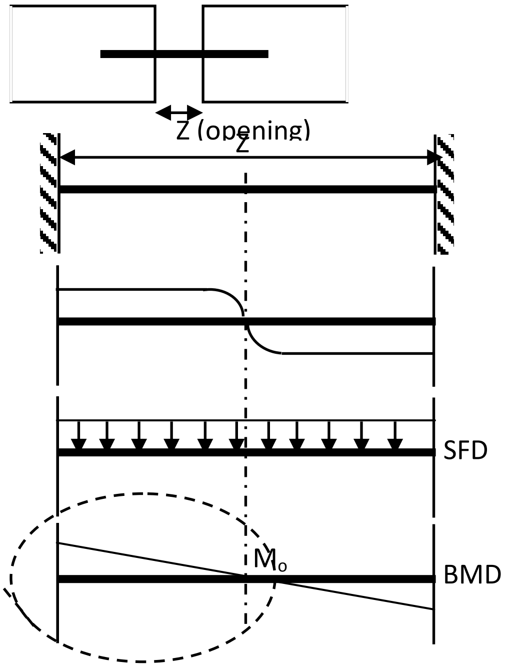
P_t; transfer load

y; deflection .

M_o; Bending Moment on dowel at face of concrete.

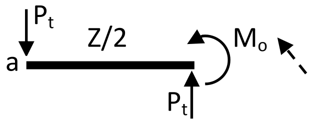
مساحة مقطع الكونكريت تكون اكبر من مساحة مقطع الحديد (مقدار قابلية المادة للمقاومة للكونكريت تكون اكبر من الحديد)

If the joint width opening is designated (Z) and since the concrete is very stiff compared with steel bar, the bending moment at the dowel concrete interface can be calculated as follows;



Shear Force Diagram (SFD)
نفرض ان اجهادات القص تتوزع بصورة منتظمة

Bending Moment Diagram
العزم المتولد من القص



$$\sum M_o = 0 \; +\uparrow$$

$$M_o + P_t * \tfrac{z}{2} = 0 \Rightarrow M_o = -P_t * \tfrac{z}{2}$$

Winkler σ=ky bearing pressure
اين يحدث اقصى تشوة في المنطقة التي يسلط عليها اقصى حمل في منطقة التثبيت

at x=0 $\Rightarrow$ y_o=Max. deflection

$$y = \frac{e^{-\beta * 0}}{2\beta^3 \, EI}\left[P_t * Cos(0) - \beta * Mo\left[Cos(0) - Sin(0)\right]\right] \Rightarrow \sin(0) \,\&\, \cos(0) = 0$$

$$y = \frac{1}{2\beta^3 \, EI}\left[P_t - \beta * Mo\right] = \frac{1}{2\beta^3 \, EI}\left[P_t + \beta * P_t * \tfrac{z}{2}\right]$$

$$Max.\text{ deflection at dowel } \Rightarrow y_o = \frac{P_t}{4\beta^3 \, EI}\left[2 + \beta * z\right]$$

$$\sigma_{Max.} = ky_o = \frac{k * P_t}{4\beta^3 \, EI}\left[2 + \beta * z\right]$$

K; Modulus of subgrade reaction.
$\sigma_{Max.}$; Max. Bearing pressure on concrete at face.

Values of K range between 300 000 and 1 500 000 psi, since β varies as $\sqrt[4]{k}$, large change in the modulus don't effect the stress calculated greatly thus the use of $1.5*10^6$ pci appears to be warranted.

قيمة k عالية ولكن تاثيرها قليل لكون تاثيرها يكون تحت الجذر الرابع.

Dowel Group Action

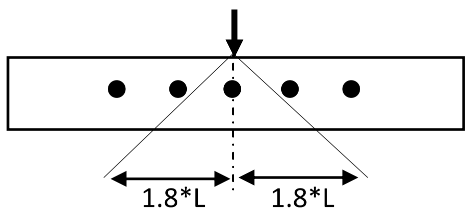
استخدام مجموعة من الحديد لاسوء حالة وهي عندما يكون الحمل في الحافة فماذا نعمل

Friberg $\Rightarrow$ If a series of dowel bars are design ,the dowel bar immediately under the applied load caries fall capacity decreasing to zero at a distance of (1.8*L) from this dowel.

Where

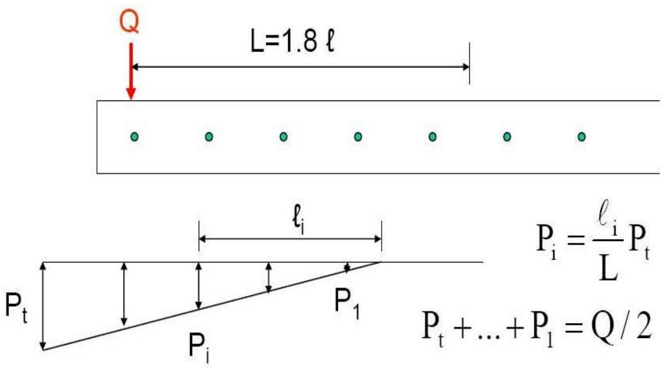
L ; radius of relative stiffness of the slab $\sqrt[4]{\dfrac{E * h^3}{12 * k(1-\mu^2)}}$

h; uniform thickness of slab.



The deflection profile within 1.8*L can be assumed to be a straight line. The load transferred to each dowel of a group is a function of the deflection at location of the dowel.

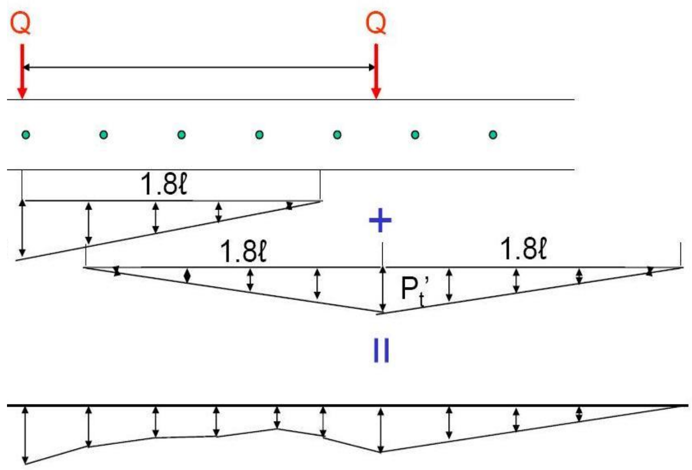
توزيع الحمل يكون مشابة لتوزيع التشوة الناتج منة (المنطقة الذي يحدث فيها تشوةاكبر يكون مسلط عليها حمل اكبر لذلك نفرض توزيع يتوزع بصورة خطية خلال مسافة 1.8*L)



$$P_i = \frac{\ell_i}{L} P_t$$

$$P_t + \ldots + P_1 = Q/2$$

Load Diagrams



$$\ell = \left[\frac{Eh^3}{12(1-v^2)k}\right]^{0.25}$$

E = elastic modulus of slab
v = Poisson's ratio of slab (typically 0.15)
H = thickness of slab
k = modulus of subgrade reaction

- Load transfer across joints (theoretically , if the dowel is 100 % efficient , the dowel will transfer one-half at the applied load from one slab to another)

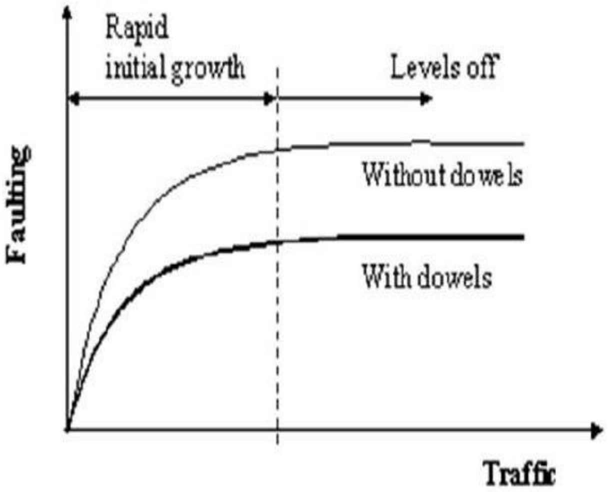
اذا كانت المفاصل مرتبطة بصورة جيدة و بكفاءة 100 % فكل واحد ينقل 50 % من الحمل.

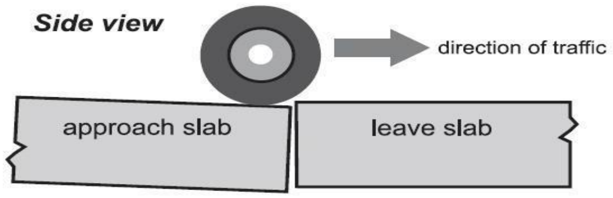
- The reduction in load transfer resulting , can be assume to be about 5-10 % , thus the design load transfer should be 45 % of the design load must cases.

Load Transfer Efficiency

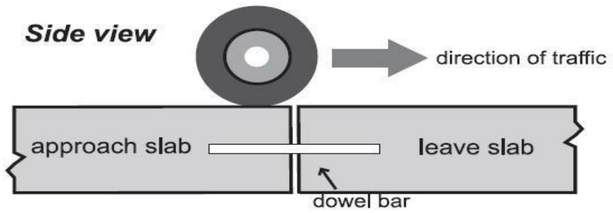
When a traffic load is applied near a joint in a PCC pavement, both loaded and unloaded slabs deflect because a portion of the load applied to the loaded slab is transferred to the unloaded slab. As a result, deflections and stresses in the loaded slab may be significantly less than if, instead of a joint with another slab, there was a free edge. The magnitude of reduction in stresses and deflections by a joint compared to a free edge depends on the joint's LTE.

عندما يكون الحمل المسلط بالقرب من المفصل فكلا اللوحين المحمل و الغير محمل يحدث فيهما هطول لان جزء من الحمل ينقل الى اللوح الثاني وهذا يعتمد على كفاءة الجزء الذي ينقل الحمل ويمكن قياس كفاءة هذا الجزء من المعادلة ادناة





Without dowels, the slab on either side of a joint tends to move up and down; that is, there is little "load transfer" at the joint.



Dowels help provide load transfer at the joint.

This efficiency depends on several factors, including **temperature** (which affects joint opening), joint spacing, number and magnitude of load applications, foundation support, aggregate particle angularity, and the presence of mechanical load transfer devices.

كفاءة الجزء الناقل تعتمد على درجة الحرارة الذي تؤثر على فتحة المفصل والمسافات بين المفاصل وعدد ومقدار الحمل المسلط و كذلك طريقة اسناد الطريق وخشونة الركام المستخدم واستخدام حديد ناقل الحمل ام لا

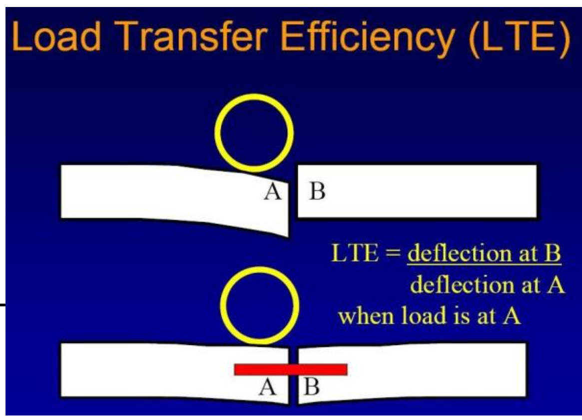
LTE at the joint is determined based on the ratio of the maximum deflection at the joint of the loaded slab and the deflection of the unloaded slab measured right across the joint from the maximum deflection. Two equations for the deflection LTE are used most often:

$$LTE = \frac{du}{d1} * 100$$

where

d_u : deflection at the joint or crack of the unloaded side

d_l : deflection at the joint or crack of the loaded side



Length of Embedment (L_{emb}) الطول المطمور بالكونكريت

Tests have indicated that for 3/4 inch round dowel bar , the length of embedment (L_{emb}) should be about (8d) where d is diameter of the dowel bar . مقدار الطول المغمور يساوي 8 مرات القطر

$L_{\text{length of dowel bar}}$ = 16*d+Z where Z ; joint spacing , joint width or joint opening)

The ACI committee on standard design bar recommended , dowels length 18 inch long for highway

المنظمة الامريكية للكونكريت تقترح استخدام طول 18 انج

For airport dowels ; 18 " length for 1 " diameter , 20 " length for 1.25 " diameter and 24 " length for 1.5 diameter

Allowable bearing stresses recommendation from ACI committee is 325 psi مقدار قوة السحق الذي يتحملها الكونكريت

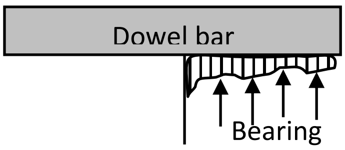
$$f_b = \left(\frac{4-b}{3}\right) * f_c^{\backslash}$$

where; b : dowel bar diameter (inch)

$f_c^{\backslash}$; ultimate compression strength of concrete(psi).

f_b; allowable bearing stresses (psi)

$$\sigma_{Max.\,\text{bearing (dowel bar)}} = \frac{p_{t} * K}{4 * \beta^3 * EI}\left[2 + \beta\, z\right]$$



Example :

A concrete pavement show in the following figure, having a radius of stiffness (ℓ) =60 in is subjected to an axle load of 30 kips carried on a single axle with single tire in each side, distance between is 5 ft . Determine the size of dowels for a contraction joint width of 0.25 in the load transfer is 45 % of the applied load, dowel spacing =12 in, dowel support Modulus (K) 1.5*10^6 pci, E_{steel}(Modulus of elasticity of steel)= 29*10^6 psi , ultimate compression of concrete ($f_c^{\backslash}$)= 3000 psi.

Solution

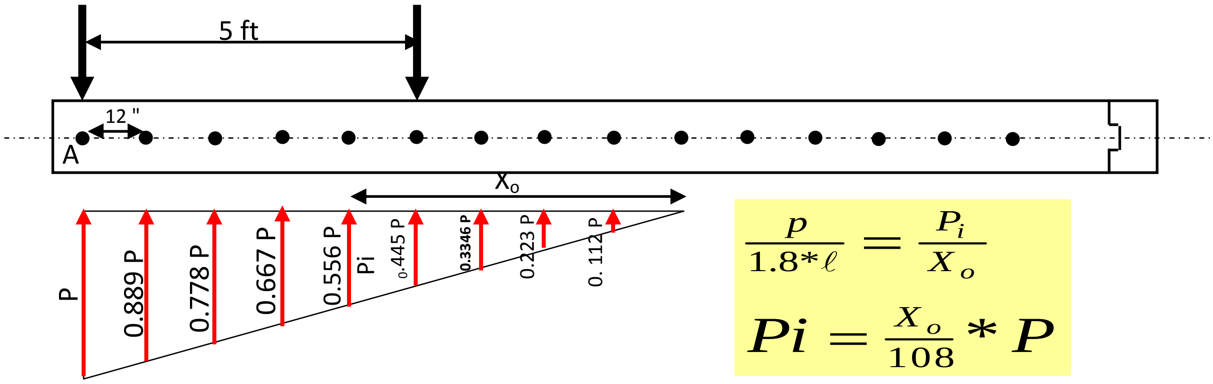
$$\ell = \sqrt[4]{\frac{Eh^3}{12K(1-\mu^2)}} = 60''$$

ملاحظات مهمة:-

1) دائما نأخذ الاسؤ وهي الحالة التي يكون فيها الحمل مسلط في الحافة.

2)الحمل يتوزع على مسافة ℓ (radius of relative stiffness)

$$\textit{Distribution}\ \text{distance} = 1.8 * \ell = 108'' = 9\ \text{ft}$$



$$\frac{p}{1.8*\ell} = \frac{P_i}{X_o}$$

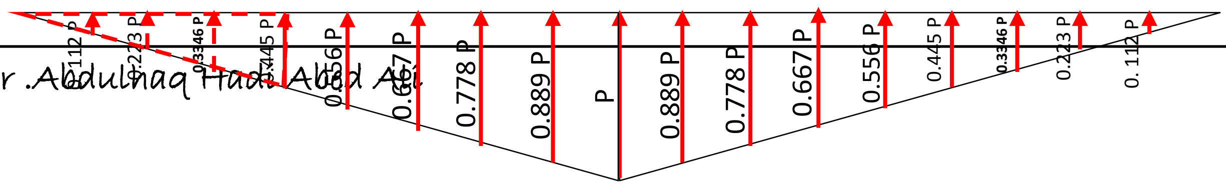
$$Pi = \frac{X_o}{108} * P$$

Load transfer at (A)

$$\sum P_o = P_t(P_A\%T)$$

$$\sum P_i = P*[0.889 + 0.778 + 0.667 + 0.556 + 0.445 + 0.334 + 0.223 + 0.112 + 1] = 5.004\ \text{P}$$

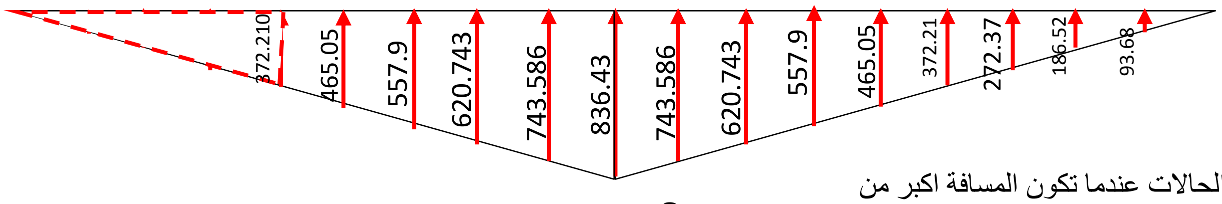
$$15.5*0.45 = \sum P_i \Rightarrow 6.975 = 5.004\ \text{P} \ \Rightarrow\ \text{P} = 1.394\ \text{kip} \ \ \Rightarrow \text{P} = 1394\ \text{lb}$$



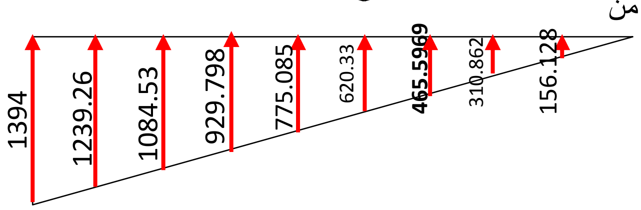
نأخذ لغرض التصميم

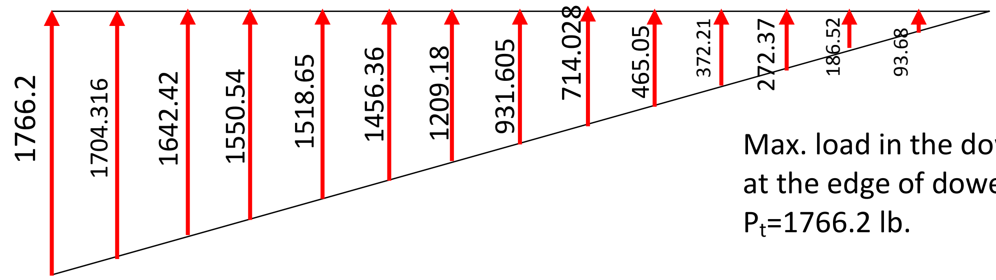
$$\sum P_i = P*[2*0.556+2*0.445+2*0.667+2*778+2*0.889+1+0.334+0.223+0.112] = 8.339*P$$

$$\sum F_y = 0 \Rightarrow 15.5*0.45 = P*8.339 \quad \Rightarrow P = 0.83643\,\text{kip} \quad \Rightarrow 836.43\,\text{lb}$$



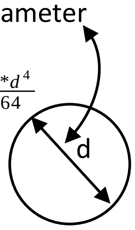
في بعض الحالات عندما تكون المسافة اكبر من 1.8*L فنهمل تأثير الحمل في نقطة B على A





Dowel diameter

$I = \frac{\pi * d^4}{64}$



Max. load in the dowel bar at the edge of dowel P_t=1766.2 lb.

في التصميم ناخذ اسو حمل ونحسب اشقد يعمل اجهادات سحق bearing في الكونكريت

$$\sigma_b = k*y_o \Rightarrow y_o = \frac{P_t}{4\beta^3 EI}(2+\beta z) \Rightarrow \beta = \sqrt[4]{\frac{k*b}{4EI}} \quad \Rightarrow b = ? \quad \text{solve by tryand error}$$

Try No.1: assume b (diameter of dowel bar)= 1 in

$$I = \frac{\pi * d^4}{64} \Rightarrow I_x = I_y = 0.0491*d^4$$

$$\beta = \sqrt[4]{\frac{k*b}{4EI}} = \beta = \sqrt[4]{\frac{1.5*10^6*1}{4*29*10^6*0.0491*1^4}} = 0.716 \ \frac{1}{\text{in}}$$

$$\text{Max. deflection} \Rightarrow y_o = \frac{P_t}{4\beta^3 EI}(2+\beta z) = \frac{17662}{4(0.716)^3*29*10^6*0.0491*(1)^4}(2+0.716*0.25) = 0.001838 \ \text{in}$$

$$\text{Max. Bearing stresses} \quad \text{Max. } \sigma_b = 1.5*10^6*0.001838 = 2757\ \text{psi}$$

$$\text{check with allowable bearing stress} \quad f_b = (\tfrac{4-b}{3})*f_c' \Rightarrow (\tfrac{4-1}{3})*3000 = 3000 \ \text{psi}$$

$$\therefore \text{Max. } \sigma_b < f_b \qquad \text{ok}$$

using dowel bar with 1 in

Figure 1.1 Dowel and tie bar baskets placed in preparation for slipform paving

Modulus of subgrade Reaction (K)

P_s (subgrade reaction)=K*w

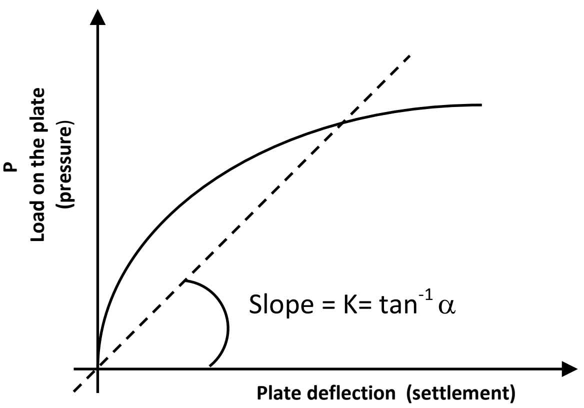
K= modulus of subgrade reaction, spring element, density liquid constant (K=P_s / w, psi/ in (pci))

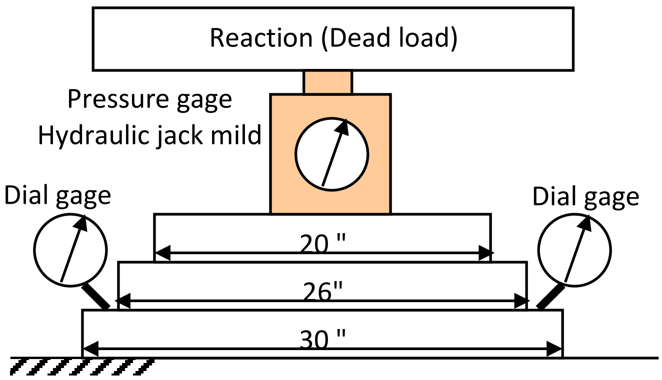
Should be determined by means, plate bearing tests (plate load test); in-situ test)

يتم تسليط حمل معين على مساحة محددة وحساب الاجهادات المسلطة على التربة او التبليط ويتم حساب الهبوط الذي يحث نتجة تسليط الحمل.ويتم اجراء هذ الفحص بطريقتين هما:

1) AASHTO designation T221-66 (1974).
   Standard Methods for repetitive static plate load tests of soil and flexible pavement components for used in evolution and design of air port and highway pavement.
   يسلط حمل معين في البداية ونحسب التشوة الناتج منه ومن ثم يرفع الحمل ويسلط حمل اكبر ويحسب التشوة الناتج منه وهكذا .
2) AASHTO designation T222-78.
   Standard of Method for non- repetitive static plate load tests of soil and flexible pavement components for used in evolution and design of air port and highway pavement.
   يسلط الحمل كله من البداية ويحسب التشوة الناتج منه خلال فترات زمنية مختلفة.

هي عبارة عن لوحة من الحديد دائرية الشكل قطرها 30 انج تستخدم مقدار تحمل التربة او التبليط الاسفلتي و الكونكريتي في الطرق و المطارات ومن ثم يتم تسليط حمل من خلال جك وحساب التشوة الناتج من الحمل المسلط (نرسم العلاقة بين التشوة والحمل)





نلاحظ من المخطط اعلاه ان التشوة في التربة ليس خطيا" ولغرض تحويل هذة العلاقة الى علاقة خطية يتم استخدام

Assumed linear relationship between unit load and deflection

1) USA K= 10 psi / Δ_{10}

Δ_{10} ; deflection corresponding to 10 psi pressure.

2) England K=$P_{0.05"}$/ 0.05"

DESIGN EXAMPLE

Consider a 40 KN (9000 lbf) wheel load applied to a 250 mm (10 in.) thick concrete pavement slab with a compressive strength of 48 MPa (7000 psi). The pavement slab rests on a subbase having a modulus of subgrade reaction equal to 27 MPa/m (100 pci). Assuming a joint width of 6 mm (0.25 in.), determine the required spacing and diameter for GFRP dowels with the following properties:

modulus of elasticity = 41 GPa (6 x 106 psi , shear modulus = 3.3 GPa (476,000 psi ,dowel bar length = 460 mm (18 in.).

Lecture No. 6 Equivalent Single-Wheel Load (ESWL)

The study of ESWL for dual wheels was first initialed during World War II when the *B-29* bombers were introduced into combat missions because the design criteria for flexible airport pavements then available were based on single-wheel load. دراسة حمل العجلة المكافئة بدات في الحب العالمية الثانية عند تصميم المطارات

The ESWL is defined as the load on a single tire that will cause an equal magnitude of a reselected parameter (stress, strain, deflection or distress) at a given location within a specific pavement system so that resulting from a multiple-wheel load at the same location within the pavement structure. Depending upon the procedure selected either the tire pressure or contact area of the ESWL may be equal to that of one tire of the multiple-gear assembly.

عبارة عن الحمل المسلط على اطار خيالي واحد والذي يسبب نفس المقدار من التطورات (نسبة الاجهادات في نقطة معينة – كافة انواع الاجهادات العمودية و الافقية و القص) في الموقع المحدد للتبليط (السطح او الاساس او ما تحت الاساس) وحمل العجلة المكافئة تكون لـه علاقة بالاطار الاصلي (1- ضغط الهواء المحصور في الاطار الخيالي = ضغط الهواء المحصور في الاطار الاصلي , 2- مساحة التماس للعجلة المكافئة = مساحة التماس للاطار الاصلي).

وهي عملية تعويض عن منظومة من الاطارات بمنظومة مبسطة (اطار واحد) – في المركبات الكبيرة تستخدم اكثر من اطار على جانبي المحور وذلك لزيادة مساحة التماس بين الاطار والطريق لتقليل الضغط الناشئ من الحمل فالضغط الناتج من المحور المتعدد الاطارات يكون اقل من تاثير الحمل الناشئ من اطار واحد

The ESWL can be determined from the theoretically calculated or experimentally measured stress, strain or deflection. It can be also be determined from pavement distress and performance such as the large, scale *WASHO* and *AASHO* road tests . حمل العجلة المكافئة يمكن حسابه نظريا او عمليا

Methods for measuring (ESWL) in Flexible Pavement

1) Equal Vertical Stress Criterion.

Boyd and foster, presented a semirational method for determining ESWL based on a theoretical consideration of the vertical stress in elastic half – space.

This method assumes that the ESWL varies with the pavement thickness as shown in figure .

1-For thickness ≤ d/2

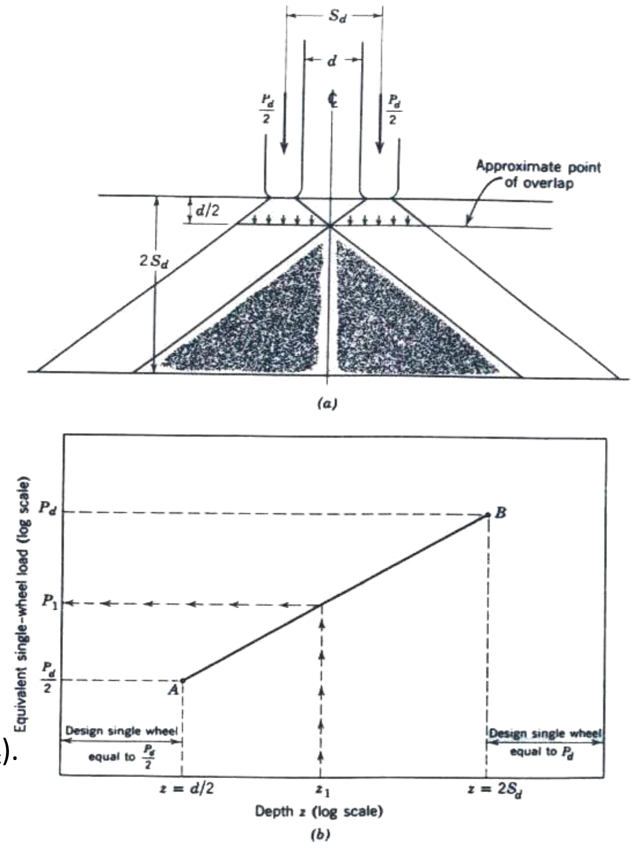
The ESWL = Pd/2 (indicating that the subgrade vertical stresses caused by two wheels do not overlap).

2-For thickness > 2Sd

The ESWL=Pd Total load (indicating that the subgrade stresses due to the two wheels overlap completely).

3-The ESWL for any intermediate thickness can be easily determined by assuming a straight-line relationship between pavement thickness and wheel load on logarithmic scales.

For mere convenient to compute the ESWL by



(a) (b)

$$Log(ESWL) = LogP_d + \frac{0.301*\log\left(\frac{2z}{d}\right)}{\log\left(\frac{4S_d}{d}\right)}$$

Where

P_d=The total load on the dual tire.

d= The clearance distance between dual tires ($d=S_d-2a_c$).

S_d= The center to center spacing between dual tires.

a_c= Redious of circular tire pavement contact area.

2. Equal Vertical Interface Deflection Criterion:

Foster and Ahlvin (35), was developed a new method, in this method the pavement systems considered as a homogenous half–space and the vertical deflection at a depth equal to the thickness of pavement can be obtained from *Boussinesq's* solution. A single- wheel load that has the same contact radios ($a=a_e$) as one of the dual wheels and results in a maximum deflection equal to that caused by the dual wheels is the ESWL.

Max interface deflection can be measured by using *Huang* theory.

$$W = \frac{P*a}{E1} * F$$

$$(Wo)_{max} = \frac{P*a}{E1} * \left(\sum_{i=1}^{n} Fi \right)_{max}$$

Where:

$(Wo)_{max}$ = Max. Interface deflection due to multiple wheel loads at the computation point (o).

F_i= Interface deflection factor corresponding to the i^{th} tire.

E_1 = Modulus of elastically of subgrade soil.

n = No. of tires in the multiple wheel assembly

$$(We)_{max} = \frac{Pe*ae}{E1} * Fe_{\not\subset}$$

Where:

$(We)_{max}$ = Max deflection due to ESWL at center of load.

P_e = Tire pressure corresponding to (ESWL)

a_e = Radios of circular tire pavement contact area for the (ESWL)

$Fe_{\not\subset}$ =Interface deflection factor corresponding to (ESWL)

To obtain the same deflection $(Wo)_{max} = (We)_{max}$

$$\frac{P*a}{E1} * \left(\sum_{i=1}^{n} Fi \right) = \frac{Pe*ae}{E1} * Fe_{\not\subset}$$

1) For equal contact area concept ($a=a_e$)

$$ESWL = P * \frac{\left(\sum_{i=n}^{n} F_i \right)_{max}}{Fe_{\not\subset}}$$

2) For equal contact pressure concept ($P=P_e$)

$$ESWL = P * \left[\frac{\left(\sum_{i=n}^{n} F_i \right)_{max}}{Fe_{\not\subset}} \right]^2$$

Where:

P= Wheel load

The ESWL for layered systems is greater than that for a homogenous half-space, *Huang*, suggested the use of layered theory and presented a simple chart for determining ESWL based on the interface deflection of two layered systems, the chart gives a load factor (L) defined as:

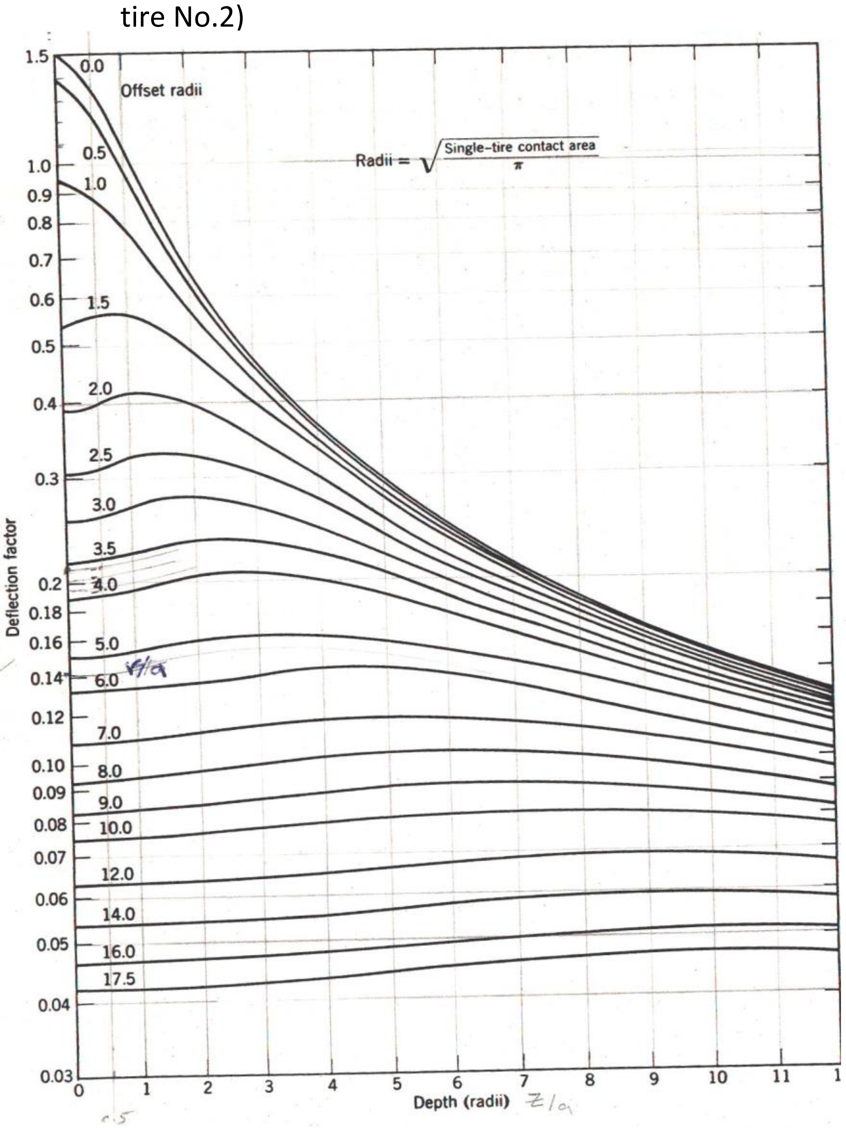
$$L = \frac{Total\ load}{ESWL} = \frac{2P_d}{P_s}$$

$$ESWL = \frac{2P_d}{L}$$

Example:

Calculate the ESWL versus depth relationship for the C_5A aircraft gear assembly shown in figure below by one layer interface deflection theory and equal contact area concept, assume for the analysis that the Max. deflection accurse at the center of wheel No.2 (point o) knowing that this load is uniformly distribution upon all tires and the load on each tire is uniformly distribution upon its corresponding circular contact area.

Note:-that the load on each tire=30 kips ; contact radius for each tire = 9.54 in (Max. deflection at center of tire No.2)



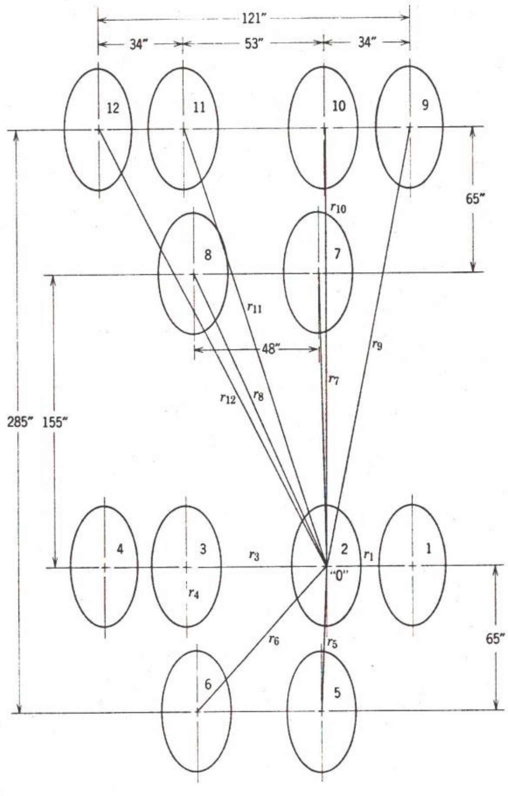


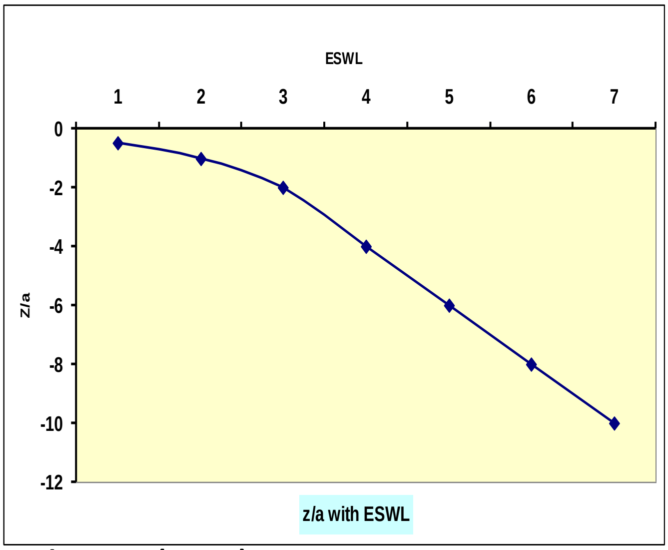
Figure 4.7. C5A gear layout.

Depth ratio z/a	Tire No.												Sum F_i	Sum Fe⊄	ESWL
	1	2	3	4	5	6	7	8	9	10	11	12			
	Radial distance to point "o" r_i (inch)														
	34	0	53	87	65	82.3	155	163	222.6	220	226.3	236.6			

		r/ a ratio													
		3.6	0	5.6	9.1	6.8	8.6	16.3	17.1	22.4	23.1	23.2	24.7			
F_I (figure 5-5)	0.5	0.215	1.35	0.141	0.083	0.112	0.088	0.046	0.043	0.027	0.028	0.027	0.025	2.185	1.35	48.84
	1	0.22	1.06	0.145	0.084	0.114	0.089	0.046	0.043	0.027	0.028	0.027	0.025	1.916	1.062	54.2
	2	0.23	0.67	0.15	0.085	0.116	0.09	0.047	0.044	0.027	0.028	0.027	0.025	1.539	0.67	68.8
	4	0.22	0.36	0.152	0.088	0.122	0.094	0.048	0.045	0.027	0.028	0.027	0.025	1.236	0.364	101.9
	6	0.184	0.25	0.145	0.089	0.12	0.093	0.049	0.046	0.027	0.028	0.027	0.025	1.083	0.247	131.5
	8	0.158	0.186	0.13	0.086	0.114	0.093	0.05	0.047	0.027	0.028	0.027	0.025	1.000	0.186	161.3
	10	0.131	0.152	0.116	0.086	0.108	0.089	0.051	0.049	0.027	0.028	0.027	0.025	0.86	0.149	177.2

The relationship between z/a and ESWL are shown in the following figure;

اذا كانت المسافة بين اطار واخربعيدة نسبيا فان الاجهادات تتضائل بينهما لذلك نهمل تاثير احدهما على الاخر



Equivalency factors (AASHTO Factor) Equivalent Wheel Load Factor (EWLF).

The damage per pass caused to a specific pavement system by the vehicle in question relative to the damage per pass of an arbitrarily selected standard vehicle moving on the same pavement system.

النسبة بين مقدار التلف الذي تسببه مركبة على الطريق الى التلف الذي تسببه مركبة قياسية.

$$dj = \frac{1}{Nf_j}$$

Nfj ; No. of repetitions caused to failure for the j th vehicle.

dj ; damage per pass for the j th vehicle. التلف الناتج من مرور المركبة

$$ds = \frac{1}{Nf_s}$$

Nfs; No. of repetitions caused to failure for the standard vehicle.

ds; damage per pass for the standard vehicle. التلف الناتج من مرور المركبة القياسية

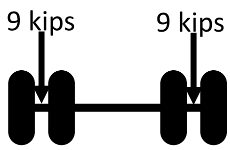
EWLF يمثل النسبة بين مقدار التلف الذي يسببه مرور مركبة عادية الى مقدار التلف الذي تسببه مركبة قياسية

$$EWLF = Fj = \frac{dj}{ds}$$

Fj ; equivalency factor for j th vehicle.

$$EWLF = Fj = \frac{dj}{ds} = \frac{\frac{1}{Nf_j}}{\frac{1}{Nf_s}} = \frac{Nf_s}{Nf_j}$$

Standard Axle



18 kips single axle load (single axle with dual tires at each end.

Standard single axle wheel often AASHTO

$$EWLF = Fj = \frac{Nf_{18kips}}{Nf_j}$$

e EWLF

1) pavement type (Rigid or flexible pavement)
2) type of axle (single axle, tandem axle, triple axle, floating tandem axle, 16 wheel tandem axle).
3) Magnitude of axle load (kips)- [2-40 for single and 10-48 for tandem)
4) Terminal level of serviceability (P_t) or failure point [2 for secondary , 2.5 for primary and 3 for Majer way or highway

TABLE 62.7 AASHTO Load Equivalency Factors for Flexible Pavements

Axle Load (kips)	Pavement Structural Number (SN)					
	1	2	3	4	5	6
(a) Single Axles and p_t of 2.5						
2	.0004	.0004	.0003	.0002	.0002	.0002
4	.003	.004	.004	.003	.002	.002
6	.011	.017	.017	.013	.010	.009
8	.032	.047	.051	.041	.034	.031

Axle Load (kips)	Pavement Structural Number (SN)					
	1	2	3	4	5	6
86	75.7	69.1	54.5	42.3	38.8	41.7
88	84.3	76.9	60.6	46.8	42.6	45.6
90	93.7	85.4	67.1	51.7	46.8	49.7
(c) Triple Axles and p_t of 2.5						
2	.0000	.0000	.0000	.0000	.0000	.0000
4	.0002	.0002	.0002	.0001	.0001	.0001
6	.0006	.0007	.0005	.0004	.0003	.0003
8	.001	.002	.001	.001	.001	.001

TABLE 62.8 AASHTO Load Equivalency Factors for Rigid Pavements

Axle Load (kips)	Slab Thickness, D (inches) 6	7	8	9	10	11	12	13	14
	(a) Single Axles and p_t of 2.5								
2	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
4	.003	.002	.002	.002	.002	.002	.002	.002	.002
6	.012	.011	.010	.010	.010	.010	.010	.010	.010
8	.039	.035	.033	.032	.032	.032	.032	.032	.032
10	.097	.089	.084	.082	.081	.080	.080	.080	.080
12	.203	.189	.181	.176	.175	.174	.174	.173	.173
14	.376	.360	.347	.341	.338	.337	.336	.336	.336

TABLE 62.8 (continued) AASHTO Load Equivalency Factors for Rigid Pavements

Axle Load (kips)	Slab Thickness, *D* (inches) 6	7	8	9	10	11	12	13	14
56	15.0	13.8	13.6	14.2	15.2	16.2	16.8	17.3	17.5
58	17.5	16.0	15.7	16.3	17.5	18.6	19.5	20.1	20.4
60	20.3	18.5	18.1	18.7	20.0	21.4	22.5	23.2	23.6
62	23.5	21.4	20.8	21.4	22.8	24.4	25.7	26.7	27.3
64	27.0	24.6	23.8	24.4	25.8	27.7	29.3	30.5	31.3
66	31.0	28.1	27.1	27.6	29.2	31.3	33.2	34.7	35.7
68	35.4	32.1	30.9	31.3	32.9	35.2	37.5	39.3	40.5
70	40.3	36.5	35.0	35.3	37.0	39.5	42.1	44.3	45.9

Example ; Convert the following axle load repetitions into equivalent standard single axle load repetitions for the cars of rigid pavement with D=10 in and P_t=2.5.

Single axle		Tandem axle	
Axle load (kips)	repetitions	Axle load (kips)	repetitions
18	1000	40	1000

28	10000	46	10000
40	30000	48	30000

Solution using above table to determining Fj

Total No. of apply axle = 82 000

Single axle			Tandem axle		
Axle load (kips)	Equivalency Factor Fj	Equivalent repetitions	Axle load (kips)	Equivalency Factor Fj	Equivalent repetitions
18	1	1000*1=1000	40	3.87	1000*3.87=3870
28	6.61	10000*6.61=66100	46	6.90	10000*6.9=69000
40	27.91	30000*27.91=837300	48	8.21	30000*8.21=246300
total		904400	total		319170

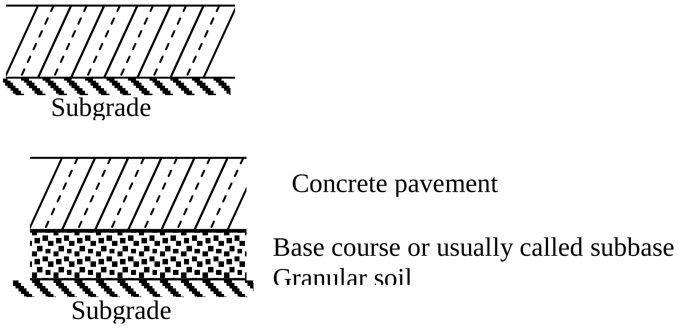
Total No. of equivalent single axle load repetitions = 904400+319170= 1223570

Lecture No. 7 Design of Rigid Highway Pavement

Concrete pavement is called rigid pavement are made of Portland cement concrete and may or may not have a base course between the pavement and subgrade as a general rule the concrete exclusive of the base is referred to as a pavement.

التبليط الكونكريتي يدعى بالتبليط الصلد يصنع من السمنت البورتلاندي قد يحتوي اولا يحتوي طبقة الاساس بين التبليط و ماتحت الاساس بصورة عامة لاتوضع طبقة اساس (حيث ان الغرض منها هو انشائي لمنع ظاهرة النضح لان الكونكريت له القابلية العالية لمقاومة الانحناء و لايحتاج الى اسناد).

Portland cement concrete slab=pavement



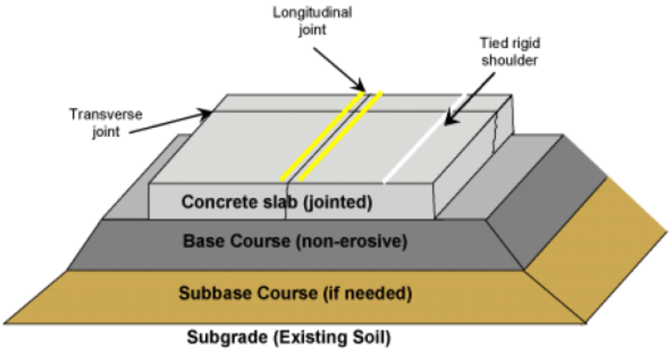
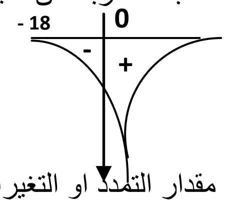


Figure 2-3. Typical section for a rigid pavement.

Purpose of Base Course for Rigid Pavement

1) Control of pumping. السيطرة على ظاهرة النضح
2) Control of frost action. ابعاد التربة عن تأثير الجو الخارجي للتقليل من ظاهرة انجماد او تبخر الماء الموجود بالتربة
3) Drainage يجب ابعاد الماء عن التبليط
   1) rigid – نمد طبقة الاساس الى خارج التبليط ونستخدم فيها مواد ذات نفاذية وبميل كبير لتسريع التصريف
   2) Flexible – نمد طبقة ماتحت الاساس الى الخارج وبأعطاء ميل قوي لتصريف الماء الى الخارج



4) Control of shrink and swell of the temperature. مقدار التمدد او التغيرات الحجمية الذي تحدث نتجة التغير بدرجات الحرارة او حركة الماء.
5) Control to expedition of construction.
6) The base lends some structured capacity to the pavement; however its contribution to the load carrying capacity is relatively miner.

طبقة الاساس تضيف نسبة قليلة لقابلية تحمل المنشأ.

Concrete Pavement Types;

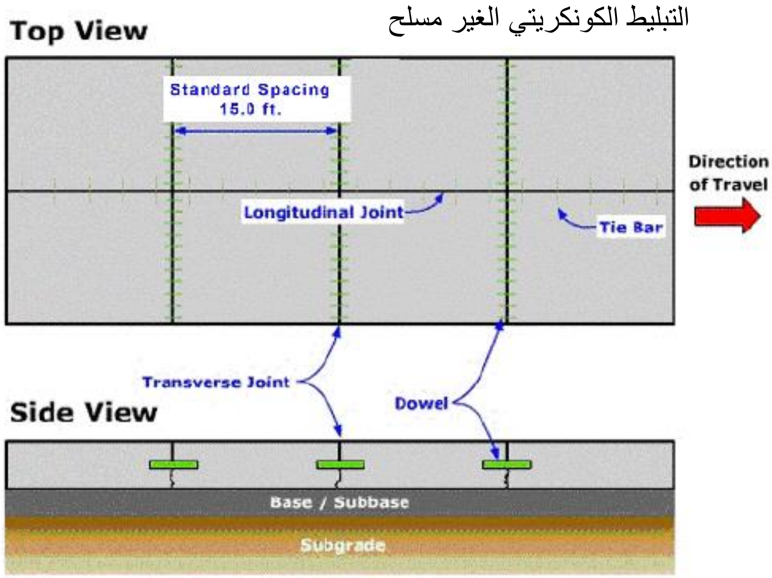
1) Plain Concrete Pavement (PCC) التبليط الكونكريتي الغير مسلح

Contains no steel for crack control except at the longitudinal joint's (Tie bar)

و هو التبليط الكونكريتي الذي لايستخدم فية حديد التسليح لغرض السيطرة على الشقوق عدا التسليح بالاتجاه الطولي .

must relatively close contraction joint spacing – short small contraction

هذا النوع يحتوي على عدد كبير من المفاصل للسيطرة على الشقوق.

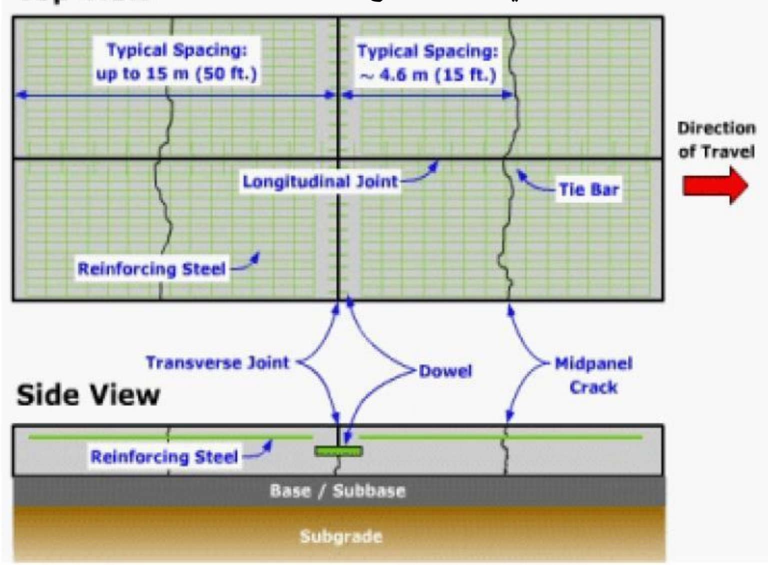


2) Simply Reinforced Concrete Pavements (SRCP)

Generally have relatively wide spacing of contraction joints and as a rule provision for load transfer across the joints using dowel bars, wire mesh steel is used between joints for crack control (Tie + Dowel + Mesh)

لغرض زيادة طول اللوح الكونكريتي وتقليل المفاصل نستخدم حديد التسليح.

التبليط الكونكريتي بسيط التسليح



3) Continuously Reinforcement Concrete Pavement (CRCP)

التبليط الكونكريتي المسلح المستمر

Contain relatively high percent of steel (0.6 % and higher) so that no joint for the contraction and same expansion joints.

نستخدم نسبة عالية من الحديد لتقليل المفاصل

CRCP provides joint-free design. The formation of transverse cracks at relatively close intervals is a distinctive characteristic of CRCP. These cracks are held tightly by the reinforcement and should be of no concern as long as the cracks are uniformly spaced, do not spell excessively, and a uniform non-erosive base is provided.

التبليط الخالي من المفاصل (الشقوق العرضية تكون بمسافات متعاقبة و هذة الشقوق تكون مسلحة

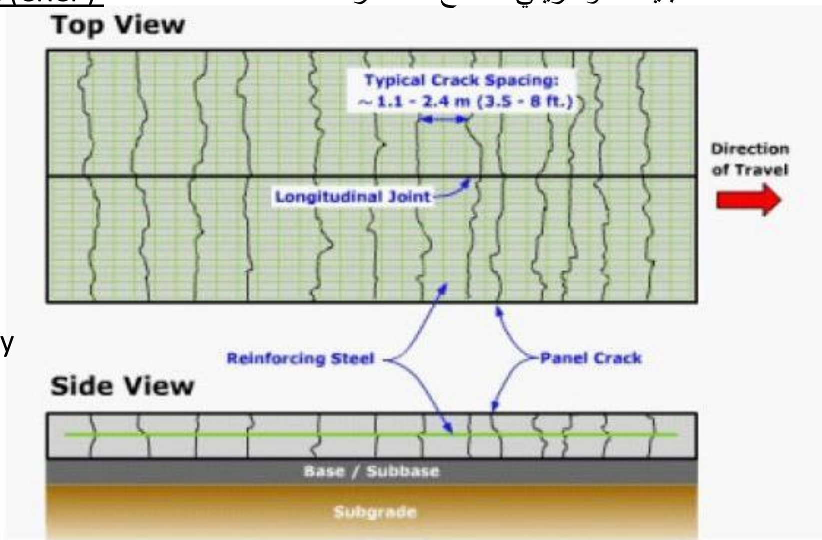
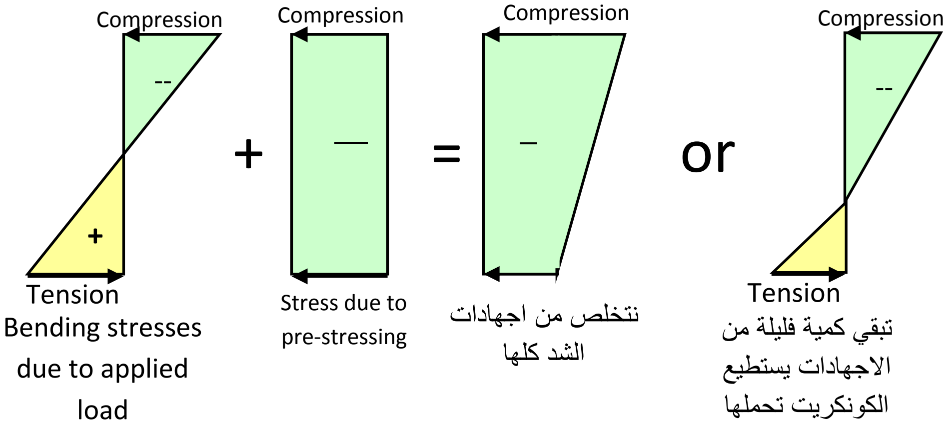


Figure 2-4. Continuously Reinforced Concrete Pavement.

4) Pre-stress Concrete Pavement (PCP) or Post-tensioned Concrete Pavements

التبليط الكونكريتي المسبق الجهد

is one in which a permanent and essentially horizontal compressive stresses has been introduced prior to the application of any wheel loads.



تكون عملية سحب الحديد لامتصاص اجهادات الشد ويجعل لامتصاص اجهادات الشد وجعل الكونكريت يقاوم اجهادات الضغط الناتجة من السحب ففي هذة الحالة يكون المقطع كله يقاوم اجهادات الضغط و بالتالي يقلل من المفاصل عدا المفاصل الانشائية فتزيد من عمر التبليط و نقلل من فترة الصيانة

Design Methods

1) PCA method (Portland cement Association)- Design based on the Fatigue

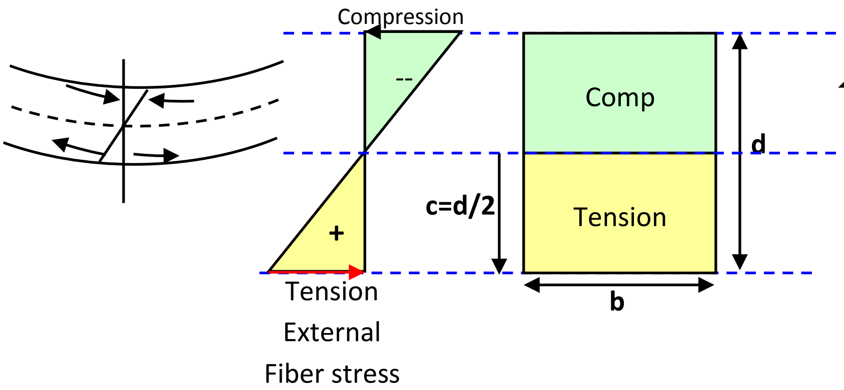
اي مادة يسلط عليها حمل فان قابليتها على التحمل سوف تقل مع زيادة عدد تكرار الحمل فاذا كان الحمل المسلط اكبر فسوف يحدث اجهاد اكبر وبالتالي سوف تقل عدد الضربات اللازمة لفشله (سوف يفشل بعدد قليل من الضربات) ولكن اذا كان الحمل المسلط قليل فالاجهاد قليل فنحتاج الى ضربات اكبر للفشل وهذه تمثل مقاومة الكلال (Fatigue Strength). وهذه تعتمد على عدة عوامل منها (خواص الكونكريت و نسبة الاجهاد)

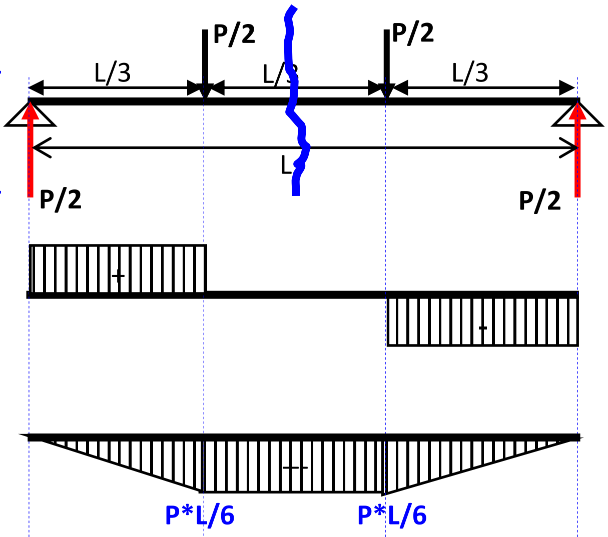
Number of load repetitions (life of concrete under repeated loads) = f(Properties of concrete , stress level – stress ratio). For the usually case, average fatigue data that cover a verity of concrete are used. The major factor that determines the life of concrete under repeated loading is magnitude of load.

لغرض تحديد خواص الكونكريت نستخدم معدل التعب والعامل الرئيسي هو ايجاد عمر الكونكريت تحت الحمل المتكرر

$$\text{Stress Ratio} = \frac{\text{Actual stress due to load}}{\text{Strength of Concrete(Max. Tensile Strength at the External Fiber - Modulus of Rapture - MR)}}$$

Modulus of Rapture





Modulus of Rapture should be determined from third point Loading test. Max. Tensile stress in bending

$$MR = \frac{M_{max}}{I} * c \quad \text{where } I = \frac{bd^3}{12}$$

for fraction within middle third

1) اذا حدث الفشل في الثلث الوسطي

$$MR = \frac{\frac{P*L}{6}}{\frac{bd^3}{12}} * \frac{d}{2} = \frac{P*L}{b*d^2}$$

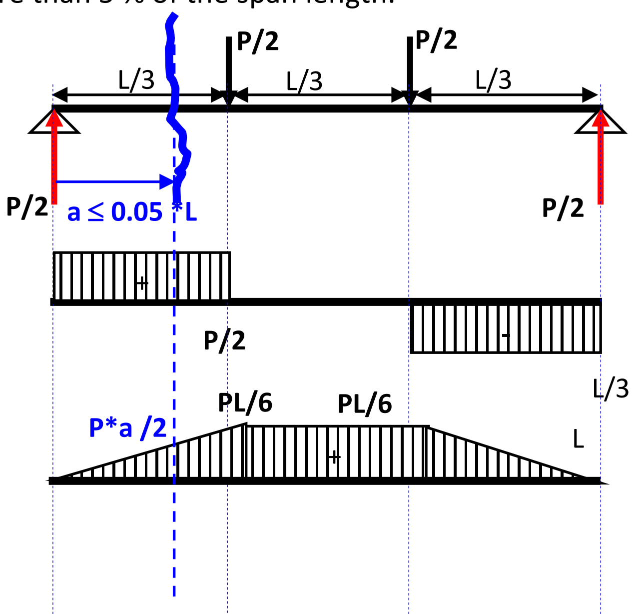
2) اذا حدث الفشل خارج الثلث الوسطي

a) if the fracture occurs outside the middle third by not more than 5 % of the span length.

$$MR = \frac{\frac{P*a}{2}}{\frac{bd^3}{12}} * \frac{d}{2} = \frac{3*P*a}{b*d^2}$$

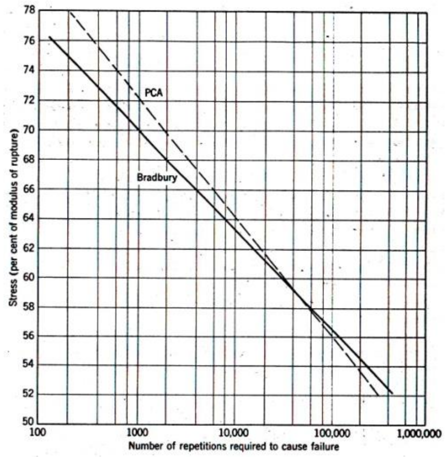
a ; distance between the line of fraction to the nearest support measured a long the center line of the bottom surface of the beam.

b) if the fracture occurs outside the middle third by more than 5 % of the span length.
(Neglect test and repeated test)



Actual stress using Westergoard at edge

- **Stress Ratio** using the following figure



1) Fatigue curves for plain concrete in flexure

المنظمة الامريكية للكونكريت قامت بمجموعة من الفحوص لتحديد نسبة الاجهادات على السمنت

2) Using the following tables to determined stress ratio.

Stress ratio %	Allowable repetition N
0.51	400 000
0.6	32 000
0.7	2 000
0.8	120
0.85	30

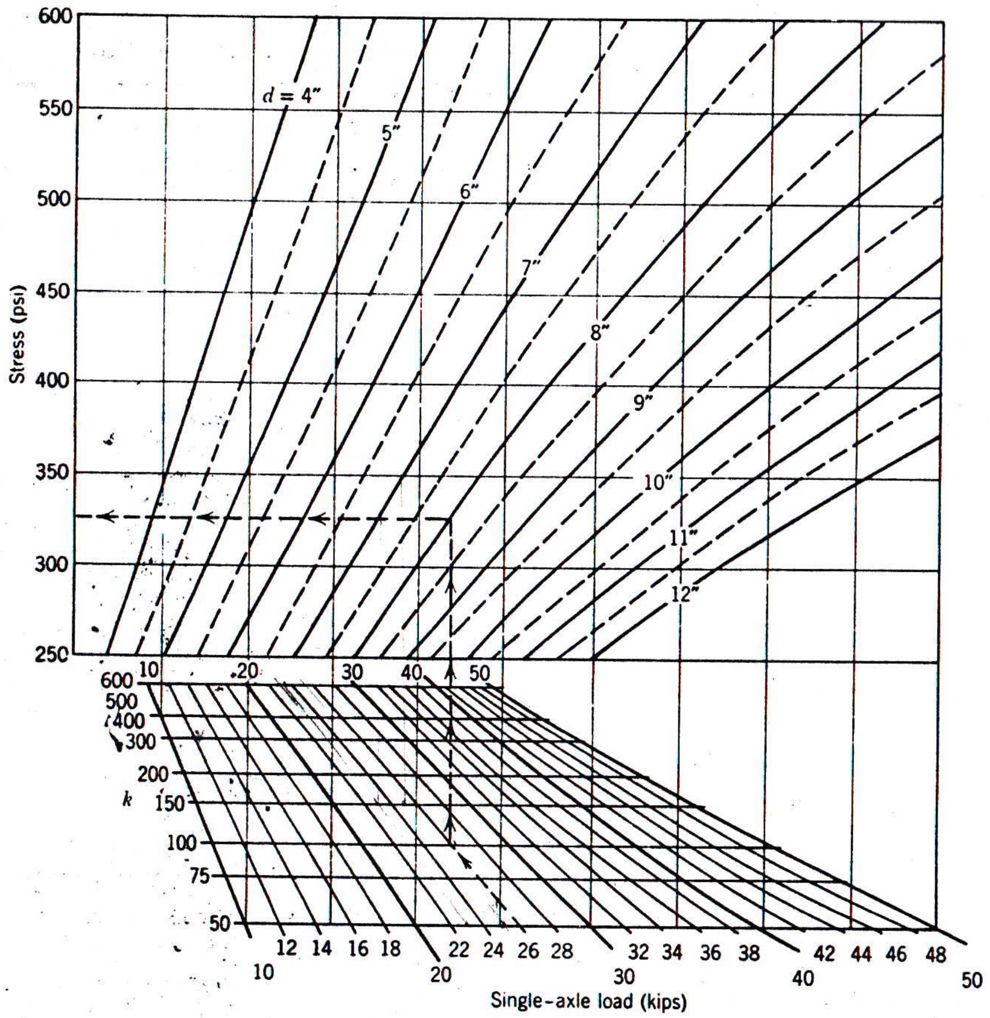
3) Percent of Fatigue used by axle load or vehicle or aircraft = N_i/N_a

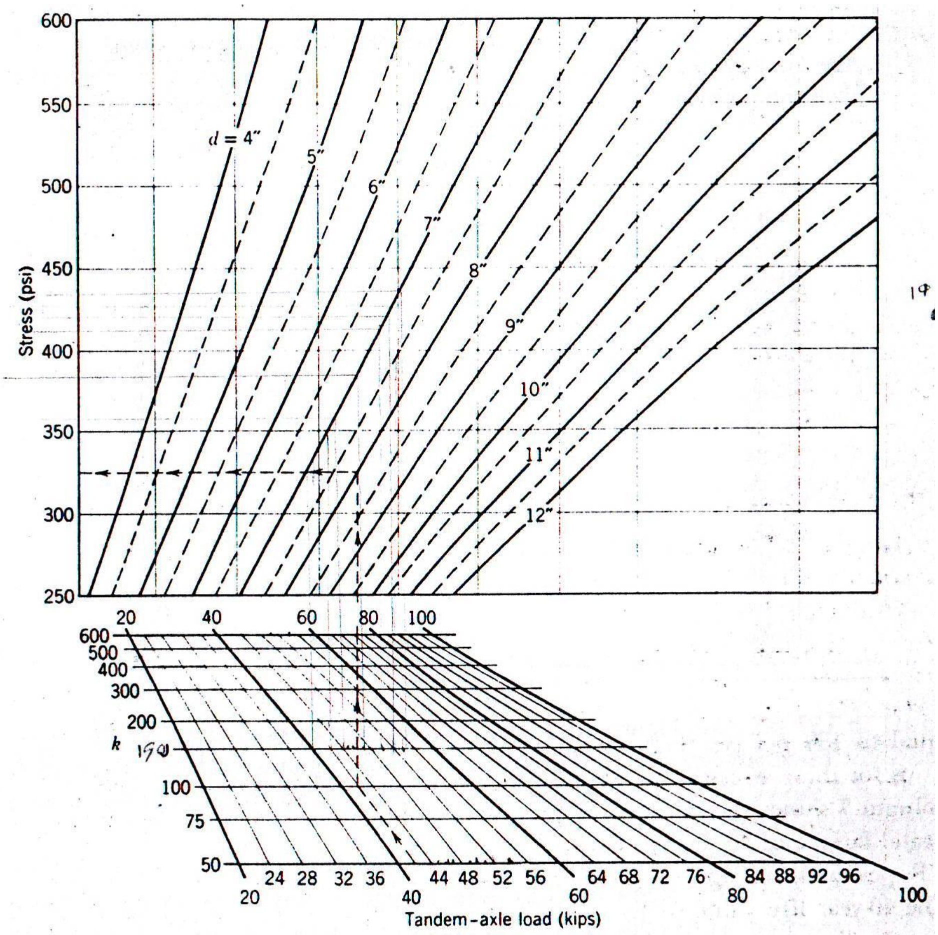
N_i= actual repetition of i^{th} load.

N_a= allowable repetition of i^{th} load using tables and figures = No. of repetitions to failure corresponding to the i^{th} load.

$$\sum \frac{N_i}{N_a} * 100 \le 100 - 110 \text{ theo or } 100 - 125\,\% \text{ par}$$

هذة الطريقة تعتمد على عدد الضربات اللازمة للفشل لذلك يجب تحديد عدد المركبات المارة خلال الفترة التصميمية وخاصة الثقيلة منها من خلال استخدام معامل نمو المركبات وعلى فرض بان الحمل المسلط يكون ستاتيك ولكن نتجة لعدم انتظام الطريق يحدث اهتزازات تضاف كعدد الى العدد الكلي.





Note:-

1) In this method is necessary to make are estimation of the traffic growth on the facility.

زيادة عدد المركبات خلال الفترة التصميمية باستخدام معامل نمو المركبات.

2) Ti is recommended that the actual wheel or axle loads that will be applied to the pavement by increasing a factor of 20% (static*1.2) this increase account for increase of load due to impact.

نفرض بان عدد الاهتزازات تعادل 20% من عدد الضربات المسلطة وهي تضاف.

3) Experience has shown that Sum of fatigue used can approved (125%) without serious problems.
4) The 28 day MR are used for thickness design of streets and roads.

فحص MR يحري لاغراض الطرق كل 28 يوم و المطارات كل 90 يوم .

Example

It is desired to investigate the design of concrete pavement for the condition given below using the Portland cement association method.

ADT in both direction = 400 veh/day.
Traffic distribution =50% in each direction.
% of truck= 20% from ADT.
Annual growth= 2% (truck distribution I following table).
Design life (period)=40 year.
Modulus of subgrade reaction (K)=150 pci.
Modulus of Rupture (MR)=650 psi.
Thickness of pavement=7 inch.
Impact factor= 1.2

TABLE 17.2. Truck Axle Distribution

Axle Load Group (thousand pounds)	No. Axles per 100 Trucks on the Road	
	Single Axles	Tandem Axles
12–14	8.0	
14–16	7.3	
16–18	6.1	
18–20	5.4	
20–22	3.2	5.2
22–24		7.6
24–26		8.4
26–28		9.0
28–30		11.2
30–32		9.4
32–34		1.8
34–36		1.4
36–38		0.9
38–40		1.0
40–42		0.1
42–44		0.1
44–46		0.1

Solution

في بداية التصميم نفرض سمك ونجد نسبة الاجهاد الذي يتحمله هذا السمك فاذا كان حسب المواصفات (100-125 %) فيكون الاختيار

If thickness > 100-125 % ---- increased thickness.

If thickness < 100-125 % ---- decreased thickness.

في البداية نحدد عدد اللوريات خلال اليوم الواحد ونستخدم معامل النمو growth factor لايجاد عدد اللوريات المتوقعة للفترة التصميمية 40 سنة.

TABLE 17.3. Yearly Rates of Traffic Growth and Corresponding Projection Factors[a]

Yearly Rate of Traffic Growth (percent)	Projection Factor for 20 Years[b]	Weighted Average Projection Factor for 40 Years[c]
1	1.2	1.2
$1\frac{1}{2}$	1.3	1.3
2	1.5	1.5
$2\frac{1}{2}$	1.6	1.7
3	1.8	1.9
$3\frac{1}{2}$	2.0	2.2
4	2.2	2.5
$4\frac{1}{2}$	2.4	2.8
5	2.7	3.2
$5\frac{1}{2}$	2.9	3.6
6	3.2	4.1

[a] From Portland Cement Association.

[b] Values of $(1 + i)^{20}$.

[c] $\frac{\sum_1^{40} (1 + i)^n}{40}$

where i = growth rate (% per year).

Total trucks to consider for a design life of 40 years

ADT/direction =400*0.5=200 vpdpdir

No. of trucks/dir/day= ADT/dir*T%

=400*0.2=40 vpd.

No. of trucks during design period

= growth factor* No. of trucks/day

= 1.5*40=60 vpd/dir

Total No. of trucks during period=60*365*40=876000 T/40 year

الان نجد عدد الضربات الذي يولدة كل نوع من المحاور

Axle type	Axle load(kip)	Approximation repetition	Effective axle load =1.2*static axle (kip)	Stress psi Design chart	Stress ratio Stress/MR	Allowable repetition using (Table17-1)	% of fatigue = $N_i/N_{allowable}$
single	21	28032	25.2	350	0.54	180000	(28032/18000)*100=15.6 %
	19	47304	22.8	325	0.5	∞	0 %
tandem	45	876	54	435	0.67	4500	19.5 %
	43	876	51.6	415	0.64	11000	8 %
	41	876	49.2	410	0.63	14000	6.3 %
	39	8760	46.8	390	0.6	32000	27.4 %
	37	7864	44.4	375	0.58	57000	13.8 %
	35	12264	42.0	350	0.54	18000	7 %
	33	15768	39.6	325	0.5	∞	0 %
	31	82344	37.2	310	0.47	∞	0%
						Sum of % fatigue	97.6 %
						Less than 100-120 %	ok

أذن الفرضية صحيحة (عند استخدام هذا السمك ونتجه لأنواع المركبات المارة أعلاه والذي يسلط عدد من الضربات تولد اجهادات تناسب ضرباتها هذه وهذه الاجهادات تسبب تعب للمادة الذي يعبر عنها بعدد الضربات التي يتحملها .

يمكن استخدام هذه الطريقة لإيجاد

1) عدد الضربات التي سوف يفشل عندها التبليط.

2) التصميم (السمك اللازم لتحمل الاجهادات الناتجة من الضربات).

Approximation repetition

$$\frac{876000}{100} * 3.2 = 28032$$

$$\frac{876000}{100} * 5.4 = 47304$$

2) **AASHTO Guide for Design of Pavement Structures, 1986/ 1993**

The current AASHTO design method for pavement structures includes a number of improvements, based on mechanistic principles, to those procedures which existed prior to the publication of the 1986/1993 AASHTO Guides. There are two design equations, one each for asphaltic concrete and PCC pavements. The previous design procedures were, for the most part, empirical and were based directly on the results obtained from the AASHO Road Test. In some instances, such as in the case of rigid pavement design, these results were supplemented with then-existing theory and pavement design methods.

In the procedures presented in the new guide, an attempt is made to incorporate some mechanistic-based principles. In the characterization of material properties for design, for example, mechanistic-based parameters are used in place of empirical parameters, such as the CBR. Characterization of pavement materials for use in this procedure is based on the resilient modulus. The method of determining the damaging effect of seasonal variations on the pavement design also incorporates some mechanistic principles, as do the procedures for establishing the coefficients for drainage and load transfer.

1. Introduction

Empirical design based on the AASHO road test:

- Over 200 test sections JPCP (15’ spacing) and JRPC (40’ spacing)
- Subbase type: untreated gravel/sand with plastic fines
- Subbase thickness; 0 to 9 inches
- Subgrade soil: silty-clay (A-6)
- Monitored PSI w/ load applications – developed regression eqn’s
- Number of load applications: 1,114,000

The basic design equation is

$$\log(W_{18}) = (Z_R \cdot S_0) + 7.35 \cdot \log(D+1) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{4.5-1.5}\right)}{1 + \frac{1.624 \cdot 10^7}{(D+1)^{8.46}}} + (4.22 - 0.32 \cdot p_t) \cdot \log\left[\frac{S_c \cdot C_d \cdot (D^{0.75} - 1.132)}{215.63 \cdot J \cdot \left[D^{0.75} - \frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]}\right]$$

Where:

W_{18}; predicted No. of 18 kips equivalent single axle load.

Z_R; standard normal deviate.

S_o; combined standard error of the traffic predication and performances predication.

D; thickness (inch) of pavement slab.

ΔPSI; difference between the initial design serviceability (P_o=4.5) and design terminal level of serviceability (P_t=1.5) , ΔPSI=P_o- P_t=4.5-1.5= 3

$S_c^{\backslash}$;Modulus of rapture(psi) for Portland cement concrete used are specific project.

J; load transfer coefficient used to adjust for load transfer characterizes of a specific design.

C_d; drainage coefficient.

E_c; Modulus of elasticity (psi) for Portland cement concrete

K; modulus of subgrade reaction (pci).

Z_R based on reliability (probability that any particular type of distress as combination of distress manifestations will remain below or within the permissible level during the design life.

احتمالية ظهور الفشل بحيث تكون اقل من المسموح به خلال الفترة التصميمية (درجة الثقة بالتصميم) كلما تزداد الثقة بالتصميم تزداد Z_R كمقدار فقط ولكنها تقلل من عدد الضربات القياسية.

Reliability %	Standard normal deviate (ZR)
50	0
90	-1.282
99	-2.327
99.99	-3.750

Suggested levels of reliability for various functional classification

أذن درجة الثقة تعتمد على أهمية الطريق فكلما تزداد أهمية الطريق تزداد درجة الثقة

Functional classification	Recommended level of reliability	
	urban	rural
Freeway and expressway	85-99.9	80-99.9
Principle arterials	80-99	75-95
Collectors	80-95	75-95
local	50-80	50-80

S_o; overall standard deviation

1) The estimated overall standard deviation for the case where the variance of the projected future traffic is considered S_o= 0.34 for rigid pavement and S_o=0.44 for flexible pavement.
2) The estimated overall standard deviation for the case where the variance of the projected future traffic is not considered S_o= 0.39 for rigid pavement and S_o=0.49 for flexible pavement.
3) for rigid pavement the range of (S_o= 0.3-0.4) and (0.4-0.5) for flexible pavement.

C_d; Drainage coefficient based on the;

1) quality of drainage نوعية التصريف السطحي
2) percent of time during the year ,the pavement structure is exposed to the moisture levels approaching saturation. نسبة الوقت الذي نتوقع فيه التبليط يكون مغمور بالماء (مشبع بالماء ويعتمد على
   1) معدل سقوط الأمطار خلال السنة average yearly rainfall
   2) ارتفاع منسوب المياه الجوفية خلال السنة ground water table

TABLE 62.16 Recommended Value of Drainage Coefficient, C_d, for Rigid Pavement Design

Quality of Drainage	Percent of Time Pavement Structure Is Exposed to Moisture Levels Approaching Saturation: Less than 1%	1–5%	5–25%	Greater than 25%
Excellent	1.25–1.20	1.20–1.15	1.15–1.10	1.10
Good	1.20–1.15	1.15–1.10	1.10–1.00	1.00
Fair	1.15–1.10	1.10–1.00	1.00–0.90	0.90
Poor	1.10–1.00	1.00–0.90	0.90–0.80	0.80
Very Poor	1.00–0.90	0.90–0.80	0.80–0.70	0.70

Source: AASHTO. 1993. *AASHTO Guides for Design of Pavement Structures*. Copyright 1993 by the American Association of State Highway and Transportation Officials, Washington, D.C. Used by permission.

Excellent ; water removed within 2 hr.

Good ; water removed within 1 day.

Fair ; water removed within 1 week.

Poor ; water removed within 1 month.

Very Poor; water without drain.

Load transfer coefficient factor (J) يستخدم هذا الثابت لتوضيح كيفية الربط مع الأكتاف

- Ability to transfer loads across joints and cracks
- Lower J - better performance/less conservative

قابلية نقل الحمل خلال المفاصل و الشقوق
كلما تكون J قليلة فان الكفاءة تزداد وتقل

TABLE 62.15 Recommended Load Transfer Coefficient for Various Pavement Types and Design Conditions

Shoulder	Asphalt		Tied P.C.C.	
Load Transfer Device	Yes	No	Yes	No
Pavement Type				
Plain Jointed and Jointed reinforced	3.2	3.8–4.4	2.5–3.1	3.6–4.2
CRCP	2.9–3.2	N/A	2.3–2.9	N/A

Loss of Support Factor (LS);
Take into account for the potential loss of support arising from subbase erosion and or differentional movement معامل فقدان الإسناد يؤخذ نتجه لتأكل أو تعرية طبقة ما تحت الأساس أو الحركة الأفقية.
It is treated in the actual design procedure by domination the effective or composite k-values (Modulus of subgrade reaction) based on the size of the void that way develop beneath the slab.
الفقدان بالإسناد ويعبر عنه خلال خطوات التصميم بقيمة k والذي يعتمد على حجم الفراغات الناتجة تحت السقف (مناطق حدوث الفقدان بالإسناد هي أركان اللوحة)

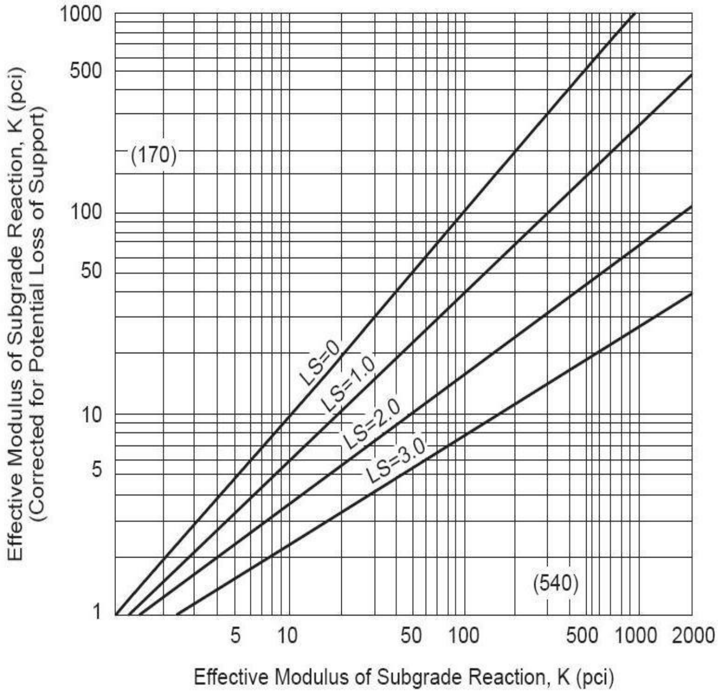
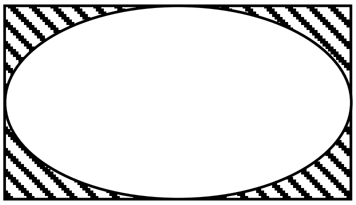


TABLE 62.14 Typical Ranges of Loss of Support (*LS*) Factors for Various Types of Materials

Type of Material	Loss of Support (*LS*)
Cement treatment granular base (E = 1,000,000 to 2,000,000 psi)	0.0–1.0
Cement aggregate mixtures (E = 500,000 to 1,000,000 psi)	0.0–1.0
Asphalt treated base (E = 350,000 to 1,000,000 psi)	0.0–1.0
Bituminous stabilized mixtures (E = 40,000 to 300,000 psi)	0.0–1.0
Lime stabilized (E = 20,000 to 70,000 psi)	1.0–3.0
Unbound granular materials (E = 15,000 to 45,000 psi)	1.0–3.0
Fine-grained or natural subgrade materials (E = 3000 to 40,000 psi)	2.0–3.0



مناطق حدوث الفقدان بالإسناد هي أركان اللوح

Example:
Determine the thickness of the rigid pavement (Jointed Concrete Pavement) under the following conditions using AASHTO guide.

K=72 pci	S_o=9
$E_c=5*10^6$ psi	R=90% (ZR=-1.645)
$S_c^{\backslash}$=650 psi	P_o=4.2
J=3.2	P_t=2.5

C_d=1.0 W_{18}=5.1*10^6 (18 kip ESAL)

Solution by equation

$$\log(W_{18}) = (Z_R \cdot S_0) + 7.35 \cdot \log(D+1) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{4.5-1.5}\right)}{1+\frac{1.624\cdot 10^7}{(D+1)^{8.46}}} + (4.22 - 0.32 \cdot p_t)\cdot \log\left[\frac{S_c \cdot C_d \cdot (D^{0.75} - 1.132)}{215.63 \cdot J \cdot \left[D^{0.75} - \frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]}\right]$$

Function lift (F_L)= Function Right (F_R)

$F_L = log_{10}(W_{18}) = log_{10}(5.1 * 10^6) = 6.70757$

ΔPSI(loss in the serviceability)=P_t-P_o=4.2-2.5=1.7

Z_R= standard normal division based on the Reliability from table= -1.645

S_o=Overall standard division (combined standard error of the traffic and performance).

D= thickness of the pavement slab(in).

$S_c^{\prime}$= Modulus of Rapture for PCC.

C_d= Drainage coefficient factor.

J= load transfer coefficient factor.

Trial No.1 assumed D=9 inch

$$F_R = -1.645 * 0.29 + 7.35 * log_{10}(9+1) - 0.06 + \frac{log_{10}\left[\frac{1.7}{3}\right]}{1+\frac{1.624*10^7}{(9+1)^{8.46}}} +$$

$$(4.12 - 0.32 * 2.5) * log_{10}\left[\frac{650 * 1 * ((9)^{0.75} - 1.132}{215.63 * 3.2\left[(9)^{0.75} - \frac{18.42}{\left(\frac{5*10^6}{72}\right)^{0.25}}\right]}\right] = 6.49167 < F_L \qquad Not\ Good$$

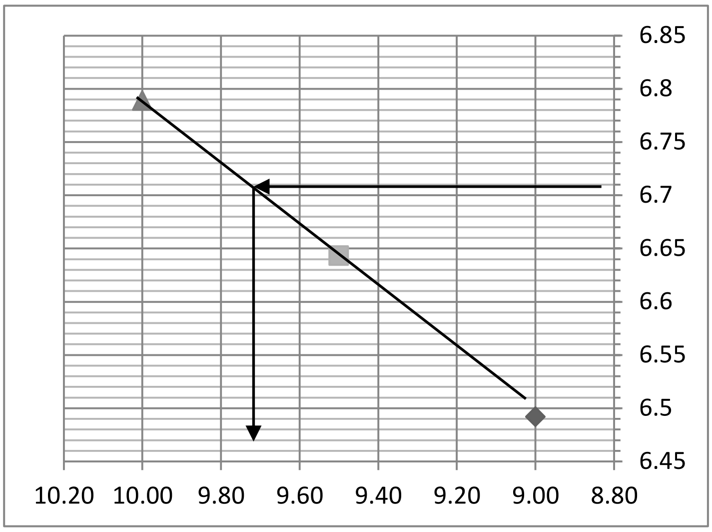
Trial No.2 Assumed D=10 inch

Using above equation F_R=6.78872 > F_L(6.70757) ok

Trail No.3 Assumed D= 9.5 inch

Using above equation F_R=6.6429 ≈ F_L(6.70757) very Good

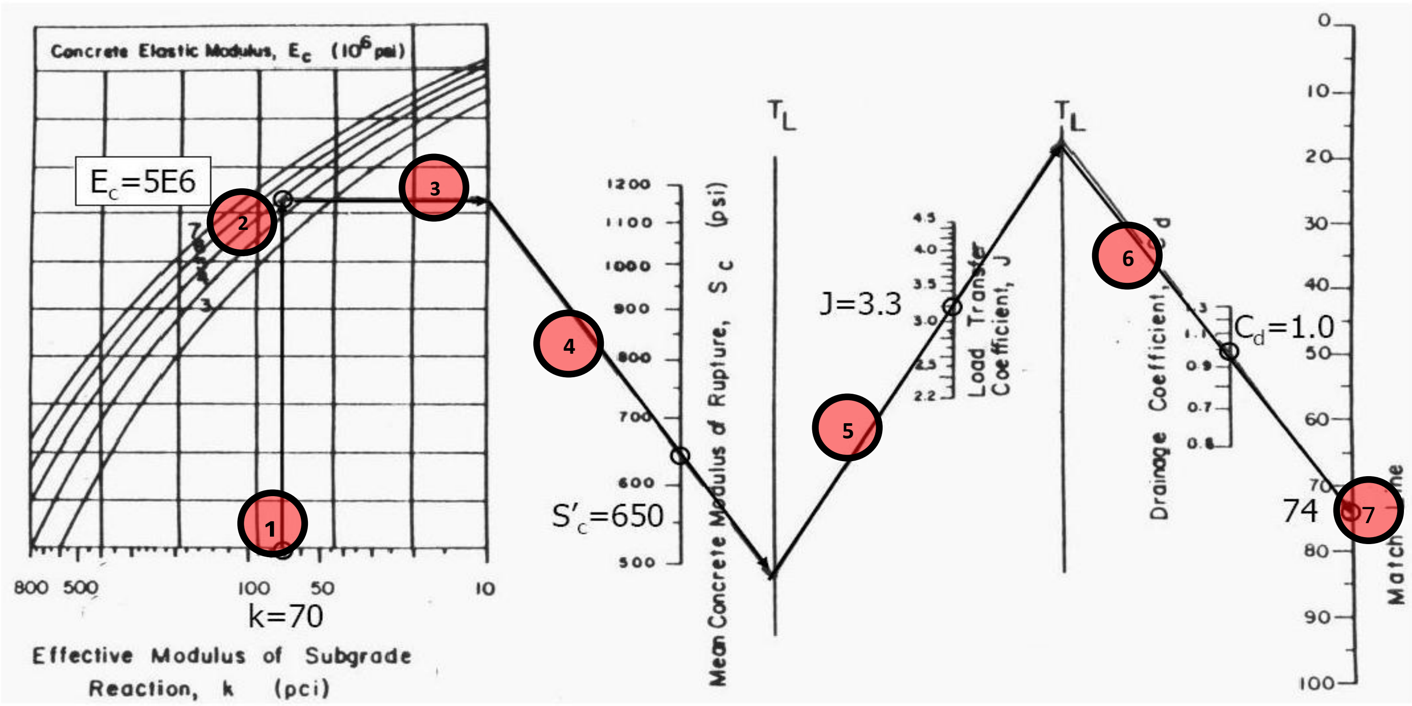
نرسم العلاقة بين قيم السمك و قيم الطرف الأيمن والأيسر ونجد قيمة السمك الذي يحقق تساوي الطرفين الأيمن والأيسر.



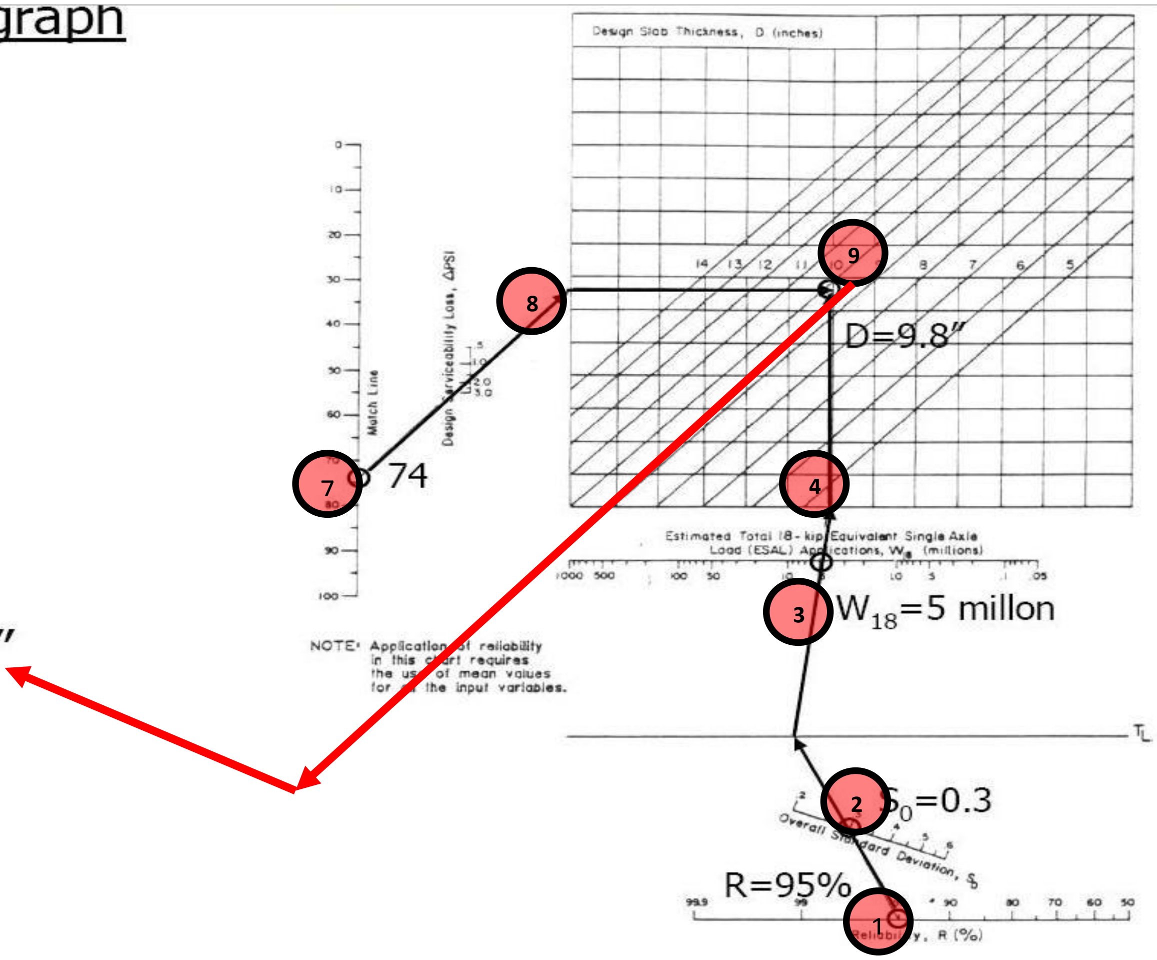
من الشكل أعلاه نلاحظ بان قيمة السمك الذي يساوي الطرفين هو
9.75 inch

Solution by using AASHTO Design chart.

4.2 Nomograph



4.2 Nomograph



Use D=10″

Lecture No. 8 Design of Flexible Highway Pavement

1) AASHTO Guide 1993 Design method.
General equation for design is;

$$\log_{10} W_{18} = -ZS_o + 9.36 \log_{10}(SN+1) - 0.20 + \frac{\log_{10}\left[\frac{\Delta PSI_w}{PSI_o - 1.5}\right]}{0.4 + \frac{1094}{(SN+1)^{5.19}}} + 2.32 \log_{10} M_r - 8.07$$

Where:

W_{18}; predicted No. of 18 kips equivalent single axle load.
Z_R; standard normal deviate.
S_o; combined standard error of the traffic predication and performances predication.
D; thickness (inch) of pavement slab.
ΔPSI; difference between the initial design serviceability (P_o) and design terminal level of serviceability (P_t).
M_r ;Resilient Modulus (psi).
SN; Structural Number indicated in total pavement thickness required.

The structural number , an abstract number related to the strength required of the total pavement structure, is the summation of the layer thicknesses multiplied by their corresponding strength coefficients. Use the following equation to determine the structural number:

$SN = a_1D_1 m_1 + a_2D_2m_2 + a_3D_3 m_3$

Where:

SN ; flexible pavement structural number
a_1, a_2, and a_3 ; coefficients of relative strength of the surface, base, and subbase materials, respectively
D_1, D_2, and D_3 ; actual thickness, in inches, of the surface, base, and subbase layers, respectively.
m_1,m_2,m_3 ; drainage coefficients for surface , base and subbase layers, respectively (assumed m_1=1)

تعتمد هذه الطريقة على قوة المواد المستخدمة بالتبليط (stiffness) وكذلك سمك الطبقات وكيفية تصريف المياه (سمك الطبقة والميل و نفاذية).

Drainage Coefficient for flexible pavement can be obtained from the following table;

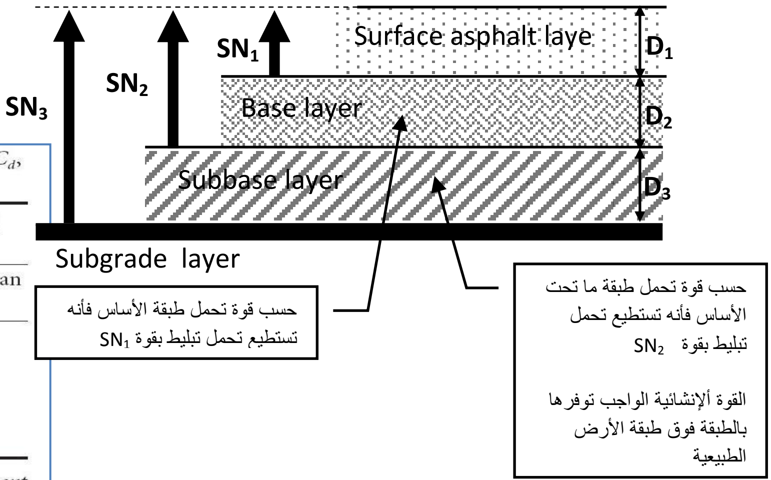


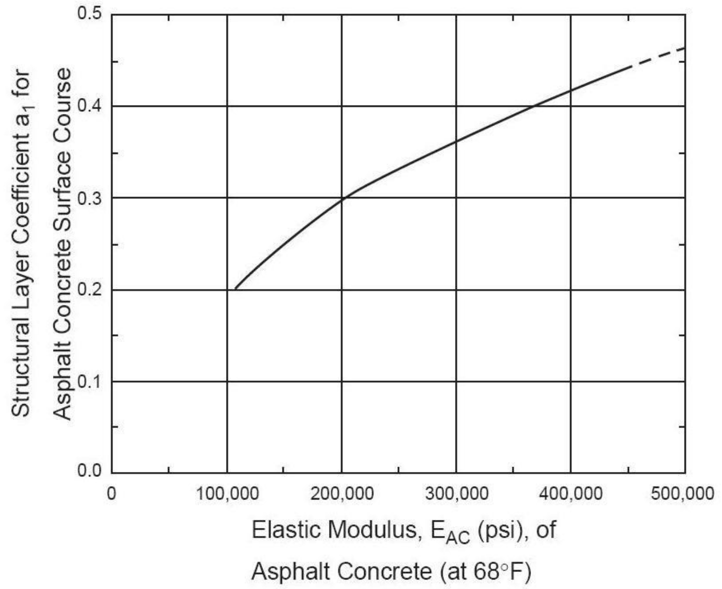
TABLE 62.16 Recommended Value of Drainage Coefficient, C_d, for Rigid Pavement Design

Quality of Drainage	Percent of Time Pavement Structure Is Exposed to Moisture Levels Approaching Saturation: Less than 1%	1–5%	5–25%	Greater than 25%
Excellent	1.25–1.20	1.20–1.15	1.15–1.10	1.10
Good	1.20–1.15	1.15–1.10	1.10–1.00	1.00
Fair	1.15–1.10	1.10–1.00	1.00–0.90	0.90
Poor	1.10–1.00	1.00–0.90	0.90–0.80	0.80
Very Poor	1.00–0.90	0.90–0.80	0.80–0.70	0.70

Source: AASHTO. 1993. *AASHTO Guides for Design of Pavement Structures.* Copyright 1993 by the American Association of State Highway and Transportation Officials, Washington, D.C. Used by permission.

Layer Coefficient عبارة عن معامل يعبر عن قوة تحمل المواد التي تستخدم بالطبقة

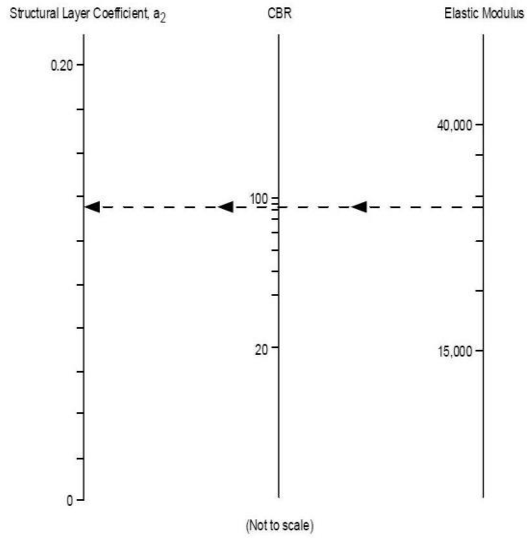
1) Surface layer coefficient (can be obtained from the following figure)

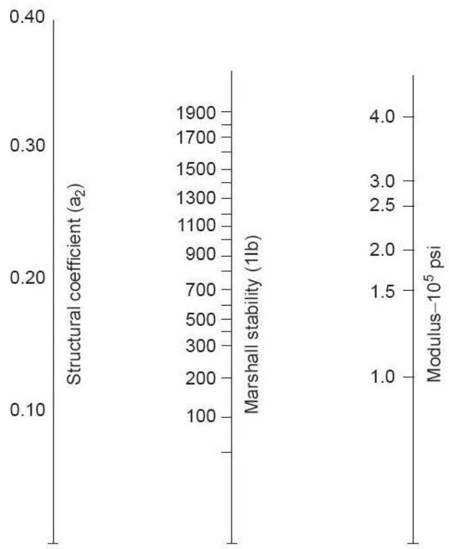


Layer Coefficient of Dense-Graded Asphalt Concrete

Elastic modulus, E_{AC} (psi) of asphalt concrete (at 68°F)	Structural layer coefficient, a_1, for asphalt concrete surface course
110,000	0.20
150,000	0.25
200,000	0.30
250,000	0.34
300,000	0.37
350,000	0.39
400,000	0.42
450,000	0.45

2) Base layer Coefficient (can be obtained from the following figure).





(b) Base Course

Layer Coefficient of Granular Base

Granular base CBR (%)	Structural layer coefficient, a_2
20	0.07
30	0.09
35	0.10
45	0.11
55	0.12
70	0.13
100	0.14

3) Subbase layer Coefficient (can be obtained from the following figure).

a) Equation ;

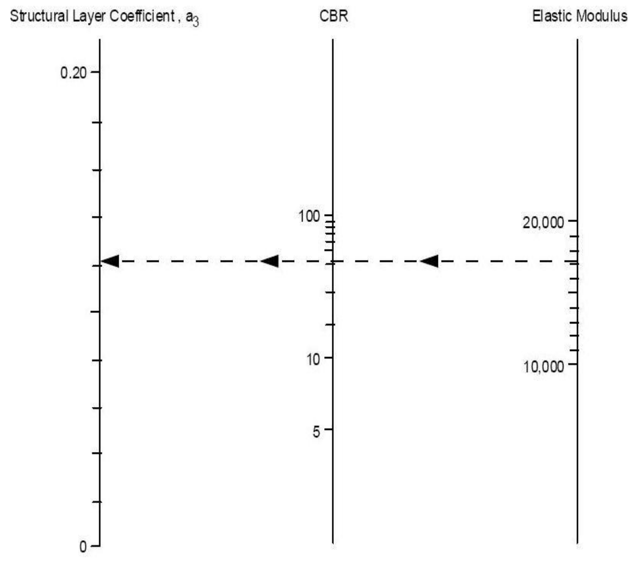
$$a_2 = 0.249(\log_{10} M_r) - 0.977$$

$$a_3 = 0.227(\log_{10} M_r) - 0.839$$

b) Chart ;

Layer Coefficient of Granular Subbase

Granular subbase CBR (%)	Structural layer coefficient, a_3
10	0.08
25	0.10
30	0.11
40	0.12
70	0.13
100	0.14



M_r; Resilience Modulus (psi) or Dynamic Elastic Modulus

Is a dynamic test response defined the ratio of the repeated axial deviate stress σ_d to the recoverable axial strain ϵ_d .($M_r = \sigma_d / \epsilon_d$)

$$M_r = 1500 * CBR\%$$

Min thickness required according to Iraqi specification (SORB) for construction or maintenance considerations are

1) Surface layer = 2 in (5 cm).
2) Base layer = 4 in(10 cm).
3) Subabse layer = 4 in (10 cm).

Example:

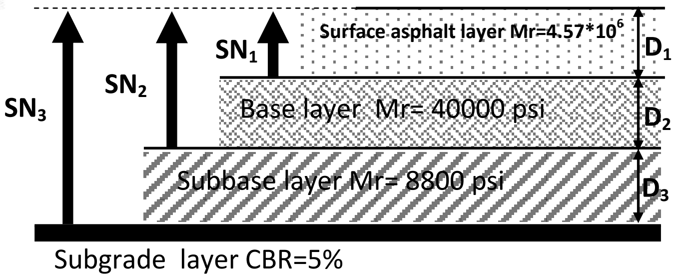
Design a flexible pavement for the following conditions;

$W_{18}=1.46*10^6$ ESAL within 20 year design period, subgrade CBR=5 %, M_r(granular subase)=E_{sb}=8800 psi , M_r(granular base) =E_{bs}=40000 psi, M_r(asphaltic concrete layer)=$E_{AC}=4.57*10^5$ psi, P_o=4.2 , P_t=2.5 , m_2=1 , m_3=0.8 , R%=90 % and S_o=0.45.

Solution;

$$\log_{10} W_{18} = -ZS_o + 9.36\log_{10}(SN+1) - 0.20 + \frac{\log_{10}\left[\frac{\Delta PSI_w}{PSI_o - 1.5}\right]}{0.4 + \frac{1094}{(SN+1)^{5.19}}} + 2.32\log_{10} M_r - 8.07$$

$SN_{total} = a_1D_1 m_1 + a_2D_2m_2 + a_3D_3 m_3$



from above figures;

a_1=0.42

a_2=0.13

a3=0.095

$\Delta PSI = P_o - P_t$= 4.2-2.5 =1.7

Using AASHTO design chart;

a) Determine structural Number
   1) Determine SN_3=3.7
   2) Determine SN_2=3.4
   3) Determine SN_1=1.2

b) Determine thickness for each layer;
   1) Calculate D_1 from $SN_1=a_1*D_1$; $1.2=0.42*D_1$ so D_1= 4.286 in ≈ 10.886 cm ≈ 11 cm > 5 cm (2 in)
      Check with min thickness required (SORB, surface layer = 2" or 5 cm) ok.
   2) Calculate D_2 from
      $SN_2 = a_1D_1 m_1+ a_2D_2m_2$; $3.4=0.42*(11/2.54)+0.13*D_2*1$; D_2 = 12.16 in ≈ 30.892 cm ≈31 cm
      Check with min thickness required (SORB, base layer= 4" or 10 cm) ok.
   3) Calculate D_3 from
      $SN_2 = a_1D_1 m_1+ a_2D_2m_2+a_3D_3m_3$; $3.7=0.42*(11/2.54)+0.13*1*(31/2.54)+0.095*0.8*D_3$
      so; D_3= 3.875 in ≈ 9.842 cm ≈ 10 cm.
      Check with min thickness required (SORB, Subbase layer= 4" or 10 cm) ok.