Example-2: Testing a heteroassociative net using the training input

We now **test the ability of the net to produce the correct output** for each of the training inputs using the activation function

$$f(x) = \begin{cases} 1 & \text{if} \quad x > 0 \\ 0 & \text{if} \quad x \le 0 \end{cases}$$

The weights are as found in Example 1.

$$W = \begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 2 \end{bmatrix}$$

Sol:

1- For p = 1 ; x = [1, 0, 0, 0]

$$net_1 = x_1w_{11} + x_2w_{21} + x_3w_{31} + x_4w_{41}$$

$$= 1(2) + 0(1) + 0(0) + 0(0) = 2$$

$$net_2 = x_1w_{12} + x_2w_{22} + x_3w_{32} + x_4w_{42}$$

$$= 1(0) + 0(0) + 0(1) + 0(2) = 0$$

$$y_1 = f(net_1) = f(2) = 1$$

$$y_2 = f(net_2) = f(0) = 0$$

This is the correct response for the first training pattern.

2- For p = 2 ; x = [1, 1, 0, 0]

$$net_1 = x_1w_{11} + x_2w_{21} + x_3w_{31} + x_4w_{41}$$

$$= 1(2) + 1(1) + 0(0) + 0(0) = 3$$

$$net_2 = x_1w_{12} + x_2w_{22} + x_3w_{32} + x_4w_{42}$$

$$= 1(0) + 1(0) + 0(1) + 0(2) = 0$$

$$y_1 = f(net_1) = f(3) = 1$$

$$y_2 = f(net_2) = f(0) = 0$$

This is the correct response for the second training pattern.

3- For p = 3 ;   x = [0, 0, 0, 1]

$$net_1 = x_1w_{11} + x_2w_{21} + x_3w_{31} + x_4w_{41}$$

$$= 0(2) + 0(1) + 0(0) + 1(0) = 0$$

$$net_2 = x_1w_{12} + x_2w_{22} + x_3w_{32} + x_4w_{42}$$

$$= 0(0) + 0(0) + 0(1) + 1(2) = 2$$

$$y_1 = f(net_1) = f(0) =0$$

$$y_2 = f(net_2) = f(2) = 1$$

This is the correct response for the third training pattern.

4- For p = 4 ;   x = [0, 0, 1, 1]

$$net_1 = x_1w_{11} + x_2w_{21} + x_3w_{31} + x_4w_{41}$$

$$= 0(2) + 0(1) + 1(0) + 1(0) = 0$$

$$net_2 = x_1w_{12} + x_2w_{22} + x_3w_{32} + x_4w_{42}$$

$$= 0(0) + 0(0) + 1(1) + 1(2) = 3$$

$$y_1 = f(net_1) = f(0) = 0$$

$$y_2 = f(net_2) = f(3) = 1$$

This is the correct response for the fourth training pattern.

The process we have just illustrated can be represented using vector-matrix notation. Note first, that the net input to any particular output unit is the (dot) product of the input (row) vector with the column of the weight matrix that has the weights for the output unit in question.

$$\mathbf{xW} = [net_1 \quad\quad net_2] \quad\quad \rightarrow \mathbf{y} = [y_1 \quad y_2]$$

$$[1 \quad 0 \quad 0 \quad 0]\begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 2 \end{bmatrix} = [\,2 \quad 0] \rightarrow [1 \quad 0]$$

or:

$$[1 \quad 0 \quad 0 \quad 0]\,.\,W = [\,2 \quad 0] \rightarrow [1 \quad 0]$$

$$[1 \quad 1 \quad 0 \quad 0]\,.\,W = [\,3 \quad 0] \rightarrow [1 \quad 0]$$

$$[0 \quad 0 \quad 0 \quad 1]\,.\,W = [\,0 \quad 2] \rightarrow [0 \quad 1]$$

$$[0 \quad 0 \quad 1 \quad 1]\,.\,W = [\,0 \quad 3] \rightarrow [0 \quad 1]$$

Note that the net has responded correctly to (has produced the desired vector of output activations for) each of the training patterns.

Example-3: Testing a heteroassociative net with input similar to the training input

The test vector x = (0, 1, 0, 0) differs from the training vector **s** = (1, 1, 0,0) only in the first component. We have Thus, the net also associates a known output pattern with this input.

$$[0 \quad 1 \quad 0 \quad 0]\,.\,W = [\,1 \quad 0] \rightarrow [1 \quad 0]$$

Example-4: Testing a heteroassociative net with input that is not similar to the training input

The test pattern (0 1, 1, 0) differs from each of the training input patterns in at least two components. We have

$$[0 \quad 1 \quad 1 \quad 0]\,.\,W = [\,1 \quad 1] \rightarrow [1 \quad 1]$$

The output is not one of the outputs with which the net was trained; in other words, the net does not recognize the pattern. In this case, we can view **x** = (0, 1, 1, **0)** as differing from the training vector **s** = (1, 1, 0, 0) in the first and third components, so that the two "mistakes" in the input pattern make it impossible for the net to recognize it. This is not surprising, since the vector could equally well be viewed as formed from **s** = (0, 0, 1, I), with "mistakes" in the second and fourth components.

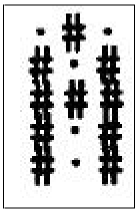
In general, a bipolar representation of our patterns is computationally preferable to a binary representation.

- One of the computational advantages of bipolar representation is that it gives us a very simple way of **expressing two different levels of noise** that may be applied to our training inputs to produce testing inputs for our net.
- We shall refer informally to these levels as **"missing data"** and **"mistakes".**
- If each of our original patterns is a sequence of ***yes*** or ***no*** responses, **"missing data"** would correspond to a response of ***unsure.*** whereas a **"mistake"** would be a response of ***yes*** when the correct response was ***no*** and vice versa. With bipolar representations, ***yes*** would be represented by **+1**, ***no*** by **-1,** and ***unsure*** by **0.**

Character recognition

Example-5: A heteroassociative net for associating letters from different fonts

A heteroassociative neural net was trained using the Hebb rule (outer products) to associate three vector pairs. The **x** vectors have 63 components, the y vectors 15. The vectors represent two-dimensional patterns. The pattern



is converted to a vector representation that is suitable for processing as follows: The #s are replaced by 1's and the dots by - l's, reading across each row (starting with the top row). The pattern shown becomes the vector

(-1, 1, -1,   1, -1, 1,   1, 1, 1,   1, -1, 1,   1, -1, 1)

Figure 2 shows the vector pairs in their original two-dimensional form.

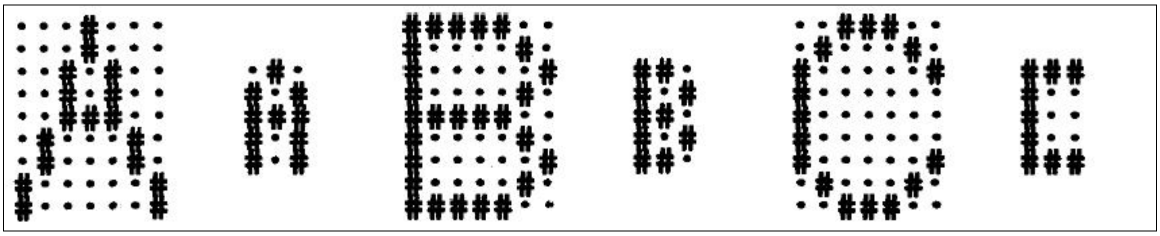


Fig.2 Training patterns for character recognition using heteroassociative net.

After training, the net was used with input patterns that were noisy versions of the training input patterns. The results are shown in Figures 3 and 4.

The noise took the form of turning pixels **"on"** that should have been **"off"** and vice versa. These are denoted as follows:

@ Pixel is now "on," but this is a mistake (noise).

0 Pixel is now "off," but this is a mistake (noise).

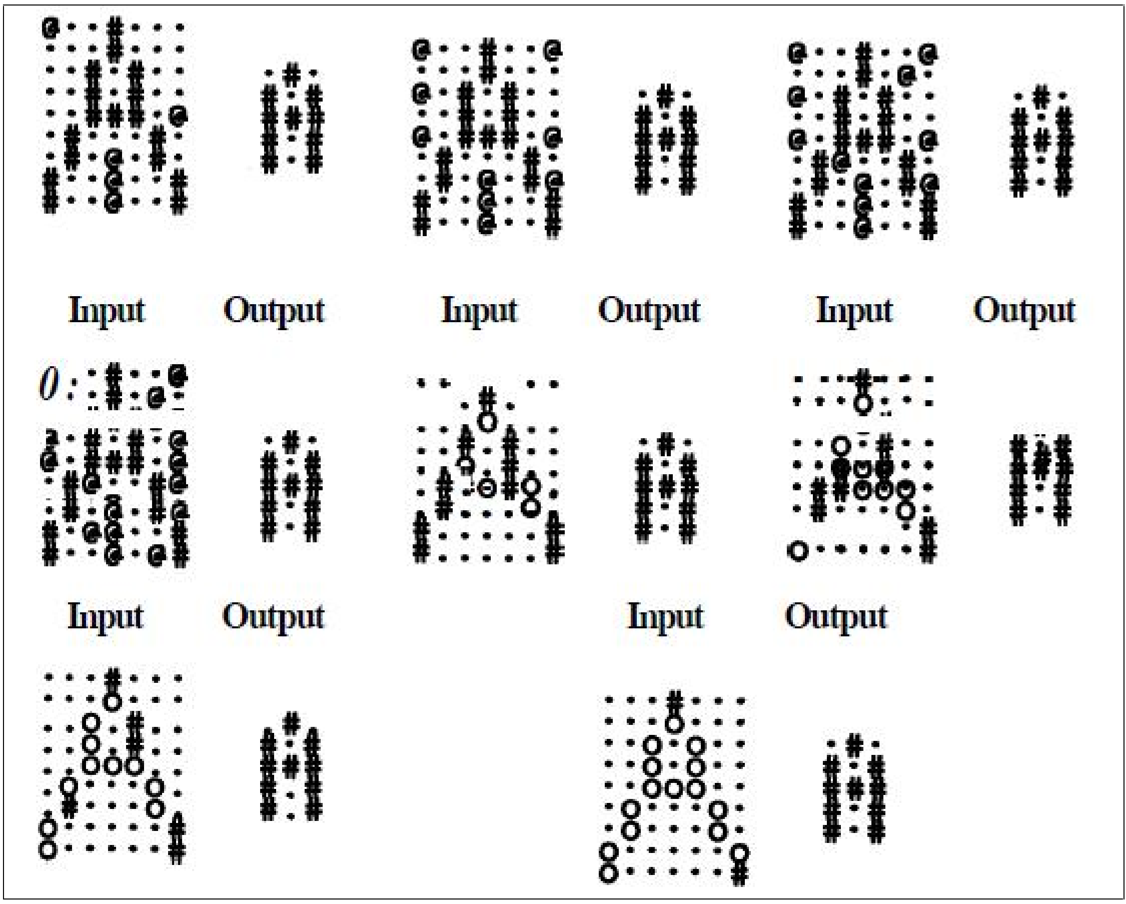


Figure 3 Response of heteroassociative net to several noisy versions of pattern A.

Figure 4 shows that the neural net can recognize the small letters that are stored in it, even when given input patterns representing the large training patterns with 30% noise.

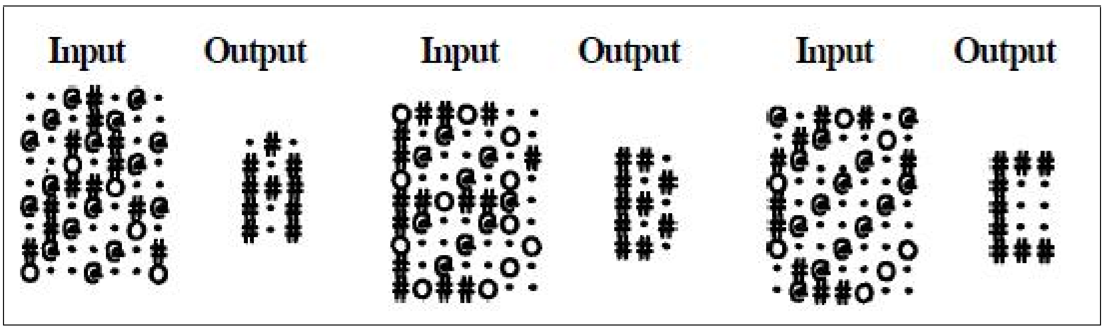


Figure 4 Response of heteroassociative net to patterns A, B, and C with mistakes in *1/3* of the components.