Karnaugh Map (K-Map)

The Karnaugh map consists of one square for each possible minterm in a function. Thus, a two-variable map has 4 squares, a three-variable map has 8 squares, and a four-variable map has 16 squares.

Three views of the two-variable map are shown below. In each, the upper right square, for example, corresponds to $A=1$ and $B=0$, minterm 2.

$\bar{A}\bar{B}$	$A\bar{B}$
$\bar{A}B$	AB

		$\overbrace{\quad}^{A}$
	m_0	m_2
B	m_1	m_3

B \ A	0	1
0	0	2
1	1	3

When we plot a function, we put a 1 in each square corresponding to a minterm that is included in the function, and put a **0** in or leave blank those squares not included in the function. For functions with don't cares, an **X** goes in the square for which the minterm is a don't care.

Ex:

B \ A	0	1
0	1	
1		1

$f(a,b) = \sum m(0,3)$

B \ A	0	1
0	1	X
1		1

$g(A,B) = \sum m(0,3) + \sum d(2)$

Three-variable maps have 8 squares, arranged in a rectangle as shown below:

AB / C	00	01	11	10
0	0	2	6	4
1	1	3	7	5

		$\bar{A}$	$\bar{A}$, B	A, B	A
AB / C		$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$	0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
C	1	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

$m_0 + m_1 : \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C = \bar{A}\bar{B}$

$m_4 + m_6 : A\bar{B}\bar{C} + AB\bar{C} = A\bar{C}$

$m_7 + m_5 : ABC + A\bar{B}C = AC$

$m_0 + m_4 : \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} = \bar{B}\bar{C}$

$m_1 + m_5 : \bar{A}\bar{B}C + A\bar{B}C = \bar{B}C$

AB / C	00	01	11	10
0	1			
1	1			

$\bar{A}\bar{B}$

AB / C	00	01	11	10
0			1	1
1				

$A\bar{C}$

AB / C	00	01	11	10
0				
1			1	1

AC

AB / C	00	01	11	10
0	1			1
1				

$\bar{B}\bar{C}$

AB / C	00	01	11	10
0				
1	1			1

$\bar{B}C$

It is sometimes more convenient to draw the map in a vertical orientation as shown below. Both versions of the map produce the same results.

BC \ A	0	1
00	0	4
01	1	5
11	3	7
10	2	6

BC \ A	0	1
00	1	
01	1	
11		
10		

$\bar{A}\bar{B}$

BC \ A	0	1
00		
01		1
11		1
10		

AC

BC \ A	0	1
00		1
01		
11		
10		1

$A\bar{C}$

The four-variable map consists of 16 squares in the 4 by 4 arrangement shown below.

As with the three-variable map, 1's in two adjacent squares correspond to a single product term.

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

	CD \ AB	00 ($\bar{A}$)	01 ($\bar{A}$)	11 (A)	10 (A)	
$\bar{C}$	00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}B\bar{C}\bar{D}$	$AB\bar{C}\bar{D}$	$A\bar{B}\bar{C}\bar{D}$	$\bar{D}$
$\bar{C}$	01	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}B\bar{C}D$	$AB\bar{C}D$	$A\bar{B}\bar{C}D$	D
C	11	$\bar{A}\bar{B}CD$	$\bar{A}BCD$	$ABCD$	$A\bar{B}CD$	D
C	10	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}BC\bar{D}$	$ABC\bar{D}$	$A\bar{B}C\bar{D}$	$\bar{D}$
		$\bar{B}$	B	B	$\bar{B}$	

Example: Read the following maps.

CD \ AB	00	01	11	10
00	1			
01	1		1	1
11	1		1	1
10	1			

$\bar{A}\bar{B} + AD$

CD \ AB	00	01	11	10
00				
01	1			1
11	1			1
10				

$\bar{B}D$

CD \ AB	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1			1

$\bar{B}\bar{D} + BD$

CD \ AB	00	01	11	10
00	1	1		
01	1	1		1
11	1	1	1	
10	1	1		

$\bar{A} + BCD + \bar{B}\bar{C}D$

CD \ AB	00	01	11	10
00	1	1	1	1
01				
11			1	
10	1	1	1	1

$\bar{D} + ABC$

example: Simplify the Boolean functions $F_1(A,B,C) = \sum(0,2,4,5,6)$

$F_2(A,B,C) = \bar{A}C + \bar{A}B + A\bar{B}C + B$

C \ AB	00	01	11	10
0	1	1	1	1
1	0	0	0	1

$F_1 = \bar{C} + A\bar{B}$

C \ AB	00	01	11	10
0		1	0	
1	1	1	1	1

$F_2 = \bar{A}B + C$

Don't Cares

Finding minimum solutions for functions with don't cares does not significantly change the method we developed before. We need to modify slightly the definitions of a prime implicant and clarify the definition of an essential prime implicant.

A Prime Implicant: is a rectangle of 1, 2, 4, 8, ... 1's or X's not included in any one larger rectangle. Thus, from the point of view of finding prime implicants, X's (don't cares) are treated as 1's.

An Essential Prime Implicant: is a prime implicant that cover at least one 1 ~~that~~ not covered by any other prime implicant (as always). Don't cares (X's) do not make a prime implicant essential.

Ex: $F(A,B,C,D) = \sum m(1,7,10,11,13) + \sum d(5,8,15)$

Find the minimum solution using Karnaugh Map.

CD \ AB	00	01	11	10
00				X
01	1	X	1	
11		1	X	1
10				1

CD \ AB	00	01	11	10
00				X
01	1	X	1	
11		1	X	1
10				1

CD \ AB	00	01	11	10
00				X
01	1	X	1	
11		1	X	1
10				1

$F = BD + \bar{A}\bar{C}D + A\bar{B}C$ //

$F = BD + \bar{A}\bar{C}D + A\bar{B}C + A\bar{B}\bar{D}$ OR

$F = BD + \bar{A}\bar{C}D + ACD + A\bar{B}\bar{D}$

Product of Sums

Finding a minimum Product of sums expression requires no new theory. The following approach is the simplest:

1. Map the complement of the function. (if there is already a map for the function, replace all 0's by 1's, all 1's by 0's and leave X's unchanged.)
2. Find the minimum sum of Products (SOP) expression for the complement of the function.
3. Use De Morgan's theorem (P11) to complement that expression, Producing a product of sums (POS) expression.

ex: $f(a,b,c,d) = \sum m(0,1,4,5,10,11,14)$

Soln $\bar{f}(a,b,c,d) = \sum m(2,3,6,7,8,9,12,13,15)$

cd \ ab	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

There are two equally good Solutions for the sum of products for $\bar{f}$:

$$* \quad \bar{f} = a\bar{c} + \bar{a}c + bcd$$

$$** \quad \bar{f} = a\bar{c} + \bar{a}c + abd$$

We can then complement the solutions for $\bar{f}$ to get the two minimum product of sums solutions for f:

$$* \quad f = (\bar{a}+c)(a+\bar{c})(\bar{b}+\bar{c}+\bar{d})$$

$$** \quad f = (\bar{a}+c)(a+\bar{c})(\bar{a}+\bar{b}+\bar{d})$$

Five-Variable Karnaugh Map

The five-variable map is shown below. It consists of 2 four-variable maps with variables A, B, C, D, and E. This means a five-variable map needs 32 (2^5) squares.

Variable A distinguishes between the two maps, as indicated on the top of the diagram. The left-hand four-variable map represents the 16 squares where $A=0$, and the other four-variable map represents the squares where $A=1$. Minterms 0 through 15 belong with $A=0$ and minterms 16 through 31 with $A=1$. Each square in the $A=0$ map is adjacent to the corresponding square in the $A=1$ map. For example, minterm 4 is adjacent to minterm 20 and minterm 15 to 31.

$A=0$

BC \ DE	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$A=1$

BC \ DE	00	01	11	10
00	16	17	19	18
01	20	21	23	22
11	28	29	31	30
10	24	25	27	26