Number Systems

A decimal number such as 7392 represents a quantity equal to 7 thousands plus 3 hundreds, plus 9 tens, plus 2 units. The thousands, hundreds, etc. are powers of 10 implied by the position of the coefficients. To be more exact, 7392 should be written as $7\times10^3 + 3\times10^2 + 9\times10^1 + 2\times10^0$

In general, a number with a decimal point is represented by a series of coefficients as follows:

$$a_5\ a_4\ a_3\ a_2\ a_1\ a_0\ .\ a_{-1}\ a_{-2}\ a_{-3}$$

The a_j coefficients are any of the 10 digits (0, 1, To distinguish between numbers of different bases, we enclose the coefficients in parentheses and write a subscript equal to the base used.

$$(4021.2)_5 = 4\times5^3 + 0\times5^2 + 2\times5^1 + 1\times5^0 + 2\times5^{-1} = (511.4)_{10}$$

The coefficient values for base 5 can be only 0, 1, 2, 3, and 4. The <u>octal</u> number system is a base-8 system that has eight digits: 0, 1, 2, 3, 4, 5, 6, 7. An example

$$(127.4)_8 = 1\times8^2 + 2\times8^1 + 7\times8^0 + 4\times8^{-1} = (87.5)_{10}$$

In the hexadecimal (base 16) number system, the first ten digits are borrowed from the decimal system. The letters A, B, C, D, E, and F are used for digits 10, 11, 12, 13, 14, and 15, respectively. An example of a hexadecimal number is

$$(B65F)_{16} = 11\times16^3 + 6\times16^2 + 5\times16^1 + 15\times16^0 = (46687)_{10}$$

In computer work, 2^{10} is reffered to as K (kilo), 2^{20} as M (mega), 2^{30} as G (giga), and 2^{40} as T (tera). Thus $4K = 2^{12} = 4096$ and $16M = 2^{24} = 16\,777\,216$.

It would save a great deal of time and effort it at least the first ten powers of 2 were memorized;

The first 20 are shown in the table below

n	2^n	n	2^n
1	2	11	2048
2	4	12	4096
3	8	13	8192
4	16	14	16384
5	32	15	32768
6	64	16	65536
7	128	17	131072
8	256	18	262144
9	512	19	524288
10	1024	20	1048576

Note that the number one less than 2^n consists of n 1's (For example, $2^4 - 1 = 1111 = 15$ and $2^5 - 1 = 11111 = 31$).

An n-bit number can represent the positive integers from 0 to $2^n - 1$. Thus, for example, 4-bit numbers have the range of 0 to 15, 8-bit numbers 0 to 255 and 16-bit numbers 0 to 65535.

Number Base Conversions

We present a general procedure for converting a decimal number to a number in base r. If the number includes a radix point, it is necessary to seperate the number into an integer part and a fraction part, since each part must be converted differently.

The conversion of a decimal integer to a number in base r is done by dividing the number and all successive quotients حواصل القسمة by r and accumulating the remainders.

Example: convert decimal 41 to binary.

Integer	Remainder	
41		
20	1	a_0
10	0	a_1
5	0	a_2
2	1	
1	0	
0	1	a_5

101001 = answer

The conversion from decimal to any base-r system is similar to this example, except that division is done by r instead of 2.

Ex: Convert decimal 746 to octal

$746/8 = 93$	rem 2	Produces	2
$93/8 = 11$	rem 5		52
$11/8 = 1$	rem 3		352
$1/8 = 0$	rem 1		$(1352)_8$ = answer

Ex: Convert $(0.6875)_{10}$ to binary.

	integer		Fraction	Coefficient
$0.6875 \times 2 =$	1	+	0.375	$a_{-1} = 1$
$0.375 \times 2 =$	0	+	0.75	$a_{-2} = 0$
$0.75 \times 2 =$	1	+	0.5	$a_{-3} = 1$
$0.5 \times 2 =$	1	+	0.0	$a_{-4} = 1$

$$(0.6875)_{10} = (0.1011)_2 = (0.a_{-1}\,a_{-2}\,a_{-3}\,a_{-4})_2$$

First 0.6875 is multiplied by 2 to give an integer and a fraction. The new fraction is multiplied by 2 to give a new integer and a new fraction. This process continued until the fraction becomes 0 or until the number of digits have sufficient accuracy.

To convert a decimal fraction to a number expressed in base r, a similar procedure is used. Multiplication is by r instead of 2, and the coefficients found from the integers may range in value from 0 to r−1 instead of 0 and 1.

Ex: Convert $(0.513)_{10}$ to octal.

$0.513 \times 8 = 4.104$ $\quad a_{-1} = 4$

$0.104 \times 8 = 0.832$ $\quad a_{-2} = 0$

$0.832 \times 8 = 6.656$ $\quad a_{-3} = 6$

$0.656 \times 8 = 5.248$ $\quad a_{-4} = 5$

$0.248 \times 8 = 1.984$ $\quad a_{-5} = 1$

$0.984 \times 8 = 7.872$ $\quad a_{-6} = 7$

The answer, to seven significant figures, is obtained from the integer part of the products

$$(0.513)_{10} = (0.406517\,---)_8$$

Octal and Hexadecimal Numbers

Since $2^3 = 8$ and $2^4 = 16$, each octal digit corresponds to three binary digits and each hexadecimal digit corresponds to four binary digits.

The conversion from binary to octal is easily accomplished by partitioning the binary number into groups of three digits each, starting from the binary point and proceeding to the left and to the right. The corresponding octal digit is then assigned to each group.

$$(\underbrace{10}_{2}\ \underbrace{110}_{6}\ \underbrace{001}_{1}\ \underbrace{101}_{5}\ \underbrace{011}_{3}\ .\ \underbrace{111}_{7}\ \underbrace{100}_{4}\ \underbrace{000}_{0}\ \underbrace{110}_{6})_2 = (26153.7406)_8$$

Conversion from binary to hexadecimal is similar, except the binary number is divided into groups of **four** digits:

$$(\underbrace{10}_{2}\ \underbrace{1100}_{C}\ \underbrace{0110}_{6}\ \underbrace{1011}_{B}\ .\ \underbrace{1111}_{F}\ \underbrace{0010}_{2})_2 = (2C6B.F2)_{16}$$

Decimal (base 10)	Binary (base 2)	octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

conversion from octal or hexadecimal to binary is done by reversing the preceding procedure. Each octal digit is converted to its three-digit binary equivalent. Similarly, each hexadecimal digit is converted to its four-digit binary equivalent.

$$(673.124)_8 = (\underbrace{110}_{6}\ \underbrace{111}_{7}\ \underbrace{011}_{3} . \underbrace{001}_{1}\ \underbrace{010}_{2}\ \underbrace{100}_{4})_2$$

$$(306.D)_{16} = (\underbrace{0011}_{3}\ \underbrace{0000}_{0}\ \underbrace{0110}_{6} . \underbrace{1101}_{D})_2$$

Complements

Complement are used in digital computers for simplifying the subtraction operation and for logical manipulation. There are two types of complements for each base-r system:

1) Radix Complement: is referred to as r's complement.

2) Diminished radix complement: as (r-1)'s complement.

When the value of the base **r** is substituted in the name, the two types are referred to as the **2's** complement and **1's** complement for binary numbers, and the **10's** complement and **9's** complement for decimal numbers.

Diminished Radix Complement

Given a number N in base r having n digits, the $(r-1)$'s Complement of N is defined as $(r^n-1)-N$.

The 9's complement of N is $(10^n-1)-N$. In this case, 10^n represents a number that consists of a single 1 followed by n 0s. (10^n-1) is a number represented by n 9s. For example, if $n=4$, we have $10^4=10000$ and $10^4-1=9999$. It follows that the 9's complement of a decimal number is obtained by <u>subtracting each digit from 9</u>.

The 9's complement of 546700 is 999999 − 546700 = 453299

The 9's complement of 012398 is 999999 − 012398 = 987601

For binary numbers, the 1's Complement is obtained by subtracting each digit (bit) from 1. This means the 1's Complement of a binary number is formed by changing 1's to 0's and 0's to 1's.

The 1's complement of 10110000 is 01001111

The 1's complement of 0101101 is 1010010

The $(r-1)$'s complement of octal or hexadecimal numbers is obtained by subtracting each digit from 7 or F (decimal 15), respectively.

Radix Complement

The r's complement is obtained by adding 1 to the (r-1)'s complement. Thus the 10's complement of decimal 2389 is 7610 + 1 = 7611, and is obtained by adding 1 to the 9's complement value.

The 2's complement of binary 101100 is 010011+1
= 010100
and is obtained by adding 1 to the 1's-complement value.

ex: Find the 2's complement for these binary numbers
1101100 and 0110111

The 2's complement of the first number will be obtained by leaving the two least significant 0's and the first 1 unchanged, and then replacing 1's with 0's and 0's with 1's in the other four most significant digits, the result will be 0010100

The 2's complement of the second number will be obtained by leaving the least significant 1 unchanged and complementing all other digits, the result will be 1001001

Worth mentioning that the complement of the complement **restores** the number to its original value.