CHAPTER

8

Radiation Heat Transfer

8-1 | INTRODUCTION

Preceding chapters have shown how conduction and convection heat transfer may be calculated with the aid of both mathematical analysis and empirical data. We now wish to consider the third mode of heat transfer—thermal radiation. *Thermal radiation* is that electromagnetic radiation emitted by a body as a result of its temperature. In this chapter, we shall first describe the nature of thermal radiation, its characteristics, and the properties that are used to describe materials insofar as the radiation is concerned. Next, the transfer of radiation through space will be considered. Finally, the overall problem of heat transfer by thermal radiation will be analyzed, including the influence of the material properties and the geometric arrangement of the bodies on the total energy that may be exchanged.

8-2 | PHYSICAL MECHANISM

There are many types of electromagnetic radiation; thermal radiation is only one. Regardless of the type of radiation, we say that it is propagated at the speed of light, 3×10^8 m/s. This speed is equal to the product of the wavelength and frequency of the radiation,

$$c = \lambda \nu$$

where

c = speed of light
λ = wavelength
ν = frequency

The unit for λ may be centimeters, angstroms ($1\ \text{Å} = 10^{-8}$ cm), or micrometers ($1\ \mu\text{m} = 10^{-6}$ m). A portion of the electromagnetic spectrum is shown in Figure 8-1. Thermal radiation lies in the range from about 0.1 to 100 μm, while the visible-light portion of the spectrum is very narrow, extending from about 0.35 to 0.75 μm.

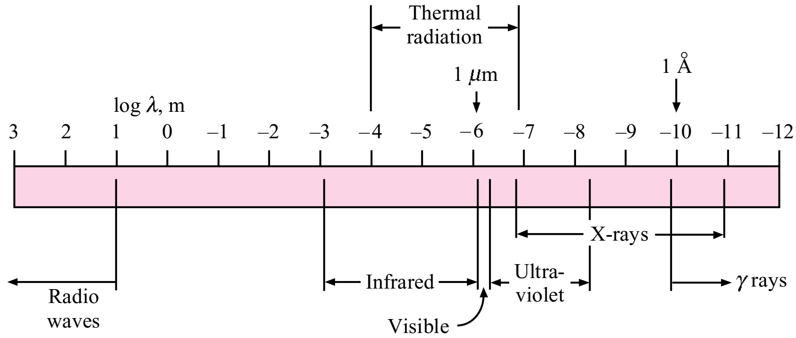
The propagation of thermal radiation takes place in the form of discrete quanta, each quantum having an energy of

$$E = h\nu \qquad \textbf{[8-1]}$$

where h is Planck's constant and has the value

$$h = 6.625 \times 10^{-34}\ \text{J} \cdot \text{s}$$

Figure 8-1 | Electromagnetic spectrum.



A very rough physical picture of the radiation propagation may be obtained by considering each quantum as a particle having energy, mass, and momentum, just as we considered the molecules of a gas. So, in a sense, the radiation might be thought of as a "photon gas" that may flow from one place to another. Using the relativistic relation between mass and energy, expressions for the mass and momentum of the "particles" could thus be derived; namely,

$$E = mc^2 = h\nu$$

$$m = \frac{h\nu}{c^2}$$

$$\text{Momentum} = c\frac{h\nu}{c^2} = \frac{h\nu}{c}$$

By considering the radiation as such a gas, the principles of quantum-statistical thermodynamics can be applied to derive an expression for the energy density of radiation per unit volume and per unit wavelength as[†]

$$u_\lambda = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1} \tag{8-2}$$

where k is Boltzmann's constant, 1.38066×10^{-23} J/molecule · K. When the energy density is integrated over all wavelengths, the total energy emitted is proportional to absolute temperature to the fourth power:

$$E_b = \sigma T^4 \tag{8-3}$$

Equation (8-3) is called the Stefan-Boltzmann law, E_b is the energy radiated per unit time and per unit area by the ideal radiator, and σ is the Stefan-Boltzmann constant, which has the value

$$\sigma = 5.669 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \quad [0.1714 \times 10^{-8}\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{R}^4]$$

where E_b is in watts per square meter and T is in degrees Kelvin. In the thermodynamic analysis the energy density is related to the energy radiated from a surface per unit time and per unit area. Thus the heated interior surface of an enclosure produces a certain energy density of thermal radiation in the enclosure. We are interested in radiant exchange with surfaces—hence the reason for the expression of radiation from a surface in terms of its temperature. The subscript b in Equation (8-3) denotes that this is the radiation from a blackbody. We call this *blackbody radiation* because materials that obey this law appear black to the eye; they appear black because they do not reflect any radiation. Thus a blackbody is

[†] See, for example, J. P. Holman, *Thermodynamics*, 4th ed. New York: McGraw-Hill, 1988, p. 350.

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also considered as one that absorbs all radiation incident upon it. E_b is called the *emissive power* of a blackbody.

It is important to note at this point that the "blackness" of a surface to thermal radiation can be quite deceiving insofar as visual observations are concerned. A surface coated with lampblack appears black to the eye and turns out to be black for the thermal-radiation spectrum. On the other hand, snow and ice appear quite bright to the eye but are essentially "black" for long-wavelength thermal radiation. Many white paints are also essentially black for long-wavelength radiation. This point will be discussed further in later sections.

8-3 | RADIATION PROPERTIES

When radiant energy strikes a material surface, part of the radiation is reflected, part is absorbed, and part is transmitted, as shown in Figure 8-2. We define the reflectivity ρ as the fraction reflected, the absorptivity α as the fraction absorbed, and the transmissivity τ as the fraction transmitted. Thus

$$\rho + \alpha + \tau = 1 \tag{8-4}$$

Most solid bodies do not transmit thermal radiation, so that for many applied problems the transmissivity may be taken as zero. Then

$$\rho + \alpha = 1$$

Two types of reflection phenomena may be observed when radiation strikes a surface. If the angle of incidence is equal to the angle of reflection, the reflection is called *specular*. On the other hand, when an incident beam is distributed uniformly in all directions after reflection, the reflection is called *diffuse*. These two types of reflection are depicted

Figure 8-2 | Sketch showing effects of incident radiation.

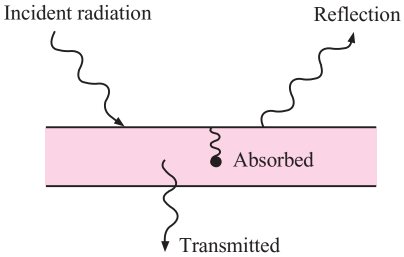


Figure 8-3 | (*a*) Specular ($\phi_1 = \phi_2$) and (*b*) diffuse reflection.

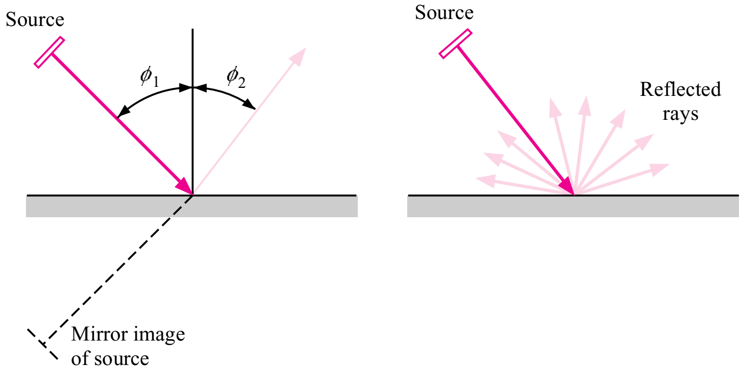
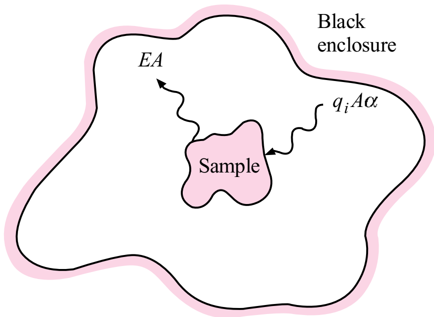


Figure 8-4 | Sketch showing model used for deriving Kirchhoff's law.



in Figure 8-3. Note that a specular reflection presents a mirror image of the source to the observer. No real surface is either specular or diffuse. An ordinary mirror is quite specular for visible light, but would not necessarily be specular over the entire wavelength range of thermal radiation. Ordinarily, a rough surface exhibits diffuse behavior better than a highly polished surface. Similarly, a polished surface is more specular than a rough surface. The influence of surface roughness on thermal-radiation properties of materials is a matter of serious concern and remains a subject for continuing research.

The emissive power of a body E is defined as the energy emitted by the body per unit area and per unit time. One may perform a thought experiment to establish a relation between the emissive power of a body and the material properties defined above. Assume that a perfectly black enclosure is available, i.e., one that absorbs all the incident radiation falling upon it, as shown schematically in Figure 8-4. This enclosure will also emit radiation according to the T^4 law. Let the radiant flux arriving at some area in the enclosure be q_i W/m^2. Now suppose that a body is placed inside the enclosure and allowed to come into temperature equilibrium with it. At equilibrium the energy absorbed by the body must be equal to the energy emitted; otherwise there would be an energy flow into or out of the body that would raise or lower its temperature. At equilibrium we may write

$$EA = q_i A\alpha \qquad \textbf{[8-5]}$$

If we now replace the body in the enclosure with a blackbody of the same size and shape and allow it to come to equilibrium with the enclosure *at the same temperature,*

$$E_b A = q_i A(1) \qquad \textbf{[8-6]}$$

since the absorptivity of a blackbody is unity. If Equation (8-5) is divided by Equation (8-6),

$$\frac{E}{E_b} = \alpha$$

and we find that the ratio of the emissive power of a body to the emissive power of a blackbody *at the same temperature* is equal to the absorptivity of the body. This ratio is defined as the *emissivity* ϵ of the body,

$$\epsilon = \frac{E}{E_b} \qquad \textbf{[8-7]}$$

so that

$$\epsilon = \alpha \qquad \textbf{[8-8]}$$

Equation (8-8) is called Kirchhoff's identity. At this point we note that the emissivities and absorptivities that have been discussed are the *total* properties of the particular material;

that is, they represent the integrated behavior of the material over all wavelengths. Real substances emit less radiation than ideal black surfaces as measured by the emissivity of the material. In reality, the emissivity of a material varies with temperature and the wavelength of the radiation.

The Gray Body

A *gray body* is defined such that the monochromatic emissivity ϵ_λ of the body is independent of wavelength. The *monochromatic emissivity* is defined as the ratio of the monochromatic emissive power of the body to the monochromatic emissive power of a blackbody at the same wavelength and temperature. Thus

$$\epsilon_\lambda = \frac{E_\lambda}{E_{b\lambda}}$$

The total emissivity of the body may be related to the monochromatic emissivity by noting that

$$E = \int_0^\infty \epsilon_\lambda E_{b\lambda}\, d\lambda \qquad \text{and} \qquad E_b = \int_0^\infty E_{b\lambda}\, d\lambda = \sigma T^4$$

so that

$$\epsilon = \frac{E}{E_b} = \frac{\int_0^\infty \epsilon_\lambda E_{b\lambda}\, d\lambda}{\sigma T^4} \qquad \textbf{[8-9]}$$

where $E_{b\lambda}$ is the emissive power of a blackbody per unit wavelength. If the gray-body condition is imposed, that is, ϵ_λ = constant, Equation (8-9) reduces to

$$\epsilon = \epsilon_\lambda \qquad \textbf{[8-10]}$$

The emissivities of various substances vary widely with wavelength, temperature, and surface condition. Some typical values of the total emissivity of various surfaces are given in Appendix A. We may note that the tabulated values are subject to considerable experimental uncertainty. A rather complete survey of radiation properties is given in Reference 14.

The functional relation for $E_{b\lambda}$ was derived by Planck by introducing the quantum concept for electromagnetic energy. The derivation is now usually performed by methods of statistical thermodynamics, and $E_{b\lambda}$ is shown to be related to the energy density of Equation (8-2) by

$$E_{b\lambda} = \frac{u_\lambda c}{4} \qquad \textbf{[8-11]}$$

or

$$E_{b\lambda} = \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1} \qquad \textbf{[8-12]}$$

where

λ = wavelength, μm

T = temperature, K

$C_1 = 3.743 \times 10^8\ \text{W} \cdot \mu\text{m}^4/\text{m}^2 \quad [1.187 \times 10^8\ \text{Btu} \cdot \mu\text{m}^4/\text{h} \cdot \text{ft}^2]$

$C_2 = 1.4387 \times 10^4\ \mu\text{m} \cdot \text{K} \quad [2.5896 \times 10^4\ \mu\text{m} \cdot {}^\circ\text{R}]$

A plot of $E_{b\lambda}$ as a function of temperature and wavelength is given in Figure 8-5*a*. Notice that the peak of the curve is shifted to the shorter wavelengths for the higher temperatures. These maximum points in the radiation curves are related by Wien's displacement law,

$$\lambda_{max} T = 2897.6\ \mu\text{m} \cdot \text{K} \quad [5215.6\ \mu\text{m} \cdot {}^\circ\text{R}] \qquad \textbf{[8-13]}$$

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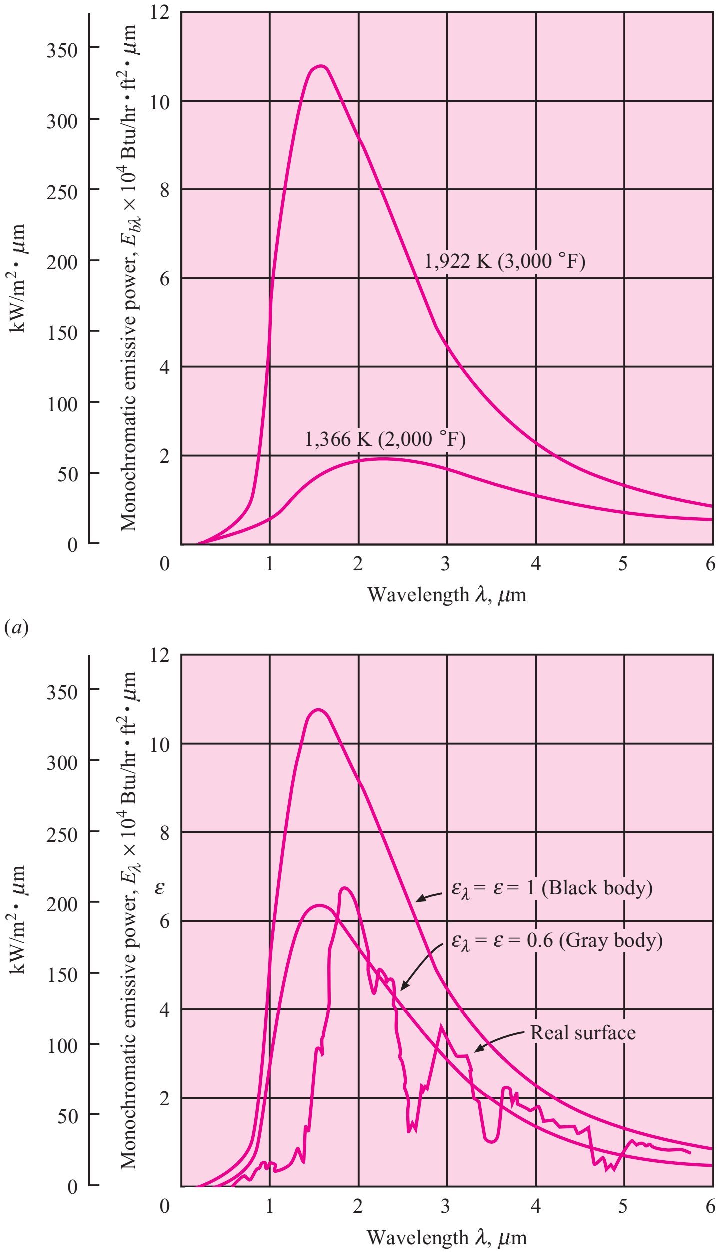
Figure 8-5 | (a) Blackbody emissive power as a function of wavelength and temperature; (b) comparison of emissive power of ideal blackbodies and gray bodies with that of a real surface.



(a)

(b)

Figure 8-5b indicates the relative radiation spectra from a blackbody at 3000°F and a corresponding ideal gray body with emissivity equal to 0.6. Also shown is a curve indicating an approximate behavior for a real surface, which may differ considerably from that of either an ideal blackbody or an ideal gray body. For analysis purposes surfaces are usually considered as gray bodies, with emissivities taken as the integrated average value.

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The shift in the maximum point of the radiation curve explains the change in color of a body as it is heated. Since the band of wavelengths visible to the eye lies between about 0.3 and 0.7 μm, only a very small portion of the radiant-energy spectrum at low temperatures is detected by the eye. As the body is heated, the maximum intensity is shifted to the shorter wavelengths, and the first visible sign of the increase in temperature of the body is a dark-red color. With further increase in temperature, the color appears as a bright red, then bright yellow, and finally white. The material also appears much brighter at higher temperatures because a larger portion of the total radiation falls within the visible range.

We are frequently interested in the amount of energy radiated from a blackbody in a certain specified wavelength range. The fraction of the total energy radiated between 0 and λ is given by

$$\frac{E_{b_{0-\lambda}}}{E_{b_{0-\infty}}} = \frac{\int_0^{\lambda} E_{b\lambda}\, d\lambda}{\int_0^{\infty} E_{b\lambda}\, d\lambda} \qquad \textbf{[8-14]}$$

Equation (8-12) may be rewritten by dividing both sides by T^5, so that

$$\frac{E_{b\lambda}}{T^5} = \frac{C_1}{(\lambda T)^5 (e^{C_2/\lambda T} - 1)} \qquad \textbf{[8-15]}$$

Now, for any specified temperature, the integrals in Equation (8-14) may be expressed in terms of the single variable λT. The ratio in Equation (8-14) is plotted in Figure 8-6 and tabulated in Table 8-1, along with the ratio in Equation (8-15). If the radiant energy emitted

Figure 8-6 | Fraction of blackbody radiation in wavelength interval.

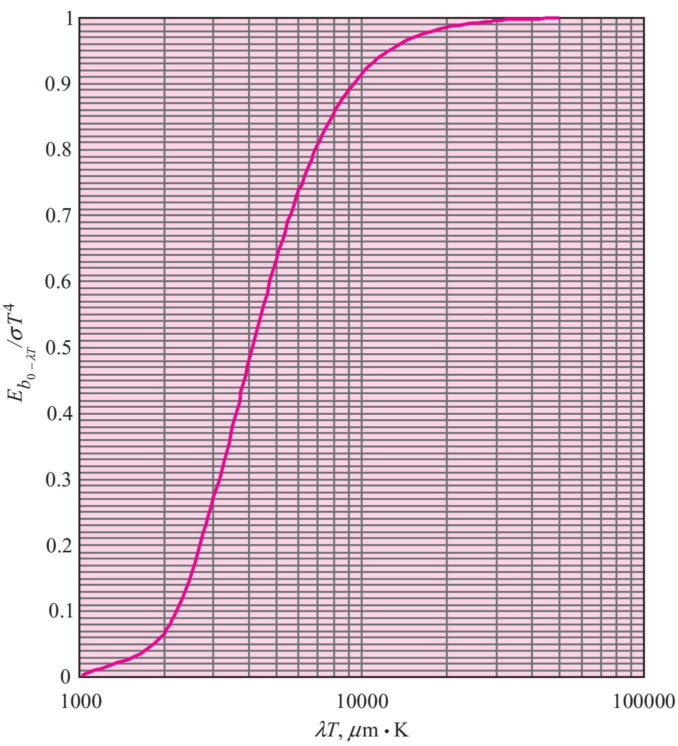

Table 8-1 | Radiation functions.

λT $\mu m \cdot K$	$E_{b\lambda}/T^5$ $\frac{W}{m^2 \cdot K^5 \cdot \mu m} \times 10^{11}$	$\frac{E_{b_{0-\lambda T}}}{\sigma T^4}$	λT $\mu m \cdot K$	$E_{b\lambda}/T^5$ $\frac{W}{m^2 \cdot K^5 \cdot \mu m} \times 10^{11}$	$\frac{E_{b_{0-\lambda T}}}{\sigma T^4}$
1000	0.02110	0.00032	6300	0.42760	0.76180
1100	0.04846	0.00091	6400	0.41128	0.76920
1200	0.09329	0.00213	6500	0.39564	0.77631
1300	0.15724	0.00432	6600	0.38066	0.78316
1400	0.23932	0.00779	6700	0.36631	0.78975
1500	0.33631	0.01285	6800	0.35256	0.79609
1600	0.44359	0.01972	6900	0.33940	0.80219
1700	0.55603	0.02853	7000	0.32679	0.80807
1800	0.66872	0.03934	7100	0.31471	0.81373
1900	0.77736	0.05210	7200	0.30315	0.81918
2000	0.87858	0.06672	7300	0.29207	0.82443
2100	0.96994	0.08305	7400	0.28146	0.82949
2200	1.04990	0.10088	7500	0.27129	0.83436
2300	1.11768	0.12002	7600	0.26155	0.83906
2400	1.17314	0.14025	7700	0.25221	0.84359
2500	1.21659	0.16135	7800	0.24326	0.84796
2600	1.24868	0.18311	7900	0.23468	0.85218
2700	1.27029	0.20535	8000	0.22646	0.85625
2800	1.28242	0.22788	8100	0.21857	0.86017
2900	1.28612	0.25055	8200	0.21101	0.86396
3000	1.28245	0.27322	8300	0.20375	0.86762
3100	1.27242	0.29576	8400	0.19679	0.87115
3200	1.25702	0.31809	8500	0.19011	0.87456
3300	1.23711	0.34009	8600	0.18370	0.87786
3400	1.21352	0.36172	8700	0.17755	0.88105
3500	1.18695	0.38290	8800	0.17164	0.88413
3600	1.15806	0.40359	8900	0.16596	0.88711
3700	1.12739	0.42375	9000	0.16051	0.88999
3800	1.09544	0.44336	9100	0.15527	0.89277
3900	1.06261	0.46240	9200	0.15024	0.89547
4000	1.02927	0.48085	9300	0.14540	0.89807
4100	0.99571	0.49872	9400	0.14075	0.90060
4200	0.96220	0.51599	9500	0.13627	0.90304
4300	0.92892	0.53267	9600	0.13197	0.90541
4400	0.89607	0.54877	9700	0.12783	0.90770
4500	0.86376	0.56429	9800	0.12384	0.90992
4600	0.83212	0.57925	9900	0.12001	0.91207
4700	0.80124	0.59366	10,000	0.11632	0.91415
4800	0.77117	0.60753	10,200	0.10934	0.91813
4900	0.74197	0.62088	10,400	0.10287	0.92188
5000	0.71366	0.63372	10,600	0.09685	0.92540
5100	0.68628	0.64606	10,800	0.09126	0.92872
5200	0.65983	0.65794	11,000	0.08606	0.93184
5300	0.63432	0.66935	11,200	0.08121	0.93479
5400	0.60974	0.68033	11,400	0.07670	0.93758
5500	0.58608	0.69087	11,600	0.07249	0.94021
5600	0.56332	0.70101	11,800	0.06856	0.94270
5700	0.54146	0.71076	12,000	0.06488	0.94505
5800	0.52046	0.72012	12,200	0.06145	0.94728
5900	0.50030	0.72913	12,400	0.05823	0.94939
6000	0.48096	0.73778	12,600	0.05522	0.95139
6100	0.46242	0.74610	12,800	0.05240	0.95329
6200	0.44464	0.75410	13,000	0.04976	0.95509

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Table 8-1 | (*Continued*).

λT $\mu m \cdot K$	$E_{b\lambda}/T^5$ $\frac{W}{m^2 \cdot K^5 \cdot \mu m} \times 10^{11}$	$\frac{E_{b0-\lambda T}}{\sigma T^4}$	λT $\mu m \cdot K$	$E_{b\lambda}/T^5$ $\frac{W}{m^2 \cdot K^5 \cdot \mu m} \times 10^{11}$	$\frac{E_{b0-\lambda T}}{\sigma T^4}$
13,200	0.04728	0.95680	19,800	0.01151	0.98515
13,400	0.04494	0.95843	20,000	0.01110	0.98555
13,600	0.04275	0.95998	21,000	0.00931	0.98735
13,800	0.04069	0.96145	22,000	0.00786	0.98886
14,000	0.03875	0.96285	23,000	0.00669	0.99014
14,200	0.03693	0.96418	24,000	0.00572	0.99123
14,400	0.03520	0.96546	25,000	0.00492	0.99217
14,600	0.03358	0.96667	26,000	0.00426	0.99297
14,800	0.03205	0.96783	27,000	0.00370	0.99367
15,000	0.03060	0.96893	28,000	0.00324	0.99429
15,200	0.02923	0.96999	29,000	0.00284	0.99482
15,400	0.02794	0.97100	30,000	0.00250	0.99529
15,600	0.02672	0.97196	31,000	0.00221	0.99571
15,800	0.02556	0.97288	32,000	0.00196	0.99607
16,000	0.02447	0.97377	33,000	0.00175	0.99640
16,200	0.02343	0.97461	34,000	0.00156	0.99669
16,400	0.02245	0.97542	35,000	0.00140	0.99695
16,600	0.02152	0.97620	36,000	0.00126	0.99719
16,800	0.02063	0.97694	37,000	0.00113	0.99740
17,000	0.01979	0.97765	38,000	0.00103	0.99759
17,200	0.01899	0.97834	39,000	0.00093	0.99776
17,400	0.01823	0.97899	40,000	0.00084	0.99792
17,600	0.01751	0.97962	41,000	0.00077	0.99806
17,800	0.01682	0.98023	42,000	0.00070	0.99819
18,000	0.01617	0.98081	43,000	0.00064	0.99831
18,200	0.01555	0.98137	44,000	0.00059	0.99842
18,400	0.01496	0.98191	45,000	0.00054	0.99851
18,600	0.01439	0.98243	46,000	0.00049	0.99861
18,800	0.01385	0.98293	47,000	0.00046	0.99869
19,000	0.01334	0.98340	48,000	0.00042	0.99877
19,200	0.01285	0.98387	49,000	0.00039	0.99884
19,400	0.01238	0.98431	50,000	0.00036	0.99890
19,600	0.01193	0.98474			

between wavelengths λ_1 and λ_2 is desired, then

$$E_{b_{\lambda 1-\lambda 2}} = E_{b_{0-\infty}} \left(\frac{E_{b_{0-\lambda 2}}}{E_{b_{0-\infty}}} - \frac{E_{b_{0-\lambda 1}}}{E_{b_{0-\infty}}} \right) \qquad \textbf{[8-16]}$$

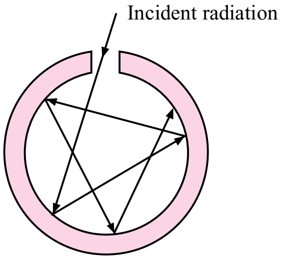
where $E_{b_{0-\infty}}$ is the total radiation emitted over all wavelengths,

$$E_{b_{0-\infty}} = \sigma T^4 \qquad \textbf{[8-17]}$$

and is obtained by integrating the Planck distribution formula of Equation (8-12) over all wavelengths.

Solar radiation has a spectrum approximating that of a blackbody at 5800 K. Ordinary window glass transmits radiation up to about 2.5 μm. Consulting Table 8-1 for $\lambda T = (2.5)(5800) = 14{,}500\ \mu m \cdot K$, we find the fraction of the solar spectrum below 2.5μm to be about 0.97. Thus the glass transmits most of the solar radiation incident upon it. In contrast, room radiation at about 300 K below 2.5 μm has $\lambda T = (2.5)(300) = 750\ \mu m \cdot K$, and only a minute fraction (less than 0.001 percent) of this radiation would be transmitted

Figure 8-7 | Method of constructing a blackbody enclosure.



through the glass. The glass, which is essentially transparent for visible light, is almost totally opaque for thermal radiation emitted at ordinary room temperatures.

Construction of a Blackbody

The concept of a blackbody is an idealization; that is, a perfect blackbody does not exist—all surfaces reflect radiation to some extent, however slight. A blackbody may be approximated very accurately, however, in the following way. A cavity is constructed, as shown in Figure 8-7, so that it is very large compared with the size of the opening in the side. An incident ray of energy is reflected many times on the inside before finally escaping from the side opening. With each reflection there is a fraction of the energy absorbed corresponding to the absorptivity of the inside of the cavity. After the many absorptions, practically all the incident radiation at the side opening is absorbed. It should be noted that the cavity of Figure 8-7 behaves approximately as a blackbody emitter as well as an absorber.

EXAMPLE 8-1 Transmission and Absorption in a Glass Plate

A glass plate 30 cm square is used to view radiation from a furnace. The transmissivity of the glass is 0.5 from 0.2 to 3.5 μm. The emissivity may be assumed to be 0.3 up to 3.5 μm and 0.9 above that. The transmissivity of the glass is zero, except in the range from 0.2 to 3.5 μm. Assuming that the furnace is a blackbody at 2000°C, calculate the energy absorbed in the glass and the energy transmitted.

■ Solution

$$T = 2000°\text{C} = 2273 \text{ K}$$
$$\lambda_1 T = (0.2)(2273) = 454.6 \ \mu\text{m}\cdot\text{K}$$
$$\lambda_2 T = (3.5)(2273) = 7955.5 \ \mu\text{m}\cdot\text{K}$$
$$A = (0.3)^2 = 0.09 \text{ m}^2$$

From Table 8-1

$$\frac{E_{b0-\lambda 1}}{\sigma T^4} = 0 \qquad \frac{E_{b0-\lambda 2}}{\sigma T^4} = 0.85443$$

$$\sigma T^4 = (5.669 \times 10^{-8})(2273)^4 = 1513.3 \text{ kW/m}^2$$

Total incident radiation is

$$0.2 \ \mu\text{m} < \lambda < 3.5 \ \mu\text{m} = (1.5133 \times 10^6)(0.85443 - 0)(0.3)^2$$
$$= 116.4 \text{ kW} \quad [3.97 \times 10^5 \text{ Btu/h}]$$

$$\text{Total radiation transmitted} = (0.5)(116.4) = 58.2 \text{ kW}$$

$$\text{Radiation absorbed} = \begin{cases} (0.3)(116.4) = 34.92 \text{ kW} & \text{for } 0 < \lambda < 3.5 \ \mu\text{m} \\ (0.9)(1 - 0.85443)(1513.3)(0.09) = 17.84 \text{ kW} & \text{for } 3.5 \ \mu\text{m} < \lambda < \infty \end{cases}$$

$$\text{Total radiation absorbed} = 34.92 + 17.84 = 52.76 \text{ kW} \quad [180{,}000 \text{ Btu/h}]$$

8-4 | RADIATION SHAPE FACTOR

Consider two black surfaces A_1 and A_2, as shown in Figure 8-8. We wish to obtain a general expression for the energy exchange between these surfaces when they are maintained at different temperatures. The problem becomes essentially one of determining the amount of

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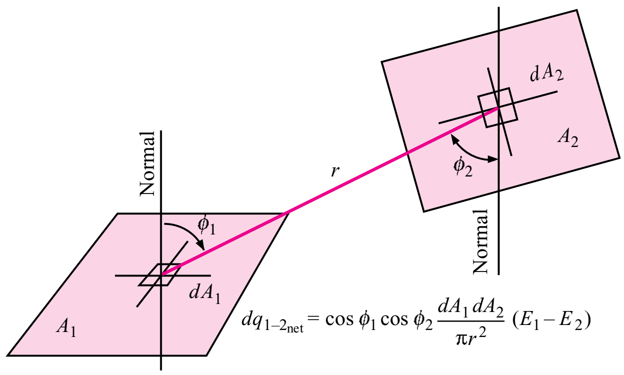
Figure 8-8 | Sketch showing area elements used in deriving radiation shape factor.



energy that leaves one surface and reaches the other. To solve this problem the *radiation shape factors* are defined as

$$F_{1-2} = \text{fraction of energy leaving surface 1 that reaches surface 2}$$
$$F_{2-1} = \text{fraction of energy leaving surface 2 that reaches surface 1}$$
$$F_{i-j} = \text{fraction of energy leaving surface } i \text{ that reaches surface } j$$

Other names for the radiation shape factor are *view factor, angle factor,* and *configuration factor*. The energy leaving surface 1 and arriving at surface 2 is

$$E_{b1} A_1 F_{12}$$

and the energy leaving surface 2 and arriving at surface 1 is

$$E_{b2} A_2 F_{21}$$

Since the surfaces are black, all the incident radiation will be absorbed, and the net energy exchange is

$$E_{b1} A_1 F_{12} - E_{b2} A_2 F_{21} = Q_{1-2}$$

If both surfaces are at the same temperature, there can be no heat exchange, that is, $Q_{1-2} = 0$. Also, for $T_1 = T_2$

$$E_{b1} = E_{b2}$$

so that

$$A_1 F_{12} = A_2 F_{21} \qquad \textbf{[8-18]}$$

The net heat exchange is therefore

$$Q_{1-2} = A_1 F_{12}(E_{b1} - E_{b2}) = A_2 F_{21}(E_{b1} - E_{b2}) \qquad \textbf{[8-19]}$$

Equation (8-18) is known as a reciprocity relation, and it applies in a general way for any two surfaces i and j:

$$A_i F_{ij} = A_j F_{ji} \qquad \textbf{[8-18}a\textbf{]}$$

Although the relation is derived for black surfaces, it holds for other surfaces also as long as diffuse radiation is involved.

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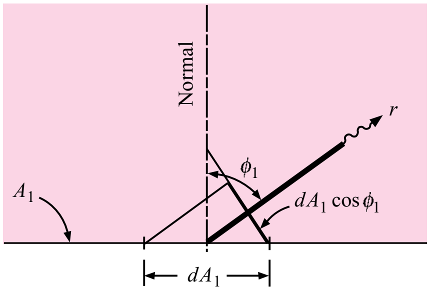
Figure 8-9 | Elevation view of area shown in Figure 8-8.



We now wish to determine a general relation for F_{12} (or F_{21}). To do this, we consider the elements of area dA_1 and dA_2 in Figure 8-8. The angles ϕ_1 and ϕ_2 are measured between a normal to the surface and the line drawn between the area elements r. The projection of dA_1 on the line between centers is

$$dA_1 \cos\phi_1$$

This may be seen more clearly in the elevation drawing shown in Figure 8-9. We assume that the surfaces are diffuse, that is, that the intensity of the radiation is the same in all directions. The intensity is the radiation emitted per unit area and per unit of solid angle in a certain specified direction. So, in order to obtain the energy emitted by the element of area dA_1 in a certain direction, we must multiply the intensity by the projection of dA_1 in the specified direction. Thus the energy leaving dA_1 in the direction given by the angle ϕ_1 is

$$I_b \, dA_1 \cos\phi_1 \qquad \textbf{[a]}$$

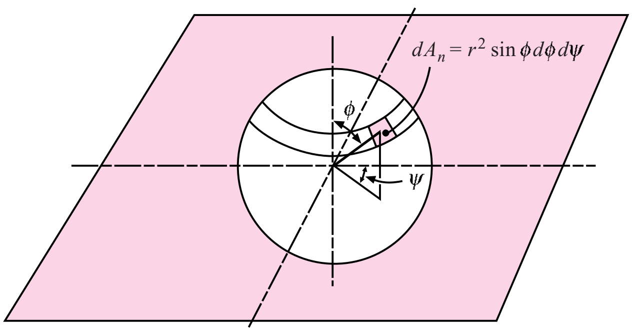
where I_b is the blackbody intensity. The radiation arriving at some area element dA_n at a distance r from A_1 would be

$$I_b \, dA_1 \cos\phi_1 \frac{dA_n}{r^2} \qquad \textbf{[b]}$$

where dA_n is constructed normal to the radius vector. The quantity dA_n/r^2 represents the solid angle subtended by the area dA_n. The intensity may be obtained in terms of the emissive power by integrating expression (b) over a hemisphere enclosing the element of area dA_1. In a spherical coordinate system like that in Figure 8-10,

$$dA_n = r^2 \sin\phi \, d\psi \, d\phi$$

Figure 8-10 | Spherical coordinate system used in derivation of radiation shape factor.



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Then

$$E_b\, dA_1 = I_b\, dA_1 \int_0^{2\pi} \int_0^{\pi/2} \sin\phi \cos\phi\, d\phi\, d\psi$$
$$= \pi I_b\, dA_1$$

so that

$$E_b = \pi I_b \qquad \textbf{[8-20]}$$

We may now return to the energy-exchange problem indicated in Figure 8-8. The area element dA_n is given by

$$dA_n = \cos\phi_2\, dA_2$$

so that the energy leaving dA_1 that arrives at dA_2 is

$$dq_{1-2} = E_{b1} \cos\phi_1 \cos\phi_2 \frac{dA_1\, dA_2}{\pi r^2}$$

That energy leaving dA_2 and arriving at dA_1 is

$$dq_{2-1} = E_{b2} \cos\phi_2 \cos\phi_1 \frac{dA_2\, dA_1}{\pi r^2}$$

and the net energy exchange is

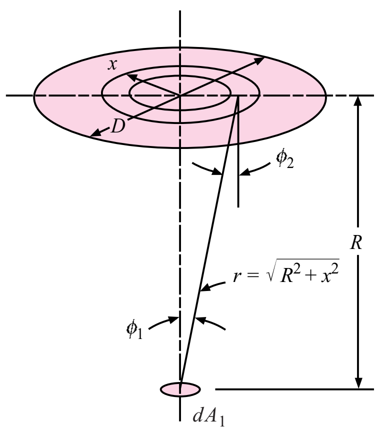
$$q_{\text{net}_{1-2}} = (E_{b1} - E_{b2}) \int_{A_2} \int_{A_1} \cos\phi_1 \cos\phi_2 \frac{dA_1 dA_2}{\pi r^2} \qquad \textbf{[8-21]}$$

The integral is either A_1F_{12} or A_2F_{21} according to Equation (8-19). To evaluate the integral, the specific geometry of the surfaces A_1 and A_2 must be known. We shall work out an elementary problem and then present the results for more complicated geometries in graphical and equation form.

Consider the radiation from the small area dA_1 to the flat disk A_2, as shown in Figure 8-11. The element of area dA_2 is chosen as the circular ring of radius x. Thus

$$dA_2 = 2\pi x\, dx$$

Figure 8-11 | Radiation from a small-area element to a disk.



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We note that $\phi_1 = \phi_2$ and apply Equation (8-21), integrating over the area A_2:

$$dA_1\, F_{dA_1-A_2} = dA_1 \int_{A_2} \cos^2\phi_1 \frac{2\pi x\, dx}{\pi r^2}$$

Making the substitutions

$$r = (R^2 + x^2)^{1/2} \qquad \text{and} \qquad \cos\phi_1 = \frac{R}{(R^2+x^2)^{1/2}}$$

we have

$$dA_1\, F_{dA_1-A_2} = dA_1 \int_0^{D/2} \frac{2R^2 x\, dx}{(R^2+x^2)^2}$$

Performing the integration gives

$$dA_1\, F_{dA_1-A_2} = -dA_1 \left(\frac{R^2}{R^2+x^2}\right)\Bigg]_0^{D/2} = dA_1 \frac{D^2}{4R^2+D^2}$$

so that

$$F_{dA_1-A_2} = \frac{D^2}{4R^2+D^2} \qquad \textbf{[8-22]}$$

The calculation of shape factors may be extended to more complex geometries, as described in References 3, 5, 24, and 32; 32 gives a very complete catalog of analytical relations and graphs for shape factors. For our purposes we give only the results of a few geometries as shown in Figures 8-12 through 8-16. The analytical relations for these geometries are given in Table 8-2.

Figure 8-12 | Radiation shape factor for radiation between parallel rectangles.

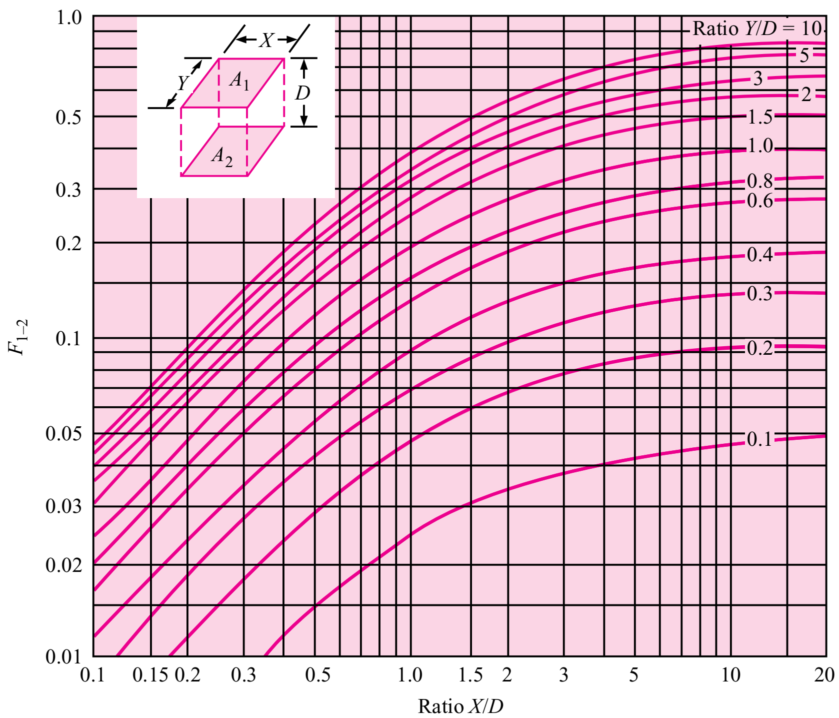
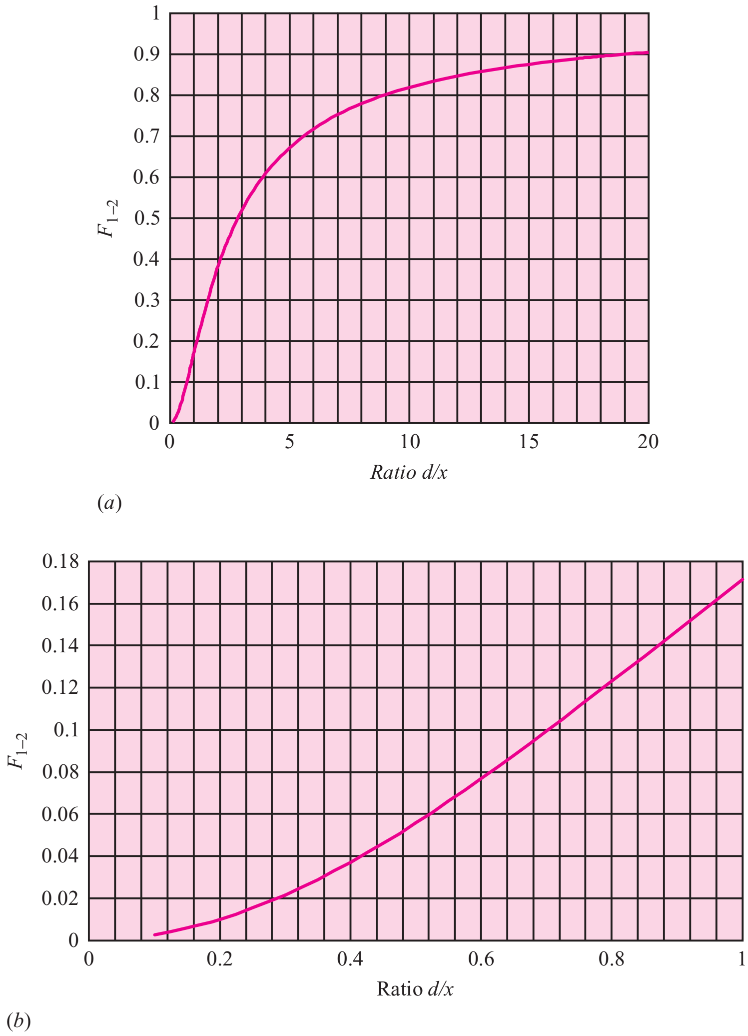


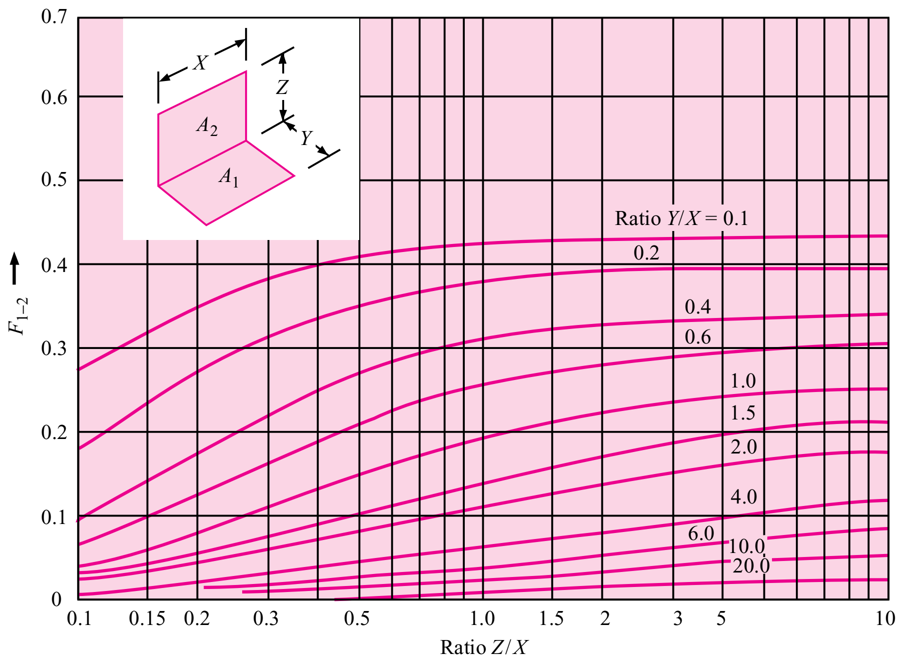
Figure 8-13 | Radiation shape factor for radiation between parallel equal coaxial disks.



Real-Surface Behavior

Real surfaces exhibit interesting deviations from the ideal surfaces described in the preceding paragraphs. Real surfaces, for example, are not perfectly diffuse, and hence the intensity of emitted radiation is not constant over all directions. The directional-emittance characteristics of several types of surfaces are shown in Figure 8-17. These curves illustrate the characteristically different behavior of electric conductors and nonconductors. Conductors emit more energy in a direction having a large azimuth angle. This behavior may be satisfactorily explained with basic electromagnetic wave theory, and is discussed in Reference 24. As a result of this basic behavior of conductors and nonconductors, we may anticipate the appearance of a sphere which is heated to incandescent temperatures, as shown in Figure 8-18. An electric conducting sphere will appear bright around the rim since more energy is emitted at large angles ϕ. A sphere constructed of a nonconducting material will have the opposite behavior and will appear bright in the center and dark around the edge.

Figure 8-14 | Radiation shape factor for radiation between perpendicular rectangles with a common edge.



Reflectance and absorptance of thermal radiation from real surfaces are a function not only of the surface itself but also of the surroundings. These properties are dependent on the direction and wavelength of the incident radiation. But the distribution of the intensity of incident radiation with wavelength may be a very complicated function of the temperatures and surface characteristics of all the surfaces that incorporate the surroundings. Let us denote the total incident radiation on a surface per unit time, per unit area, and per unit wavelength as G_λ. Then the total absorptivity will be given as the ratio of the total energy absorbed to the total energy incident on the surface,
or

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda \, d\lambda}{\int_0^\infty G_\lambda \, d\lambda} \qquad \textbf{[8-23]}$$

If we are fortunate enough to have a gray body such that $\epsilon_\lambda = \epsilon =$ constant, this relation simplifies considerably. It may be shown that Kirchoff's law [Equation(8-8)] may be written for monochromatic radiation as

$$\epsilon_\lambda = \alpha_\lambda \qquad \textbf{[8-24]}$$

Therefore, for a gray body, α_λ = constant, and Equation (8-23) expresses the result that the total absorptivity is also constant and independent of the wavelength distribution of incident radiation. Furthermore, since the emissivity and absorptivity are constant over all wavelengths for a gray body, they must be independent of temperature as well. Unhappily, real surfaces are not always "gray" in nature, and significant errors may ensue by assuming gray-body behavior. On the other hand, analysis of radiation exchange using real-surface behavior is so complicated that the ease and simplification of the gray-body assumption is justified by the practical utility it affords. References 10, 11, and 24 present comparisons of heat-transfer calculations based on both gray and nongray analyses.

Figure 8-15 | Radiation shape factors for two concentric cylinders of finite length. (*a*) Outer cylinder to itself; (*b*) outer cylinder to inner cylinder.

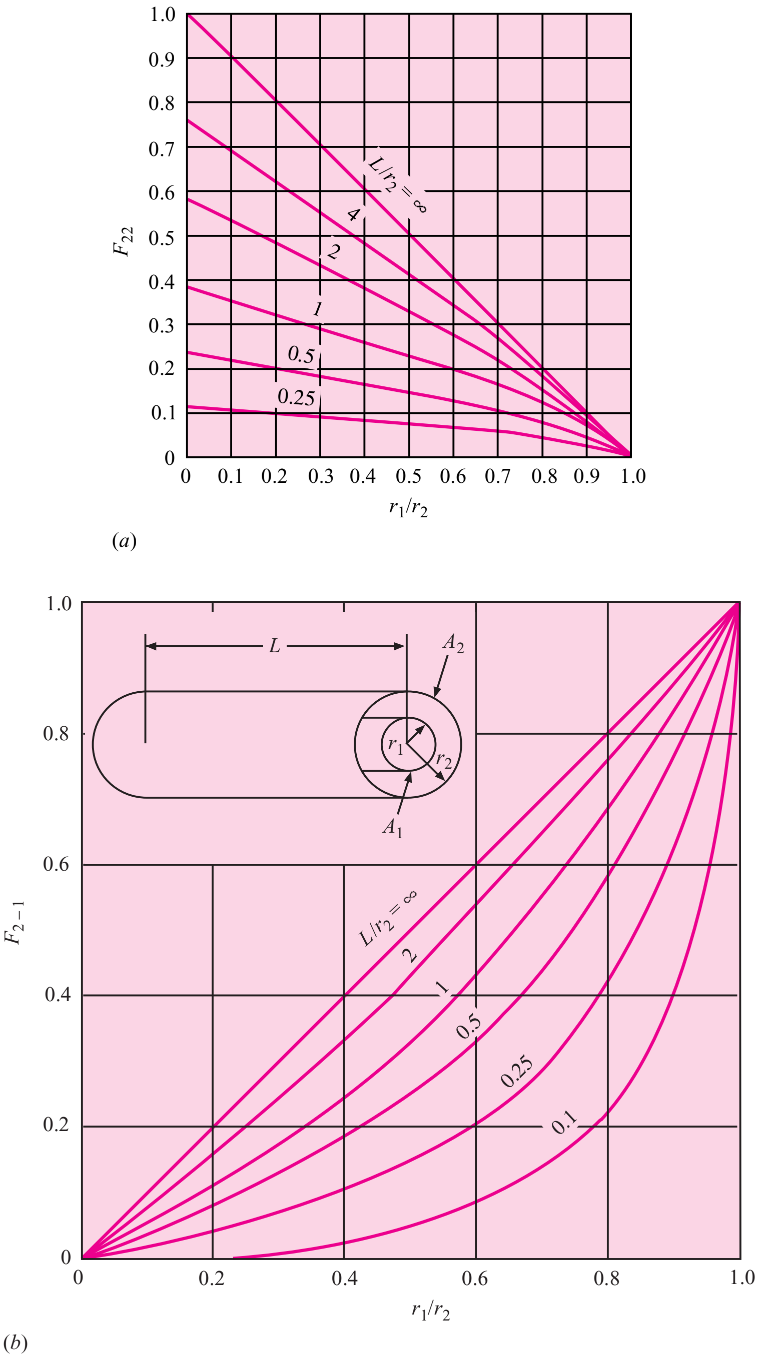


Figure 8-16 | Radiation shape factor for radiation between two parallel coaxial disks.

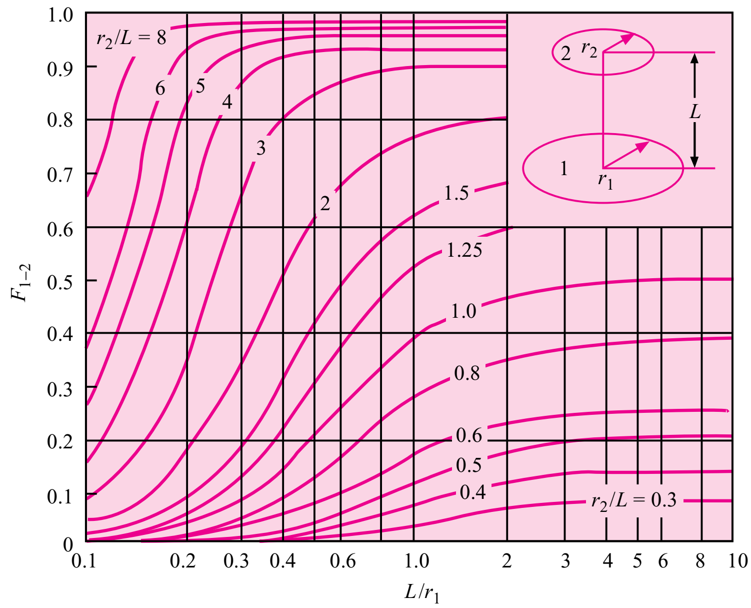


Table 8-2 | Radiation shape factor relations.

Geometry	Shape factor
1. Parallel, equal rectangles (Fig. 8-12) $x = X/D$, $y = Y/D$	$F_{1-2} = (2/\pi xy)\{\ln[(1+x^2)(1+y^2)/(1+x^2+y^2)]^{1/2} + x(1+y^2)^{1/2}\tan^{-1}[x/(1+y^2)^{1/2}] + y(1+x^2)^{1/2}\tan^{-1}[y/(1+x^2)^{1/2}] - x\tan^{-1}x - y\tan^{-1}y\}$
2. Parallel, equal, coaxial disks (Fig. 8-13) $R = d/2x$, $X = (2R^2+1)/R^2$	$F_{1-2} = [X - (X^2-4)^{1/2}]/2$
3. Perpendicular rectangles with a common edge (Fig. 8-14) $H = Z/X$, $W = Y/X$	$F_{1-2} = (1/\pi W)\big(W\tan^{-1}(1/W) + H\tan^{-1}(1/H) - (H^2+W^2)^{1/2}\tan^{-1}[1/(H^2+W^2)^{1/2}] + (1/4)\ln\{[(1+W^2)(1+H^2)/(1+W^2+H^2)] \times [W^2(1+W^2+H^2)/(1+W^2)(W^2+H^2)]^{W^2} \times [H^2(1+H^2+W^2)/(1+H^2)(H^2+W^2)]^{H^2}\}\big)$
4. Finite, coaxial cylinders (Fig. 8-15) $X = r_2/r_1$, $Y = L/r_1$, $A = X^2+Y^2-1$, $B = Y^2-X^2+1$	$F_{2-1} = (1/X) - (1/\pi X)\{\cos^{-1}(B/A) - (1/2Y)[(A^2+4A-4X^2+4)^{1/2}\cos^{-1}(B/XA) + B\sin^{-1}(1/X) - \pi A/2]\}$ $F_{2-2} = 1 - (1/X) + (2/\pi X)\tan^{-1}[2(X^2-1)^{1/2}/Y] - (Y/2\pi X)\big([\sqrt{(4X^2+Y^2)}/Y]\sin^{-1}\{[4(X^2-1)+(Y/X)^2(X^2-2)]/[Y^2+4(X^2-1)]\} - \sin^{-1}[(X^2-2)/X^2] + (\pi/2)[(4X^2+Y^2)^{1/2}/Y - 1]\big)$
5. Parallel, coaxial disks (Fig. 8-16) $R_1 = r_1/L$, $R_2 = r_2/L$, $X = 1 + (1+R_2^2)/R_1^2$	$F_{1-2} = \{X - [X^2 - 4(R_2/R_1)^2]^{1/2}\}/2$

Figure 8-17 | Typical directional behavior of emissivity for conductors and nonconductors. ϵ_ϕ is emissivity at angle ϕ measured from normal to surface. Nonconductor curves are for (*a*) wet ice, (*b*) wood, (*c*) glass, (*d*) paper, (*e*) clay, (*f*) copper oxide, and (*g*) aluminum oxide.

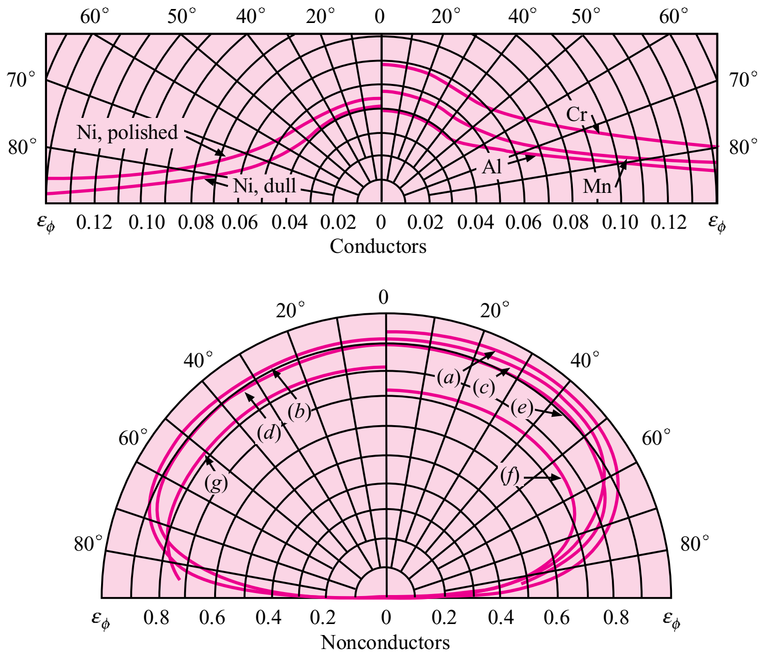
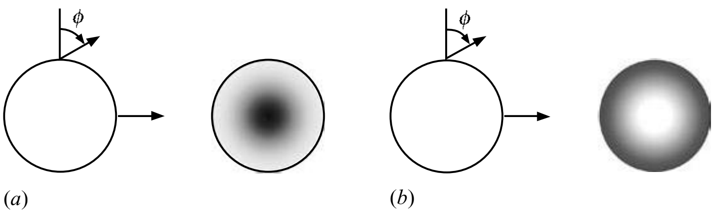


Figure 8-18 | Effect of directional emittance on appearance of an incandescent sphere: (*a*) electrical conductor; (*b*) electrical nonconductor.



Heat Transfer Between Black Surfaces

EXAMPLE 8-2

Two parallel black plates 0.5 by 1.0 m are spaced 0.5 m apart. One plate is maintained at 1000°C and the other at 500°C. What is the net radiant heat exchange between the two plates?

■ Solution

The ratios for use with Figure 8-12 are

$$\frac{Y}{D} = \frac{0.5}{0.5} = 1.0 \qquad \frac{X}{D} = \frac{1.0}{0.5} = 2.0$$

so that $F_{12} = 0.285$. The heat transfer is calculated from

$$\begin{aligned} q &= A_1 F_{12}(E_{b1} - E_{b2}) = \sigma A_1 F_{12}(T_1^4 - T_2^4) \\ &= (5.669 \times 10^{-8})(0.5)(0.285)(1273^4 - 773^4) \\ &= 18.33 \text{ kW} \quad [62{,}540 \text{ Btu/h}] \end{aligned}$$

8-5 | RELATIONS BETWEEN SHAPE FACTORS

Some useful relations between shape factors may be obtained by considering the system shown in Figure 8-19. Suppose that the shape factor for radiation from A_3 to the combined area $A_{1,2}$ is desired. This shape factor must be given very simply as

$$F_{3-1,2} = F_{3-1} + F_{3-2} \qquad \textbf{[8-25]}$$

that is, the total shape factor is the sum of its parts. We could also write Equation (8-25) as

$$A_3 F_{3-1,2} = A_3 F_{3-1} + A_3 F_{3-2} \qquad \textbf{[8-26]}$$

and making use of the reciprocity relations

$$A_3 F_{3-1,2} = A_{1,2} F_{1,2-3}$$
$$A_3 F_{3-1} = A_1 F_{1-3}$$
$$A_3 F_{3-2} = A_2 F_{2-3}$$

the expression could be rewritten

$$A_{1,2} F_{1,2-3} = A_1 F_{1-3} + A_2 F_{2-3} \qquad \textbf{[8-27]}$$

which simply states that the total radiation arriving at surface 3 is the sum of the radiations from surfaces 1 and 2. Suppose we wish to determine the shape factor F_{1-3} for the surfaces in Figure 8-20 in terms of known shape factors for perpendicular rectangles with a common edge. We may write

$$F_{1-2,3} = F_{1-2} + F_{1-3}$$

in accordance with Equation (8-25). Both $F_{1-2,3}$ and F_{1-2} may be determined from Figure 8-14, so that F_{1-3} is easily calculated when the dimensions are known. Now consider the somewhat more complicated situation shown in Figure 8-21. An expression for the shape factor F_{1-4} is desired in terms of known shape factors for perpendicular rectangles

Figure 8-19 | Sketch showing some relations between shape factors.

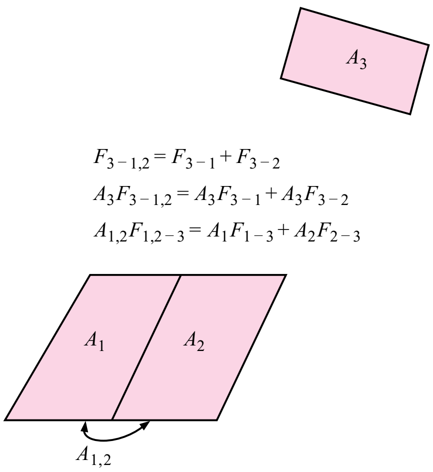


Figure 8-20

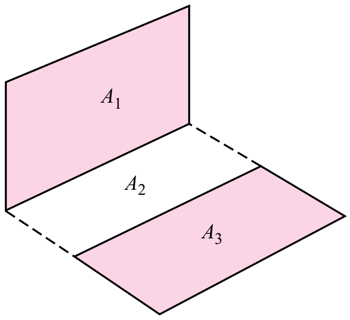
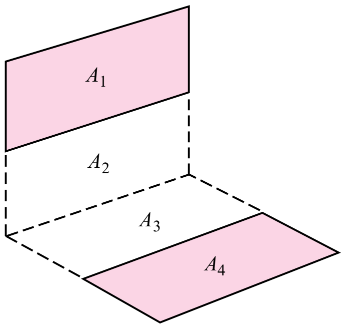


Figure 8-21



with a common edge. We write

$$A_{1,2}F_{1,2-3,4} = A_1F_{1-3,4} + A_2F_{2-3,4} \quad \textbf{[a]}$$

in accordance with Equation (8-25). Both $F_{1,2-3,4}$ and $F_{2-3,4}$ can be obtained from Figure 8-14, and $F_{1-3,4}$ may be expressed

$$A_1F_{1-3,4} = A_1F_{1-3} + A_1F_{1-4} \quad \textbf{[b]}$$

Also

$$A_{1,2}F_{1,2-3} = A_1F_{1-3} + A_2F_{2-3} \quad \textbf{[c]}$$

Solving for A_1F_{1-3} from (c), inserting this in (b), and then inserting the resultant expression for $A_1F_{1-3,4}$ in (a) gives

$$A_{1,2}F_{1,2-3,4} = A_{1,2}F_{1,2-3} - A_2F_{2-3} + A_1F_{1-4} + A_2F_{2-3,4} \quad \textbf{[d]}$$

Notice that all shape factors except F_{1-4} may be determined from Figure 8-14. Thus

$$F_{1-4} = \frac{1}{A_1}(A_{1,2}F_{1,2-3,4} + A_2F_{2-3} - A_{1,2}F_{1,2-3} - A_2F_{2-3,4}) \quad \textbf{[8-28]}$$

In the foregoing discussion the tacit assumption has been made that the various bodies do not see themselves, that is,

$$F_{11} = F_{22} = F_{33} = 0 \cdots$$

To be perfectly general, we must include the possibility of concave curved surfaces, which may then see themselves. The general relation is therefore

$$\sum_{j=1}^{n} F_{ij} = 1.0 \quad \textbf{[8-29]}$$

where F_{ij} is the fraction of the total energy leaving surface i that arrives at surface j. Thus for a three-surface enclosure we would write

$$F_{11} + F_{12} + F_{13} = 1.0$$

and F_{11} represents the fraction of energy leaving surface 1 that strikes surface 1. A certain amount of care is required in analyzing radiation exchange between curved surfaces.

Figure 8-22 | Generalized perpendicular-rectangle arrangement.

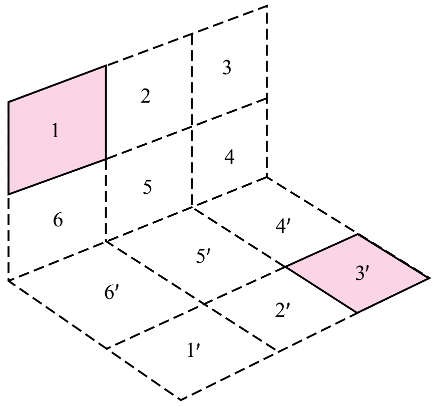
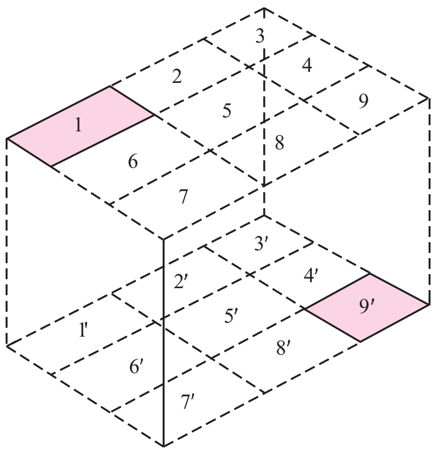


Figure 8-23 | Generalized parallel-rectangle arrangement.



Hamilton and Morgan [5] have presented generalized relations for parallel and perpendicular rectangles in terms of shape factors which may be obtained from Figures 8-12 and 8-14. The two situations of interest are shown in Figures 8-22 and 8-23. For the perpendicular rectangles of Figure 8-22 it can be shown that the following reciprocity relations apply [5]:

$$A_1 F_{13'} = A_3 F_{31'} = A_{3'} F_{3'1} = A_{1'} F_{1'3} \qquad \textbf{[8-30]}$$

By making use of these reciprocity relations, the radiation shape factor $F_{13'}$ may be expressed by

$$\begin{aligned} A_1 F_{13'} = \tfrac{1}{2}\big[&K_{(1,2,3,4,5,6)^2} - K_{(2,3,4,5)^2} - K_{(1,2,5,6)^2} + K_{(4,5,6)^2} - K_{(4,5,6)-(1',2',3',4',5',6')} \\ &- K_{(1,2,3,4,5,6)-(4',5',6')} + K_{(1,2,5,6)-(5',6')} + K_{(2,3,4,5)-(4',5')} + K_{(5,6)-(1',2',5',6')} \\ &+ K_{(4,5)-(2',3',4',5')} + K_{(2,5)^2} - K_{(2,5)-5'} - K_{(5,6)^2} - K_{(4,5)^2} - K_{5-(2',5')} + K_{5^2}\big] \end{aligned} \qquad \textbf{[8-31]}$$

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where the K terms are defined by

$$K_{m-n} = A_m F_{m-n} \qquad \textbf{[8-32]}$$

$$K_{(m)^2} = A_m F_{m-m'} \qquad \textbf{[8-33]}$$

The generalized parallel-rectangle arrangement is depicted in Figure 8-23. The reciprocity relations that apply to this situation are given in Reference 5 as

$$A_1 F_{19'} = A_3 F_{37'} = A_9 F_{91'} = A_7 F_{73'} \qquad \textbf{[8-34]}$$

Making use of these relations, it is possible to derive the shape factor $F_{19'}$ as

$$\begin{aligned} A_1 F_{19'} = \tfrac{1}{4}\big[&K_{(1,2,3,4,5,6,7,8,9)^2} - K_{(1,2,5,6,7,8)^2} - K_{(2,3,4,5,8,9)^2} - K_{(1,2,3,4,5,6)^2} \\ &+ K_{(1,2,5,6)^2} + K_{(2,3,4,5)^2} + K_{(4,5,8,9)^2} - K_{(4,5)^2} - K_{(5,8)^2} - K_{(5,6)^2} \\ &- K_{(4,5,6,7,8,9)^2} + K_{(5,6,7,8)^2} + K_{(4,5,6)^2} + K_{(2,5,8)^2} - K_{(2,5)^2} + K_{(5)^2}\big] \end{aligned} \qquad \textbf{[8-35]}$$

The nomenclature for the K terms is the same as given in Equations (8-32) and (8-33).

Shape-Factor Algebra for Open Ends of Cylinders

EXAMPLE 8-3

Two concentric cylinders having diameters of 10 and 20 cm have a length of 20 cm. Calculate the shape factor between the open ends of the cylinders.

Solution

We use the nomenclature of Figure 8-15 for this problem and designate the open ends as surfaces 3 and 4. We have $L/r_2 = 20/10 = 2.0$ and $r_1/r_2 = 0.5$; so from Figure 8-15 or Table 8-2 we obtain

$$F_{21} = 0.4126 \qquad F_{22} = 0.3286$$

Using the reciprocity relation [Equation (8-18)] we have

$$A_1 F_{12} = A_2 F_{21} \qquad \text{and} \qquad F_{12} = (d_2/d_1)F_{21} = (20/10)(0.4126) = 0.8253$$

For surface 2 we have

$$F_{21} + F_{22} + F_{23} + F_{24} = 1.0$$

From symmetry $F_{23} = F_{24}$ so that

$$F_{23} = F_{24} = \left(\tfrac{1}{2}\right)(1 - 0.4126 - 0.3286) = 0.1294$$

Using reciprocity again,

$$A_2 F_{23} = A_3 F_{32}$$

and

$$F_{32} = \frac{\pi(20)(20)}{\pi(20^2 - 10^2)/4} 0.1294 = 0.6901$$

We observe that $F_{11} = F_{33} = F_{44} = 0$ and for surface 3

$$F_{31} + F_{32} + F_{34} = 1.0 \qquad [a]$$

So, if F_{31} can be determined, we can calculate the desired quantity F_{34}. For surface 1

$$F_{12} + F_{13} + F_{14} = 1.0$$

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and from symmetry $F_{13} = F_{14}$ so that

$$F_{13} = \left(\frac{1}{2}\right)(1 - 0.8253) = 0.0874$$

Using reciprocity gives

$$A_1 F_{13} = A_3 F_{31}$$
$$F_{31} = \frac{\pi(10)(20)}{\pi(20^2 - 10^2)/4} 0.0874 = 0.233$$

Then, from Equation (a)

$$F_{34} = 1 - 0.233 - 0.6901 = 0.0769$$

EXAMPLE 8-4 Shape-Factor Algebra for Truncated Cone

A truncated cone has top and bottom diameters of 10 and 20 cm and a height of 10 cm. Calculate the shape factor between the top surface and the side and also the shape factor between the side and itself.

Solution

We employ Figure 8-16 for solution of this problem and take the nomenclature as shown, designating the top as surface 2, the bottom as surface 1, and the side as surface 3. Thus, the desired quantities are F_{23} and F_{33}. We have $L/r_1 = 10/10 = 1.0$ and $r_2/L = 5/10 = 0.5$. Thus, from Figure 8-16

$$F_{12} = 0.12$$

From reciprocity [Equation (8-18)]

$$A_1 F_{12} = A_2 F_{21}$$
$$F_{21} = (20/10)^2(0.12) = 0.48$$

and

$$F_{22} = 0$$

so that

$$F_{21} + F_{23} = 1.0$$

and

$$F_{23} = 1 - 0.48 = 0.52$$

For surface 3,

$$F_{31} + F_{32} + F_{33} = 1.0 \qquad [a]$$

so we must find F_{31} and F_{32} in order to evaluate F_{33}. Since $F_{11} = 0$, we have

$$F_{12} + F_{13} = 1.0 \qquad \text{and} \qquad F_{13} = 1 - 0.12 = 0.88$$

and from reciprocity

$$A_1 F_{13} = A_3 F_{31} \qquad [b]$$

The surface area of the side is

$$A_3 = \pi(r_1 + r_2)\left[(r_1 - r_2)^2 + L^2\right]^{1/2}$$
$$= \pi(5 + 10)(5^2 + 10^2)^{1/2} = 526.9 \text{ cm}^2$$

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So, from Equation (b)

$$F_{31} = \frac{\pi(10^2)}{526.9}0.88 = 0.525$$

A similar procedure applies with surface 2 so that

$$F_{32} = \frac{\pi(5)^2}{526.9}0.52 = 0.0775$$

Finally, from Equation (a)

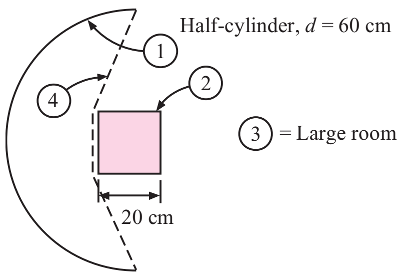
$$F_{33} = 1 - 0.525 - 0.0775 = 0.397$$

Shape-Factor Algebra for Cylindrical Reflector

EXAMPLE 8-5

The long circular half-cylinder shown in Figure Example 8-5 has a diameter of 60 cm and a square rod 20 by 20 cm placed along the geometric centerline. Both are surrounded by a large enclosure. Find F_{12}, F_{13}, and F_{11} in accordance with the nomenclature in the figure.

Figure Example 8-5



■ Solution

From symmetry we have

$$F_{21} = F_{23} = 0.5 \qquad [a]$$

In general, $F_{11} + F_{12} + F_{13} = 1.0$. To aid in the analysis we create the fictitious surface 4 shown as the dashed line. For this surface, $F_{41} = 1.0$. Now, all radiation leaving surface 1 will arrive either at 2 or at 3. Likewise, this radiation will arrive at the imaginary surface 4, so that

$$F_{14} = F_{12} + F_{13} \qquad [b]$$

From reciprocity,

$$A_1 F_{14} = A_4 F_{41}$$

The areas are, for unit length,

$$A_1 = \pi d/2 = \pi(0.6)/2 = 0.942$$

$$A_4 = 0.2 + (2)[(0.1)^2 + (0.2)^2]^{1/2} = 0.647$$

$$A_2 = (4)(0.2) = 0.8$$

so that

$$F_{14} = \frac{A_4}{A_1}F_{41} = \frac{(0.647)(1.0)}{0.942} = 0.686 \qquad [c]$$

We also have, from reciprocity,

$$A_2F_{21} = A_1F_{12}$$

so

$$F_{12} = \frac{A_2}{A_1}F_{21} = \frac{(0.8)(0.5)}{0.942} = 0.425 \qquad [d]$$

Combining (b), (c), and (d) gives

$$F_{13} = 0.686 - 0.425 = 0.261$$

Finally,

$$F_{11} = 1 - F_{12} - F_{13} = 1 - 0.425 - 0.261 = 0.314$$

This example illustrates how one may make use of clever geometric considerations to calculate the radiation shape factors.

8-6 | HEAT EXCHANGE BETWEEN NONBLACKBODIES

The calculation of the radiation heat transfer between black surfaces is relatively easy because all the radiant energy that strikes a surface is absorbed. The main problem is one of determining the geometric shape factor, but once this is accomplished, the calculation of the heat exchange is very simple. When nonblackbodies are involved, the situation is much more complex, for all the energy striking a surface will not be absorbed; part will be reflected back to another heat-transfer surface, and part may be reflected out of the system entirely. The problem can become complicated because the radiant energy can be reflected back and forth between the heat-transfer surfaces several times. The analysis of the problem must take into consideration these multiple reflections if correct conclusions are to be drawn.

We shall assume that all surfaces considered in our analysis are diffuse, gray, and uniform in temperature and that the reflective and emissive properties are constant over all the surface. Two new terms may be defined:

G = irradiation
= total radiation incident upon a surface per unit time and per unit area

J = radiosity
= total radiation that leaves a surface per unit time and per unit area

In addition to the assumptions stated above, we shall also assume that the radiosity and irradiation are uniform over each surface. This assumption is not strictly correct, even for ideal gray diffuse surfaces, but the problems become exceedingly complex when this analytical restriction is not imposed. Sparrow and Cess [10] give a discussion of such problems. As shown in Figure 8-24, the radiosity is the sum of the energy emitted and the energy reflected when no energy is transmitted, or

$$J = \epsilon E_b + \rho G \qquad [8\text{-}36]$$

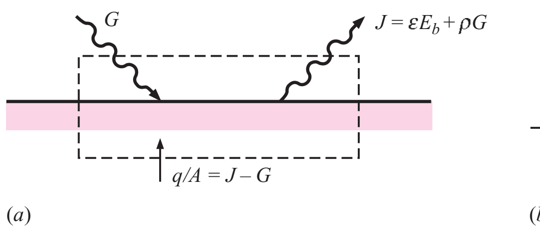
where ϵ is the emissivity and E_b is the blackbody emissive power. Since the transmissivity is assumed to be zero, the reflectivity may be expressed as

$$\rho = 1 - \alpha = 1 - \epsilon$$

so that

$$J = \epsilon E_b + (1 - \epsilon)G \qquad [8\text{-}37]$$

Figure 8-24 | (*a*) Surface energy balance for opaque material; (*b*) element representing "surface resistance" in the radiation-network method.



The net energy leaving the surface is the difference between the radiosity and the irradiation:

$$\frac{q}{A} = J - G = \epsilon E_b + (1-\epsilon)G - G$$

Solving for G in terms of J from Equation (8-37),

$$q = \frac{\epsilon A}{1-\epsilon}(E_b - J)$$

or

$$q = \frac{E_b - J}{(1-\epsilon)/\epsilon A} \qquad \textbf{[8-38]}$$

At this point we introduce a very useful interpretation for Equation (8-38). If the denominator of the right side is considered as the surface resistance to radiation heat transfer, the numerator as a potential difference, and the heat flow as the "current," then a network element could be drawn as in Figure 8-24(*b*) to represent the physical situation. This is the first step in the network method of analysis originated by Oppenheim [20].

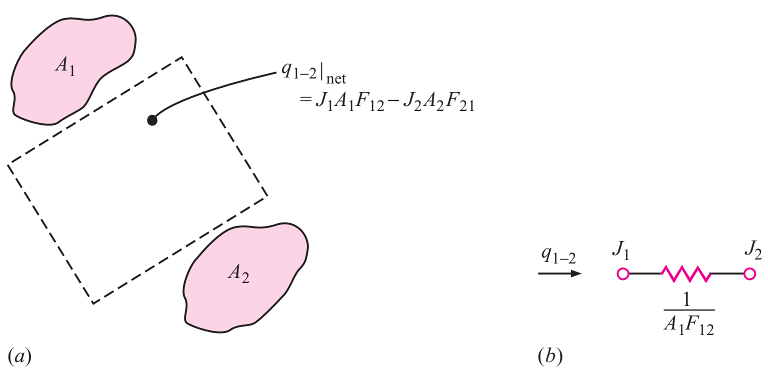
Now consider the exchange of radiant energy by two surfaces, A_1 and A_2, shown in Figure 8-25. Of that total radiation leaving surface 1, the amount that reaches surface 2 is

$$J_1 A_1 F_{12}$$

and of that total energy leaving surface 2, the amount that reaches surface 1 is

$$J_2 A_2 F_{21}$$

Figure 8-25 | (*a*) Spatial energy exchange between two surfaces; (*b*) element representing "space resistance" in the radiation-network method.



The net interchange between the two surfaces is

$$q_{1-2} = J_1 A_1 F_{12} - J_2 A_2 F_{21}$$

But

$$A_1 F_{12} = A_2 F_{21}$$

so that

$$q_{1-2} = (J_1 - J_2) A_1 F_{12} = (J_1 - J_2) A_2 F_{21}$$

or

$$q_{1-2} = \frac{J_1 - J_2}{1/A_1 F_{12}} \qquad \textbf{[8-39]}$$

We may thus construct a network element that represents Equation (8-39), as shown in Figure 8-25*b*. The two network elements shown in Figures 8-24 and 8-25 represent the essentials of the radiation-network method. To construct a network for a particular radiation heat-transfer problem we need only connect a "surface resistance" $(1-\epsilon)/\epsilon A$ to each surface and a "space resistance" $1/A_i F_{ij}$ between the radiosity potentials. For example, two surfaces that exchange heat with each other *and nothing else* would be represented by the network shown in Figure 8-26. In this case the net heat transfer would be the overall potential difference divided by the sum of the resistances:

$$q_{\text{net}} = \frac{E_{b1} - E_{b2}}{(1-\epsilon_1)/\epsilon_1 A_1 + 1/A_1 F_{12} + (1-\epsilon_2)/\epsilon_2 A_2}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{(1-\epsilon_1)/\epsilon_1 A_1 + 1/A_1 F_{12} + (1-\epsilon_2)/\epsilon_2 A_2} \qquad \textbf{[8-40]}$$

A network for a three-body problem is shown in Figure 8-27. In this case each of the bodies exchanges heat with the other two. The heat exchange between body 1 and body 2 would be

$$q_{1-2} = \frac{J_1 - J_2}{1/A_1 F_{12}}$$

and that between body 1 and body 3,

$$q_{1-3} = \frac{J_1 - J_3}{1/A_1 F_{13}}$$

To determine the heat flows in a problem of this type, the values of the radiosities must be calculated. This may be accomplished by performing standard methods of analysis used in dc circuit theory. The most convenient method is an application of Kirchhoff's current law to the circuit, which states that the sum of the currents entering a node is zero. Example 8-6 illustrates the use of the method for the three-body problem.

Figure 8-26 | Radiation network for two surfaces that see each other and nothing else.

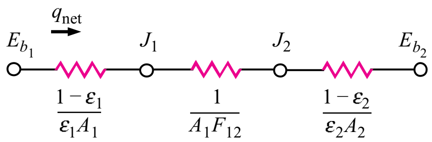
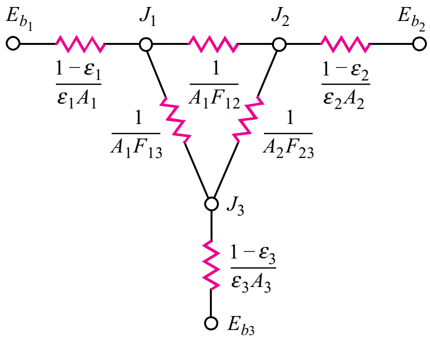


Figure 8-27 | Radiation network for three surfaces that see each other and nothing else.

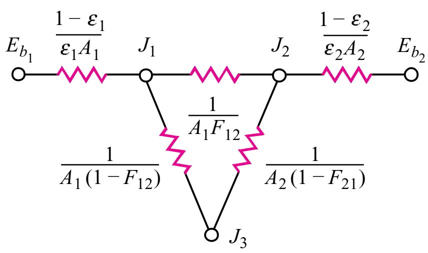


Insulated Surfaces and Surfaces with Large Areas

As we have seen, $(E_b - J)$ represents the potential difference for heat flow through the surface resistance $(1-\epsilon)/\epsilon A$. If a surface is perfectly insulated, or re-radiates all the energy incident upon it, it has zero heat flow and the potential difference across the surface resistance is zero, resulting in $J = E_b$. *But, the insulated surface does not have zero surface resistance.* In effect, the J node in the network is *floating,* that is, it does not draw any current. On the other hand, a surface with a very large area $(A \to \infty)$ has a surface resistance approaching zero, which makes it behave like a blackbody with $\epsilon = 1.0$. It, too, will have $J = E_b$ because of the zero surface resistance. Thus, these two cases—insulated surface and surface with a large area—both have $J = E_b$, but for entirely different reasons. We will make use of these special cases in several examples.

A problem that may be easily solved with the network method is that of two flat surfaces exchanging heat with one another but connected by a third surface that does not exchange heat, i.e., one that is perfectly insulated. This third surface nevertheless influences the heat-transfer process because it absorbs and re-radiates energy to the other two surfaces that exchange heat. The network for this system is shown in Figure 8-28. Notice that node J_3 is not connected to a radiation surface resistance because surface 3 does not exchange energy. A surface resistance $(1-\epsilon)/\epsilon A$ exists, but because there is no heat current flow there is no

Figure 8-28 | Radiation network for two plane or convex surfaces enclosed by a third surface that is nonconducting but re-radiating (insulated).



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potential difference, and $J_3 = E_{b_3}$. Notice also that the values for the space resistances have been written

$$F_{13} = 1 - F_{12}$$
$$F_{23} = 1 - F_{21}$$

since surface 3 completely surrounds the other two surfaces. For the *special case where surfaces 1 and 2 are convex*, that is, they do not see themselves and $F_{11} = F_{22} = 0$, Figure 8-28 is a simple series-parallel network that may be solved for the heat flow as

$$q_{net} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\dfrac{A_1 + A_2 - 2A_1 F_{12}}{A_2 - A_1 (F_{12})^2} + \left(\dfrac{1}{\epsilon_1} - 1\right) + \dfrac{A_1}{A_2}\left(\dfrac{1}{\epsilon_2} - 1\right)} \qquad \textbf{[8-41]}$$

where the reciprocity relation

$$A_1 F_{12} = A_2 F_{21}$$

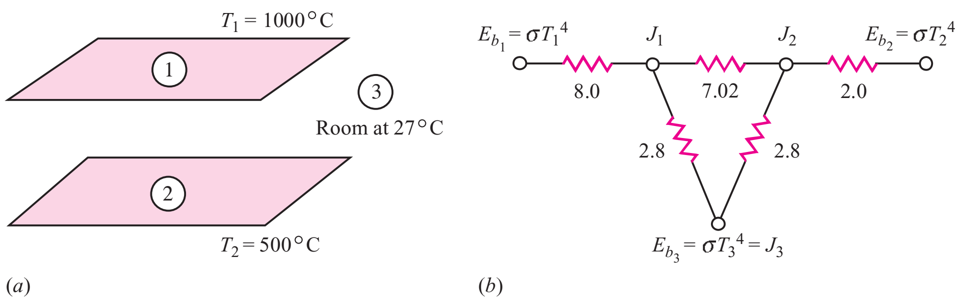
has been used to simplify the expression. *It is to be noted again that Equation (8-41) applies only to surfaces that do not see themselves; that is,* $F_{11} = F_{22} = 0$. If these conditions do not apply, one must determine the respective shape factors and solve the network accordingly. Example 8-7 gives an appropriate illustration of a problem involving an insulated surface.

This network, and others that follow, assume that the only heat exchange is by radiation. Conduction and convection are neglected for now.

EXAMPLE 8-6 Hot Plates Enclosed by a Room

Two parallel plates 0.5 by 1.0 m are spaced 0.5 m apart, as shown in Figure Example 8-6. One plate is maintained at 1000°C and the other at 500°C. The emissivities of the plates are 0.2 and 0.5, respectively. The plates are located in a very large room, the walls of which are maintained at 27°C. The plates exchange heat with each other and with the room, but only the plate surfaces facing each other are to be considered in the analysis. Find the net transfer to each plate and to the room.

Figure Example 8-6 | (*a*) Schematic. (*b*) Network.



■ Solution

This is a three-body problem, the two plates and the room, so the radiation network is shown in Figure 8-27. From the data of the problem

$$T_1 = 1000°\text{C} = 1273\ \text{K} \qquad A_1 = A_2 = 0.5\ \text{m}^2$$
$$T_2 = 500°\text{C} = 773\ \text{K} \qquad \epsilon_1 = 0.2$$
$$T_3 = 27°\text{C} = 300\ \text{K} \qquad \epsilon_2 = 0.5$$

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Because the area of the room A_3 is very large, the resistance $(1-\epsilon_3)/\epsilon_3 A_3$ may be taken as zero and we obtain $E_{b_3} = J_3$. The shape factor F_{12} was given in Example 8-2:

$$F_{12} = 0.285 = F_{21}$$
$$F_{13} = 1 - F_{12} = 0.715$$
$$F_{23} = 1 - F_{21} = 0.715$$

The resistances in the network are calculated as

$$\frac{1-\epsilon_1}{\epsilon_1 A_1} = \frac{1-0.2}{(0.2)(0.5)} = 8.0 \qquad \frac{1-\epsilon_2}{\epsilon_2 A_2} = \frac{1-0.5}{(0.5)(0.5)} = 2.0$$
$$\frac{1}{A_1 F_{12}} = \frac{1}{(0.5)(0.285)} = 7.018 \qquad \frac{1}{A_1 F_{13}} = \frac{1}{(0.5)(0.715)} = 2.797$$
$$\frac{1}{A_2 F_{23}} = \frac{1}{(0.5)(0.715)} = 2.797$$

Taking the resistance $(1-\epsilon_3)/\epsilon_3 A_3$ as zero, we have the network as shown. To calculate the heat flows at each surface we must determine the radiosities J_1 and J_2. The network is solved by setting the sum of the heat currents entering nodes J_1 and J_2 to zero:

node J_1:

$$\frac{E_{b_1} - J_1}{8.0} + \frac{J_2 - J_1}{7.018} + \frac{E_{b_3} - J_1}{2.797} = 0 \qquad \text{[a]}$$

node J_2:

$$\frac{J_1 - J_2}{7.018} + \frac{E_{b_3} - J_2}{2.797} + \frac{E_{b_2} - J_2}{2.0} = 0 \qquad \text{[b]}$$

Now

$$E_{b_1} = \sigma T_1^4 = 148.87 \text{ kW/m}^2 \qquad [47{,}190 \text{ Btu/h} \cdot \text{ft}^2]$$
$$E_{b_2} = \sigma T_2^4 = 20.241 \text{ kW/m}^2 \qquad [6416 \text{ Btu/h} \cdot \text{ft}^2]$$
$$E_{b_3} = \sigma T_3^4 = 0.4592 \text{ kW/m}^2 \qquad [145.6 \text{ Btu/h} \cdot \text{ft}^2]$$

Inserting the values of E_{b_1}, E_{b_2}, and E_{b_3} into Equations (*a*) and (*b*), we have two equations and two unknowns J_1 and J_2 that may be solved simultaneously to give

$$J_1 = 33.469 \text{ kW/m}^2 \qquad J_2 = 15.054 \text{ kW/m}^2$$

The total heat lost by plate 1 is

$$q_1 = \frac{E_{b_1} - J_1}{(1-\epsilon_1)/\epsilon_1 A_1} = \frac{148.87 - 33.469}{8.0} = 14.425 \text{ kW}$$

and the total heat lost by plate 2 is

$$q_2 = \frac{E_{b_2} - J_2}{(1-\epsilon_2)/\epsilon_2 A_2} = \frac{20.241 - 15.054}{2.0} = 2.594 \text{ kW}$$

The total heat received by the room is

$$q_3 = \frac{J_1 - J_3}{1/A_1 F_{13}} + \frac{J_2 - J_3}{1/A_2 F_{23}}$$
$$= \frac{33.469 - 0.4592}{2.797} + \frac{15.054 - 0.4592}{2.797} = 17.020 \text{ kW} \qquad [58{,}070 \text{ Btu/h}]$$

From an overall-balance standpoint we must have

$$q_3 = q_1 + q_2$$

because the net energy lost by both plates must be absorbed by the room.

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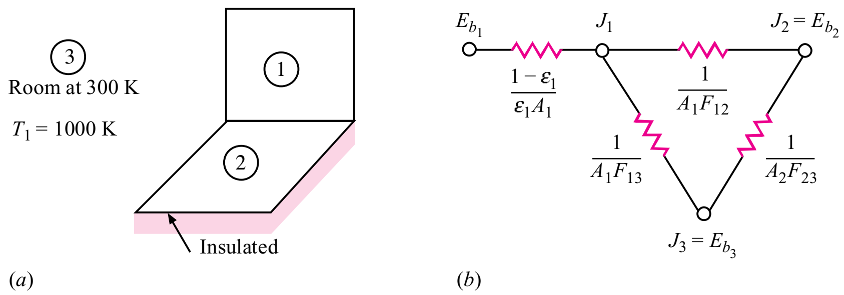
EXAMPLE 8-7 Surface in Radiant Balance

Two rectangles 50 by 50 cm are placed perpendicularly with a common edge. One surface has $T_1 = 1000$ K, $\epsilon_1 = 0.6$, while the other surface is insulated and in radiant balance with a large surrounding room at 300 K. Determine the temperature of the insulated surface and the heat lost by the surface at 1000 K.

Figure Example 8-7 | (*a*) Schematic. (*b*) Network.



■ **Solution**

Although this problem involves two surfaces that exchange heat and one that is insulated or reradiating, Equation (8-41) may not be used for the calculation because one of the heat-exchanging surfaces (the room) is not convex. The radiation network is shown in Figure Example 8-7 where surface 3 is the room and surface 2 is the insulated surface. Note that $J_3 = E_{b_3}$ because the room is large and $(1 - \epsilon_3)/\epsilon_3 A_3$ approaches zero. Because surface 2 is insulated it has zero heat transfer and $J_2 = E_{b_2}$. J_2 "floats" in the network and is determined from the overall radiant balance. From Figure 8-14 the shape factors are

$$F_{12} = 0.2 = F_{21}$$

Because $F_{11} = 0$ and $F_{22} = 0$ we have

$$F_{12} + F_{13} = 1.0 \qquad \text{and} \qquad F_{13} = 1 - 0.2 = 0.8 = F_{23}$$

$$A_1 = A_2 = (0.5)^2 = 0.25 \text{ m}^2$$

The resistances are

$$\frac{1-\epsilon_1}{\epsilon_1 A_1} = \frac{0.4}{(0.6)(0.25)} = 2.667$$

$$\frac{1}{A_1 F_{13}} = \frac{1}{A_2 F_{23}} = \frac{1}{(0.25)(0.8)} = 5.0$$

$$\frac{1}{A_1 F_{12}} = \frac{1}{(0.25)(0.2)} = 20.0$$

We also have

$$E_{b_1} = (5.669 \times 10^{-8})(1000)^4 = 5.669 \times 10^4 \text{ W/m}^2$$

$$J_3 = E_{b_3} = (5.669 \times 10^{-8})(300)^4 = 459.2 \text{ W/m}^2$$

The overall circuit is a series-parallel arrangement and the heat transfer is

$$q = \frac{E_{b_1} - E_{b_3}}{R_{\text{equiv}}}$$

We have

$$R_{\text{equiv}} = 2.667 + \frac{1}{\frac{1}{5} + 1/(20 + 5)} = 6.833$$

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and

$$q = \frac{56{,}690 - 459.2}{6.833} = 8.229 \text{ kW} \quad [28{,}086 \text{ Btu/h}]$$

This heat transfer can also be written

$$q = \frac{E_{b_1} - J_1}{(1 - \epsilon_1)/\epsilon_1 A_1}$$

Inserting the values we obtain

$$J_1 = 34{,}745 \text{ W/m}^2$$

The value of J_2 is determined from proportioning the resistances between J_1 and J_3, so that

$$\frac{J_1 - J_2}{20} = \frac{J_1 - J_3}{20 + 5}$$

and

$$J_2 = 7316 = E_{b_2} = \sigma T_2^4$$

Finally, we obtain the temperature of the insulated surface as

$$T_2 = \left(\frac{7316}{5.669 \times 10^{-8}}\right)^{1/4} = 599.4 \text{ K} \quad [619^\circ\text{F}]$$

■ Comment

Note, once again, that we have made use of the $J = E_b$ relation in two instances in this example, but for two different reasons. $J_2 = E_{b_2}$ because surface 2 is insulated and there is zero current flow through the surface resistance, while $J_3 = E_{b_3}$ because the surface resistance for surface 3 approaches zero as $A_3 \to \infty$.

8-7 | INFINITE PARALLEL SURFACES

When two infinite parallel planes are considered, A_1 and A_2 are equal; and the radiation shape factor is unity since all the radiation leaving one plane reaches the other. The network is the same as in Figure 8-26, and the heat flow per unit area may be obtained from Equation (8-40) by letting $A_1 = A_2$ and $F_{12} = 1.0$. Thus

$$\frac{q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_2 - 1} \quad \textbf{[8-42]}$$

When two long concentric cylinders as shown in Figure 8-29 exchange heat we may again apply Equation (8-40). Rewriting the equation and noting that $F_{12} = 1.0$,

$$q = \frac{\sigma A_1(T_1^4 - T_2^4)}{1/\epsilon_1 + (A_1/A_2)(1/\epsilon_2 - 1)} \quad \textbf{[8-43]}$$

The area ratio A_1/A_2 may be replaced by the diameter ratio d_1/d_2 when cylindrical bodies are concerned.

Convex Object in Large Enclosure

Equation (8-43) is particularly important when applied to the limiting case of a convex object completely enclosed by a very large concave surface. In this instance $A_1/A_2 \to 0$

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Figure 8-29 | Radiation exchange between two cylindrical surfaces.

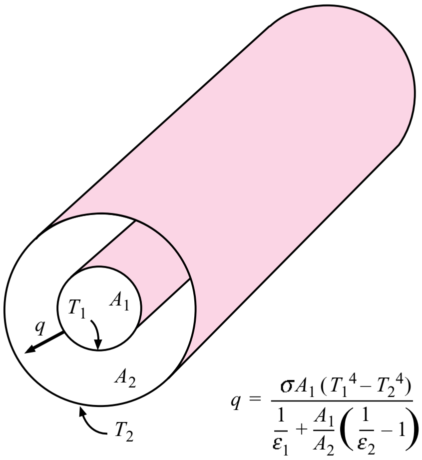
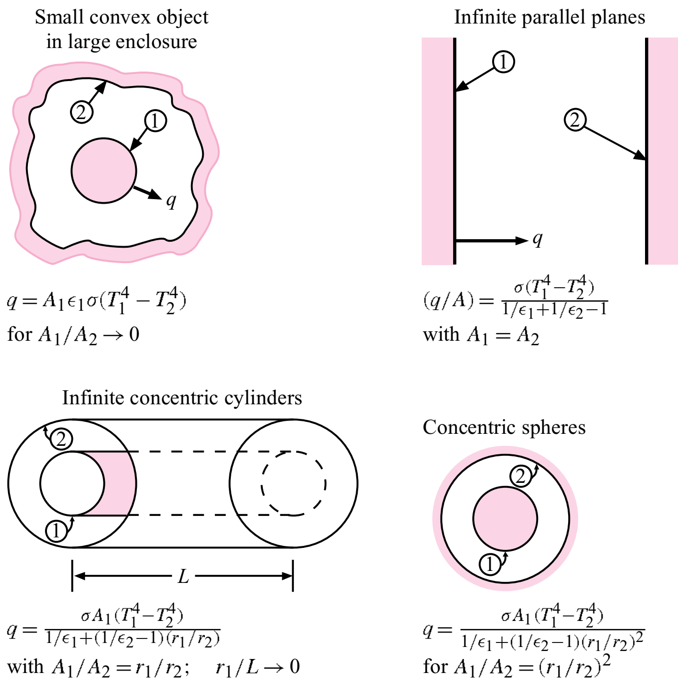


Figure 8-30 | Radiation heat transfer between simple two-body diffuse, gray surfaces. In all cases $F_{12} = 1.0$.



and the following simple relation results:

$$q = \sigma A_1 \epsilon_1 (T_1^4 - T_2^4) \qquad \textbf{[8-43}\boldsymbol{a}\textbf{]}$$

This equation is readily applied to calculate the radiation-energy loss from a hot object in a large room.

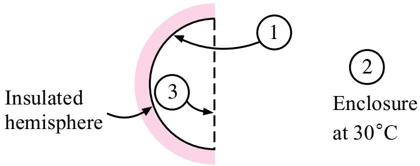
Some of the radiation heat-transfer cases for simple two-body problems are summarized in Figure 8-30. In this figure, both surfaces are assumed to be gray and diffuse.

Open Hemisphere in Large Room

EXAMPLE 8-8

The 30-cm-diameter hemisphere in Figure Example 8-8 is maintained at a constant temperature of 500°C and insulated on its back side. The surface emissivity is 0.4. The opening exchanges radiant energy with a large enclosure at 30°C. Calculate the net radiant exchange.

Figure Example 8-8



■ Solution

This is an object completely surrounded by a large enclosure but the inside surface of the sphere is *not convex;* that is, it sees itself, and therefore we are *not* permitted to use Equation (8-43*a*). In the figure we take the inside of the sphere as surface 1 and the enclosure as surface 2. We also create an imaginary surface 3 covering the opening. We actually have a two-surface problem (surfaces 1 and 2) and therefore may use Equation (8-40) to calculate the heat transfer. Thus,

$$
\begin{aligned}
E_{b_1} &= \sigma T_1^4 = \sigma(773)^4 = 20{,}241 \text{ W/m}^2 \\
E_{b_2} &= \sigma T_2^4 = \sigma(303)^4 = 478 \text{ W/m}^2 \\
A_1 &= 2\pi r^2 = (2)\pi(0.15)^2 = 0.1414 \text{ m}^2 \\
\frac{1-\epsilon_1}{\epsilon_1 A_1} &= \frac{0.6}{(0.4)(0.1414)} = 10.61 \\
A_2 &\to \infty
\end{aligned}
$$

so that

$$\frac{1-\epsilon_2}{\epsilon_2 A_2} \to 0$$

Now, at this point we recognize that all of the radiation leaving surface 1 that will eventually arrive at enclosure 2 will also hit the imaginary surface 3 (i.e., $F_{12} = F_{13}$). We also recognize that

$$A_1 F_{13} = A_3 F_{31}$$

But, $F_{31} = 1.0$ so that

$$F_{13} = F_{12} = \frac{A_3}{A_1} = \frac{\pi r^2}{2\pi r^2} = 0.5$$

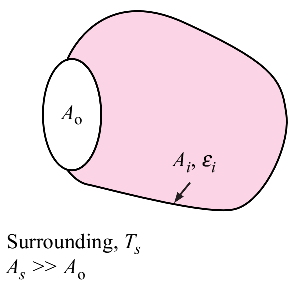
Then $1/A_1 F_{12} = 1/(0.1414)(0.5) = 14.14$ and we can calculate the heat transfer by inserting the quantities in Equation (8-40):

$$q = \frac{20{,}241 - 478}{10.61 + 14.14 + 0} = 799 \text{ W}$$

Apparent Emissivity of a Cavity

Consider the cavity shown in Figure 8-31 having an internal concave surface area A_i and emissivity ϵ_i radiating out through the opening with area A_o. The cavity exchanges radiant energy with a surrounding at T_s having an area that is large compared to the area of the opening. We want to determine a relationship for an apparent emissivity of the opening in terms of the above variables. If one considers the imaginary surface A_o covering the

Figure 8-31 | Apparent emissivity of cavity.



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opening and exchanging heat with A_i we have

$$F_{oi} = 1.0$$

and, from reciprocity,

$$A_o F_{oi} = A_i F_{io}$$

But, $F_{io} = F_{is}$ so that

$$A_i F_{is} = A_o \qquad \textbf{[8-44]}$$

The net radiant exchange of surface A_i with the large enclosure A_s is given by

$$q_{i-s} = (E_{bi} - E_{bs})/[(1-\epsilon_i)/\epsilon_i A_i + 1/A_i F_{is}] \qquad \textbf{[8-45]}$$

and the net radiant energy exchange of an imaginary surface A_o having an apparent emissivity ϵ_a with the large surroundings is given by Equation (8-43*a*) as

$$q_{o-s} = \epsilon_a A_o (E_{bi} - E_{bs}) \qquad \textbf{[8-46]}$$

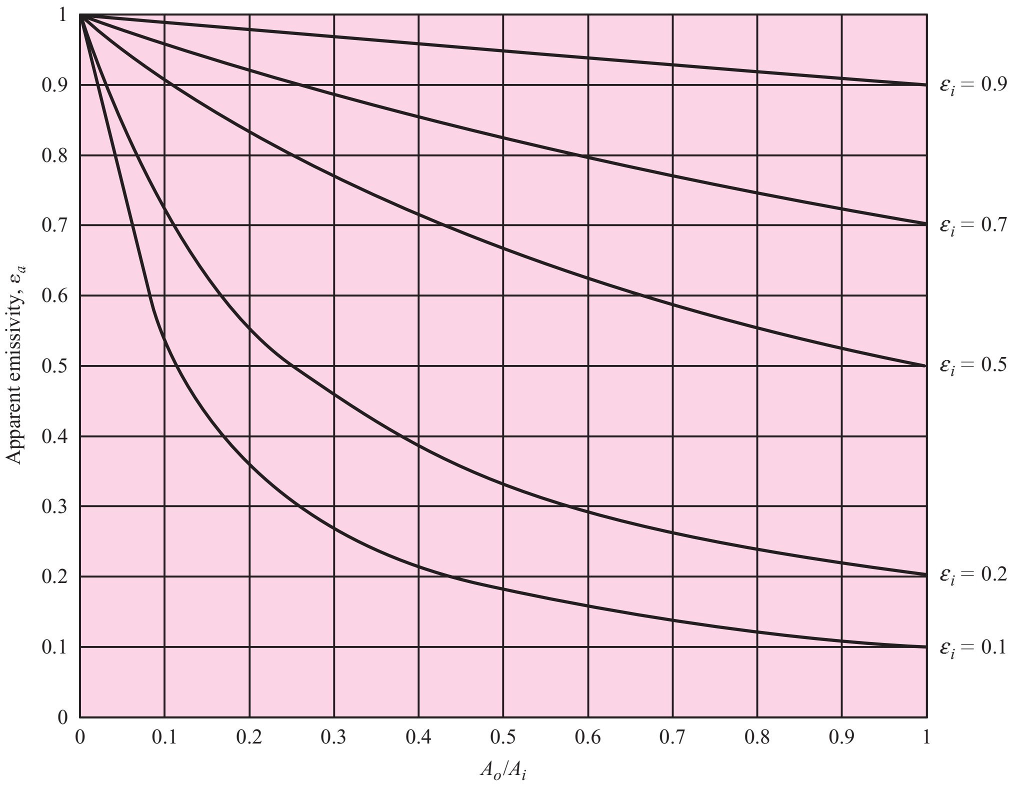
for A_o at the same temperature at the cavity surface A_i. Substituting (8-44) in (8-45) and equating (8-45) and (8-46) gives, after algebraic manipulation,

$$\epsilon_a = \epsilon_i A_i / [A_o + \epsilon_i (A_i - A_o)] \qquad \textbf{[8-47]}$$

We can observe the following behavior for ϵ_a in limiting cases:

$$\epsilon_a = \epsilon_i \quad \text{for } A_o = A_i \text{ or no cavity at all}$$

Figure 8-32 | Apparent emissivity of cavity.



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and

$$\epsilon_a \to 1.0 \quad \textit{for } A_i \gg A_o$$

or a very large cavity. A plot of Equation (8-47) is given in Figure 8-32.

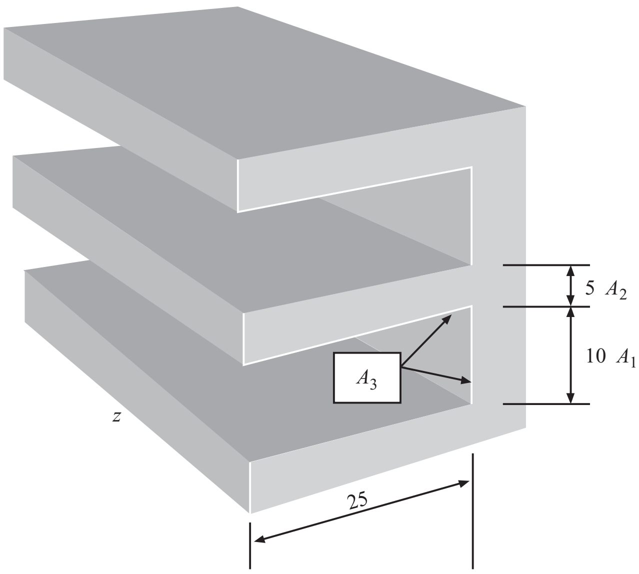
The apparent emissivity concept may also be used to analyze transient problems that admit to the lumped capacity approximation. Such an example is discussed in Appendix D, section D-6. In addition, an example is given of multiple lumped capacity formulation applied to the heating of a box of electronic components exchanging energy by convection and radiation with an enclosure.

Effective Emissivity of Finned Surface

EXAMPLE 8-9

A repeating finned surface having the relative dimensions shown in Figure Example 8-9 is utilized to produce a higher effective emissivity than that for a flat surface alone. Calculate the effective emissivity of the combination of fin tip and open cavity for surface emissivities of 0.2, 0.5, and 0.8.

Figure Example 8-9



■ Solution

For unit depth in the *z*-dimension we have

$$A_1 = 10, \;\; A_2 = 5, \;\; A_3 = (2)(25) + 10 = 60$$

The apparent emissivity of the open cavity area A_1 is given by Equation (8-47) as

$$\epsilon_{a1} = \epsilon A_3 / [A_1 + \epsilon(A_3 - A_1)] = 60\epsilon/(10 + 50\epsilon) \qquad \textbf{[a]}$$

For constant *surface emissivity* the emitted energy from the total erea $A_1 + A_2$ is

$$(\epsilon_{a1} A_1 + \epsilon A_2) E_b \qquad \textbf{[b]}$$

and the energy emitted per unit area for that total area is

$$[(\epsilon a_1 A_1 + \epsilon A_2)/(A_1 + A_2)] E_b \qquad \textbf{[c]}$$

The coefficient of E_b is the effective emissivity, ϵ_{eff} of the combination of the flat surface and open cavity. Inserting Equation (a) in (c) gives the following numerical values:

For $\epsilon = 0.2$ $\quad \epsilon_{\text{eff}} = 0.4667$
For $\epsilon = 0.5$ $\quad \epsilon_{\text{eff}} = 0.738$
For $\epsilon = 0.8$ $\quad \epsilon_{\text{eff}} = 0.907$

One could employ these effective values to calculate the radiation performance of such a finned surface in conjunction with applicable radiation properties of surrounding surfaces.

8-8 | RADIATION SHIELDS

One way of reducing radiant heat transfer between two particular surfaces is to use materials that are highly reflective. An alternative method is to use radiation shields between the heat-exchange surfaces. These shields do not deliver or remove any heat from the overall system; they only place another resistance in the heat-flow path so that the overall heat transfer is retarded. Consider the two parallel infinite planes shown in Figure 8-33a. We have shown that the heat exchange between these surfaces may be calculated with Equation (8-42). Now consider the same two planes, but with a radiation shield placed between them, as in Figure 8-33b. The heat transfer will be calculated for this latter case and compared with the heat transfer without the shield.

Since the shield does not deliver or remove heat from the system, the heat transfer between plate 1 and the shield must be precisely the same as that between the shield and plate 2, and this is the overall heat transfer. Thus

$$\left(\frac{q}{A}\right)_{1-3} = \left(\frac{q}{A}\right)_{3-2} = \frac{q}{A}$$

$$\frac{q}{A} = \frac{\sigma(T_1^4 - T_3^4)}{1/\epsilon_1 + 1/\epsilon_3 - 1} = \frac{\sigma(T_3^4 - T_2^4)}{1/\epsilon_3 + 1/\epsilon_2 - 1} \qquad \textbf{[8-48]}$$

The only unknown in Equation (8-48) is the temperature of the shield T_3. Once this temperature is obtained, the heat transfer is easily calculated. If the emissivities of all three surfaces are equal, that is, $\epsilon_1 = \epsilon_2 = \epsilon_3$, we obtain the simple relation

$$T_3^4 = \frac{1}{2}(T_1^4 + T_2^4) \qquad \textbf{[8-49]}$$

Figure 8-33 | Radiation between parallel infinite planes with and without a radiation shield.

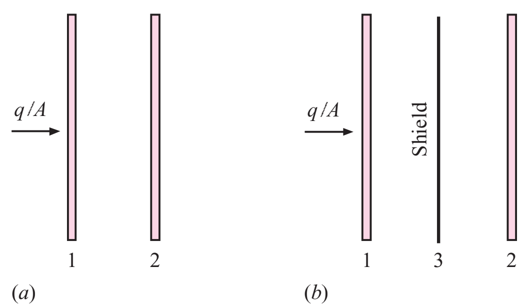


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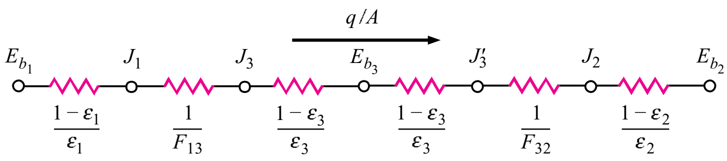
Figure 8-34 | Radiation network for two parallel planes separated by one radiation shield.



and the heat transfer is

$$\frac{q}{A} = \frac{\frac{1}{2}\sigma(T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_3 - 1}$$

But since $\epsilon_3 = \epsilon_2$, we observe that this heat flow is just one-half of that which would be experienced if there were no shield present. The radiation network corresponding to the situation in Figure 8-33*b* is given in Figure 8-34.

By inspecting the network in Figure 8-34, we see that the radiation heat transfer is impeded by the insertion of three resistances more than would be present with just two surfaces facing each other: an extra space resistance and two extra surface resistances for the shield. The higher the reflectivity of the shield (i.e., the smaller its emissivity), the greater will be the surface resistances inserted. Even for a black shield, with $\epsilon = 1$ and zero surface resistance, there will still be an extra space resistance inserted in the network. As a result, insertion of any surface that intercepts the radiation path will always cause some reduction in the heat-transfer rate, regardless of its surface emissive properties.

Multiple-radiation-shield problems may be treated in the same manner as that outlined above. When the emissivities of all surfaces are different, the overall heat transfer may be calculated most easily by using a series radiation network with the appropriate number of elements, similar to the one in Figure 8-34. If the emissivities of all surfaces are equal, a rather simple relation may be derived for the heat transfer when the surfaces may be considered as infinite parallel planes. Let the number of shields be n. Considering the radiation network for the system, all the "surface resistances" would be the same since the emissivities are equal. There would be two of these resistances for each shield and one for each heat-transfer surface. There would be $n+1$ "space resistances," and these would all be unity since the radiation shape factors are unity for the infinite parallel planes. The total resistance in the network would thus be

$$R(n \text{ shields}) = (2n+2)\frac{1-\epsilon}{\epsilon} + (n+1)(1) = (n+1)\left(\frac{2}{\epsilon} - 1\right)$$

The resistance when no shield is present is

$$R(\text{no shield}) = \frac{1}{\epsilon} + \frac{1}{\epsilon} - 1 = \frac{2}{\epsilon} - 1$$

We note that the resistance with the shields in place is $n+1$ times as large as when the shields are absent. Thus

$$\left(\frac{q}{A}\right)_{\substack{\text{with}\\\text{shields}}} = \frac{1}{n+1}\left(\frac{q}{A}\right)_{\substack{\text{without}\\\text{shields}}} \qquad \textbf{[8-50]}$$

if the temperatures of the heat-transfer surfaces are maintained the same in both cases. The radiation-network method may also be applied to shield problems involving cylindrical systems. In these cases the proper area relations must be used in formulating the resistance elements.

Notice that the analyses above, dealing with infinite parallel planes, have been carried out on a per-unit-area basis because all areas are the same.

EXAMPLE 8-10 Heat-Transfer Reduction with Parallel-Plate Shield

Two very large parallel planes with emissivities 0.3 and 0.8 exchange heat. Find the percentage reduction in heat transfer when a polished-aluminum radiation shield ($\epsilon = 0.04$) is placed between them.

Solution

The heat transfer without the shield is given by

$$\frac{q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_2 - 1} = 0.279\sigma(T_1^4 - T_2^4)$$

The radiation network for the problem with the shield in place is shown in Figure 8-34. The resistances are

$$\frac{1-\epsilon_1}{\epsilon_1} = \frac{1-0.3}{0.3} = 2.333$$

$$\frac{1-\epsilon_3}{\epsilon_3} = \frac{1-0.04}{0.04} = 24.0$$

$$\frac{1-\epsilon_2}{\epsilon_2} = \frac{1-0.8}{0.8} = 0.25$$

The total resistance with the shield is

$$2.333 + (2)(24.0) + (2)(1) + 0.25 = 52.583$$

and the heat transfer is

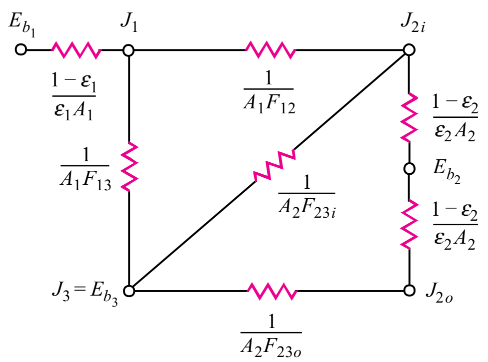
$$\frac{q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{52.583} = 0.01902\sigma(T_1^4 - T_2^4)$$

so that the heat transfer is *reduced* by 93.2 percent.

EXAMPLE 8-11 Open Cylindrical Shield in Large Room

The two concentric cylinders of Example 8-3 have $T_1 = 1000$ K, $\epsilon_1 = 0.8$, $\epsilon_2 = 0.2$ and are located in a large room at 300 K. The outer cylinder is in radiant balance. Calculate the temperature of the outer cylinder and the total heat lost by the inner cylinder.

Figure Example 8-11



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Solution

The network for this problem is shown in Figure Example 8-11. The room is designated as surface 3 and $J_3 = E_{b_3}$, because the room is very large (i.e., its surface resistance is very small). In this problem we must consider the inside and outside of surface 2 and thus have subscripts i and o to designate the respective quantities. The shape factors can be obtained from Example 8-3 as

$$F_{12} = 0.8253 \qquad F_{13} = 0.1747$$
$$F_{23i} = (2)(0.1294) = 0.2588 \qquad F_{23o} = 1.0$$

Also,

$$A_1 = \pi(0.1)(0.2) = 0.06283 \text{ m}^2$$
$$A_2 = \pi(0.2)(0.2) = 0.12566 \text{ m}^2$$
$$E_{b_1} = (5.669 \times 10^{-8})(1000)^4 = 5.669 \times 10^4 \text{ W/m}^2$$
$$E_{b_3} = (5.669 \times 10^{-8})(300)^4 = 459.2 \text{ W/m}^2$$

and the resistances may be calculated as

$$\frac{1-\epsilon_1}{\epsilon_1 A_1} = 3.979 \qquad \frac{1-\epsilon_2}{\epsilon_2 A_2} = 31.83$$
$$\frac{1}{A_1 F_{12}} = 19.28 \qquad \frac{1}{A_2 F_{23i}} = 30.75$$
$$\frac{1}{A_2 F_{23o}} = 7.958 \qquad \frac{1}{A_1 F_{13}} = 91.1$$

The network could be solved as a series-parallel circuit to obtain the heat transfer, but we will need the radiosities anyway, so we set up three nodal equations to solve for J_1, J_{2i}, and J_{2o}. We sum the currents into each node and set them equal to zero:

node J_1: $$\frac{E_{b_1} - J_1}{3.979} + \frac{E_{b_3} - J_1}{91.1} + \frac{J_{2i} - J_1}{19.28} = 0$$

node J_{2i}: $$\frac{J_1 - J_{2i}}{19.28} + \frac{E_{b_3} - J_{2i}}{30.75} + \frac{J_{2o} - J_{2i}}{(2)(31.83)} = 0$$

node J_{2o}: $$\frac{E_{b_3} - J_{2o}}{7.958} + \frac{J_{2i} - J_{2o}}{(2)(31.83)} = 0$$

These equations have the solution

$$J_1 = 49{,}732 \text{ W/m}^2$$
$$J_{2i} = 26{,}444 \text{ W/m}^2$$
$$J_{2o} = 3346 \text{ W/m}^2$$

The heat transfer is then calculated from

$$q = \frac{E_{b_1} - J_1}{(1-\epsilon_1)/\epsilon_1 A_1} = \frac{56{,}690 - 49{,}732}{3.979} = 1749 \text{ W} \quad [5968 \text{ Btu/h}]$$

From the network we see that

$$E_{b_2} = \frac{J_{2i} + J_{2o}}{2} = \frac{26{,}444 + 3346}{2} = 14{,}895 \text{ W/m}^2$$

and

$$T_2 = \left(\frac{14{,}895}{5.669 \times 10^{-8}}\right)^{1/4} = 716 \text{ K} \quad [829°\text{F}]$$

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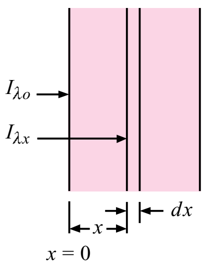
If the outer cylinder had not been in place acting as a "shield" the heat loss from cylinder 1 could have been calculated from Equation (8-43*a*) as

$$q = \epsilon_1 A_1 (E_{b_1} - E_{b_3})$$
$$= (0.8)(0.06283)(56{,}690 - 459.2) = 2826 \text{ W} \quad [9644 \text{ Btu/h}]$$

8-9 | GAS RADIATION

Radiation exchange between a gas and a heat-transfer surface is considerably more complex than the situations described in the preceding sections. Unlike most solid bodies, gases are in many cases transparent to radiation. When they absorb and emit radiation, they usually do so only in certain narrow wavelength bands. Some gases, such as N_2, O_2, and others of nonpolar symmetrical molecular structure, are essentially transparent at low temperatures, while CO_2, H_2O, and various hydrocarbon gases radiate to an appreciable extent.

Figure 8-35 | Absorption in a gas layer.



The absorption of radiation in gas layers may be described analytically in the following way, considering the system shown in Figure 8-35. A monochromatic beam of radiation having an intensity I_λ impinges on the gas layer of thickness dx. The decrease in intensity resulting from absorption in the layers is assumed to be proportional to the thickness of the layer and the intensity of radiation at that point. Thus

$$dI_\lambda = -a_\lambda I_\lambda \, dx \qquad \textbf{[8-51]}$$

where the proportionality constant a_λ is called the *monochromatic absorption coefficient*. Integrating this equation gives

$$\int_{I_{\lambda 0}}^{I_{\lambda x}} \frac{dI_\lambda}{I_\lambda} = \int_0^x -a_\lambda \, dx$$

or

$$\frac{I_{\lambda x}}{I_{\lambda 0}} = e^{-a_\lambda x} \qquad \textbf{[8-52]}$$

Equation (8-52) is called Beer's law and represents the familiar exponential-decay formula experienced in many types of radiation analyses dealing with absorption. In accordance with our definitions in Section 8-3, the monochromatic transmissivity will be given as

$$\tau_\lambda = e^{-\alpha_\lambda x} \qquad \textbf{[8-53]}$$

If the gas is nonreflecting, then

$$\tau_\lambda + \alpha_\lambda = 1$$

and

$$\alpha_\lambda = 1 - e^{-\alpha_\lambda x} \qquad \textbf{[8-54]}$$

As we have mentioned, gases frequently absorb only in narrow wavelength bands. For example, water vapor has an absorptivity of about 0.7 between 1.4 and 1.5 μm, about 0.8 between 1.6 and 1.8 μm, about 1.0 between 2.6 and 2.8 μm, and about 1.0 between 5.5 and 7.0 μm. As we have seen in Equation (8-54), the absorptivity will also be a function of the thickness of the gas layer, and there is a temperature dependence as well.

The calculation of gas-radiation properties is quite complicated, and References 23 to 25 should be consulted for detailed information.

8-10 | RADIATION NETWORK FOR AN ABSORBING AND TRANSMITTING MEDIUM

The foregoing discussions have shown the methods that may be used to calculate radiation heat transfer between surfaces separated by a completely transparent medium. The radiation-network method is used to great advantage in these types of problems.

Many practical problems involve radiation heat transfer through a medium that is both absorbing and transmitting. The various glass substances are one example of this type of medium; gases are another. Some approximate transmissivities or glass substances over the wavelength range of $0.5\ \mu\text{m} < \lambda < 2.5\ \mu\text{m}$ are given in Table 8-3.

Keeping in mind the complications involved with the band absorption characteristics of gases, we shall now examine a simplified radiation network method for analyzing transmitting absorbing systems.

To begin, let us consider a simple case, that of two nontransmitting surfaces that see each other and nothing else. In addition, we let the space between these surfaces be occupied by a transmitting and absorbing medium. The practical problem might be that of two large planes separated by either an absorbing gas or a transparent sheet of glass or plastic. The situation is shown schematically in Figure 8-36. The transparent medium is designated by the subscript m. We make the assumption that the medium is nonreflecting and that Kirchhoff's identity applies, so that

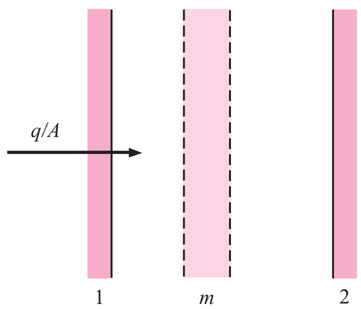
$$\alpha_m + \tau_m = 1 = \epsilon_m + \tau_m \quad \textbf{[8-55]}$$

The assumption that the medium is nonreflecting is a valid one when gases are considered. For glass or plastic plates this is not necessarily true, and reflectivities of the order

Table 8-3 | Approximate transmissivities for glasses at 20°C.

Glass		**τ (0.5 μm < λ < 2.5 μm)**
Soda lime glass		
Thickness =	1.6 mm	0.9
=	6.4 mm	0.75
=	9.5 mm	0.7
=	12.7 mm	0.65
Aluminum silicate, thickness =	12.7 mm	0.85
Borosilicate =	12.7 mm	0.8
Fused silica =	12.7 mm	0.85
Pyrex =	12.7 mm	0.65

Figure 8-36 | Radiation system consisting of a transmitting medium between two planes.



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of 0.1 are common for many glass substances. In addition, the transmissive properties of glasses are usually limited to a narrow wavelength band between about 0.2 and 4 μm. Thus the analysis that follows is highly idealized and serves mainly to furnish a starting point for the solution of problems in which transmission of radiation must be considered. Other complications with gases are mentioned later in the discussion. When both reflection and transmission must be taken into account, the analysis techniques discussed in Section 8-12 must be employed.

Returning to the analysis, we note that the medium can emit and transmit radiation from one surface to the other. Our task is to determine the network elements to use in describing these two types of exchange processes. The transmitted energy may be analyzed as follows. The energy leaving surface 1 that is transmitted through the medium and arrives at surface 2 is

$$J_1 A_1 F_{12} \tau_m$$

and the energy leaving surface 2 and arrives at surface 1 is

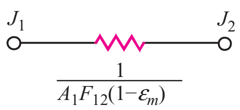
$$J_2 A_2 F_{21} \tau_m$$

The net exchange in the transmission process is therefore

$$q_{1-2_{\text{transmitted}}} = A_1 F_{12} \tau_m (J_1 - J_2)$$

$$q_{1-2_{\text{transmitted}}} = \frac{J_1 - J_2}{1/A_1 F_{12}(1-\epsilon_m)} \qquad \textbf{[8-56]}$$

Figure 8-37 | Network element for transmitted radiation through medium.



and the network element that may be used to describe this process is shown in Figure 8-37.

Now consider the exchange process between surface 1 and the transmitting medium. Since we have assumed that this medium is nonreflecting, the energy leaving the medium (other than the transmitted energy, which we have already considered) is precisely the energy emitted by the medium

$$J_m = \epsilon_m E_{bm}$$

And of the energy leaving the medium, the amount which reaches surface 1 is

$$A_m F_{m1} J_m = A_m F_{m1} \epsilon_m E_{bm}$$

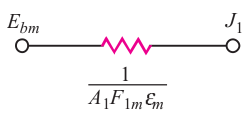
Of that energy leaving surface 1, the quantity that reaches the transparent medium is

$$J_1 A_1 F_{1m} \alpha_m = J_1 A_1 F_{1m} \epsilon_m$$

At this point we note that absorption in the medium means that the incident radiation has "reached" the medium. Consistent with the above relations, the net energy exchange between the medium and surface 1 is the difference between the amount emitted by the medium toward surface 1 and that absorbed which emanated from surface 1. Thus

$$q_{m-1_{\text{net}}} = A_m F_{m1} \epsilon_m E_{bm} - J_1 A_1 F_{1m} \epsilon_m$$

Figure 8-38 | Network element for radiation exchange between medium and surface.



Using the reciprocity relation

$$A_1 F_{1m} = A_m F_{m1}$$

we have

$$q_{m-1_{\text{net}}} = \frac{E_{bm} - J_1}{1/A_1 F_{1m} \epsilon_m} \qquad \textbf{[8-57]}$$

This heat-exchange process is represented by the network element shown in Figure 8-38. The total network for the physical situation of Figure 8-36 is shown in Figure 8-39.

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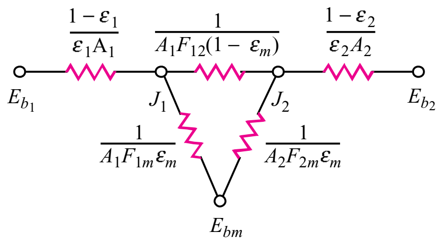
Figure 8-39 | Total radiation network for system of Figure 8-36.



If the transport medium is maintained at some fixed temperature, then the potential E_{bm} is fixed according to

$$E_{bm} = \sigma T_m^4$$

On the other hand, if no net energy is delivered to the medium, then E_{bm} becomes a floating node, and its potential is determined by the other network elements.

In reality, the radiation shape factors F_{1-2}, F_{1-m}, and F_{2-m} are unity for this example, so that the expression for the heat flow could be simplified to some extent; however, these shape factors are included in the network resistances for the sake of generality in the analysis.

When the practical problem of heat exchange between gray surfaces through an absorbing gas is encountered, the major difficulty is that of determining the transmissivity and emissivity of the gas. These properties are functions not only of the temperature of the gas, but also of the thickness of the gas layer; that is, thin gas layers transmit more radiation than thick layers. The usual practical problem almost always involves more than two heat-transfer surfaces, as in the simple example given above. As a result, the transmissivities between the various heat-transfer surfaces can be quite different, depending on their geometric orientation. Since the temperature of the gas will vary, the transmissive and emissive properties will vary with their location in the gas. One way of handling this situation is to divide the gas body into layers and set up a radiation network accordingly, letting the potentials of the various nodes "float," and thus arriving at the gas-temperature distribution. Even with this procedure, an iterative method must eventually be employed because the radiation properties of the gas are functions of the unknown "floating potentials." Naturally, if the temperature of the gas is uniform, the solution is much easier.

We shall not present the solution of a complex gas-radiation problem since the tedious effort required for such a solution is beyond the scope of our present discussion; however, it is worthwhile to analyze a two-layer transmitting system in order to indicate the general scheme of reasoning that might be applied to more complex problems.

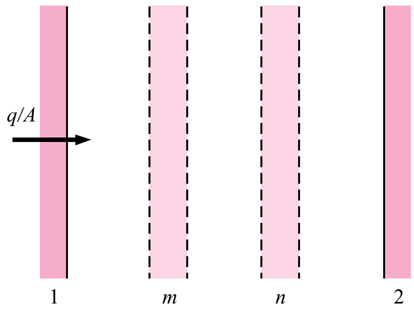
Consider the physical situation shown in Figure 8-40. Two radiating and absorbing surfaces are separated by two layers of transmitting and absorbing media. These two layers might represent two sheets of transparent media, such as glass, or they might represent the division of a separating gas into two parts for purposes of analysis. We designate the two transmitting and absorbing layers with the subscripts m and n. The energy exchange between surface 1 and m is given by

$$q_{1-m} = A_1 F_{1m} \epsilon_m J_1 - A_m F_{m1} \epsilon_m E_{bm} = \frac{J_1 - E_{bm}}{1/A_1 F_{1m} \epsilon_m} \qquad \textbf{[8-58]}$$

and that between surface 2 and n is

$$q_{2-n} = A_1 F_{2n} \epsilon_n J_2 - A_n F_{n2} \epsilon_n E_{bn} = \frac{J_2 - E_{bn}}{1/A_2 F_{2n} \epsilon_n} \qquad \textbf{[8-59]}$$

Figure 8-40 | Radiation system consisting of two transmitting layers between two planes.



Of that energy leaving surface 1, the amount arriving at surface 2 is

$$q_{1-2} = A_1 F_{12} J_1 \tau_m \tau_n = A_1 F_{12} J_1 (1-\epsilon_m)(1-\epsilon_n)$$

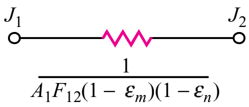
and of that energy leaving surface 2, the amount arriving at surface 1 is

$$q_{2-1} = A_2 F_{21} J_2 \tau_n \tau_m = A_2 F_{12} J_2 (1-\epsilon_n)(1-\epsilon_m)$$

so that the net energy exchange by transmission between surfaces 1 and 2 is

$$q_{1-2_{\text{transmitted}}} = A_1 F_{12}(1-\epsilon_m)(1-\epsilon_n)(J_1 - J_2) = \frac{J_1 - J_2}{1/A_1 F_{12}(1-\epsilon_m)(1-\epsilon_n)} \quad \textbf{[8-60]}$$

Figure 8-41 | Network element for transmitted radiation between planes.



and the network element representing this transmission is shown in Figure 8-41. Of that energy leaving surface 1, the amount that is absorbed in n is

$$q_{1-n} = A_1 F_{1n} J_1 \tau_m \epsilon_n = A_1 F_{1n} J_1 (1-\epsilon_m)\epsilon_n$$

Also,

$$q_{n-1} = A_n F_{n1} J_n \tau_m = A_n F_{n1} \epsilon_n E_{bn} (1-\epsilon_m)$$

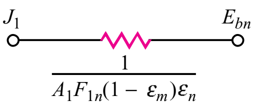
since

$$J_n = \epsilon_n E_{bn}$$

The net exchange between surface 1 and n is therefore

$$q_{1-n_{\text{net}}} = A_1 F_{1n}(1-\epsilon_m)\epsilon_n (J_1 - E_{bn}) = \frac{J_1 - E_{bn}}{1/A_1 F_{1n}(1-\epsilon_m)\epsilon_n} \quad \textbf{[8-61]}$$

Figure 8-42 | Network element for transmitted radiation for medium n to plane 1.



and the network element representing this situation is shown in Figure 8-42. In like manner, the net exchange between surface 2 and m is

$$q_{2-m_{\text{net}}} = \frac{J_2 - E_{bm}}{1/A_2 F_{2m}(1-\epsilon_n)\epsilon_m}$$

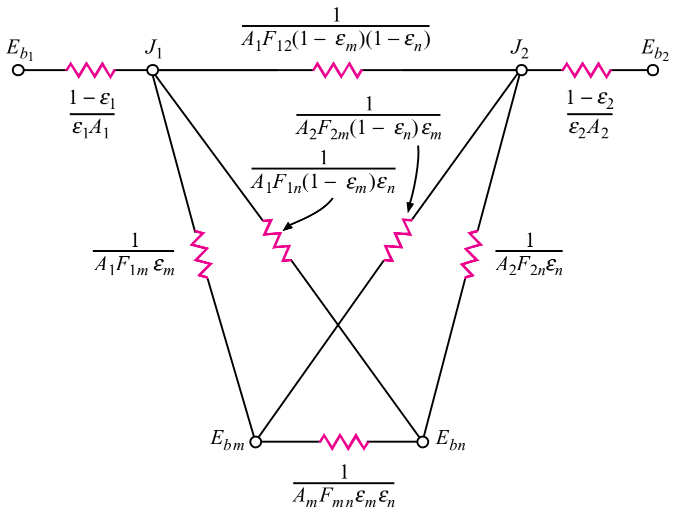
Of that radiation leaving m, the amount absorbed in n is

$$q_{m-n} = J_m A_m F_{mn} \alpha_n = A_m F_{mn} \epsilon_m \epsilon_n E_{bm}$$

and

$$q_{n-m} = A_n F_{nm} \epsilon_n \epsilon_m E_{bn}$$

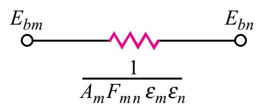
Figure 8-44 | Total radiation network for system of Figure 8-40.



so that the net energy exchange between m and n is

$$q_{m-n_{\text{net}}} = A_m F_{mn} \epsilon_m \epsilon_n (E_{bm} - E_{bn}) = \frac{E_{bm} - E_{bn}}{1/A_m F_{mn} \epsilon_m \epsilon_n} \qquad \textbf{[8-62]}$$

Figure 8-43 | Network element for radiation exchange between two transparent layers.



and the network element representing this energy transfer is given in Figure 8-43.

The final network for the entire heat-transfer process is shown in Figure 8-44, with the surface resistances added. If the two transmitting layers m and n are maintained at given temperatures, the solution to the network is relatively easy to obtain because only two unknown potentials J_1 and J_2 need be determined to establish the various heat-flow quantities. In this case the two transmitting layers will either absorb or lose a certain quantity of energy, depending on the temperature at which they are maintained.

When no net energy is delivered to the transmitting layers, nodes E_{bm} and E_{bn} must be left "floating" in the analysis; and for this particular system four nodal equations would be required for a solution of the problem.

EXAMPLE 8-12 Network for Gas Radiation Between Parallel Plates

Two large parallel planes are at $T_1 = 800$ K, $\epsilon_1 = 0.3$, $T_2 = 400$ K, $\epsilon_2 = 0.7$ and are separated by a gray gas having $\epsilon_g = 0.2$, $\tau_g = 0.8$. Calculate the heat-transfer rate between the two planes and the temperature of the gas using a radiation network. Compare with the heat transfer without presence of the gas.

Solution

The network shown in Figure 8-39 applies to this problem. All the shape factors are unity for large planes and the various resistors can be computed on a unit-area basis as

$$\frac{1-\epsilon_1}{\epsilon_1} = \frac{0.7}{0.3} = 2.333 \qquad \frac{1}{F_{12}(1-\epsilon_g)} = \frac{1}{1-0.2} = 1.25$$

$$\frac{1-\epsilon_2}{\epsilon_2} = \frac{0.3}{0.7} = 0.4286 \qquad \frac{1}{F_{1g}\epsilon_g} = \frac{1}{F_{2g}\epsilon_g} = \frac{1}{0.2} = 5.0$$

$$E_{b_1} = \sigma T_1^4 = 23{,}220 \text{ W/m}^2 \qquad E_{b_2} = \sigma T_2^4 = 1451 \text{ W/m}^2$$

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The equivalent resistance of the center "triangle" is

$$R = \frac{1}{1/1.25 + 1/(5.0 + 5.0)} = 1.1111$$

The total heat transfer is then

$$\frac{q}{A} = \frac{E_{b_1} - E_{b_2}}{\sum R} = \frac{23{,}200 - 1451}{2.333 + 1.111 + 0.4286} = 5616 \text{ W/m}^2$$

If there were no gas present the heat transfer would be given by Equation (8-42):

$$\frac{q}{A} = \frac{23{,}200 - 1451}{1/0.3 + 1/0.7 - 1} = 5781 \text{ W/m}^2$$

The radiosities may be computed from

$$\frac{q}{A} = (E_{b_1} - J_1)\left(\frac{\epsilon_1}{1 - \epsilon_1}\right) = (J_2 - E_{b_2})\left(\frac{\epsilon_2}{1 - \epsilon_2}\right) = 5616 \text{ W/m}^2$$

which gives $J_1 = 10{,}096$ W/m^2 and $J_2 = 3858$ W/m^2. For the network E_{bg} is just the mean of these values

$$E_{bg} = \frac{1}{2}(10{,}096 + 3858) = 6977 = \sigma T_g^4$$

so that the temperature of the gas is

$$T_g = 592.3 \text{ K}$$

8-11 | RADIATION EXCHANGE WITH SPECULAR SURFACES

All the preceding discussions have considered radiation exchange between diffuse surfaces. In fact, the radiation shape factors defined by Equation (8-21) hold only for diffuse radiation because the radiation was assumed to have no preferred direction in the derivation of this relation. In this section we extend the analysis to take into account some simple geometries containing surfaces that may have a specular type of reflection. No real surface is completely diffuse or completely specular. We shall assume, however, that all the surfaces to be considered *emit* radiation diffusely but that they may *reflect* radiation partly in a specular manner and partly in a diffuse manner. We therefore take the reflectivity to be the sum of a specular component and a diffuse component:

$$\rho = \rho_s + \rho_D \qquad \textbf{[8-63]}$$

It is still assumed that Kirchhoff's identity applies so that

$$\epsilon = \alpha = 1 - \rho \qquad \textbf{[8-64]}$$

The net heat lost by a surface is the difference between the energy emitted and absorbed:

$$q = A(\epsilon E_b - \alpha G) \qquad \textbf{[8-65]}$$

We define the *diffuse radiosity* J_D as the total *diffuse* energy leaving the surface per unit area and per unit time, or

$$J_D = \epsilon E_b + \rho_D G \qquad \textbf{[8-66]}$$

Solving for the irradiation G from Equation (8-66) and inserting in Equation (8-65) gives

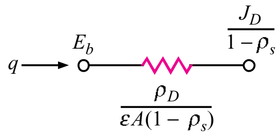
$$q = \frac{\epsilon A}{\rho_D}[E_b(\epsilon + \rho_D) - J_D]$$

or, written in a different form,

$$q = \frac{E_b - J_D/(1-\rho_s)}{\rho_D/[\epsilon A(1-\rho_s)]} \qquad \textbf{[8-67]}$$

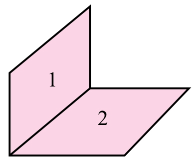
where $1 - \rho_s$ has been substituted for $\epsilon + \rho_D$. It is easy to see that Equation (8-67) may be represented with the network element shown in Figure 8-45. A quick inspection will show that this network element reduces to that in Figure 8-24 for the case of a surface that reflects in only a diffuse manner (i.e., for $\rho_s = 0$).

Figure 8-45 | Network element representing Equation (8-67).



Now let us compute the radiation exchange between two specular-diffuse surfaces. For the moment, we assume that the surfaces are oriented as shown in Figure 8-46. In this arrangement any diffuse radiation leaving surface 1 that is specularly reflected by 2 will not be reflected directly back to 1. This is an important point, for in eliminating such reflections we are considering only the *direct* diffuse exchange between the two surfaces. In subsequent paragraphs we shall show how the specular reflections must be analyzed. For the surfaces in Figure 8-46 the *diffuse* exchanges are given by

Figure 8-46



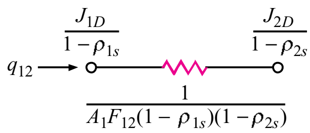
$$q_{1\to 2} = J_{1D}A_1F_{12}(1-\rho_{2s}) \qquad \textbf{[8-68]}$$

$$q_{2\to 1} = J_{2D}A_2F_{21}(1-\rho_{1s}) \qquad \textbf{[8-69]}$$

Equation (8-68) expresses the diffuse radiation leaving 1 that arrives at 2 *and* that may contribute to a diffuse radiosity of surface 2. The factor $1 - \rho_s$ represents the fraction absorbed plus the fraction reflected diffusely. The inclusion of this factor is important because we are considering only diffuse direct exchange, and thus must leave out the specular-reflection contribution for now. The net exchange is given by the difference between Equations (8-68) and (8-69), according to Reference 21.

$$q_{12} = \frac{J_{1D}/(1-\rho_{1s}) - J_{2D}/(1-\rho_{2s})}{1/[A_1F_{12}(1-\rho_{1s})(1-\rho_{2s})]} \qquad \textbf{[8-70]}$$

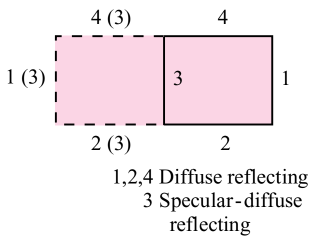
Figure 8-47 | Network element representing Equation (8-70).



The network element representing Equation (8-70) is shown in Figure 8-47.

To analyze specular reflections we utilize a technique presented in References 12 and 13. Consider the enclosure with four long surfaces shown in Figure 8-48. Surfaces 1, 2, and 4 reflect diffusely, while surface 3 has both a specular and a diffuse component of reflection. The dashed lines represent mirror images of the surfaces 1, 2, and 4 in surface 3. (A specular reflection produces a mirror image.) The nomenclature 2 (3) designates the mirror image of surface 2 in mirror 3.

Figure 8-48 | System with one specular-diffuse surface.



Now consider the radiation leaving 2 that arrives at 1. There is a direct diffuse radiation of

$$(q_{2\to 1})^{\text{direct}}_{\text{diffuse}} = J_2A_2F_{21} \qquad \textbf{[8-71]}$$

Part of the diffuse radiation from 2 is specularly reflected in 3 and strikes 1. This specularly reflected radiation acts like *diffuse* energy coming from the image surface 2 (3). Thus we may write

$$(q_{2\to 1})^{\text{specular}}_{\text{reflected}} = J_2A_{2(3)}F_{2(3)1}\rho_{3s} \qquad \textbf{[8-72]}$$

The radiation shape factor $F_{2(3)1}$ is the one between surface 2 (3) and surface 1. The reflectivity ρ_{3s} is inserted because only this fraction of the radiation gets to 1. Of course,

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$A_2 = A_{2(3)}$. We now have

$$q_{2\to 1} = J_2 A_2 (F_{21} + \rho_{3s} F_{2(3)1}) \qquad \textbf{[8-73]}$$

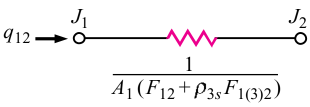
Similar reasoning leads to

$$q_{1\to 2} = J_1 A_1 (F_{12} + \rho_{3s} F_{1(3)2}) \qquad \textbf{[8-74]}$$

Combining Equations (8-73) and (8-74) and making use of the reciprocity relation $A_1 F_{12} = A_2 F_{21}$ gives

$$q_{12} = \frac{J_1 - J_2}{1/[A_1(F_{12} + \rho_{3s} F_{1(3)2})]} \qquad \textbf{[8-75]}$$

Figure 8-49 | Network element for Equation (8-75).



The network element represented by Equation (8-75) is shown in Figure 8-49.

Analogous network elements may be developed for radiation between the other surfaces in Figure 8-48, so that the final complete network becomes as shown in Figure 8-50. It is to be noted that the elements connecting to J_{3D} are simple modifications of the one shown in Figure 8-47 since $\rho_{1s} = \rho_{2s} = \rho_{4s} = 0$. An interesting observation can be made about this network for the case where $\rho_{3D} = 0$. In this instance surface 3 is completely specular and

$$J_{3D} = \epsilon_3 E_{b_3}$$

so that we are left with only three unknowns, J_1, J_2, and J_4, when surface 3 is completely specular-reflecting.

Now let us complicate the problem a step further by letting the enclosure have two specular-diffuse surfaces, as shown in Figure 8-51. In this case multiple images may be formed as shown. Surface 1 (3, 2) represents the image of 1 after it is viewed first through 3 and then through 2. In other words, it is the image of surface 1 (3) in mirror 2. At the same location is surface 1 (2, 3), which is the image of surface 1 (2) in mirror 3.

This problem is complicated because multiple specular reflections must be considered. Consider the exchange between surfaces 1 and 4. Diffuse energy leaving 1 can arrive at 4 in

Figure 8-50 | Complete radiation network for system in Figure 8-48.

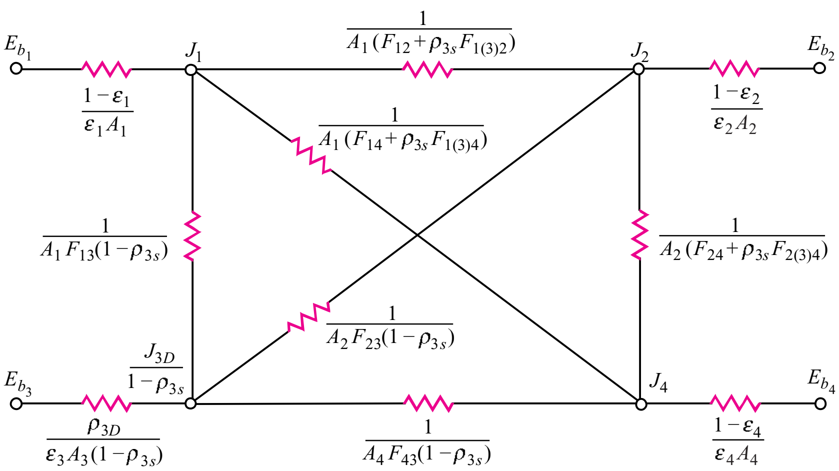
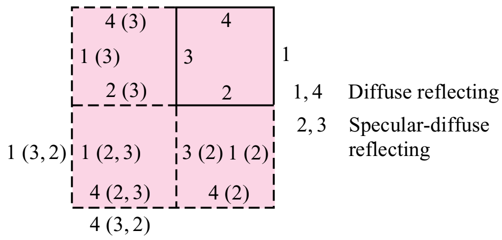


Figure 8-51 | System with two specular-diffuse surfaces.



five possible ways:

direct:	$J_1 A_1 F_{14}$	
reflection in 2 only:	$J_1 A_1 F_{1(2)4} \rho_{2s}$	
reflection in 3 only:	$J_1 A_1 F_{1(3)4} \rho_{3s}$	**[8-76]**
reflection first in 2 and then in 3:	$J_1 A_1 \rho_{3s} \rho_{2s} F_{1(2,3)4}$	
reflection first in 3 and then in 2:	$J_1 A_1 \rho_{2s} \rho_{3s} F_{1(3,2)4}$	

The last shape factor, $F_{1(3,2)4}$, is zero because surface 1 (3, 2) cannot see surface 4 when looking *through* mirror 2. On the other hand, $F_{1(2,3)4}$ is not zero because surface 1 (2, 3) can see surface 4 when looking through mirror 3. The sum of the above terms is given as

$$q_{1\to4} = J_1 A_1 (F_{14} + \rho_{2s} F_{1(2)4} + \rho_{3s} F_{1(3)4} + \rho_{3s} \rho_{2s} F_{1(2,3)4}) \qquad \textbf{[8-77]}$$

In a similar manner,

$$q_{4\to1} = J_4 A_4 (F_{41} + \rho_{2s} F_{4(2)1} + \rho_{3s} F_{4(3)1} + \rho_{3s} \rho_{2s} F_{4(3,2)1}) \qquad \textbf{[8-78]}$$

Subtracting these two equations and applying the usual reciprocity relations gives the network element shown in Figure 8-52.

Now consider the diffuse exchange between surfaces 1 and 3. Of the energy leaving 1, the amount which contributes to the diffuse radiosity of surface 3 is

$$q_{1\to3} = J_1 A_1 F_{13} (1 - \rho_{3s}) + J_1 A_1 \rho_{2s} F_{1(2)3} (1 - \rho_{3s}) \qquad \textbf{[8-79]}$$

The first term represents the direct exchange, and the second term represents the exchange after one specular reflection in mirror 2. As before, the factor $1 - \rho_{3s}$ is included to leave out of consideration the specular reflection from 3. This reflection, of course, is taken into account in other terms. The *diffuse* energy going from 3 to 1 is

$$q_{3\to1} = J_{3D} A_3 F_{31} + J_{3D} A_3 \rho_{2s} F_{3(2)1} \qquad \textbf{[8-80]}$$

The first term is the direct radiation, and the second term is that which is specularly reflected in mirror 2. Combining Equations (8-79) and (8-80) gives the network element shown in Figure 8-53.

Figure 8-52 | Network element representing exchange between surfaces 1 and 4 of Figure 8-51.

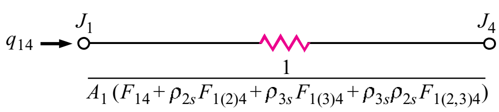


Figure 8-53 | Network element representing exchange between surfaces 1 and 3 of Figure 8-51.

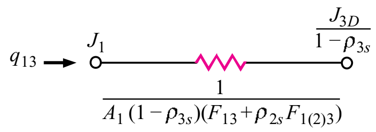
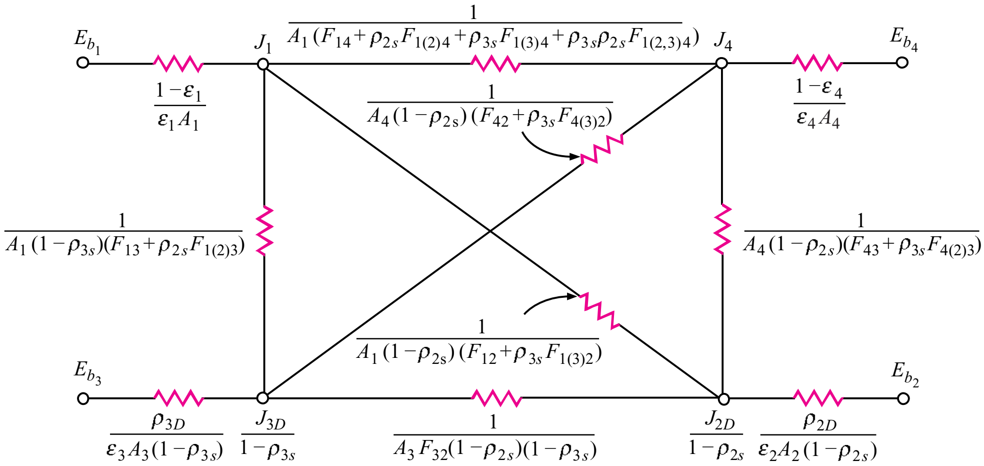


Figure 8-54 | Complete radiation network for system in Figure 8-51.



The above two elements are typical for the enclosure of Figure 8-51 and the other elements may be constructed by analogy. Thus the final complete network is given in Figure 8-54.

If both surfaces 2 and 3 are pure specular reflectors, that is,

$$\rho_{2D} = \rho_{3D} = 0$$

we have

$$J_{2D} = \epsilon_2 E_{b_2} \qquad J_{3D} = \epsilon_3 E_{b_3}$$

and the network involves only two unknowns, J_1 and J_4, under these circumstances.

We could complicate the calculation further by installing the specular surfaces opposite each other. In this case there would be an infinite number of images, and a series solution would have to be obtained; however, the series for such problems usually converge rather rapidly. The reader should consult Reference 13 for further information on this aspect of radiation exchange between specular surfaces.

8-12 | RADIATION EXCHANGE WITH TRANSMITTING, REFLECTING, AND ABSORBING MEDIA

We now consider a simple extension of the presentations in Sections 8-10 and 8-11 to analyze a medium where reflection, transmission, and absorption modes are all important.

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As in Section 8-10, we shall analyze a system consisting of two parallel diffuse planes with a medium in between that may absorb, transmit, and reflect radiation. For generality we assume that the surface of the transmitting medium may have both a specular and a diffuse component of reflection. The system is shown in Figure 8-55.

For the transmitting medium m we have

$$\alpha_m + \rho_{mD} + \rho_{ms} + \tau_m = 1 \qquad \textbf{[8-81]}$$

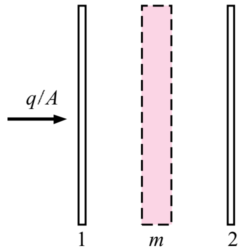


Figure 8-55 | Physical system for analysis of transmitting and reflecting layers.

Also

$$\epsilon_m = \alpha_m$$

The diffuse radiosity of a particular surface of the medium is defined by

$$J_{mD} = \epsilon_m E_{bm} + \rho_{mD} G \qquad \textbf{[8-82]}$$

where G is the irradiation on the particular surface. Note that J_{mD} no longer represents the total diffuse energy leaving a surface. Now it represents only emission and diffuse reflection. The transmitted energy will be analyzed with additional terms. As before, the heat exchange is written

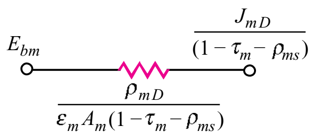
$$q = A(\epsilon E_b - \alpha G) \qquad \textbf{[8-83]}$$

Solving for G from Equation (8-82) and making use of Equation (8-81) gives

$$q = \frac{E_{bm} - J_{mD}/(1-\tau_m-\rho_{ms})}{\rho_{mD}/[\epsilon_m A_m(1-\tau_m-\rho_{ms})]} \qquad \textbf{[8-84]}$$

The network element representing Equation (8-84) is shown in Figure 8-56. This element is quite similar to the one shown in Figure 8-45, except that here we must take the transmissivity into account.

Figure 8-56 | Network element representing Equation (8-84).



The transmitted heat exchange between surfaces 1 and 2 is the same as in Section 8-10; that is,

$$q = \frac{J_1 - J_2}{1/A_1F_{12}\tau_m} \qquad \textbf{[8-85]}$$

The heat exchange between surface 1 and m is computed in the following way. Of that energy leaving surface 1, the amount that arrives at m and contributes to the diffuse radiosity of m is

$$q_{1\to m} = J_1 A_1 F_{1m}(1-\tau_m-\rho_{ms}) \qquad \textbf{[8-86]}$$

The diffuse energy leaving m that arrives at 1 is

$$q_{m\to 1} = J_{mD} A_m F_{m1} \qquad \textbf{[8-87]}$$

Subtracting (8-87) from (8-86) and using the reciprocity relation

$$A_1F_{1m} = A_mF_{m1}$$

gives

$$q_{1m} = \frac{J_1 - J_{mD}/(1-\tau_m-\rho_{ms})}{1/[A_1F_{1m}(1-\tau_m-\rho_{ms})]} \qquad \textbf{[8-88]}$$

The network element corresponding to Equation (8-89) is quite similar to the one shown in Figure 8-50. An equation similar to Equation (8-89) can be written for the radiation exchange between surface 2 and m. Finally, the complete network may be drawn as in Figure 8-57. It is to be noted that J_{mD} represents the diffuse radiosity of the left side of m, while J'_{mD} represents the diffuse radiosity of the right side of m.

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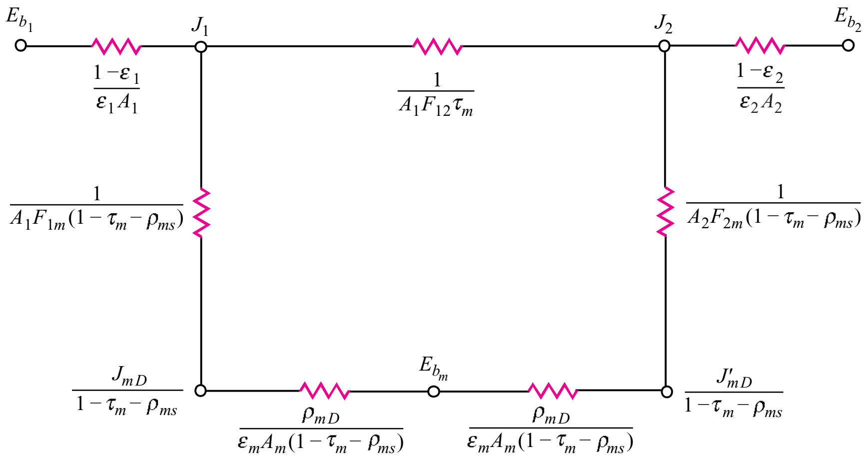
Figure 8-57 | Complete radiation network for system in Figure 8-55.



If m is maintained at a fixed temperature, then J_1 and J_2 must be obtained as a solution to nodal equations for the network. On the other hand, if no net energy is delivered to m, then E_{bm} is a floating node, and the network reduces to a simple series-parallel arrangement. In this latter case the temperature of m must be obtained by solving the network for E_{bm}.

We may extend the analysis a few steps further by distinguishing between specular and diffuse transmission. A specular transmission is one where the incident radiation goes "straight through" the material, while a diffuse transmission is encountered when the incident radiation is scattered in passing through the material, so that it emerges from the other side with a random spatial orientation. As with reflected energy, the assumption is made that the transmissivity may be represented with a specular and a diffuse component:

$$\tau = \tau_s + \tau_D \qquad \textbf{[8-89]}$$

The diffuse radiosity is still defined as in Equation (8-82), and the net energy exchange with a transmitting surface is given by Equation (8-84). The analysis of transmitted energy exchange with other surfaces must be handled somewhat differently, however.

Consider, for example, the arrangement in Figure 8-58. The two diffuse opaque surfaces are separated by a specular-diffuse transmitting and reflecting plane. For this example all planes are assumed to be infinite in extent. The *specular*-transmitted exchange between surfaces 1 and 3 may be calculated immediately with

$$(q_{13})_{\text{specular-transmitted}} = \frac{J_1 - J_3}{1/A_1F_{13}\tau_{2s}} \qquad \textbf{[8-90]}$$

The *diffuse*-transmitted exchange between 1 and 3 is a bit more complicated. The energy leaving 1 that is transmitted diffusely through 2 is

$$J_1A_1F_{12}\tau_{2D}$$

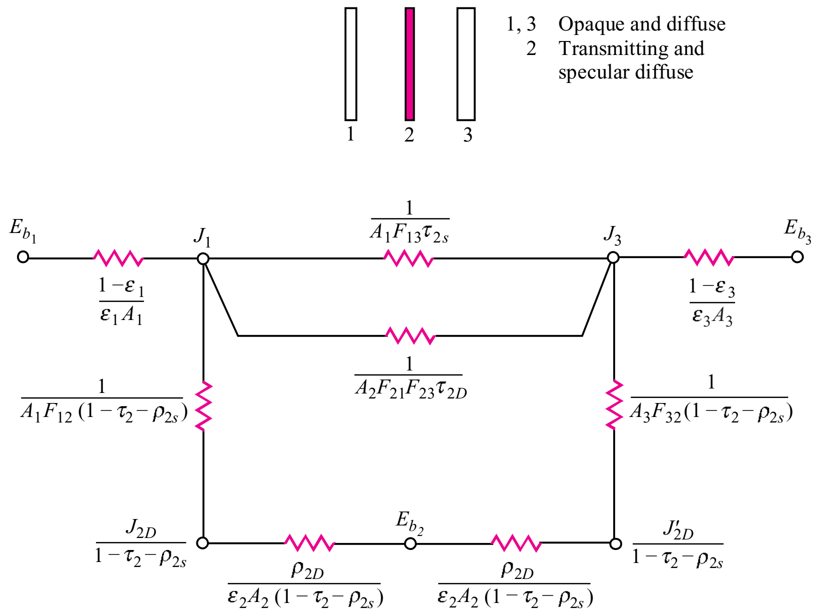
Of this amount transmitted through 2, the amount that arrives at 3 is

$$(q_{13})_{\text{diffuse-transmitted}} = J_1A_1F_{12}\tau_{2D}F_{23} \qquad \textbf{[8-91]}$$

Similarly, the amount leaving 3 that is diffusely transmitted to 1 is

$$(q_{31})_{\text{diffuse-transmitted}} = J_3A_3F_{32}\tau_{2D}F_{21} \qquad \textbf{[8-92]}$$

Figure 8-58 | Radiation network for infinite parallel planes separated by a transmitting specular-diffuse plane.



Now, by making use of the reciprocity relations, $A_1F_{12} = A_2F_{21}$ and $A_3F_{32} = A_2F_{23}$, subtraction of Equation (8-92) from Equation (8-91) gives

$$(q_{13})_{\text{net diffuse–transmitted}} = \frac{J_1 - J_3}{1/A_1F_{21}F_{23}\tau_{2D}} \qquad \textbf{[8-93]}$$

Apparent Emissivity of Cavity with Transparent Cover

Using similar reasoning to that which enabled us to arrive at a relation for the apparent emissivity of a cavity in Equation (8-47), we may consider the effect a transparent covering may have on ϵ_a. The covered cavity is indicated in Figure 8-59 with the characteristics of the cover described by

$$\epsilon_2 + \rho_2 + \tau_2 = 1.0$$

The corresponding radiation network for this cavity exchanging heat with a large surrounding at T_s is shown in Figure 8-60. As in Equation (8-47), we define the apparent emissivity

Figure 8-59 | Cavity with semitransparent covering.

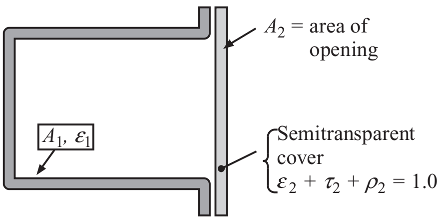
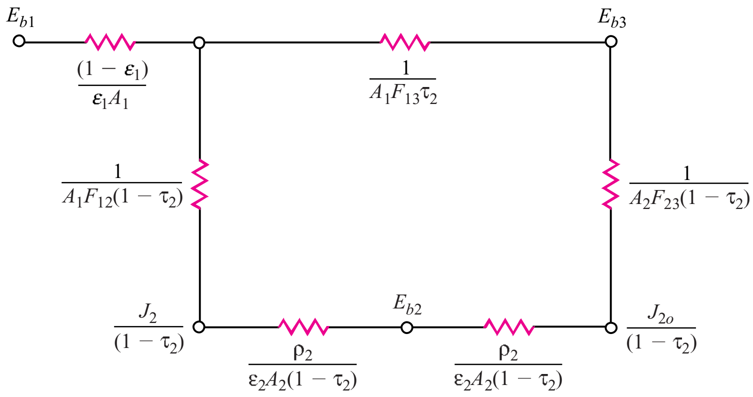


Figure 8-60 | Radiation network for cavity with partially transparent cover.



of the cavity in terms of the net radiation exchange with the surroundings as

$$q = \epsilon_a A_2(E_{b1} - E_{bs}) \quad \textbf{[8-94]}$$

The shape factors in the radiation network are determined as

$$F_{21} = 1, \quad A_1F_{12} = A_2F_{21} = A_2, \quad F_{23} = 1, \quad A_2F_{23} = A_2$$

But, $F_{12} = F_{13}$ so that $A_1F_{13} = A_2$. The heat exchange is determined from the network as

$$q = (E_{b1} - E_{bs})/\Sigma R \quad \textbf{[8-95]}$$

where ΣR is the equivalent resistance for the series parallel network. Performing the necessary algebraic manipulation to evaluate ΣR, and equating the heat transfers in (8–95) and (8–94) gives the relation for the apparent emissivity as

$$\epsilon_a/(\tau_2 + \epsilon_2/2) = K/[(A_2/A_1)(1-\epsilon_1) + K] \quad \textbf{[8-96]}$$

where

$$K = \epsilon_1/(\tau_2 + \epsilon_2/2) \quad \textbf{[8-96}a\textbf{]}$$

We may note the following behavior for three limiting conditions.

1. For $\tau_2 \to 1$, we have an open cavity and the behavior approaches that described by Equation (8-47).
2. For $\tau_2 \to 1$ and $A_2 = A_1$, we have neither cavity nor cover and $\epsilon_a \to \epsilon_1$.
3. For $A_1 \gg A_2$ we have a very large cavity with $\epsilon_a \to \tau_2 + \epsilon_2/2$.

The behavior of ϵ_a is displayed graphically in Figure 8-61.

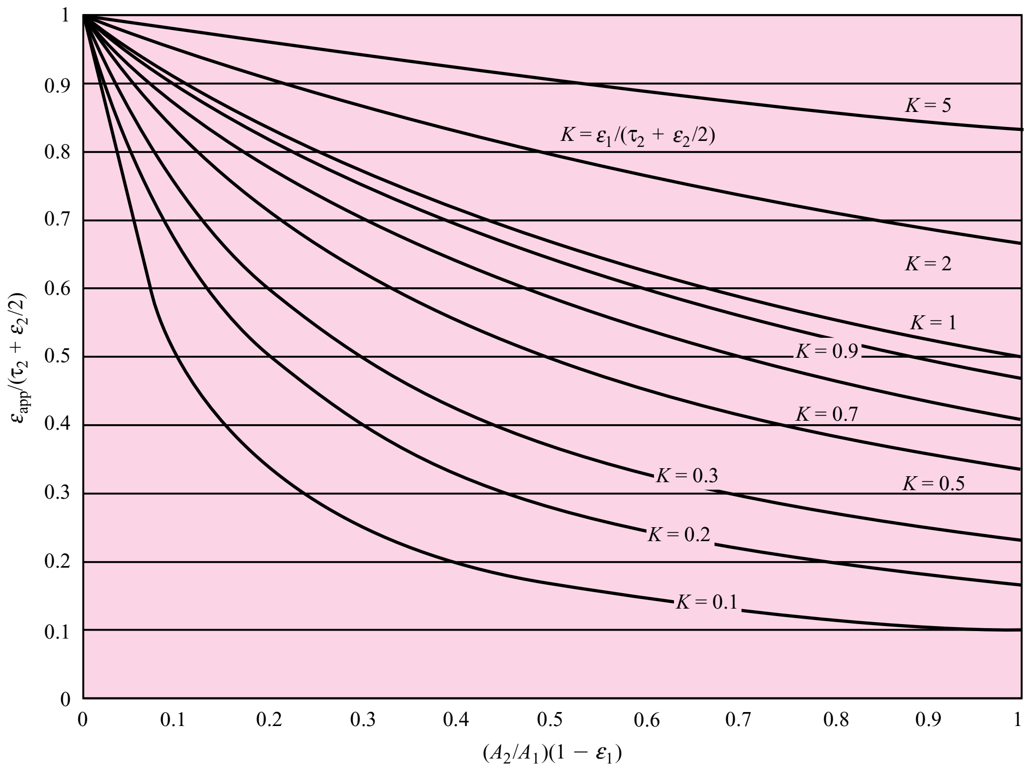
EXAMPLE 8-13 Cavity with Transparent Cover

The rectangular cavity between the fins of Example 8-9 has $\epsilon_1 = 0.5$ along with a cover placed over the opening with the properties

$$\tau_2 = 0.5 \quad \rho_2 = 0.1 \quad \epsilon_2 = 0.4$$

Calculate the apparent emissivity of the covered opening.

Figure 8-61 | Apparent emissivity of cavity with partially transparent cover.



■ **Solution**

Per unit depth in the z direction we have $A_1 = 225 + 25 + 10 = 60$ and $A_2 = 10$. We may evaluate K from Equation (8-96a)

$$K = 0.5/(0.5 + 0.4/2) = 5/7$$

The value of ϵ_a is then computed from Equation (8-96) as

$$\epsilon_a = (0.5 + 0.4/2)(5/7)/[(10/60)(1 - 0.5) + 5/7] = 0.6269$$

If there were no cover present, the value of ϵ_a would be given by Equation (8-47) as

$$\epsilon_a = (0.5)(60)/[10 + (0.5)(60 - 10)] = 0.8571$$

Obviously, the presence of the cover reduces the heat transfer for values of $\tau_2 < 1.0$.

Transmitting and Reflecting System for Furnace Opening

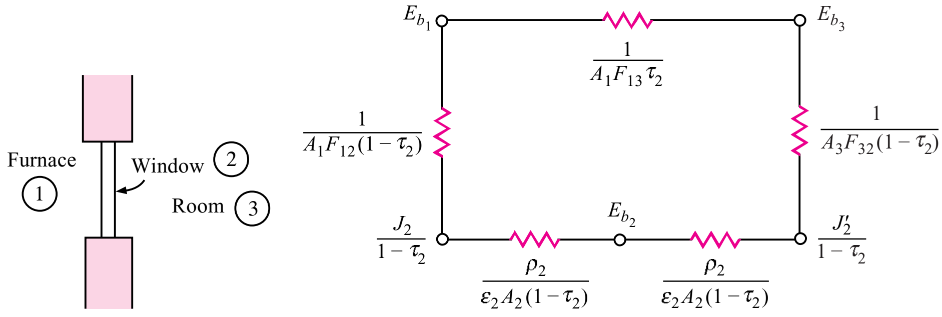
EXAMPLE 8-14

A furnace at 1000°C has a small opening in the side that is covered with a quartz window having the following properties:

$0 < \lambda < 4\ \mu m$	$\tau = 0.9$	$\epsilon = 0.1$	$\rho = 0$
$4 < \lambda < \infty$	$\tau = 0$	$\epsilon = 0.8$	$\rho = 0.2$

The interior of the furnace may be treated as a blackbody. Calculate the radiation lost through the quartz window to a room at 30°C. Diffuse surface behavior is assumed.

Figure Example 8-14



■ Solution

The diagram for this problem is shown in Figure Example 8-14. Because the room is large it may be treated as a blackbody also. We shall analyze the problem by calculating the heat transfer for each wavelength band and then adding them together to obtain the total. The network for each band is a modification of Figure 8-57, as shown here for black furnace and room. We shall make the calculation for unit area; then

$$A_1 = A_2 = A_3 = 1.0$$

$$F_{12} = 1.0 \qquad F_{13} = 1.0 \qquad F_{32} = 1.0$$

The total emissive powers are

$$E_{b_1} = (5.669 \times 10^{-8})(1273)^4 = 1.4887 \times 10^5 \text{ W/m}^2$$

$$E_{b_3} = (5.669 \times 10^{-8})(303)^4 = 477.8 \text{ W/m}^2$$

To determine the fraction of radiation in each wavelength band, we calculate

$$\lambda T_1 = (4)(1273) = 5092\ \mu\text{m}\cdot\text{K}$$

$$\lambda T_3 = (4)(303) = 1212\ \mu\text{m}\cdot\text{K}$$

Consulting Table 8-1, we find

$$E_{b_1}(0-4\ \mu\text{m}) = 0.6450 E_{b_1} = 96{,}021 \text{ W/m}^2$$

$$E_{b_3}(0-4\ \mu\text{m}) = 0.00235 E_{b_3} = 1.123 \text{ W/m}^2$$

$$E_{b_1}(4-\infty) = (1-0.6450)E_{b_1} = 52{,}849 \text{ W/m}^2$$

$$E_{b_3}(4-\infty) = (1-0.00235)E_{b_3} = 476.7 \text{ W/m}^2$$

We now apply these numbers to the network for the two wavelength bands, with unit areas. $0 < \lambda < 4\ \mu\text{m}$ *band:*

$$\frac{1}{F_{13}\tau_2} = \frac{1}{0.9} \qquad \frac{1}{F_{32}(1-\tau_2)} = \frac{1}{0.1} = \frac{1}{F_{12}(1-\tau_2)}$$

$$\frac{\rho_2}{\epsilon_2(1-\tau_2)} = 0$$

The net heat transfer from the network is then

$$q = \frac{E_{b_1} - E_{b_3}}{R_{\text{equiv}}} = \frac{96{,}021 - 1.123}{1.0526} = 91{,}219 \text{ W/m}^2 \qquad 0 < \lambda < 4\ \mu\text{m}$$

$4\ \mu\text{m} < \lambda < +\infty$ *band:*

$$\frac{1}{F_{13}\tau_2} = \infty \qquad \frac{1}{F_{32}(1-\tau_2)} = \frac{1}{F_{12}(1-\tau_2)} = 1.0$$

$$\frac{\rho_2}{\epsilon_2(1-\tau_2)} = \frac{0.2}{0.8} = 0.25$$

The net heat transfer from the network is

$$q = \frac{E_{b_1} - E_{b_3}}{1 + 0.25 + 0.25 + 1} = \frac{52{,}849 - 476.7}{2.5} = 20{,}949 \text{ W/m}^2 \qquad 4 < \lambda < \infty$$

The total heat loss is then

$$q_{\text{total}} = 91{,}219 + 20{,}949 = 112{,}168 \text{ W/m}^2 \qquad [35{,}560 \text{ Btu/h} \cdot \text{ft}^2]$$

With no window at all, the heat transfer would have been the difference in blackbody emissive powers,

$$q - E_{b_1} - E_{b_3} = 1.4887 \times 10^5 - 477.8 = 1.4839 \times 10^5 \text{ W/m}^2 \qquad [47{,}040 \text{ Btu/h} \cdot \text{ft}^2]$$

8-13 | FORMULATION FOR NUMERICAL SOLUTION

The network method that we have used to analyze radiation problems is an effective artifice for visualizing radiant exchange between surfaces. For simple problems that do not involve too many surfaces, the network method affords a solution that can be obtained quite easily. When many heat-transfer surfaces are involved, it is to our advantage to formalize the procedure for writing the nodal equations. For this procedure we consider only opaque, gray, diffuse surfaces. The reader should consult Reference 10 for information on transmitting and specular surfaces. The radiant-energy balance on a particular opaque surface can be written

$$\text{Net heat lost by surface} = \text{energy emitted} - \text{energy absorbed}$$

or on a unit-area basis with the usual gray-body assumptions,

$$\frac{q}{A} = \epsilon E_b - \alpha G$$

Considering the ith surface, the total irradiation is the sum of all irradiations G_j from the other j surfaces. Thus, for $\epsilon = \alpha$,

$$\frac{q_i}{A_i} = \epsilon_i \left(E_{b_i} - \sum_j G_j \right) \qquad \textbf{[8-97]}$$

But, the irradiations can be expressed by

$$A_j J_j F_{ji} = G_j A_i \qquad \textbf{[8-98]}$$

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From reciprocity, we have

$$A_j F_{ji} = A_i F_{ij}$$

so that we can combine the equations to give

$$\frac{q_i}{A_i} = \epsilon_i \left(E_{b_i} - \sum_j F_{ij} J_j \right) \qquad \textbf{[8-99]}$$

The heat transfer at each surface is then evaluated in terms of the radiosities J_j. These parameters are obtained by recalling that the heat transfer can also be expressed as

$$\frac{q_i}{A_i} = J_i - G_i = J_i - \sum_j F_{ij} J_j \qquad \textbf{[8-100]}$$

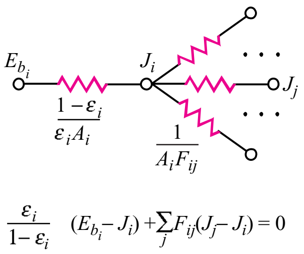
Combining Equations (8-99) and (8-100) gives

$$J_i - (1 - \epsilon_i) \sum_j F_{ij} J_j = \epsilon_i E_{b_i} \qquad \textbf{[8-101]}$$

In the equations above it must be noted that the summations must be performed over *all* surfaces in the enclosure. For a three-surface enclosure, with $i = 1$, the summation would then become

$$\sum_j F_{ij} J_j = F_{11} J_1 + F_{12} J_2 + F_{13} J_3$$

Figure 8-62 | Radiation network balance on node J_i.



Of course, if surface 1 is convex, $F_{11} = 0$ and some simplification could be effected.

The nodal equations for the radiosities may also be derived from the nodes in the network formulation, as indicated in Figure 8-62. At each J_i node an energy balance gives

$$\frac{\epsilon_i}{1 - \epsilon_i}(E_{b_i} - J_i) + \sum_j F_{ij}(J_j - J_i) = 0 \qquad \textbf{[8-102]}$$

Again, an equation will be obtained for each J_i that is entirely equivalent to Equation (8-101). Once all the equations are written out they can be expressed in the matrix form

$$[A][J] = [C] \qquad \textbf{[8-103]}$$

where

$$[A] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1i} \\ a_{21} & a_{22} & \cdots & a_{2i} \\ \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{ii} \end{bmatrix} \qquad [J] = \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_i \end{bmatrix} \qquad [C] = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_i \end{bmatrix}$$

The solution for the radiosities is found by obtaining the inverse to $[A]$ such that

$$[J] = [A]^{-1}[C]$$

The inverse $[A]^{-1}$ is written as

$$[A]^{-1} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1i} \\ b_{21} & \cdot & \cdots & \cdot \\ \cdots & \cdots & \cdots & \cdots \\ b_{i1} & b_{i2} & \cdots & b_{ii} \end{bmatrix}$$

so that the unknown radiosities are written as

$$J_1 = b_{11}C_1 + b_{12}C_2 + \cdots + b_{1i}C_i$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$J_i = b_{i1}C_1 + b_{i2}C_2 + \cdots + b_{ii}C_i$$

Standard computer subroutines are available to obtain the inverse matrix and perform the final calculations of the J_i. The heat-transfer rate at each ith surface having an area A_i is then calculated from

$$\frac{q_i}{A_i} = \frac{\epsilon_i}{1-\epsilon_i}(E_{b_i} - J_i) \qquad \textbf{[8-104]}$$

In formulating the nodal equations one must take note of the consequence of Equation (8-104) for an *insulated* surface (i.e., one for which there is no net heat transfer). Equation (8-110) thus requires that

$$E_{b_i} = J_i \qquad \text{for insulated surface} \qquad \textbf{[8-105]}$$

From a practical point of view, a Gauss-Seidel iteration scheme may be the most efficient numerical procedure to follow in solving the set of equations for the J_i's. For the Gauss-Seidel scheme the above equations must be organized in explicit form for J_i. Solving for J_i in Equation (8-101) and breaking out the F_{ii} term gives

$$J_i = (1-\epsilon_i)\sum_{j\neq i} F_{ij}J_j + (1-\epsilon_i)F_{ii}J_i + \epsilon_i E_{b_i}$$

$$J_i = \frac{1}{1 - F_{ii}(1-\epsilon_i)}\left[(1-\epsilon_i)\sum_{j\neq i} F_{ij}J_j + \epsilon_i E_{b_i}\right] \qquad \textbf{[8-106]}$$

For a surface in radiant equilibrium, $q_i/A_i = 0$ and $J_i = E_{b_i}$ may be substituted into Equation (8-106) to give

$$J_i = \frac{1}{1-F_{ii}}\sum_{j\neq i} F_{ij}J_j \qquad \text{for } \frac{q_i}{A_i} = 0 \qquad \textbf{[8-107]}$$

If the problem formulation is to include a specified heat flux q_i/A_i at one of the ith surfaces, we can solve for E_{b_i} from Equation (8-104) to give

$$E_{b_i} = J_i + \frac{1-\epsilon_i}{\epsilon_i}\frac{q_i}{A_i} \qquad \textbf{[8-108]}$$

Substituting this value into Equation (8-101) and then solving for J_i give

$$J_i = \frac{1}{1-F_{ii}}\left(\sum_{j\neq i} F_{ij}J_j + \frac{q_i}{A_i}\right) \qquad \textbf{[8-109]}$$

In many cases the radiation solution must take conduction and convection into account at the ith surface. The appropriate energy balance is then, for steady state,

Heat conducted into surface + heat convected into surface
= radiant heat *lost* from surface

or

$$q_{\text{cond},i} + q_{\text{conv},i} = q_{i,\text{rad}} \qquad \textbf{[8-110]}$$

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This energy balance may then be used in conjunction with Equation (8-109) to obtain the proper nodal equation for J_i.

While the above formulations may appear rather cumbersome at first glance they are easily solved by computer, with either matrix inversion or iteration. For many practical radiation problems, the number of equations is small and programmable calculators may be employed for solution. In most cases one will not know the surface properties (ϵ_i) within better than a few percent, so an iterative solution need not be carried out to unreasonable limits of precision.

In summary, we outline the computational procedure to be followed for numerical solution of radiation heat transfer between diffuse, gray surfaces.

1. Evaluate F_{ij} and ϵ_i for all surfaces.
2. Evaluate E_{b_i} for all surfaces with specified temperature.
3. Formulate nodal equations for the J_i using:
   a. Equation (8-106) for surfaces with specified T_i.
   b. Equation (8-107) for surfaces in radiant balance ($J_i = E_{b_i}$).
   c. Equation (8-109) for surfaces with specified q_i.
4. Solve the equations for the J_i's.
5. Compute the q_i's and T_i's, using:
   a. q_i from Equation (8-104) for gray surfaces and Equation (8-99) for black surfaces with specified T_i.
   b. T_i from $J_i = E_{b_i} = \sigma T_i^4$ for surfaces in radiant balance.
   c. T_i using E_{b_i} obtained from Equation (8-108) for surfaces with specified q_i.

Of course, the above equations may be put in the following form if direct matrix inversion is preferred over an iteration scheme:

$$J_i[1 - F_{ii}(1-\epsilon_i)] - (1-\epsilon_i)\sum_{j\neq i} F_{ij}J_j = \epsilon_i E_{b_i} \quad \textbf{[8-106}\textbf{\textit{a}}\textbf{]}$$

$$J_i(1 - F_{ii}) - \sum_{j\neq i} F_{ij}J_j = 0 \quad \textbf{[8-107}\textbf{\textit{a}}\textbf{]}$$

$$J_i(1 - F_{ii}) - \sum_{j\neq i} F_{ij}J_j = \frac{q_i}{A_i} \quad \textbf{[8-109}\textbf{\textit{a}}\textbf{]}$$

Computation of the surface temperatures and heat transfers is the same as in step 5 above.

Insulated Surfaces and Surfaces with Large Areas

We have seen in the application of the network method that an insulated surface acts as if it were perfectly reflective with $\epsilon \to 0$, and thus $J = E_b$. We may note that if one takes $\epsilon = 0$ in Equation (8-106*a*), Equation (8-107*a*) will result. When the system of equations is solved for the J's, one may thus obtain the temperature of an insulated surface from $T = (E_b/\sigma)^{1/4} = (J/\sigma)^{1/4}$. When a surface with a very large area compared to other surfaces in the system is involved, it behaves like a blackbody with $\epsilon \to 1.0$ because of its low surface resistance. If such a large surface is concave, it will behave as if $F_{ii} \to 1.0$ and all the $F_{ij} \to 0$.

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Numerical Solution for Enclosure

EXAMPLE 8-15

The geometry of Example 8-5 is used for radiant exchange with a large enclosure. Surface 2 is diffuse with $\epsilon = 0.5$ while surface 1 is perfectly insulated. $T_2 = 1000$ K, and $T_3 = 300$ K. Calculate the heat lost to the large room per unit length of surface 2, using the numerical formulation. Also calculate the temperature of the insulated surface.

Solution

For unit length we have:

$$E_{b_2} = \sigma T_2^4 = 5.669 \times 10^4 \qquad E_{b_3} = \sigma T_3^4 = 459$$

$$A_1 = (4)(0.2) = 0.8 \text{ m}^2/\text{m} \qquad A_2 = \pi(0.60)/2 = 0.94 \text{ m}^2/\text{m}$$

We will use the numerical formulation. We find from Example 8-5, using the nomenclature of the figure, $F_{11} = 0.314$, $F_{12} = 0.425$, $F_{13} = 0.261$, $F_{21} = 0.5$, $F_{22} = 0$, $F_{23} = 0.5$, $F_{31} \to 0$, $F_{32} \to 0$, $F_{33} \to 1.0$. We now write the equations. Surface 1 is insulated so we use Equation (8-107*a*):

$$J_1(1 - 0.314) - 0.425 J_2 - 0.261 J_3 = 0$$

Surface 2 is constant temperature so we use Equation (8-106*a*):

$$J_2(1 - 0) - (1 - 0.5)[0.5 J_1 + 0.5 J_3] = (0.5)(56{,}690)$$

Because surface 3 is so large,

$$J_3 = E_{b_3} = 459 \text{ W/m}^2$$

Rearranging the equations gives

$$0.686 J_1 - 0.425 J_2 = 119.8$$

$$-0.25 J_1 + J_2 = 28{,}460$$

which have the solutions

$$J_1 = 21{,}070 \text{ W/m}^2$$

$$J_2 = 33{,}727 \text{ W/m}^2$$

The heat transfer is thus

$$q = \frac{E_{b_2} - J_2}{\dfrac{1 - \epsilon_2}{\epsilon_2 A_2}} = \frac{56{,}690 - 33{,}727}{(1 - 0.5)/(0.5)(4)(0.2)} = 18{,}370 \text{ W/m length}$$

Because surface 1 is insulated, $J_1 = E_{b_1}$, and we could calculate the temperature as

$$T_1 = \left(\frac{21{,}070}{5.669 \times 10^{-8}}\right)^{1/4} = 781 \text{ K}$$

For this problem a solution using the network method might be simpler because it involves only a simple series–parallel circuit.

Numerical Solutions for Parallel Plates

EXAMPLE 8-16

Two 1-m-square surfaces are separated by a distance of 1 m with $T_1 = 1000$ K, $T_2 = 400$ K, $\epsilon_1 = 0.8, \epsilon_2 = 0.5$. Obtain the numerical solutions for this system when (*a*) the plates are surrounded by a large room at 300 K and (*b*) the surfaces are connected by a re-radiating wall perfectly insulated on its outer surface. Part (*a*) of this example is identical in principle to the problem that is solved by the network method in Example 8-6.

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Solution

Consulting Figure 8-12, we obtain

$$F_{12} = 0.2 \qquad F_{21} = 0.2 \qquad F_{11} = 0 = F_{22}$$
$$F_{13} = 0.8 \qquad F_{23} = 0.8$$
$$A_1 = A_2 = 1 \text{ m}^2$$

(surface 3 is the surroundings or insulated surface). For part (*a*)

$$E_{b_1} = \sigma T_1^4 = 56.69 \text{ kW/m}^2 \quad [17{,}970 \text{ Btu/h} \cdot \text{ft}^2]$$
$$E_{b_2} = \sigma T_2^4 = 1.451 \text{ kW/m}^2$$
$$E_{b_3} = \sigma T_3^4 = 0.459 \text{ kW/m}^2$$

Because $A_3 \to \infty$, F_{31} and F_{32} must approach zero since $A_1 F_{13} = A_3 F_{31}$ and $A_2 F_{23} = A_3 F_{32}$. The nodal equations are written in the form of Equation (8-107):

surface 1: $$J_1 - (1 - \epsilon_1)(F_{11} J_1 + F_{12} J_2 + F_{13} J_3) = \epsilon_1 E_{b_1}$$

surface 2: $$J_2 - (1 - \epsilon_2)(F_{21} J_1 + F_{22} J_2 + F_{23} J_3) = \epsilon_2 E_{b_2} \qquad \textbf{[a]}$$

surface 3: $$J_3 - (1 - \epsilon_3)(F_{31} J_1 + F_{32} J_2 + F_{33} J_3) = \epsilon_3 E_{b_3}$$

Because F_{31} and F_{32} approach zero, F_{33} must be 1.0.

Inserting the numerical values for the various terms, we have

$$J_1 - (1 - 0.8)[(0) J_1 + (0.2) J_2 + (0.8) J_3] = (0.8)(56.69)$$
$$J_2 - (1 - 0.5)[(0.2) J_1 + (0) J_2 + (0.8) J_3] = (0.5)(1.451) \qquad \textbf{[b]}$$
$$J_3 - (1 - \epsilon_3)[(0) J_1 + (0) J_2 + (1.0) J_3] = \epsilon_3 (0.459)$$

The third equation yields $J_3 = 0.459 \text{ kW/m}^2 \cdot \text{K}$. Because the room is so large it acts like a hohlraum, or blackbody. But *it does not have zero heat transfer*.

Finally, the equations are written in compact form as

$$J_1 - 0.04 J_2 - 0.16 J_3 = 45.352$$
$$-0.1 J_1 + J_2 - 0.4 J_3 = 0.7255 \qquad \textbf{[c]}$$
$$J_3 = 0.459$$

Of course, there only remain two unknowns, J_1 and J_2, in this set.

For part (*b*), A_3 for the enclosing wall is 4.0 m^2, and we set $J_3 = E_{b_3}$ because surface 3 is insulated. From reciprocity we have

$$A_1 F_{13} = A_3 F_{31} \qquad F_{31} = \frac{(1.0)(0.8)}{4.0} = 0.2$$
$$A_2 F_{23} = A_3 F_{32} \qquad F_{32} = \frac{(1.0)(0.8)}{4.0} = 0.2$$

Then from $F_{31} + F_{32} + F_{33} = 1.0$ we have $F_{33} = 0.6$.

The set of equations in (*a*) still applies, so we insert the numerical values to obtain (with $J_3 = E_{b_3}$)

$$J_1 - (1 - 0.8)[(0) J_1 + (0.2) J_2 + (0.8) J_3] = (0.8)(56.69)$$
$$J_2 - (1 - 0.5)[(0.2) J_1 + (0) J_2 + (0.8) J_3] = (0.5)(1.451) \qquad \textbf{[d]}$$
$$J_3 - (1 - \epsilon_3)[(0.2) J_1 + (0.2) J_2 + (0.6) J_3] = \epsilon_3 J_3$$

Notice that the third equation of set (*d*) can be written as

$$J_3(1 - \epsilon_3) - (1 - \epsilon_3)[(0.2) J_1 + (0.2) J_2 + (0.6) J_3] = 0$$

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so that the $1 - \epsilon_3$ term drops out, and we obtain our final set of equations as

$$\begin{aligned} J_1 - 0.04J_2 - 0.16J_3 &= 45.352 \\ -0.1J_1 + J_2 - 0.4J_3 &= 0.7255 \\ -0.2J_1 - 0.2J_2 + 0.4J_3 &= 0 \end{aligned} \qquad [e]$$

To obtain the heat transfers the set of equations is first solved for the radiosities. For set (*c*),

$$\begin{aligned} J_1 &= 45.644 \text{ kW/m}^2 \\ J_2 &= 5.474 \text{ kW/m}^2 \\ J_3 &= 0.459 \text{ kW/m}^2 \end{aligned}$$

The heat transfers are obtained from Equation (8-104):

$$q_1 = \frac{A_1\epsilon_1}{1-\epsilon_1}(E_{b_1} - J_1) = \frac{(1.0)(0.8)}{1-0.8}(56.69 - 45.644) = 44.184 \text{ kW} \quad [150{,}760 \text{ Btu/h}]$$

$$q_2 = \frac{A_2\epsilon_2}{1-\epsilon_2}(E_{b_2} - J_2) = \frac{(1.0)(0.5)}{1-0.5}(1.451 - 5.474) = -4.023 \text{ kW} \quad [-13{,}730 \text{ Btu/h}]$$

The net heat *absorbed* by the room is the algebraic sum of q_1 and q_2 or

$$q_{3,\text{ absorbed}} = 44.184 - 4.023 = 40.161 \text{ kW} \quad [137{,}030 \text{ Btu/h}]$$

For part (*b*) the solutions to set (*e*) are

$$J_1 = 51.956 \text{ kW/m}^2 \qquad J_2 = 20.390 \text{ kW/m}^2 \qquad J_3 = 36.173 \text{ kW/m}^2$$

The heat transfers are

$$q_1 = \frac{A_1\epsilon_1}{1-\epsilon_1}(E_{b_1} - J_1) = \frac{(1.0)(0.8)}{1-0.8}(56.69 - 51.965) = 18.936 \text{ kW}$$

$$q_2 = \frac{A_2\epsilon_2}{1-\epsilon_2}(E_{b_2} - J_2) = \frac{(1.0)(0.5)}{1-0.5}(1.451 - 20.390) = -18.936 \text{ kW}$$

Of course, these heat transfers should be equal in magnitude with opposite sign because the insulated wall exchanges no heat. The temperature of the insulated wall is obtained from

$$J_3 = E_{b_3} = \sigma T_3^4 = 36.173 \text{ kW/m}^2$$

and

$$T_3 = 894 \text{ K} \quad [621°\text{C}, 1150°\text{F}]$$

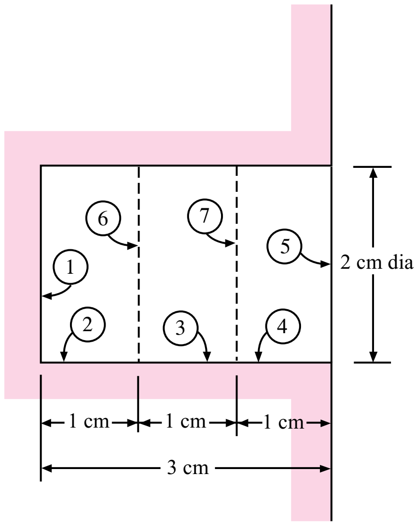
Radiation from a Hole with Variable Radiosity

EXAMPLE 8-17

To further illustrate the radiation formulation for numerical solution we consider the circular hole 2 cm in diameter and 3 cm deep, as shown in Figure Example 8-17. The hole is machined in a large block of metal, which is maintained at 1000°C and has a surface emissivity of 0.6. The temperature of the large surrounding room is 20°C. A simple approach to this problem would assume the radiosity uniform over the entire heated internal surface. In reality, the radiosity varies over the surface, and we break it into segments 1 (bottom of the hole), 2, 3, and 4 (sides of the hole) for analysis.

The large room acts like a blackbody at 20°C, so for analysis purposes we can assume the hole is covered by an imaginary black surface 5 at 20°C. We set the problem up for a numerical solution for the radiosities and then calculate the heat-transfer rates. After that, we shall examine an insulated-surface case for this same geometry.

Figure Example 8-17



■ Solution

All the shape factors can be obtained with the aid of Figure 8-13 and the imaginary disk surfaces 6 and 7. We have

$$E_{b_1} = \sigma T_1^4 = (5.669 \times 10^{-8})(1273)^4 = 1.48874 \times 10^5 \text{ W/m}^2$$
$$= E_{b_2} = E_{b_3} = E_{b_4}$$
$$E_{b_5} = \sigma T_5^4 = (5.669 \times 10^{-8})(293)^4 = 417.8 \text{ W/m}^2$$
$$\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = 0.6 \qquad \epsilon_5 = 1.0$$
$$A_1 = A_5 = \pi(1)^2 = \pi \text{ cm}^2 = A_6 = A_7$$
$$A_2 = A_3 = A_4 = \pi(2)(1) = 2\pi$$
$$F_{11} = F_{55} = 0 \qquad F_{16} = 0.37 \qquad F_{17} = 0.175 \qquad F_{15} = 0.1$$
$$F_{12} = 1 - F_{16} = 0.63 = F_{54}$$
$$F_{13} = F_{16} - F_{17} = 0.195 = F_{53}$$
$$F_{14} = F_{17} - F_{15} = 0.075 = F_{52}$$
$$F_{21} = F_{26} = F_{16}\frac{A_1}{A_2} = 0.315 = F_{45} = F_{36} = F_{37}$$
$$F_{22} = 1 - F_{21} - F_{26} = 0.37 = F_{33} = F_{44}$$
$$F_{31} = F_{13}\frac{A_1}{A_3} = 0.0975$$
$$F_{32} = F_{36} - F_{31} = 0.2175 = F_{34} = F_{43} = F_{23}$$
$$F_{27} = F_{26} - F_{23} = F_{21} - F_{23} = 0.0975 = F_{46}$$
$$F_{41} = F_{14}\frac{A_1}{A_4} = 0.0375 = F_{25}$$
$$F_{42} = F_{46} - F_{41} = 0.06 = F_{24}$$

The equations for the radiosities are now written in the form of Equation (8-106), noting that $F_{11} = 0$ and $J_5 = E_{b_5}$:

$$J_1 = (1 - \epsilon_1)(F_{12}J_2 + F_{13}J_3 + F_{14}J_4 + F_{15}E_{b_5}) + \epsilon_1 E_{b_1}$$
$$J_2 = \frac{1}{1 - F_{22}(1 - \epsilon_2)}[(1 - \epsilon_2)(F_{21}J_1 + F_{23}J_3 + F_{24}J_4 + F_{25}E_{b_5}) + \epsilon_2 E_{b_2}]$$

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$$J_3 = \frac{1}{1 - F_{33}(1-\epsilon_3)}[(1-\epsilon_3)(F_{31}J_1 + F_{32}J_2 + F_{34}J_4 + F_{35}E_{b_5}) + \epsilon_3 E_{b_3}]$$
$$J_4 = \frac{1}{1 - F_{44}(1-\epsilon_4)}[(1-\epsilon_4)(F_{41}J_1 + F_{42}J_2 + F_{43}J_3 + F_{45}E_{b_5}) + \epsilon_4 E_{b_4}]$$

When all the numerical values are inserted, we obtain

$$J_1 = 0.252J_2 + 0.078J_3 + 0.03J_4 + 89{,}341$$
$$J_2 = 0.1479J_1 + 0.1021J_3 + 0.02817J_4 + 104{,}848$$
$$J_3 = 0.04577J_1 + 0.1021J_2 + 0.1021J_4 + 104{,}859$$
$$J_4 = 0.01761J_1 + 0.02817J_2 + 0.1021J_3 + 104{,}902$$

These equations may be solved to give

$$J_1 = 1.4003 \times 10^5 \text{ W/m}^2$$
$$J_2 = 1.4326 \times 10^5 \text{ W/m}^2$$
$$J_3 = 1.3872 \times 10^5 \text{ W/m}^2$$
$$J_4 = 1.2557 \times 10^5 \text{ W/m}^2$$

The heat transfers can be calculated from Equation (8-104):

$$q_i = \frac{\epsilon_i A_i}{1-\epsilon_i}(E_{b_i} - J_i)$$
$$q_1 = \frac{(0.6)(\pi \times 10^{-4})}{1-0.6}(1.4887 - 1.4003)(10^5) = 4.1658 \text{ W}$$
$$q_2 = \frac{(0.6)(2\pi \times 10^{-4})}{1-0.6}(1.4887 - 1.4326)(10^5) = 5.2873 \text{ W}$$
$$q_3 = \frac{(0.6)(2\pi \times 10^{-4})}{1-0.6}(1.4887 - 1.3872)(10^5) = 9.5661 \text{ W}$$
$$q_4 = \frac{(0.6)(2\pi \times 10^{-4})}{1-0.6}(1.4887 - 1.2557)(10^5) = 21.959 \text{ W}$$

The total heat transfer is the sum of these four quantities or

$$q_{\text{total}} = 40.979 \text{ W} \qquad [139.8 \text{ Btu/h}]$$

It is of interest to compare this heat transfer with the value we would obtain by assuming uniform radiosity on the hot surface. We would then have a two-body problem with

$$A_1 = \pi + 3(2\pi) = 7\pi \text{ cm}^2 \qquad A_5 = \pi \qquad F_{51} = 1.0 \qquad \epsilon_1 = 0.6 \qquad \epsilon_5 = 1.0$$

The heat transfer is then calculated from Equation (8-43), with appropriate change of nomenclature:

$$q = \frac{(E_{b_1} - E_{b_5})A_5}{1/\epsilon_5 + (A_5/A_1)(1/\epsilon_1 - 1)} = \frac{(\pi \times 10^{-4})(1.4887 \times 10^5 - 417.8)}{1 + \left(\frac{1}{7}\right)(1/0.6 - 1)}$$
$$= 42.581 \text{ W} \qquad [145.3 \text{ Btu/h}]$$

Thus, the simple assumption of uniform radiosity gives a heat transfer that is 3.9 percent above the value obtained by breaking the hot surface into four parts for the calculation. This indicates that the uniform-radiosity assumption we have been using is a rather good one for engineering calculations.

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Let us now consider the case where surface 1 is still radiating at 1000°C with $\epsilon = 0.6$ but the side walls 2, 3, and 4 are insulated. The radiation is still to the large room at 20°C. The nodal equation for J_1 is the same as before but now the equations for J_2, J_3, and J_4 must be written in the form of Equation (8-107). When that is done and the numerical values are inserted, we obtain

$$J_1 = 0.252J_2 + 0.078J_3 + 0.03J_4 + 89{,}341$$
$$J_2 = 0.5J_1 + 0.3452J_3 + 0.09524J_4 + 24.869$$
$$J_3 = 0.1548J_1 + 0.3452J_2 + 0.3452J_4 + 64.66$$
$$J_4 = 0.05952J_1 + 0.0952J_2 + 0.3452J_3 + 208.9$$

When these equations are solved, we obtain

$$J_1 = 1.1532 \times 10^5 \text{ W/m}^2 \quad [36{,}560 \text{ Btu/h} \cdot \text{ft}^2]$$
$$J_2 = 0.81019 \times 10^5 \text{ W/m}^2$$
$$J_3 = 0.57885 \times 10^5 \text{ W/m}^2$$
$$J_4 = 0.34767 \times 10^5 \text{ W/m}^2$$

The heat transfer at surface 1 is

$$q_1 = \frac{\epsilon_1 A_1}{1-\epsilon_1}(E_{b_1} - J_1) = \frac{(0.6)(\pi \times 10^{-4})}{1-0.6}(1.4887 - 1.1532)(10^5)$$
$$= 15.81 \text{ W} \quad [53.95 \text{ Btu/h}]$$

The temperatures of the insulated surface elements are obtained from

$$J_i = E_{b_i} = \sigma T_i^4$$
$$T_2 = 1093 \text{ K} = 820°\text{C} \quad [1508°\text{F}]$$
$$T_3 = 1005 \text{ K} = 732°\text{C} \quad [1350°\text{F}]$$
$$T_4 = 895 \text{ K} = 612°\text{C} \quad [1134°\text{F}]$$

It is of interest to compare the heat transfer calculated above with that obtained by assuming surfaces 2, 3, and 4 uniform in temperature and radiosity. Equation (8-41) applies for this case:

$$q = \frac{A_1(E_{b_1} - E_{b_5})}{\dfrac{A_1 + A_2 + 2A_1F_{15}}{A_5 - A_1(F_{15})^2} + \left(\dfrac{1}{\epsilon_1} - 1\right) + \dfrac{A_1}{A_5}\left(\dfrac{1}{\epsilon_5} - 1\right)}$$

and

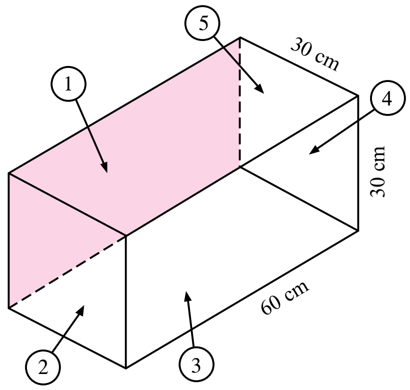
$$q = \frac{(\pi \times 10^{-4})(1.4887 \times 10^5 - 417.8)}{\dfrac{\pi + \pi - 2\pi(0.1)}{\pi - \pi(0.1)^2} + \dfrac{1}{0.6} - 1} = 18.769 \text{ W} \quad [64.04 \text{ Btu/h}]$$

In this case the assumption of uniform radiosity at the insulated surface gives an overall heat transfer with surface 1 (bottom of hole) that is 18.7 percent too high.

EXAMPLE 8-18 Heater with Constant Heat Flux and Surrounding Shields

In Figure Example 8-18, an electric heater is installed in surface 1 such that a constant heat flux of 100 kW/m^2 is generated at the surface. The four surrounding surfaces are in radiant balance with surface 1 and a large surrounding room at 20°C. The surface properties are $\epsilon_1 = 0.8$ and $\epsilon_2 = \epsilon_3 = \epsilon_4 = \epsilon_5 = 0.4$. Determine the temperatures of all surfaces. The back side of surface 1 is insulated. Repeat the calculation assuming surfaces 2, 3, 4, and 5 are just one surface uniform in temperature.

Figure Example 8-18



■ Solution

In reality, surfaces 2, 3, 4, and 5 have *two* surfaces each; an inside and an outside surface. We thus have *nine* surfaces plus the room, so a 10-body problem is involved. Of course, from symmetry we can see that $T_2 = T_4$ and $T_3 = T_5$, but we set up the problem in the general numerical formulation. We designate the large room as surface 6 and it behaves as if $\epsilon_6 = 1.0$. So, it is as if the opening were covered with a black surface at 20°C. The shape factors of the inside surfaces are obtained from Figures 8-12 and 8-14:

$$F_{16} = F_{61} = 0.285 \qquad F_{13} = F_{15} = 0.24 = F_{31} = F_{51}$$
$$F_{12} = F_{14} = 0.115 \qquad F_{24} = F_{42} = 0.068$$
$$F_{35} = F_{53} = 0.285 \qquad F_{32} = F_{52} = F_{34} = 0.115$$
$$F_{25} = F_{23} = F_{45} = F_{43} = F_{21} = F_{41} = F_{26} = F_{46} = 0.23$$
$$F_{11} = F_{22} = F_{33} = F_{44} = F_{55} = 0$$

For the *outside* surfaces,

$$F'_{26} = F'_{36} = F'_{46} = F'_{56} = 1.0$$

where the primes indicate the outside surfaces. We shall also use primes to designate the radiosities of the outside surfaces. For the room, $J_6 = E_{b_6} = (5.669 \times 10^{-8})(293)^4 = 417.8\ \text{W/m}^2$.

For surface 1 with constant heat flux, we use Equation (8-109*a*) and write

$$J_1 - (F_{12}J_2 + F_{13}J_3 + F_{14}J_4 + F_{15}J_5 + F_{16}J_6) = 1.0 \times 10^5 \qquad [a]$$

Because of the radiant balance condition we have

$$(J_2 - E_{b_2})\frac{\epsilon_2 A_2}{1-\epsilon_2} = (E_{b_2} - J'_2)\frac{\epsilon_2 A_2}{1-\epsilon_2}$$

and

$$E_{b_2} = \frac{J_2 + J'_2}{2} \qquad [b]$$

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where the prime designates the outside radiosity. A similar relation applies for surfaces 3, 4, and 5. Thus, we can use Equation (8-106*a*) for *inside* surface 2

$$J_2 - (1-\epsilon_2)(F_{21}J_1 + F_{23}J_3 + F_{24}J_4 + F_{25}J_5 + F_{26}J_6) = \frac{\epsilon_2}{2}(J_2 + J_2') \qquad [c]$$

and for the *outside* surface 2

$$J_2' - (1-\epsilon_2)(F_{26}'J_6) = \frac{\epsilon_2}{2}(J_2 + J_2') \qquad [d]$$

Equations like (*c*) and (*d*) are written for surfaces 3, 4, and 5 also, and with the shape factors and emissivities inserted the following set of equations is obtained:

$$J_1 - 0.115J_2 - 0.24J_3 - 0.115J_4 - 0.24J_5 = 1.0012 \times 10^5$$
$$-0.138J_1 + 0.8J_2 - 0.2J_2' - 0.138J_3 - 0.0408J_4 - 0.138J_5 = 57.66$$
$$0.2J_2 - 0.8J_2' = -250.68$$
$$-0.144J_1 - 0.069J_2 + 0.8J_3 - 0.2J_3' - 0.069J_4 - 0.05J_5 = 60.16$$
$$0.2J_3 - 0.8J_3' = -250.68$$
$$-0.138J_1 - 0.0408J_2 - 0.138J_3 + 0.8J_4 - 0.2J_4' - 0.138J_5 = 57.66$$
$$0.2J_4 - 0.8J_4' = -250.68$$
$$-0.144J_1 - 0.069J_2 - 0.057J_3 - 0.069J_4 + 0.8J_5 - 0.2J_5' = 60.16$$
$$0.2J_5 - 0.8J_5' = -250.68$$

We thus have nine equations and nine unknowns, which may be solved to give

$$J_1 = 1.24887 \times 10^5 \ \text{W/m}^2$$
$$J_2 = J_4 = 37{,}549$$
$$J_2' = J_4' = 9701$$
$$J_3 = J_5 = 33{,}605$$
$$J_3' = J_5' = 8714$$

The temperatures are thus computed from Equation (*b*):

$$E_{b_2} = \frac{37{,}549 + 9701}{2} = 23{,}625 \qquad T_2 = T_4 = 803.5 \text{ K}$$
$$E_{b_3} = \frac{33{,}605 + 8714}{2} = 21{,}160 \qquad T_3 = T_5 = 781.6 \text{ K}$$

For surface 1 we observed that

$$\frac{q}{A} = \frac{\epsilon}{1-\epsilon}(E_{b_1} - J_1)$$

so that

$$E_{b_1} = \frac{(1.0 \times 10^5)(1-0.8)}{0.8} + 1.24887 \times 10^5 = 1.49887 \times 10^5$$

and

$$T_1 = 1275 \text{ K}$$

We note again that we could have observed the symmetry of the problem and set $J_2 = J_4$, $J_2' = J_4'$, and so on. By so doing, we could have had only five equations with five unknowns.

Surfaces 2, 3, 4, and 5 as one surface

We now go back and take surfaces 2, 3, 4, and 5 as one surface, which we choose to call surface 7. The shape factors are then

$$F_{16} = F_{61} = 0.285 \qquad F_{17} = 1 - 0.285 = 0.715$$
$$A_1 = 2.0 \qquad A_7 = 6.0$$

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Thus

$$F_{71} = (0.715)\left(\frac{2}{6}\right) = 0.2383 = F_{76}$$

$$F_{77} = 1 - (2)(0.2383) = 0.5233 \qquad F'_{76} = 1.0$$

Then for surface 1 we use Equation (8-109*a*) to obtain

$$J_1 - (F_{17}J_7 + F_{16}J_6) = 1.0 \times 10^5$$

Using $E_{b_7} = (J_7 + J'_7)/2$, we have for the *inside* of surface 7

$$J_7[1 - F_{77}(1-\epsilon_7)] - (1-\epsilon_7)(F_{71}J_1 + F_{76}J_6) = \frac{\epsilon_7}{2}(J_7 + J'_7)$$

while for the *outside* we have

$$J'_7 - (1-\epsilon_7)F'_{76}J_6 = \frac{\epsilon_7}{2}(J_7 + J'_7)$$

When the numerical values are inserted, we obtain the set of three equations:

$$J_1 - 0.715J_7 = 1.0012 \times 10^5$$
$$-0.143J_1 + 0.486J_7 - 0.2J'_7 = 59.74$$
$$0.2J_7 - 0.8J'_7 = -250.68$$

which has the solution

$$J_1 = 1.31054 \times 10^5 \text{ W/m}^2$$
$$J_7 = 43{,}264$$
$$J'_7 = 11{,}129$$

The temperatures are then calculated as before:

$$E_{b_7} = \frac{43{,}264 + 11{,}129}{2} = 27{,}197 \qquad T_7 = 832.2 \text{ K}$$

$$E_{b_1} = \frac{(1.0 \times 10^5)(1-0.8)}{0.8} + 1.31054 \times 10^5 = 1.65054 \times 10^5$$
$$T_1 = 1306 \text{ K}$$

So, there is about a 30 K temperature difference between the two methods.

Comment

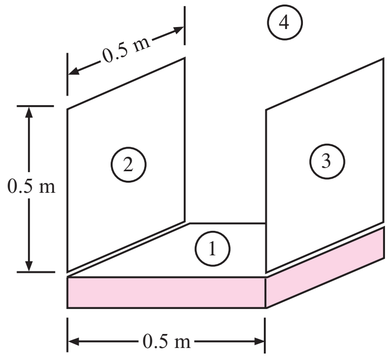
With such a small difference between the solutions we may conclude that the extra complexity of choosing each surface at a different radiosity is probably not worth the effort, particularly when one recognizes the uncertainties that are present in the surface emissivities. *This points out that our assumptions of uniform irradiation and radiosity, though strictly not correct, give answers that are quite satisfactory.*

Numerical Solution for Combined Convection and Radiation (Nonlinear System)

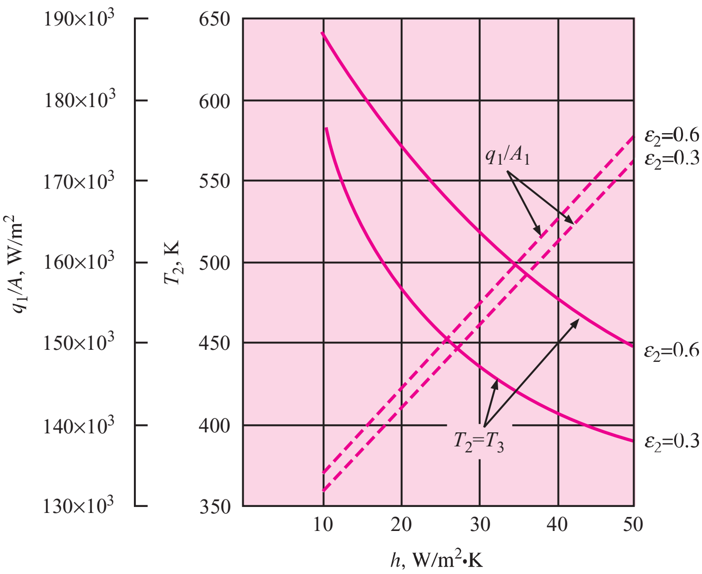
EXAMPLE 8-19

A 0.5 by 0.5 m plate is maintained at 1300 K and exposed to a convection and radiation surrounding at 300 K. Attached to the top are two radiation shields 0.5 by 0.5 m as shown in Figure Example 8-19(a). The convection heat-transfer coefficient for all surfaces is 50 W/m$^2\cdot$K, and $\epsilon_1 = 0.8$, $\epsilon_2 = 0.3 = \epsilon_3$. Determine the total heat lost from the 1300 K surface and the temperature of the shields.

Figure Example 8-19



(*a*)



(*b*)

■ Solution

This example illustrates how it is possible to handle convection-radiation problems with the numerical formulation and an iterative computational procedure. Nomenclature is shown in the figure. Using Figures 8-12 and 8-14, we can evaluate the shape factors as

$$F_{12} = F_{13} = 0.2 \qquad F_{14} = 1 - 0.2 - 0.2 = 0.6$$
$$F_{23} = F_{32} = 0.2 \qquad F_{24L} = F_{34R} = 1.0$$
$$F_{21} = F_{12} = F_{31} = 0.2 \qquad F_{24R} = F_{34L} = 0.6$$
$$F_{11} = F_{22} = F_{33} = 0$$
$$J_{2R} = J_{3L} \qquad J_{2L} = J_{3R} \qquad \text{from symmetry}$$
$$J_4 = E_{b_4}$$

We now use Equation (8-105) to obtain a relation for J_1:

$$J_1 = (1 - \epsilon_i)[F_{12}J_{2R} + F_{13}J_{3L} + F_{14}J_4] + \epsilon_1 E_{b_1}$$

But $J_{2R} = J_{3L}$ and $F_{12} = F_{13}$ so that

$$J_1 = (1 - \epsilon_1)(2F_{13}J_{2R} + F_{14}J_4) + \epsilon_1 E_{b_1} \quad [a]$$

We use Equation (8-108) for the overall energy balance on surface 2:

$$2h(T_\infty - T_2) = \frac{\epsilon_2}{1 - \epsilon_2}(E_{b_2} - J_{2R}) + \frac{\epsilon_2}{1 - \epsilon_2}(E_{b_2} - J_{2L})$$

$$= \frac{\epsilon_2}{1 - \epsilon_2}(2E_{b_2} - J_{2R} - J_{2L}) \quad [b]$$

Equation (8-105) is used for surface J_{2R}.

$$J_{2R} = (1 - \epsilon_2)(F_{21}J_1 + F_{23}J_{3L} + F_{24R}J_4) + \epsilon_2 E_{b_2}$$

But $J_{2R} = J_{3L}$ so that

$$J_{2R} = \frac{1}{1 - (1 - \epsilon_2)F_{23}}[(1 - \epsilon_2)(F_{21}J_1 + F_{24R}J_4) + \epsilon_2 E_{b_2}] \quad [c]$$

For surface J_{2L} the equation is

$$J_{2L} = (1 - \epsilon_2)(F_{24L}J_4) + \epsilon_2 E_{b_2} \quad [d]$$

Equation (b) is nonlinear in E_b so we must use an iterative method to solve the set. Such procedures are described in Reference 34. Applying the iteration technique, we obtain the final solution set as

$$J_1 = 1.3135 \times 10^5 \quad J_{2R} = 22{,}051$$

$$J_{2L} = 710 \quad E_{b_2} = 1275 \quad T_2 = 386.6 \text{ K}$$

The total heat flux lost by surface 1 is

$$\frac{q_1}{A_1} = h(T_1 - T_\infty) + (E_{b_1} - J_1)\frac{\epsilon_1}{1 - \epsilon_1}$$

$$= 1.7226 \times 10^5 \text{ W/m}^2 \quad [e]$$

For a 0.5 by 0.5 m surface the heat lost is thus

$$q_1 = (1.7226 \times 10^5)(0.5)^2 = 43{,}065 \text{ W}$$

Other cases may be computed, and the influence that h and ϵ_2 have on the results is shown in the accompanying figure.

Comment

This example illustrates how nonlinear equations resulting from combined convection and radiation can be solved with an iterative procedure.

8-14 | SOLAR RADIATION

Solar radiation is a form of thermal radiation having a particular wavelength distribution. Its intensity is strongly dependent on atmospheric conditions, time of year, and the angle of incidence for the sun's rays on the surface of the earth. At the outer limit of the atmosphere the total solar irradiation when the earth is at its mean distance from the sun is 1395 W/m^2. This number is called the *solar constant* and is subject to modification upon collection of more precise experimental data.

Not all the energy expressed by the solar constant reaches the surface of the earth, because of strong absorption by carbon dioxide and water vapor in the atmosphere. The