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CHAPTER

5

Principles of Convection

5-1 | INTRODUCTION

The preceding chapters have considered the mechanism and calculation of conduction heat transfer. Convection was considered only insofar as it related to the boundary conditions imposed on a conduction problem. We now wish to examine the methods of calculating convection heat transfer and, in particular, the ways of predicting the value of the convection heat-transfer coefficient h. The subject of convection heat transfer requires an energy balance along with an analysis of the fluid dynamics of the problems concerned. Our discussion in this chapter will first consider some of the simple relations of fluid dynamics and boundary-layer analysis that are important for a basic understanding of convection heat transfer. Next, we shall impose an energy balance on the flow system and determine the influence of the flow on the temperature gradients in the fluid. Finally, having obtained a knowledge of the temperature distribution, the heat-transfer rate from a heated surface to a fluid that is forced over it may be determined.

Our development in this chapter is primarily analytical in character and is concerned only with forced-convection flow systems. Subsequent chapters will present empirical relations for calculating forced-convection heat transfer and will also treat the subjects of natural convection and boiling and condensation heat transfer.

5-2 | VISCOUS FLOW

Consider the flow over a flat plate as shown in Figures 5-1 and 5-2. Beginning at the leading edge of the plate, a region develops where the influence of viscous forces is felt. These viscous forces are described in terms of a shear stress τ between the fluid layers. If this stress is assumed to be proportional to the normal velocity gradient, we have the defining equation for the viscosity,

$$\tau = \mu \frac{du}{dy} \qquad \textbf{[5-1]}$$

The constant of proportionality μ is called the *dynamic viscosity.* A typical set of units is newton-seconds per square meter; however, many sets of units are used for the viscosity, and care must be taken to select the proper group that will be consistent with the formulation at hand.

The region of flow that develops from the leading edge of the plate in which the effects of viscosity are observed is called the *boundary layer*. Some arbitrary point is used to

Figure 5-1 | Sketch showing different boundary-layer flow regimes on a flat plate.

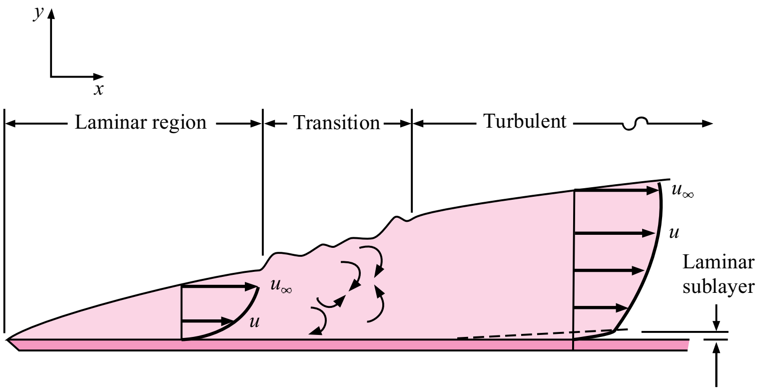
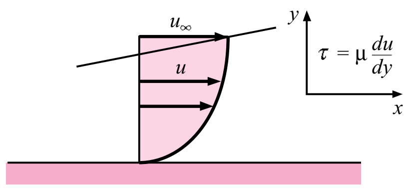


Figure 5-2 | Laminar velocity profile on a flat plate.



designate the y position where the boundary layer ends; this point is usually chosen as the y coordinate where the velocity becomes 99 percent of the free-stream value.

Initially, the boundary-layer development is laminar, but at some critical distance from the leading edge, depending on the flow field and fluid properties, small disturbances in the flow begin to become amplified, and a transition process takes place until the flow becomes turbulent. The turbulent-flow region may be pictured as a random churning action with chunks of fluid moving to and fro in all directions.

The transition from laminar to turbulent flow occurs when

$$\frac{u_\infty x}{\nu} = \frac{\rho u_\infty x}{\mu} > 5 \times 10^5$$

where

u_∞ = free-stream velocity, m/s

x = distance from leading edge, m

$\nu = \mu/\rho$ = kinematic viscosity, m^2/s

This particular grouping of terms is called the Reynolds number, and is dimensionless if a consistent set of units is used for all the properties:

$$\mathrm{Re}_x = \frac{u_\infty x}{\nu} \qquad [5\text{-}2]$$

Although the critical Reynolds number for transition on a flat plate is usually taken as 5×10^5 for most analytical purposes, the critical value in a practical situation is strongly dependent on the surface-roughness conditions and the "turbulence level" of the free stream. The normal range for the beginning of transition is between 5×10^5 and 10^6. With very large

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disturbances present in the flow, transition may begin with Reynolds numbers as low as 10^5, and for flows that are very free from fluctuations, it may not start until $\mathrm{Re} = 2 \times 10^6$ or more. In reality, the transition process is one that covers a range of Reynolds numbers, with transition being complete and with developed turbulent flow usually observed at Reynolds numbers twice the value at which transition began.

The relative shapes for the velocity profiles in laminar and turbulent flow are indicated in Figure 5-1. The laminar profile is approximately parabolic, while the turbulent profile has a portion near the wall that is very nearly linear. This linear portion is said to be due to a laminar sublayer that hugs the surface very closely. Outside this sublayer the velocity profile is relatively flat in comparison with the laminar profile.

The physical mechanism of viscosity is one of momentum exchange. Consider the laminar-flow situation. Molecules may move from one lamina to another, carrying with them a momentum corresponding to the velocity of the flow. There is a net momentum transport from regions of high velocity to regions of low velocity, thus creating a force in the direction of the flow. This force is the viscous-shear stress, which is calculated with Equation (5-1).

The rate at which the momentum transfer takes place is dependent on the rate at which the molecules move across the fluid layers. In a gas, the molecules would move about with some average speed proportional to the square root of the absolute temperature since, in the kinetic theory of gases, we identify temperature with the mean kinetic energy of a molecule. The faster the molecules move, the more momentum they will transport. Hence we should expect the viscosity of a gas to be approximately proportional to the square root of temperature, and this expectation is corroborated fairly well by experiment. The viscosities of some typical fluids are given in Appendix A.

In the turbulent-flow region, distinct fluid layers are no longer observed, and we are forced to seek a somewhat different concept for viscous action. A qualitative picture of the turbulent-flow process may be obtained by imagining macroscopic chunks of fluid transporting energy and momentum instead of microscopic transport on the basis of individual molecules. Naturally, we should expect the larger mass of the macroscopic elements of fluid to transport more energy and momentum than the individual molecules, and we should also expect a larger viscous-shear force in turbulent flow than in laminar flow (and a larger thermal conductivity as well). This expectation is verified by experiment, and it is this larger viscous action in turbulent flow which causes the flat velocity profile indicated in Figure 5-1.

Consider the flow in a tube as shown in Figure 5-3. A boundary layer develops at the entrance, as shown. Eventually the boundary layer fills the entire tube, and the flow is said to be fully developed. If the flow is laminar, a parabolic velocity profile is experienced, as shown in Figure 5-3*a*. When the flow is turbulent, a somewhat blunter profile is observed, as in Figure 5-3*b*. In a tube, the Reynolds number is again used as a criterion for laminar and turbulent flow. For

$$\mathrm{Re}_d = \frac{u_m d}{\nu} > 2300 \qquad \textbf{[5-3]}$$

the flow is usually observed to be turbulent d is the tube diameter.

Again, a range of Reynolds numbers for transition may be observed, depending on the pipe roughness and smoothness of the flow. The generally accepted range for transition is

$$2000 < \mathrm{Re}_d < 4000$$

although laminar flow has been maintained up to Reynolds numbers of 25,000 in carefully controlled laboratory conditions.

The continuity relation for one-dimensional flow in a tube is

$$\dot{m} = \rho u_m A \qquad \textbf{[5-4]}$$

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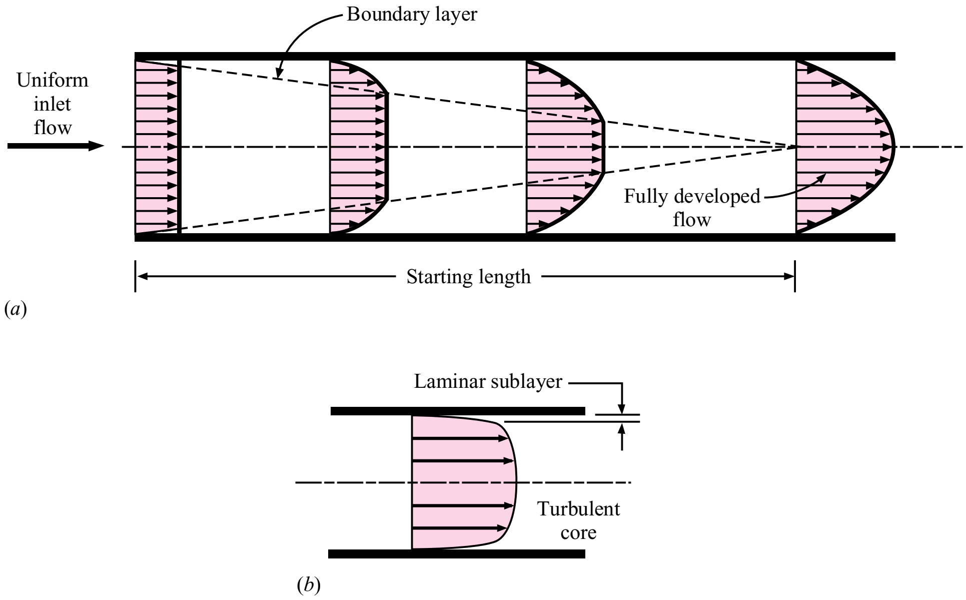
Figure 5-3 | Velocity profile for (*a*) laminar flow in a tube and (*b*) turbulent tube flow.



where

$\dot{m}$ = mass rate of flow
u_m = mean velocity
A = cross-sectional area

We define the mass velocity as

$$\text{Mass velocity} = G = \frac{\dot{m}}{A} = \rho u_m \qquad \textbf{[5-5]}$$

so that the Reynolds number may also be written

$$\text{Re}_d = \frac{Gd}{\mu} \qquad \textbf{[5-6]}$$

Equation (5-6) is sometimes more convenient to use than Equation (5-3).

5-3 | INVISCID FLOW

Although no real fluid is inviscid, in some instances the fluid may be treated as such, and it is worthwhile to present some of the equations that apply in these circumstances. For example, in the flat-plate problem discussed above, the flow at a sufficiently large distance from the plate will behave as a nonviscous flow system. The reason for this behavior is that the velocity gradients normal to the flow direction are very small, and hence the viscous-shear forces are small.

If a balance of forces is made on an element of incompressible fluid and these forces are set equal to the change in momentum of the fluid element, the Bernoulli equation for flow along a streamline results:

$$\frac{p}{\rho} + \frac{1}{2}\frac{V^2}{g_c} = \text{const} \qquad \textbf{[5-7}a\textbf{]}$$

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or, in differential form,

$$\frac{dp}{\rho} + \frac{V\,dV}{g_c} = 0 \qquad \textbf{[5-7}b\textbf{]}$$

where

$\rho =$ fluid density, kg/m^3
$p =$ pressure at particular point in flow, Pa
$V =$ velocity of flow at that point, m/s

The Bernoulli equation is sometimes considered an energy equation because the $V^2/2g_c$ term represents kinetic energy and the pressure represents potential energy; however, it must be remembered that these terms are derived on the basis of a dynamic analysis, so that the equation is fundamentally a dynamic equation. In fact, the concept of kinetic energy is based on a dynamic analysis.

When the fluid is compressible, an energy equation must be written that will take into account changes in internal thermal energy of the system and the corresponding changes in temperature. For a one-dimensional flow system this equation is the steady-flow energy equation for a control volume,

$$i_1 + \frac{1}{2g_c}V_1^2 + Q = i_2 + \frac{1}{2g_c}V_2^2 + Wk \qquad \textbf{[5-8]}$$

where i is the enthalpy defined by

$$i = e + pv \qquad \textbf{[5-9]}$$

and where

$e =$ internal energy
$Q =$ heat added to control volume
$Wk =$ net external work done in the process
$v =$ specific volume of fluid

(The symbol i is used to denote the enthalpy instead of the customary h to avoid confusion with the heat-transfer coefficient.) The subscripts 1 and 2 refer to entrance and exit conditions to the control volume. To calculate pressure drop in compressible flow, it is necessary to specify the equation of state of the fluid, for example, for an ideal gas,

$$p = \rho RT \qquad \Delta e = c_v \Delta T \qquad \Delta i = c_p \Delta T$$

The gas constant for a particular gas is given in terms of the universal gas constant $\Re$ as

$$R = \frac{\Re}{M}$$

where M is the molecular weight and $\Re = 8314.5$ J/kg · mol · K. For air, the appropriate ideal-gas properties are

$$R_{\text{air}} = 287 \text{ J/kg}\cdot\text{K} \qquad c_{p,\text{air}} = 1.005 \text{ kJ/kg}\cdot{}^\circ\text{C} \qquad c_{v,\text{air}} = 0.718 \text{ kJ/kg}\cdot{}^\circ\text{C}$$

To solve a particular problem, we must also specify the process. For example, reversible adiabatic flow through a nozzle yields the following familiar expressions relating the properties at some point in the flow to the Mach number and the stagnation properties, i.e., the

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properties where the velocity is zero:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/(\gamma-1)}$$

where

T_0, p_0, ρ_0 = stagnation properties
γ = ratio of specific heats c_p/c_v
M = Mach number

$$M = \frac{V}{a}$$

where a is the local velocity of sound, which may be calculated from

$$a = \sqrt{\gamma g_c R T} \qquad \textbf{[5-10]}$$

for an ideal gas.[†] For air behaving as an ideal gas this equation reduces to

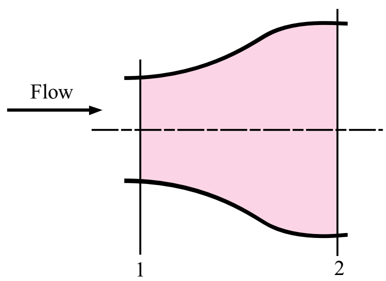
$$a = 20.045\sqrt{T} \quad \text{m/s} \qquad \textbf{[5-11]}$$

where T is in degrees Kelvin.

EXAMPLE 5-1 Water Flow in a Diffuser

Water at 20°C flows at 8 kg/s through the diffuser arrangement shown in Figure Example 5-1. The diameter at section 1 is 3.0 cm, and the diameter at section 2 is 7.0 cm. Determine the increase in static pressure between sections 1 and 2. Assume frictionless flow.

Figure Example 5-1



■ **Solution**

The flow cross-sectional areas are

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi(0.03)^2}{4} = 7.069 \times 10^{-4} \text{ m}^2$$

$$A_2 = \frac{\pi d_2^2}{4} = \frac{\pi(0.07)^2}{4} = 3.848 \times 10^{-3} \text{ m}^2$$

[†]The isentropic flow formulas are derived in Reference 7, p. 629.

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The density of water at 20°C is 1000 kg/m^3, and so we may calculate the velocities from the mass-continuity relation

$$u = \frac{\dot{m}}{\rho A}$$

$$u_1 = \frac{8.0}{(1000)(7.069 \times 10^{-4})} = 11.32 \text{ m/s} \quad [37.1 \text{ ft/s}]$$

$$u_2 = \frac{8.0}{(1000)(3.848 \times 10^{-3})} = 2.079 \text{ m/s} \quad [6.82 \text{ ft/s}]$$

The pressure difference is obtained from the Bernoulli equation (5-7*a*):

$$\frac{p_2 - p_1}{\rho} = \frac{1}{2g_c}(u_1^2 - u_2^2)$$

$$p_2 - p_1 = \frac{1000}{2}[(11.32)^2 - (2.079)^2]$$

$$= 61.91 \text{ kPa} \quad [8.98 \text{ lb/in}^2 \text{ abs}]$$

Isentropic Expansion of Air — EXAMPLE 5-2

Air at 300°C and 0.7 MPa pressure is expanded isentropically from a tank until the velocity is 300 m/s. Determine the static temperature, pressure, and Mach number of the air at the high-velocity condition. $\gamma = 1.4$ for air.

Solution

We may write the steady-flow energy equation as

$$i_1 = i_2 + \frac{u_2^2}{2g_c}$$

because the initial velocity is small and the process is adiabatic. In terms of temperature,

$$c_p(T_1 - T_2) = \frac{u_2^2}{2g_c}$$

$$(1005)(300 - T_2) = \frac{(300)^2}{(2)(1.0)}$$

$$T_2 = 255.2°\text{C} = 528.2 \text{ K} \quad [491.4°\text{F}]$$

We may calculate the pressure from the isentropic relation

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}$$

$$p_2 = (0.7)\left(\frac{528.2}{573}\right)^{3.5} = 0.526 \text{ MPa} \quad [76.3 \text{ lb/in}^2 \text{ abs}]$$

The velocity of sound at condition 2 is

$$a_2 = (20.045)(528.2)^{1/2} = 460.7 \text{ m/s} \quad [1511 \text{ ft/s}]$$

so that the Mach number is

$$M_2 = \frac{u_2}{a_2} = \frac{300}{460.7} = 0.651$$

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5-4 | LAMINAR BOUNDARY LAYER ON A FLAT PLATE

Consider the elemental control volume shown in Figure 5-4. We derive the equation of motion for the boundary layer by making a force-and-momentum balance on this element. To simplify the analysis we assume:

1. The fluid is incompressible and the flow is steady.
2. There are no pressure variations in the direction perpendicular to the plate.
3. The viscosity is constant.
4. Viscous-shear forces in the y direction are negligible.

We apply Newton's second law of motion,

$$\sum F_x = \frac{d(mV)_x}{d\tau}$$

The above form of Newton's second law of motion applies to a system of constant mass. In fluid dynamics it is not usually convenient to work with elements of mass; rather, we deal with elemental control volumes such as that shown in Figure 5-4, where mass may flow in or out of the different sides of the volume, which is fixed in space. For this system the force balance is then written

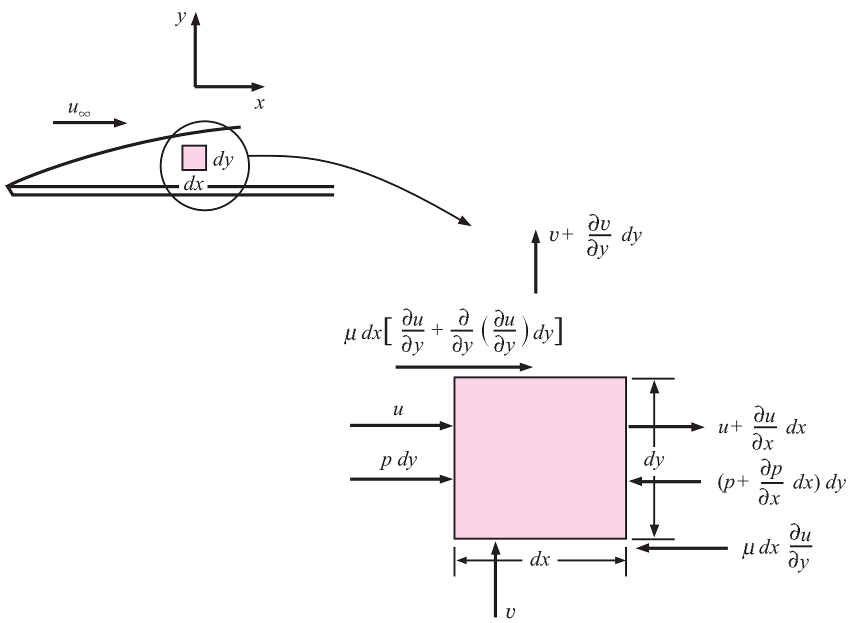
$$\sum F_x = \text{increase in momentum flux in } x \text{ direction}$$

The momentum flux in the x direction is the product of the mass flow through a particular side of the control volume and the x component of velocity at that point.

The mass entering the left face of the element per unit time is

$$\rho u\, dy$$

Figure 5-4 | Elemental control volume for force balance on laminar boundary layer.



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if we assume unit depth in the z direction. Thus the momentum flux entering the left face per unit time is

$$\rho u\, dy\, u = \rho u^2\, dy$$

The mass flow leaving the right face is

$$\rho\left(u + \frac{\partial u}{\partial x}\, dx\right) dy$$

and the momentum flux leaving the right face is

$$\rho\left(u + \frac{\partial u}{\partial x}\, dx\right)^2 dy$$

The mass flow entering the bottom face is

$$\rho v\, dx$$

and the mass flow leaving the top face is

$$\rho\left(v + \frac{\partial v}{\partial y}\, dy\right) dx$$

A mass balance on the element yields

$$\rho u\, dy + \rho v\, dx = \rho\left(u + \frac{\partial u}{\partial x}\, dx\right) dy + \rho\left(v + \frac{\partial v}{\partial y}\, dy\right) dx$$

or

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \textbf{[5-12]}$$

This is the mass continuity equation for the boundary layer.

Returning to the momentum-and-force analysis, the momentum flux *in the x direction* that enters the bottom face is

$$\rho v u\, dx$$

and the momentum *in the x direction* that leaves the top face is

$$\rho\left(v + \frac{\partial v}{\partial y}\, dy\right)\left(u + \frac{\partial u}{\partial y}\, dy\right) dx$$

We are interested only in the momentum in the x direction because the forces considered in the analysis are those in the x direction. These forces are those due to viscous shear and the pressure forces on the element. The pressure force on the left face is $p\, dy$, and that on the right is $-[p + (\partial p/\partial x)\, dx]\, dy$, so that the net pressure force in the direction of motion is

$$-\frac{\partial p}{\partial x}\, dx\, dy$$

The viscous-shear force on the bottom face is

$$-\mu \frac{\partial u}{\partial y}\, dx$$

and the shear force on the top is

$$\mu\, dx\left[\frac{\partial u}{\partial y} + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) dy\right]$$

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The net viscous-shear force in the direction of motion is the sum of the two terms:

$$\text{Net viscous-shear force} = \mu \frac{\partial^2 u}{\partial y^2}\, dx\, dy$$

Equating the sum of the viscous-shear and pressure forces to the net momentum transfer in the x direction, we have

$$\mu \frac{\partial^2 u}{\partial y^2}\, dx\, dy - \frac{\partial p}{\partial x}\, dx\, dy = \rho \left(u + \frac{\partial u}{\partial x} dx\right)^2 dy - \rho u^2\, dy$$
$$+ \rho \left(v + \frac{\partial v}{\partial y} dy\right)\left(u + \frac{\partial u}{\partial y} dy\right) dx - \rho v u\, dx$$

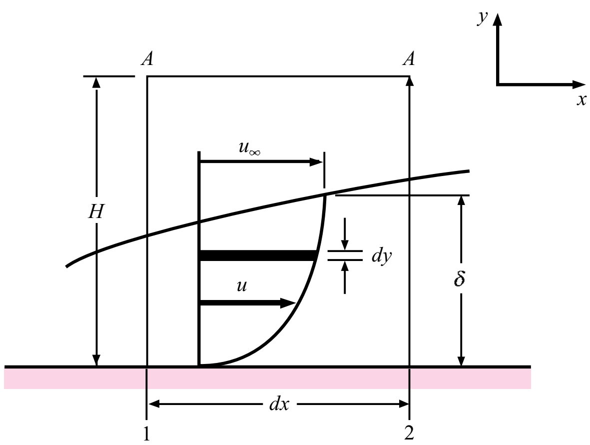
Clearing terms, making use of the continuity relation (5-12), and neglecting second-order differentials, gives

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial p}{\partial x} \qquad \textbf{[5-13]}$$

This is the momentum equation of the laminar boundary layer with constant properties. The equation may be solved exactly for many boundary conditions, and the reader is referred to the treatise by Schlichting [1] for details of the various methods employed in the solutions. In Appendix B we have included the classical method for obtaining an exact solution to Equation (5-13) for laminar flow over a flat plate. For the development in this chapter we shall be satisfied with an approximate analysis that furnishes an easier solution without a loss in physical understanding of the processes involved. The approximate method is due to von Kármán [2].

Consider the boundary-layer flow system shown in Figure 5-5. The free-stream velocity outside the boundary layer is u_∞, and the boundary-layer thickness is δ. We wish to make a momentum-and-force balance on the control volume bounded by the planes 1, 2, A-A, and the solid wall. The velocity components normal to the wall are neglected, and only those in the x direction are considered. We assume that the control volume is sufficiently high that it always encloses the boundary layer; that is, $H > \delta$.

Figure 5-5 | Elemental control volume for integral momentum analysis of laminar boundary layer.



The mass flow through plane 1 is

$$\int_0^H \rho u \, dy \tag{a}$$

and the momentum flow through plane 1 is

$$\int_0^H \rho u^2 \, dy \tag{b}$$

The momentum flow through plane 2 is

$$\int_0^H \rho u^2 \, dy + \frac{d}{dx}\left(\int_0^H \rho u^2 \, dy\right) dx \tag{c}$$

and the mass flow through plane 2 is

$$\int_0^H \rho u \, dy + \frac{d}{dx}\left(\int_0^H \rho u \, dy\right) dx \tag{d}$$

Considering the conservation of mass and the fact that no mass can enter the control volume through the solid wall, the additional mass flow in expression (d) over that in (a) must enter through plane A-A. This mass flow carries with it a momentum in the x direction equal to

$$u_\infty \frac{d}{dx}\left(\int_0^H \rho u \, dy\right) dx$$

The net momentum flow out of the control volume is therefore

$$\frac{d}{dx}\left(\int_0^H \rho u^2 \, dy\right) dx - u_\infty \frac{d}{dx}\left(\int_0^H \rho u \, dy\right) dx$$

This expression may be put in a somewhat more useful form by recalling the product formula from the differential calculus:

$$d(\eta\phi) = \eta \, d\phi + \phi \, d\eta$$

or

$$\eta \, d\phi = d(\eta\phi) - \phi \, d\eta$$

In the momentum expression given above, the integral

$$\int_0^H \rho u \, dy$$

is the ϕ function and u_∞ is the η function. Thus

$$u_\infty \frac{d}{dx}\left(\int_0^H \rho u \, dy\right) dx = \frac{d}{dx}\left(u_\infty \int_0^H \rho u \, dy\right) dx - \frac{du_\infty}{dx}\left(\int_0^H \rho u \, dy\right) dx \tag{5-14}$$

$$= \frac{d}{dx}\left(\int_0^H \rho u u_\infty \, dy\right) dx - \frac{du_\infty}{dx}\left(\int_0^H \rho u \, dy\right) dx$$

The u_∞ may be placed inside the integral since it is not a function of y and thus may be treated as a constant insofar as an integral with respect to y is concerned.

Returning to the analysis, the force on plane 1 is the pressure force pH and that on plane 2 is $[p+(dp/dx)\,dx]H$. The shear force at the wall is

$$-\tau_w\,dx = -\mu\,dx\,\frac{\partial u}{\partial y}\bigg]_{y=0}$$

There is no shear force at plane A-A since the velocity gradient is zero outside the boundary layer. Setting the forces on the element equal to the net increase in momentum and collecting terms gives

$$-\tau_w - \frac{dp}{dx}H = -\rho\frac{d}{dx}\int_0^H (u_\infty - u)u\,dy + \frac{du_\infty}{dx}\int_0^H \rho u\,dy \qquad \textbf{[5-15]}$$

This is the integral momentum equation of the boundary layer. If the pressure is constant throughout the flow,

$$\frac{dp}{dx} = 0 = -\rho u_\infty \frac{du_\infty}{dx} \qquad \textbf{[5-16]}$$

since the pressure and free-stream velocity are related by the Bernoulli equation. For the constant-pressure condition, the integral boundary-layer equation becomes

$$\rho\frac{d}{dx}\int_0^\delta (u_\infty - u)u\,dy = \tau_w = \mu\,\frac{\partial u}{\partial y}\bigg]_{y=0} \qquad \textbf{[5-17]}$$

The upper limit on the integral has been changed to δ because the integrand is zero for $y > \delta$ since $u = u_\infty$ for $y > \delta$.

If the velocity profile were known, the appropriate function could be inserted in Equation (5-17) to obtain an expression for the boundary-layer thickness. For our approximate analysis we first write down some conditions that the velocity function must satisfy:

$$u = 0 \qquad \text{at } y = 0 \qquad \textbf{[a]}$$

$$u = u_\infty \qquad \text{at } y = \delta \qquad \textbf{[b]}$$

$$\frac{\partial u}{\partial y} = 0 \qquad \text{at } y = \delta \qquad \textbf{[c]}$$

For a constant-pressure condition Equation (5-13) yields

$$\frac{\partial^2 u}{\partial y^2} = 0 \qquad \text{at } y = 0 \qquad \textbf{[d]}$$

since the velocities u and v are zero at $y = 0$. We assume that the velocity profiles at various x positions are similar; that is, they have the same functional dependence on the y coordinate. There are four conditions to satisfy. The simplest function that we can choose to satisfy these conditions is a polynomial with four arbitrary constants. Thus

$$u = C_1 + C_2 y + C_3 y^2 + C_4 y^3 \qquad \textbf{[5-18]}$$

Applying the four conditions (a) to (d),

$$\frac{u}{u_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \qquad \textbf{[5-19]}$$

Inserting the expression for the velocity into Equation (5-17) gives

$$\frac{d}{dx}\left\{\rho u_\infty^2 \int_0^\delta \left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right]\left[1 - \frac{3}{2}\frac{y}{\delta} + \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right]dy\right\} = \mu\,\frac{\partial u}{\partial y}\bigg]_{y=0}$$

$$= \frac{3}{2}\frac{\mu u_\infty}{\delta}$$

Carrying out the integration leads to

$$\frac{d}{dx}\left(\frac{39}{280}\rho u_\infty^2\,\delta\right)=\frac{3}{2}\frac{\mu u_\infty}{\delta}$$

Since ρ and u_∞ are constants, the variables may be separated to give

$$\delta\,d\delta=\frac{140}{13}\frac{\mu}{\rho u_\infty}\,dx=\frac{140}{13}\frac{\nu}{u_\infty}\,dx$$

and

$$\frac{\delta^2}{2}=\frac{140}{13}\frac{\nu x}{u_\infty}+\text{const}$$

At $x=0$, $\delta=0$, so that

$$\delta=4.64\sqrt{\frac{\nu x}{u_\infty}} \qquad \textbf{[5-20]}$$

This may be written in terms of the Reynolds number as

$$\frac{\delta}{x}=\frac{4.64}{\text{Re}_x^{1/2}}$$

where

$$\text{Re}_x=\frac{u_\infty x}{\nu} \qquad \textbf{[5-21]}$$

The exact solution of the boundary-layer equations as given in Appendix B yields

$$\frac{\delta}{x}=\frac{5.0}{\text{Re}_x^{1/2}} \qquad \textbf{[5-21}a\textbf{]}$$

Mass Flow and Boundary-Layer Thickness

EXAMPLE 5-3

Air at 27°C and 1 atm flows over a flat plate at a speed of 2 m/s. Calculate the boundary-layer thickness at distances of 20 cm and 40 cm from the leading edge of the plate. Calculate the mass flow that enters the boundary layer between $x=20$ cm and $x=40$ cm. The viscosity of air at 27°C is 1.85×10^{-5} kg/m · s. Assume unit depth in the z direction.

■ Solution

The density of air is calculated from

$$\rho=\frac{p}{RT}=\frac{1.0132\times10^5}{(287)(300)}=1.177\ \text{kg/m}^3 \quad [0.073\ \text{lb}_m/\text{ft}^3]$$

The Reynolds number is calculated as

$$\text{At } x=20\text{ cm:}\qquad \text{Re}=\frac{(1.177)(2.0)(0.2)}{1.85\times10^{-5}}=25{,}448$$

$$\text{At } x=40\text{ cm:}\qquad \text{Re}=\frac{(1.177)(2.0)(0.4)}{1.85\times10^{-5}}=50{,}897$$

The boundary-layer thickness is calculated from Equation (5-21):

$$\text{At } x=20\text{ cm:}\qquad \delta=\frac{(4.64)(0.2)}{(25{,}448)^{1/2}}=0.00582\text{ m} \quad [0.24\text{ in}]$$

$$\text{At } x=40\text{ cm:}\qquad \delta=\frac{(4.64)(0.4)}{(50{,}897)^{1/2}}=0.00823\text{ m} \quad [0.4\text{ in}]$$

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To calculate the mass flow that enters the boundary layer from the free stream between $x = 20$ cm and $x = 40$ cm, we simply take the difference between the mass flow in the boundary layer at these two x positions. At any x position the mass flow in the boundary layer is given by the integral

$$\int_0^\delta \rho u \, dy$$

where the velocity is given by Equation (5-19),

$$u = u_\infty \left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \right]$$

Evaluating the integral with this velocity distribution, we have

$$\int_0^\delta \rho u_\infty \left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \right] dy = \frac{5}{8}\rho u_\infty \delta$$

Thus the mass flow entering the boundary layer is

$$\begin{aligned} \Delta m &= \tfrac{5}{8}\rho u_\infty(\delta_{40} - \delta_{20}) \\ &= (\tfrac{5}{8})(1.177)(2.0)(0.0082 - 0.0058) \\ &= 3.531 \times 10^{-3} \text{ kg/s} \quad [7.78 \times 10^{-3} \text{ lb}_m/\text{s}] \end{aligned}$$

5-5 | ENERGY EQUATION OF THE BOUNDARY LAYER

The foregoing analysis considered the fluid dynamics of a laminar-boundary-layer flow system. We shall now develop the energy equation for this system and then proceed to an integral method of solution.

Consider the elemental control volume shown in Figure 5-6. To simplify the analysis we assume

1. Incompressible steady flow
2. Constant viscosity, thermal conductivity, and specific heat
3. Negligible heat conduction in the direction of flow (x direction), i.e.,

$$\frac{\partial T}{\partial x} \ll \frac{\partial T}{\partial y}$$

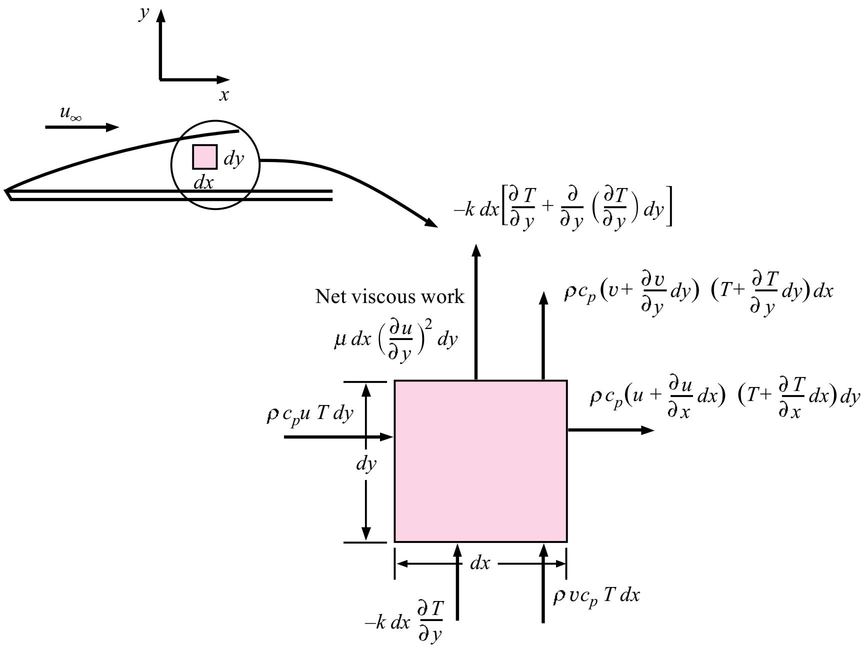
Then, for the element shown, the energy balance may be written

Energy convected in left face + energy convected in bottom face
+ heat conducted in bottom face
+ net viscous work done on element
= energy convected out right face + energy convected out top face
+ heat conducted out top face

The convective and conduction energy quantities are indicated in Figure 5-6, and the energy term for the viscous work may be derived as follows. The viscous work may be computed as a product of the net viscous-shear force and the distance this force moves in unit time. The viscous-shear force is the product of the shear-stress and the area dx,

$$\mu \frac{\partial u}{\partial y} dx$$

Figure 5-6 | Elemental volume for energy analysis of laminar boundary layer.



and the distance through which it moves per unit time in respect to the elemental control volume $dx\,dy$ is

$$\frac{\partial u}{\partial y}\,dy$$

so that the net viscous energy delivered to the element is

$$\mu\left(\frac{\partial u}{\partial y}\right)^2 dx\,dy$$

Writing the energy balance corresponding to the quantities shown in Figure 5-6, assuming unit depth in the z direction, and neglecting second-order differentials yields

$$\rho c_p\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + T\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] dx\,dy = k\frac{\partial^2 T}{\partial y^2}dx\,dy + \mu\left(\frac{\partial u}{\partial y}\right)^2 dx\,dy$$

Using the continuity relation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \textbf{[5-12]}$$

and dividing by ρc_p gives

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad \textbf{[5-22]}$$

This is the energy equation of the laminar boundary layer. The left side represents the net transport of energy into the control volume, and the right side represents the sum of the net heat conducted out of the control volume and the net viscous work done on the element. The viscous-work term is of importance only at high velocities since its magnitude will be small compared with the other terms when low-velocity flow is studied. This may be shown

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with an order-of-magnitude analysis of the two terms on the right side of Equation (5-22). For this order-of-magnitude analysis we might consider the velocity as having the order of the free-stream velocity u_∞ and the y dimension of the order of δ. Thus

$$u \sim u_\infty \qquad \text{and} \qquad y \sim \delta$$

$$\alpha \frac{\partial^2 T}{\partial y^2} \sim \alpha \frac{T}{\delta^2}$$

so that

$$\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \sim \frac{\mu}{\rho c_p}\frac{u_\infty^2}{\delta^2}$$

If the ratio of these quantities is small, that is,

$$\frac{\mu}{\rho c_p \alpha}\frac{u_\infty^2}{T} \ll 1 \qquad \textbf{[5-23]}$$

then the viscous dissipation is small in comparison with the conduction term. Let us rearrange Equation (5-23) by introducing

$$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{c_p \mu}{k}$$

where Pr is called the Prandtl number, which we shall discuss later. Equation (5-23) becomes

$$\mathrm{Pr}\frac{u_\infty^2}{c_p T} \ll 1 \qquad \textbf{[5-24]}$$

As an example, consider the flow of air at

$$u_\infty = 70 \text{ m/s} \qquad T = 20°\text{C} = 293 \text{ K} \qquad p = 1 \text{ atm}$$

For these conditions $c_p = 1005 = \text{J/kg} \cdot °\text{C}$ and $\mathrm{Pr} = 0.7$ so that

$$\mathrm{Pr}\frac{u_\infty^2}{c_p T} = \frac{(0.7)(70)^2}{(1005)(293)} = 0.012 \ll 1.0$$

indicating that the viscous dissipation is small for even this rather large flow velocity of 70 m/s. Thus, for low-velocity incompressible flow, we have

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \qquad \textbf{[5-25]}$$

In reality, our derivation of the energy equation has been a simplified one, and several terms have been left out of the analysis because they are small in comparison with others. In this way we immediately arrive at the boundary-layer approximation, without resorting to a cumbersome elimination process to obtain the final simplified relation. The general derivation of the boundary-layer energy equation is very involved and quite beyond the scope of our discussion. The interested reader should consult the books by Schlichting [1] and White [5] for more information.

There is a striking similarity between Equation (5-25) and the momentum equation for constant pressure,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \qquad \textbf{[5-26]}$$

The solution to the two equations will have exactly the same form when $\alpha = \nu$. Thus we should expect that the relative magnitudes of the thermal diffusivity and kinematic viscosity

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would have an important influence on convection heat transfer since these magnitudes relate the velocity distribution to the temperature distribution. This is exactly the case, and we shall see the role that these parameters play in the subsequent discussion.

5-6 | THE THERMAL BOUNDARY LAYER

Just as the hydrodynamic boundary layer was defined as that region of the flow where viscous forces are felt, a thermal boundary layer may be defined as that region where temperature gradients are present in the flow. These temperature gradients would result from a heat-exchange process between the fluid and the wall.

Consider the system shown in Figure 5-7. The temperature of the wall is T_w, the temperature of the fluid outside the thermal boundary layer is T_∞, and the thickness of the thermal boundary layer is designated as δ_t. At the wall, the velocity is zero, and the heat transfer into the fluid takes place by conduction. Thus the local heat flux per unit area, q'', is

$$\frac{q}{A} = q'' = -k \left.\frac{\partial T}{\partial y}\right]_{\text{wall}} \qquad \textbf{[5-27]}$$

From Newton's law of cooling [Equation (1-8)],

$$q'' = h(T_w - T_\infty) \qquad \textbf{[5-28]}$$

where h is the convection heat-transfer coefficient. Combining these equations, we have

$$h = \frac{-k(\partial T/\partial y)_{\text{wall}}}{T_w - T_\infty} \qquad \textbf{[5-29]}$$

so that we need only find the temperature gradient at the wall in order to evaluate the heat-transfer coefficient. This means that we must obtain an expression for the temperature distribution. To do this, an approach similar to that used in the momentum analysis of the boundary layer is followed.

The conditions that the temperature distribution must satisfy are

$$T = T_w \qquad \text{at } y = 0 \qquad \textbf{[a]}$$

$$\frac{\partial T}{\partial y} = 0 \qquad \text{at } y = \delta_t \qquad \textbf{[b]}$$

$$T = T_\infty \qquad \text{at } y = \delta_t \qquad \textbf{[c]}$$

Figure 5-7 | Temperature profile in the thermal boundary layer.

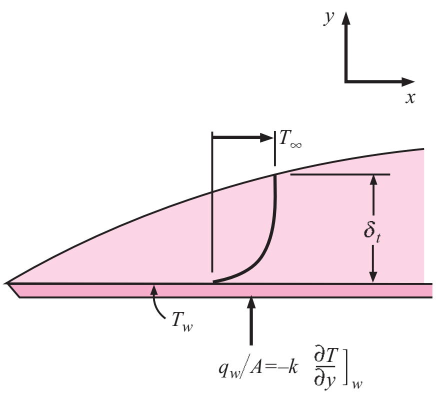
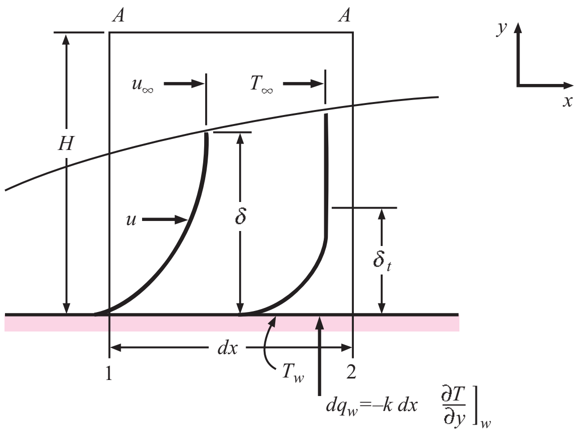


Figure 5-8 | Control volume for integral energy analysis of laminar boundary flow.



and by writing Equation (5-25) at $y = 0$ with no viscous heating we find

$$\frac{\partial^2 T}{\partial y^2} = 0 \qquad \text{at } y = 0 \tag{d}$$

since the velocities must be zero at the wall.

Conditions (*a*) to (*d*) may be fitted to a cubic polynomial as in the case of the velocity profile, so that

$$\frac{\theta}{\theta_\infty} = \frac{T - T_w}{T_\infty - T_w} = \frac{3}{2}\frac{y}{\delta_t} - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3 \tag{5-30}$$

where $\theta = T - T_w$. There now remains the problem of finding an expression for δ_t, the thermal-boundary-layer thickness. This may be obtained by an integral analysis of the energy equation for the boundary layer.

Consider the control volume bounded by the planes 1, 2, *A-A*, and the wall as shown in Figure 5-8. It is assumed that the thermal boundary layer is thinner than the hydrodynamic boundary layer, as shown. The wall temperature is T_w, the free-stream temperature is T_∞, and the heat given up to the fluid over the length dx is dq_w. We wish to make the energy balance

$$\text{Energy convected in} + \text{viscous work within element} + \text{heat transfer at wall} = \text{energy convected out} \tag{5-31}$$

The energy convected in through plane 1 is

$$\rho c_p \int_0^H uT\,dy$$

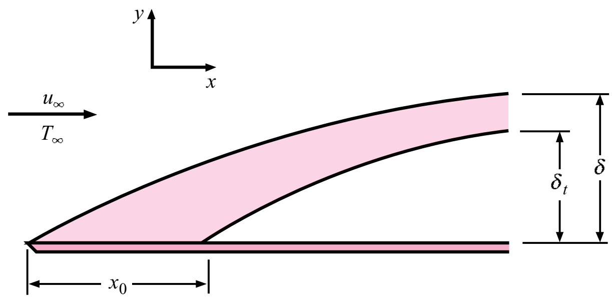
and the energy convected out through plane 2 is

$$\rho c_p \left(\int_0^H uT\,dy\right) + \frac{d}{dx}\left(\rho c_p \int_0^H uT\,dy\right) dx$$

The mass flow through plane *A-A* is

$$\frac{d}{dx}\left(\int_0^H \rho u\,dy\right) dx$$

Figure 5-9 | Hydrodynamic and thermal boundary layers on a flat plate. Heating starts at $x = x_0$.



and this carries with it an energy equal to

$$c_p T_\infty \frac{d}{dx}\left(\int_0^H \rho u \, dy\right) dx$$

The net viscous work done within the element is

$$\mu \left[\int_0^H \left(\frac{du}{dy}\right)^2 dy\right] dx$$

and the heat transfer at the wall is

$$dq_w = -k \, dx \, \frac{\partial T}{\partial y}\bigg]_w$$

Combining these energy quantities according to Equation (5-31) and collecting terms gives

$$\frac{d}{dx}\left[\int_0^H (T_\infty - T) u \, dy\right] + \frac{\mu}{\rho c_p}\left[\int_0^H \left(\frac{du}{dy}\right)^2 dy\right] = \alpha \, \frac{\partial T}{dy}\bigg]_w \qquad \textbf{[5-32]}$$

This is the integral energy equation of the boundary layer for constant properties and constant free-stream temperature T_∞.

To calculate the heat transfer at the wall, we need to derive an expression for the thermal-boundary-layer thickness that may be used in conjunction with Equations (5-29) and (5-30) to determine the heat-transfer coefficient. For now, we neglect the viscous-dissipation term; this term is very small unless the velocity of the flow field becomes very large. And the calculation of high-velocity heat transfer will be considered later.

The plate under consideration need not be heated over its entire length. The situation that we shall analyze is shown in Figure 5-9, where the hydrodynamic boundary layer develops from the leading edge of the plate, while heating does not begin until $x = x_0$.

Inserting the temperature distribution Equation (5-30) and the velocity distribution Equation (5-19) into Equation (5-32) and neglecting the viscous-dissipation term, gives

$$\begin{aligned}\frac{d}{dx}\left[\int_0^H (T_\infty - T) u \, dy\right] &= \frac{d}{dx}\left[\int_0^H (\theta_\infty - \theta) u \, dy\right] \\ &= \theta_\infty u_\infty \frac{d}{dx}\left\{\int_0^H \left[1 - \frac{3}{2}\frac{y}{\delta_t} + \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3\right]\left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right] dy\right\} \\ &= \alpha \, \frac{\partial T}{\partial y}\bigg]_{y=0} = \frac{3\alpha\theta_\infty}{2\delta_t}\end{aligned}$$

Let us assume that the thermal boundary layer is thinner than the hydrodynamic boundary layer. Then we only need to carry out the integration to $y = \delta_t$ since the integrand is zero for $y > \delta_t$. Performing the necessary algebraic manipulation, carrying out the integration, and making the substitution $\zeta = \delta_t/\delta$ yields

$$\theta_\infty u_\infty \frac{d}{dx}\left[\delta\left(\frac{3}{20}\zeta^2 - \frac{3}{280}\zeta^4\right)\right] = \frac{3}{2}\frac{\alpha\theta_\infty}{\delta\zeta} \qquad \textbf{[5-33]}$$

Because $\delta_t < \delta$, $\zeta < 1$, and the term involving ζ^4 is small compared with the ζ^2 term, we neglect the ζ^4 term and write

$$\frac{3}{20}\theta_\infty u_\infty \frac{d}{dx}(\delta\zeta^2) = \frac{3}{2}\frac{\alpha\theta_\infty}{\zeta\delta} \qquad \textbf{[5-34]}$$

Performing the differentiation gives

$$\frac{1}{10}u_\infty\left(2\delta\zeta\frac{d\zeta}{dx} + \zeta^2\frac{d\delta}{dx}\right) = \frac{\alpha}{\delta\zeta}$$

or

$$\frac{1}{10}u_\infty\left(2\delta^2\zeta^2\frac{d\zeta}{dx} + \zeta^3\delta\frac{d\delta}{dx}\right) = \alpha$$

But

$$\delta\, d\delta = \frac{140}{13}\frac{\nu}{u_\infty}dx$$

and

$$\delta^2 = \frac{280}{13}\frac{\nu x}{u_\infty}$$

so that we have

$$\zeta^3 + 4x\zeta^2\frac{d\zeta}{dx} = \frac{13}{14}\frac{\alpha}{\nu} \qquad \textbf{[5-35]}$$

Noting that

$$\zeta^2\frac{d\zeta}{dx} = \frac{1}{3}\frac{d}{dx}\zeta^3$$

we see that Equation (5-35) is a linear differential equation of the first order in ζ^3, and the solution is

$$\zeta^3 = Cx^{-3/4} + \frac{13}{14}\frac{\alpha}{\nu}$$

When the boundary condition

$$\delta_t = 0 \qquad \text{at } x = x_0$$
$$\zeta = 0 \qquad \text{at } x = x_0$$

is applied, the final solution becomes

$$\zeta = \frac{\delta_t}{\delta} = \frac{1}{1.026}\mathrm{Pr}^{-1/3}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3} \qquad \textbf{[5-36]}$$

where

$$\mathrm{Pr} = \frac{\nu}{\alpha} \qquad \textbf{[5-37]}$$

has been introduced. The ratio ν/α is called the Prandtl number after Ludwig Prandtl, the German scientist who introduced the concepts of boundary-layer theory.

When the plate is heated over the entire length, $x_0 = 0$, and

$$\frac{\delta_t}{\delta} = \zeta = \frac{1}{1.026}\mathrm{Pr}^{-1/3} \quad \textbf{[5-38]}$$

In the foregoing analysis the assumption was made that $\zeta < 1$. This assumption is satisfactory for fluids having Prandtl numbers greater than about 0.7. Fortunately, most gases and liquids fall within this category. Liquid metals are a notable exception, however, since they have Prandtl numbers of the order of 0.01.

The Prandtl number ν/α has been found to be the parameter that relates the relative thicknesses of the hydrodynamic and thermal boundary layers. The kinematic viscosity of a fluid conveys information about the rate at which momentum may diffuse through the fluid because of molecular motion. The thermal diffusivity tells us the same thing in regard to the diffusion of heat in the fluid. Thus the ratio of these two quantities should express the relative magnitudes of diffusion of momentum and heat in the fluid. But these diffusion rates are precisely the quantities that determine how thick the boundary layers will be for a given external flow field; large diffusivities mean that the viscous or temperature influence is felt farther out in the flow field. The Prandtl number is thus the connecting link between the velocity field and the temperature field.

The Prandtl number is dimensionless when a consistent set of units is used:

$$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/\rho c_p} = \frac{c_p\mu}{k} \quad \textbf{[5-39]}$$

In the SI system a typical set of units for the parameters would be μ in kilograms per second per meter, c_p in kilojoules per kilogram per Celsius degree, and k in kilowatts per meter per Celsius degree. In the English system one would typically employ μ in pound mass per hour per foot, c_p in Btu per pound mass per Fahrenheit degree, and k in Btu per hour per foot per Fahrenheit degree.

Returning now to the analysis, we have

$$h = \frac{-k(\partial T/\partial y)_w}{T_w - T_\infty} = \frac{3}{2}\frac{k}{\delta_t} = \frac{3}{2}\frac{k}{\zeta\delta} \quad \textbf{[5-40]}$$

Substituting for the hydrodynamic-boundary-layer thickness from Equation (5-21) and using Equation (5-36) gives

$$h_x = 0.332k\,\mathrm{Pr}^{1/3}\left(\frac{u_\infty}{\nu x}\right)^{1/2}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3} \quad \textbf{[5-41]}$$

The equation may be nondimensionalized by multiplying both sides by x/k, producing the dimensionless group on the left side,

$$\mathrm{Nu}_x = \frac{h_x x}{k} \quad \textbf{[5-42]}$$

called the Nusselt number after Wilhelm Nusselt, who made significant contributions to the theory of convection heat transfer. Finally,

$$\mathrm{Nu}_x = 0.332\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3} \quad \textbf{[5-43]}$$

or, for the plate heated over its entire length, $x_0 = 0$ and

$$\mathrm{Nu}_x = 0.332\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2} \quad \textbf{[5-44]}$$

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Equations (5-41), (5-43), and (5-44) express the local values of the heat-transfer coefficient in terms of the distance from the leading edge of the plate and the fluid properties. For the case where $x_0 = 0$ the average heat-transfer coefficient and Nusselt number may be obtained by integrating over the length of the plate:

$$\bar{h} = \frac{\int_0^L h_x\,dx}{\int_0^L dx} = 2h_{x=L} \qquad \text{[5-45}a\text{]}$$

For a plate where heating starts at $x = x_0$, it can be shown that the average heat transfer coefficient can be expressed as

$$\frac{\bar{h}_{x_0-L}}{h_{x=L}} = 2L\frac{1-(x_0/L)^{3/4}}{L-x_0} \qquad \text{[5-45}b\text{]}$$

In this case, the total heat transfer for the plate would be

$$q_{\text{total}} = \bar{h}_{x_0-L}(L-x_0)(T_w - T_\infty)$$

assuming the heated section is at the constant temperature T_w. For the plate heated over the entire length,

$$\overline{\text{Nu}}_L = \frac{\bar{h}L}{k} = 2\ \text{Nu}_{x=L} \qquad \text{[5-46}a\text{]}$$

or

$$\overline{\text{Nu}}_L = \frac{\bar{h}L}{k} = 0.664\ \text{Re}_L^{1/2}\text{Pr}^{1/3} \qquad \text{[5-46}b\text{]}$$

where

$$\text{Re}_L = \frac{\rho u_\infty L}{\mu}$$

The reader should carry out the integrations to verify these results.

The foregoing analysis was based on the assumption that the fluid properties were constant throughout the flow. When there is an appreciable variation between wall and free-stream conditions, it is recommended that the properties be evaluated at the so-called *film temperature* T_f, defined as the arithmetic mean between the wall and free-stream temperature,

$$T_f = \frac{T_w + T_\infty}{2} \qquad \text{[5-47]}$$

An exact solution to the energy equation is given in Appendix B. The results of the exact analysis are the same as those of the approximate analysis given above.

Constant Heat Flux

The above analysis has considered the laminar heat transfer from an isothermal surface. In many practical problems the surface *heat flux* is essentially constant, and the objective is to find the distribution of the plate-surface temperature for given fluid-flow conditions. For the constant-heat-flux case it can be shown that the local Nusselt number is given by

$$\text{Nu}_x = \frac{hx}{k} = 0.453\ \text{Re}_x^{1/2}\ \text{Pr}^{1/3} \qquad \text{[5-48]}$$

which may be expressed in terms of the wall heat flux and temperature difference as

$$\text{Nu}_x = \frac{q_w x}{k(T_w - T_\infty)} \qquad \text{[5-49]}$$

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The average temperature difference along the plate, for the constant-heat-flux condition, may be obtained by performing the integration

$$\overline{T_w - T_\infty} = \frac{1}{L}\int_0^L (T_w - T_\infty)\,dx = \frac{1}{L}\int_0^L \frac{q_w x}{k\,\mathrm{Nu}_x}\,dx$$

$$= \frac{q_w L/k}{0.6795\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}} \qquad \textbf{[5-50]}$$

or

$$q_w = \tfrac{3}{2} h_{x=L}(\overline{T_w - T_\infty})$$

In these equations q_w is the heat flux per unit area and will have the units of watts per square meter (W/m^2) in SI units or British thermal units per hour per square foot (Btu/h · ft^2) in the English system. Note that the heat flux $q_w = q/A$ is assumed constant over the entire plate surface.

Other Relations

Equation (5-44) is applicable to fluids having Prandtl numbers between about 0.6 and 50. It would not apply to fluids with very low Prandtl numbers like liquid metals or to high-Prandtl-number fluids like heavy oils or silicones. For a very wide range of Prandtl numbers, Churchill and Ozoe [9] have correlated a large amount of data to give the following relation for laminar flow on an isothermal flat plate:

$$\mathrm{Nu}_x = \frac{0.3387\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1 + \left(\dfrac{0.0468}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4}} \qquad \text{for } \mathrm{Re}_x\,\mathrm{Pr} > 100 \qquad \textbf{[5-51]}$$

For the constant-heat-flux case, 0.3387 is changed to 0.4637 and 0.0468 is changed to 0.0207. Properties are still evaluated at the film temperature.

Isothermal Flat Plate Heated Over Entire Length — EXAMPLE 5-4

For the flow system in Example 5-3 assume that the plate is heated over its entire length to a temperature of 60°C. Calculate the heat transferred in (*a*) the first 20 cm of the plate and (*b*) the first 40 cm of the plate.

Solution

The total heat transfer over a certain length of the plate is desired; so we wish to calculate average heat-transfer coefficients. For this purpose we use Equations (5-44) and (5-45), evaluating the properties at the film temperature:

$$T_f = \frac{27+60}{2} = 43.5\text{°C} = 316.5\ \text{K} \quad [110.3\text{°F}]$$

From Appendix A the properties are

$$\nu = 17.36 \times 10^{-6}\ \text{m}^2/\text{s} \quad [1.87 \times 10^{-4}\ \text{ft}^2/\text{s}]$$

$$k = 0.02749\ \text{W/m}\cdot\text{°C} \quad [0.0159\ \text{Btu/h}\cdot\text{ft}\cdot\text{°F}]$$

$$\mathrm{Pr} = 0.7$$

$$c_p = 1.006\ \text{kJ/kg}\cdot\text{°C} \quad [0.24\ \text{Btu/lb}_m\cdot\text{°F}]$$

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At $x = 20$ cm

$$\mathrm{Re}_x = \frac{u_\infty x}{\nu} = \frac{(2)(0.2)}{17.36 \times 10^{-6}} = 23{,}041$$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3}$$

$$= (0.332)(23{,}041)^{1/2}(0.7)^{1/3} = 44.74$$

$$h_x = \mathrm{Nu}_x\left(\frac{k}{x}\right) = \frac{(44.74)(0.02749)}{0.2}$$

$$= 6.15\ \mathrm{W/m^2 \cdot {}^\circ C} \quad [1.083\ \mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}]$$

The average value of the heat-transfer coefficient is twice this value, or

$$\bar{h} = (2)(6.15) = 12.3\ \mathrm{W/m^2 \cdot {}^\circ C} \quad [2.17\ \mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}]$$

The heat flow is

$$q = \bar{h}A(T_w - T_\infty)$$

If we assume unit depth in the z direction,

$$q = (12.3)(0.2)(60 - 27) = 81.18\ \mathrm{W} \quad [277\ \mathrm{Btu/h}]$$

At $x = 40$ cm

$$\mathrm{Re}_x = \frac{u_\infty x}{\nu} = \frac{(2)(0.4)}{17.36 \times 10^{-6}} = 46{,}082$$

$$\mathrm{Nu}_x = (0.332)(46{,}082)^{1/2}(0.7)^{1/3} = 63.28$$

$$h_x = \frac{(63.28)(0.02749)}{0.4} = 4.349\ \mathrm{W/m^2 \cdot {}^\circ C}$$

$$\bar{h} = (2)(4.349) = 8.698\ \mathrm{W/m^2 \cdot {}^\circ C} \quad [1.53\ \mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}]$$

$$q = (8.698)(0.4)(60 - 27) = 114.8\ \mathrm{W} \quad [392\ \mathrm{Btu/h}]$$

EXAMPLE 5-5 Flat Plate with Constant Heat Flux

A 1.0-kW heater is constructed of a glass plate with an electrically conducting film that produces a constant heat flux. The plate is 60 cm by 60 cm and placed in an airstream at 27°C, 1 atm with $u_\infty = 5$ m/s. Calculate the average temperature difference along the plate and the temperature difference at the trailing edge.

■ Solution

Properties should be evaluated at the film temperature, but we do not know the plate temperature. So for an initial calculation, we take the properties at the free-stream conditions of

$$T_\infty = 27^\circ\mathrm{C} = 300\ \mathrm{K}$$

$$\nu = 15.69 \times 10^{-6}\ \mathrm{m^2/s} \qquad \mathrm{Pr} = 0.708 \qquad k = 0.02624\ \mathrm{W/m \cdot {}^\circ C}$$

$$\mathrm{Re}_L = \frac{(0.6)(5)}{15.69 \times 10^{-6}} = 1.91 \times 10^5$$

From Equation (5-50) the average temperature difference is

$$\overline{T_w - T_\infty} = \frac{[1000/(0.6)^2](0.6)/0.02624}{0.6795(1.91 \times 10^5)^{1/2}(0.708)^{1/3}} = 240^\circ\mathrm{C}$$

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Now, we go back and evaluate properties at

$$T_f = \frac{240 + 27 + 27}{2} = 147°\text{C} = 420 \text{ K}$$

and obtain

$$\nu = 28.22 \times 10^{-6} \text{ m}^2/\text{s} \qquad \text{Pr} = 0.687 \qquad k = 0.035 \text{ W/m} \cdot °\text{C}$$

$$\text{Re}_L = \frac{(0.6)(5)}{28.22 \times 10^{-6}} = 1.06 \times 10^5$$

$$\overline{T_w - T_\infty} = \frac{[1000/(0.6)^2](0.6)/0.035}{0.6795(1.06 \times 10^5)^{1/2}(0.687)^{1/3}} = 243°\text{C}$$

At the end of the plate ($x = L = 0.6$ m) the temperature difference is obtained from Equations (5-48) and (5-50) with the constant 0.453 to give

$$(T_w - T_\infty)_{x=L} = \frac{(243.6)(0.6795)}{0.453} = 365.4°\text{C}$$

An alternate solution would be to base the Nusselt number on Equation (5-51).

Plate with Unheated Starting Length — EXAMPLE 5-6

Air at 1 atm and 300 K flows across a 20-cm-square plate at a free-stream velocity of 20 m/s. The last half of the plate is heated to a constant temperature of 350 K. Calculate the heat lost by the plate.

Solution

First we evaluate the air properties at the film temperature

$$T_f = (T_w + T_\infty)/2 = 325 \text{ K}$$

and obtain

$$v = 18.23 \times 10^{-6} \text{m}^2/\text{s} \qquad k = 0.02814 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7$$

At the trailing edge of the plate the Reynolds number is

$$\text{Re}_L = u_\infty L/v = (20)(0.2)/18.23 \times 10^{-6} = 2.194 \times 10^5$$

or, laminar flow over the length of the plate.

Heating does not start until the last half of the plate, or at a position $x_0 = 0.1$ m. The local heat-transfer coefficient for this condition is given by Equation (5-41):

$$h_x = 0.332k\,\text{Pr}^{1/3}(u_\infty/vx)^{1/2}[1 - (x_0/x)^{0.75}]^{-1/3} \qquad \textbf{[a]}$$

Inserting the property values along with $x_0 = 0.1$ gives

$$h_x = 8.6883x^{-1/2}(1 - 0.17783x^{-0.75})^{-1/3} \qquad \textbf{[b]}$$

The plate is 0.2 m wide so the heat transfer is obtained by integrating over the heated length $x_0 < x < L$

$$q = (0.2)(T_w - T_\infty)\int_{x_0=0.1}^{L=0.2} h_x dx \qquad \textbf{[c]}$$

Inserting Equation (*b*) in Equation (*c*) and performing the numerical integration gives

$$q = (0.2)(8.6883)(0.4845)(350 - 300) = 421 \text{ W} \qquad [d]$$

The average value of the heat-transfer coefficient *over the heated length* is given by

$$\bar{h} = q/(T_w - T_\infty)(L - x_0)W = 421/(350 - 300)(0.2 - 0.1)(0.2) = 421 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

where W is the width of the plate.

An easier calculation can be made by applying Equation (5-45b) to determine the average heat transfer coefficient over the heated portion of the plate. The result is

$$\bar{h} = 425.66 \text{ W/m}^2 \cdot {}^\circ\text{C} \quad \text{and} \quad q = 425.66 \text{ W}$$

which indicates, of course, only a small error in the numerical integeration.

EXAMPLE 5-7 Oil Flow Over Heated Flat Plate

Engine oil at 20°C is forced over a 20-cm-square plate at a velocity of 1.2 m/s. The plate is heated to a uniform temperature of 60°C. Calculate the heat lost by the plate.

Solution

We first evaluate the film temperature:

$$T_f = \frac{20 + 60}{2} = 40^\circ\text{C}$$

The properties of engine oil are

$$\rho = 876 \text{ kg/m}^3 \qquad \nu = 0.00024 \text{ m}^2/\text{s}$$
$$k = 0.144 \text{ W/m} \cdot {}^\circ\text{C} \qquad \text{Pr} = 2870$$

The Reynolds number is

$$\text{Re} = \frac{u_\infty L}{\nu} = \frac{(1.2)(0.2)}{0.00024} = 1000$$

Because the Prandtl number is so large we will employ Equation (5-51) for the solution. We see that h_x varies with x in the same fashion as in Equation (5-44), that is, $h_x \propto x^{-1/2}$, so that we get the same solution as in Equation (5-45) for the average heat-transfer coefficient. Evaluating Equation (5-51) at $x = 0.2$ gives

$$\text{Nu}_x = \frac{(0.3387)(1000)^{1/2}(2870)^{1/3}}{\left[1 + \left(\frac{0.0468}{2870}\right)^{2/3}\right]^{1/4}} = 152.2$$

and

$$h_x = \frac{(152.2)(0.144)}{0.2} = 109.6 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

The average value of the convection coefficient is

$$\bar{h} = (2)(109.6) = 219.2 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

so that the total heat transfer is

$$q = \bar{h}A(T_w - T_\infty) = (219.2)(0.2)^2(60 - 20) = 350.6 \text{ W}$$

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5-7 | THE RELATION BETWEEN FLUID FRICTION AND HEAT TRANSFER

We have already seen that the temperature and flow fields are related. Now we seek an expression whereby the frictional resistance may be directly related to heat transfer.

The shear stress at the wall may be expressed in terms of a friction coefficient C_f:

$$\tau_w = C_f \frac{\rho u_\infty^2}{2} \tag{5-52}$$

Equation (5-52) is the defining equation for the friction coefficient. The shear stress may also be calculated from the relation

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right]_w$$

Using the velocity distribution given by Equation (5-19), we have

$$\tau_w = \frac{3}{2}\frac{\mu u_\infty}{\delta}$$

and making use of the relation for the boundary-layer thickness gives

$$\tau_w = \frac{3}{2}\frac{\mu u_\infty}{4.64}\left(\frac{u_\infty}{\nu x}\right)^{1/2} \tag{5-53}$$

Combining Equations (5-52) and (5-53) leads to

$$\frac{C_{fx}}{2} = \frac{3}{2}\frac{\mu u_\infty}{4.64}\left(\frac{u_\infty}{\nu x}\right)^{1/2}\frac{1}{\rho u_\infty^2} = 0.323\ \mathrm{Re}_x^{-1/2} \tag{5-54}$$

The exact solution of the boundary-layer equations yields

$$\frac{C_{fx}}{2} = 0.332\ \mathrm{Re}_x^{-1/2} \tag{5-54a}$$

Equation (5-44) may be rewritten in the following form:

$$\frac{\mathrm{Nu}_x}{\mathrm{Re}_x\ \mathrm{Pr}} = \frac{h_x}{\rho c_p u_\infty} = 0.332\ \mathrm{Pr}^{-2/3}\ \mathrm{Re}_x^{-1/2}$$

The group on the left is called the Stanton number,

$$\mathrm{St}_x = \frac{h_x}{\rho c_p u_\infty}$$

so that

$$\mathrm{St}_x\ \mathrm{Pr}^{2/3} = 0.332\ \mathrm{Re}_x^{1/2} \tag{5-55}$$

Upon comparing Equations (5-54) and (5-55), we note that the right sides are alike except for a difference of about 3 percent in the constant, which is the result of the approximate nature of the integral boundary-layer analysis. We recognize this approximation and write

$$\mathrm{St}_x \mathrm{Pr}^{2/3} = \frac{C_{fx}}{2} \quad \textbf{[5-56]}$$

Equation (5-56), called the *Reynolds-Colburn analogy,* expresses the relation between fluid friction and heat transfer for laminar flow on a flat plate. The heat-transfer coefficient thus could be determined by making measurements of the frictional drag on a plate under conditions in which no heat transfer is involved.

It turns out that Equation (5-56) can also be applied to turbulent flow over a flat plate and in a modified way to turbulent flow in a tube. It does not apply to laminar tube flow. In general, a more rigorous treatment of the governing equations is necessary when embarking on new applications of the heat-transfer–fluid-friction analogy, and the results do not always take the simple form of Equation (5-56). The interested reader may consult the references at the end of the chapter for more information on this important subject. At this point, the simple analogy developed above has served to amplify our understanding of the physical processes in convection and to reinforce the notion that heat-transfer and viscous-transport processes are related at both the microscopic and macroscopic levels.

EXAMPLE 5-8 Drag Force on a Flat Plate

For the flow system in Example 5-4 compute the drag force exerted on the first 40 cm of the plate using the analogy between fluid friction and heat transfer.

Solution

We use Equation (5-56) to compute the friction coefficient and then calculate the drag force. An average friction coefficient is desired, so

$$\overline{\mathrm{St}}\, \mathrm{Pr}^{2/3} = \frac{\overline{C}_f}{2} \quad \textbf{[}\boldsymbol{a}\textbf{]}$$

The density at 316.5 K is

$$\rho = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(316.5)} = 1.115 \text{ kg/m}^3$$

For the 40-cm length

$$\overline{\mathrm{St}} = \frac{\bar{h}}{\rho c_p u_\infty} = \frac{8.698}{(1.115)(1006)(2)} = 3.88 \times 10^{-3}$$

Then from Equation (*a*)

$$\frac{\overline{C}_f}{2} = (3.88 \times 10^{-3})(0.7)^{2/3} = 3.06 \times 10^{-3}$$

The average shear stress at the wall is computed from Equation (5-52):

$$\begin{aligned} \bar{\tau}_w &= \overline{C}_f \rho \frac{u_\infty^2}{2} \\ &= (3.06 \times 10^{-3})(1.115)(2)^2 \\ &= 0.0136 \text{ N/m}^2 \end{aligned}$$

The drag force is the product of this shear stress and the area,

$$D = (0.0136)(0.4) = 5.44 \text{ mN} \quad [1.23 \times 10^{-3} \text{ lb}_f]$$

5-8 | TURBULENT-BOUNDARY-LAYER HEAT TRANSFER

Consider a portion of a turbulent boundary layer as shown in Figure 5-10. A very thin region near the plate surface has a laminar character, and the viscous action and heat transfer take place under circumstances like those in laminar flow. Farther out, at larger y distances from the plate, some turbulent action is experienced, but the molecular viscous action and heat conduction are still important. This region is called the *buffer layer.* Still farther out, the flow is fully turbulent, and the main momentum- and heat-exchange mechanism is one involving macroscopic lumps of fluid moving about in the flow. In this fully turbulent region we speak of *eddy viscosity* and *eddy thermal conductivity.* These eddy properties may be 10 to 20 times as large as the molecular values.

The physical mechanism of heat transfer in turbulent flow is quite similar to that in laminar flow; the primary difference is that one must deal with the eddy properties instead of the ordinary thermal conductivity and viscosity. The main difficulty in an analytical treatment is that these eddy properties vary across the boundary layer, and the specific variation can be determined only from experimental data. This is an important point. All analyses of turbulent flow must eventually rely on experimental data because there is no completely adequate theory to predict turbulent-flow behavior.

If one observes the instantaneous macroscopic velocity in a turbulent-flow system, as measured with a laser anemometer or other sensitive device, significant fluctuations about the mean flow velocity are observed as indicated in Figure 5-11, where $\overline{u}$ is designated as the mean velocity and u' is the *fluctuation* from the mean. The instantaneous velocity is therefore

$$u = \overline{u} + u' \qquad \textbf{[5-57]}$$

The mean value of the fluctuation u' must be zero over an extended period for steady flow conditions. There are also fluctuations in the y component of velocity, so we would write

$$v = \overline{v} + v' \qquad \textbf{[5-58]}$$

The fluctuations give rise to a turbulent-shear stress that may be analyzed by referring to Figure 5-12.

For a unit area of the plane P-P, the instantaneous turbulent mass-transport rate across the plane is $\rho v'$. Associated with this mass transport is a change in the x component of

Figure 5-10 | Velocity profile in turbulent boundary layer on a flat plate.

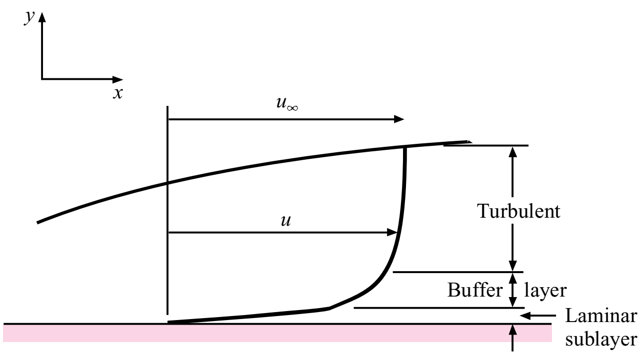


Figure 5-11 | Turbulent fluctuations with time.

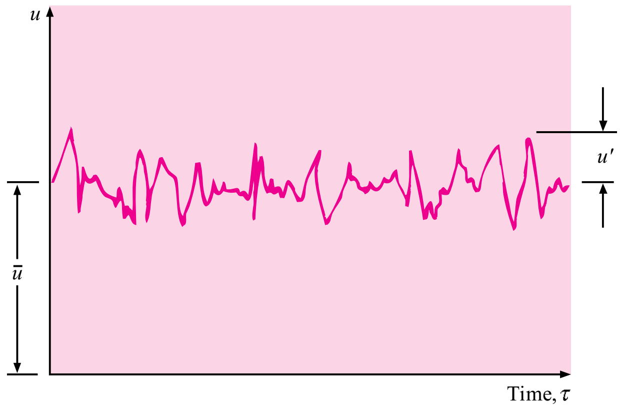
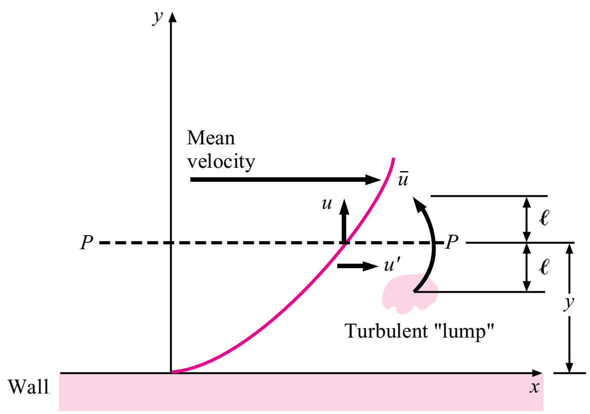


Figure 5-12 | Turbulent shear stress and mixing length.



velocity u'. The net momentum flux per unit area, in the x direction, represents the turbulent-shear stress at the plane P-P, or $\rho v' u'$. When a turbulent lump moves upward ($v' > 0$), it enters a region of higher $\bar{u}$ and is therefore likely to effect a slowing-down fluctuation in u', that is, $u' < 0$. A similar argument can be made for $v' < 0$, so that the average turbulent-shear stress will be given as

$$\tau_t = -\rho\overline{v'u'} \qquad \textbf{[5-59]}$$

We must note that even though $\overline{v'} = \overline{u'} = 0$, the average of the *fluctuation product* $\overline{u'v'}$ is *not* zero.

Eddy Viscosity and the Mixing Length

Let us define an eddy viscosity or eddy diffusivity for momentum ϵ_M such that

$$\tau_t = -\rho\overline{v'u'} = \rho\epsilon_M \frac{du}{dy} \qquad \textbf{[5-60]}$$

We have already likened the macroscopic transport of heat and momentum in turbulent flow to their molecular counterparts in laminar flow, so the definition in Equation (5-60) is a

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natural consequence of this analogy. To analyze molecular-transport problems one normally introduces the concept of *mean free path,* or the average distance a particle travels between collisions. Prandtl introduced a similar concept for describing turbulent-flow phenomena. The *Prandtl mixing length* is the distance traveled, on the average, by the turbulent lumps of fluid in a direction normal to the mean flow.

Let us imagine a turbulent lump that is located a distance ℓ above or below the plane *P-P*, as shown in Figure 5-12. These lumps of fluid move back and forth across the plane and give rise to the eddy or turbulent-shear-stress effect. At $y+\ell$ the velocity would be approximately

$$u(y+\ell) \approx u(y) + \ell \frac{\partial u}{\partial y}$$

while at $y-\ell$,

$$u(y-\ell) \approx u(y) - \ell \frac{\partial u}{\partial y}$$

Prandtl postulated that the turbulent fluctuation u' is proportional to the mean of the above two quantities, or

$$u' \approx \ell \frac{\partial u}{\partial y} \qquad \textbf{[5-61]}$$

The distance ℓ is called the Prandtl mixing length. Prandtl also postulated that v' would be of the same order of magnitude as u' so that the turbulent-shear stress of Equation (5-60) could be written

$$\tau_t = -\rho\overline{u'v'} = \rho\ell^2 \left(\frac{\partial u}{\partial y}\right)^2 = \rho\epsilon_M \frac{\partial u}{\partial y} \qquad \textbf{[5-62]}$$

The eddy viscosity ϵ_M thus becomes

$$\epsilon_M = \ell^2 \frac{\partial u}{\partial y} \qquad \textbf{[5-63]}$$

We have already noted that the eddy properties, and hence the mixing length, vary markedly through the boundary layer. Many analysis techniques have been applied over the years to take this variation into account. Prandtl's hypothesis was that the mixing length is proportional to distance from the wall, or

$$\ell = Ky \qquad \textbf{[5-64]}$$

where K is the proportionality constant. The additional assumption was made that in the near-wall region the shear stress is approximately constant so that $\tau_t \approx \tau_w$. When this assumption is used along with Equation (5-64), Equation (5-62) yields

$$\tau_w = \rho K^2 y^2 \left(\frac{\partial u}{\partial y}\right)^2$$

Taking the square root and integrating with respect to y gives

$$u = \frac{1}{K}\sqrt{\frac{\tau_w}{\rho}} \ln y + C \qquad \textbf{[5-65]}$$

where C is the constant of integration. Equation (5-65) matches very well with experimental data except in the region very close to the wall, where the laminar sublayer is present. In the sublayer the velocity distribution is essentially linear.

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Let us now quantify our earlier qualitative description of a turbulent boundary layer by expressing the shear stress as the sum of a molecular and turbulent part:

$$\frac{\tau}{\rho} = (\nu + \epsilon_M)\frac{\partial u}{\partial y} \qquad \textbf{[5-66]}$$

The so-called universal velocity profile is obtained by introducing two nondimensional coordinates

$$u^+ = \frac{u}{\sqrt{\tau_w/\rho}} \qquad \textbf{[5-67]}$$

$$y^+ = \frac{\sqrt{\tau_w/\rho y}}{\nu} \qquad \textbf{[5-68]}$$

Using these parameters and assuming $\tau \approx$ constant, we can rewrite Equation (5-66) as

$$du^+ = \frac{dy^+}{1 + \epsilon_M/\nu} \qquad \textbf{[5-69]}$$

In terms of our previous qualitative discussion, the laminar sublayer is the region where $\epsilon_M \sim 0$, the buffer layer has $\epsilon_M \sim \nu$, and the turbulent layer has $\epsilon_M \gg \nu$. Therefore, taking $\epsilon_M = 0$ in Equation (5-69) and integrating yields

$$u^+ = y^+ + c$$

At the wall, $u^+ = 0$ for $y^+ = 0$ so that $c = 0$ and

$$u^+ = y^+ \qquad \textbf{[5-70]}$$

is the velocity relation (a linear one) for the laminar sublayer. In the fully turbulent region $\epsilon_M/\nu \gg 1$. From Equation (5-65)

$$\frac{\partial u}{\partial y} = \frac{1}{K}\sqrt{\frac{\tau_w}{\rho}}\frac{1}{y}$$

Substituting this relation along with Equation (5-64) into Equation (5-63) gives

$$\epsilon_M = K\sqrt{\frac{\tau_w}{\rho}}y$$

or

$$\frac{\epsilon_m}{\nu} = Ky^+ \qquad \textbf{[5-71]}$$

Substituting this relation in Equation (5-69) for $\epsilon_M/\nu \gg 1$ and integrating gives

$$u^+ = \frac{1}{K}\ln y^+ + c \qquad \textbf{[5-72]}$$

This same *form* of equation will also be obtained for the buffer region. The limits of each region are obtained by comparing the above equations with experimental velocity measurements, with the following generally accepted constants:

$$\begin{aligned} &\text{Laminar sublayer: } 0 < y^+ < 5 && u^+ = y^+ \\ &\text{Buffer layer: } 5 < y^+ < 30 && u^+ = 5.0\ln y^+ - 3.05 \\ &\text{Turbulent layer: } 30 < y^+ < 400 && u^+ = 2.5\ln y^+ + 5.5 \end{aligned} \qquad \textbf{[5-73]}$$

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The equation set (5-73) is called the *universal velocity profile* and matches very well with experimental data; however, we should note once again that the constants in the equations must be determined from experimental velocity measurements. The satisfying point is that the simple Prandtl mixing-length model yields an equation form that fits the data so well.

Turbulent heat transfer is analogous to turbulent momentum transfer. The turbulent momentum flux postulated by Equation (5-59) carries with it a turbulent energy fluctuation proportional to the temperature gradient. We thus have, in analogy to Equation (5-62),

$$\left(\frac{q}{A}\right)_{\text{turb}} = -\rho c_p \epsilon_H \frac{\partial T}{\partial y} \qquad [5\text{-}74]$$

or, for regions where both molecular and turbulent energy transport are important,

$$\frac{q}{A} = -\rho c_p (\alpha + \epsilon_H) \frac{\partial T}{\partial y} \qquad [5\text{-}75]$$

Turbulent Heat Transfer Based on Fluid-Friction Analogy

Various analyses, similar to the one for the universal velocity profile above, have been performed to predict turbulent-boundary-layer heat transfer. The analyses have met with good success, but for our purposes the Colburn analogy between fluid friction and heat transfer is easier to apply and yields results that are in agreement with experiment and of simpler form.

In the turbulent-flow region, where $\epsilon_M \gg \nu$ and $\epsilon_H \gg \alpha$, we define the turbulent Prandtl number as

$$\text{Pr}_t = \frac{\epsilon_M}{\epsilon_H} \qquad [5\text{-}76]$$

If we can expect that the eddy momentum and energy transport will both be increased in the same proportion compared with their molecular values, we might anticipate that heat-transfer coefficients can be calculated by Equation (5-56) with the ordinary molecular Prandtl number used in the computation. In the turbulent core of the boundary layer the eddy viscosity may be as high as 100 times the molecular value experienced in the laminar sublayer, and a similar behavior is experienced for the eddy diffusivity for heat compared to the molecular diffusivity. To account for the Prandtl number effect over the entire boundary layer a weighted average is needed, and it turns out that use of $\text{Pr}^{2/3}$ works very well and matches with the laminar heat-transfer–fluid-friction analogy. We thus will base our calculations on this analogy, and we need experimental values for C_f for turbulent boundary layer flows to carry out these computations.

Schlichting [1] has surveyed experimental measurements of friction coefficients for turbulent flow on flat plates. We present the results of that survey so that they may be employed in the calculation of turbulent heat transfer with the fluid-friction–heat-transfer analogy. The *local* skin-friction coefficient is given by

$$C_{fx} = 0.0592\ \text{Re}_x^{-1/5} \qquad [5\text{-}77]$$

for Reynolds numbers between 5×10^5 and 10^7. At higher Reynolds numbers from 10^7 to 10^9 the formula of Schultz-Grunow [8] is recommended:

$$C_{fx} = 0.370(\log \text{Re}_x)^{-2.584} \qquad [5\text{-}78]$$

The *average-friction coefficient* for a flat plate with a laminar boundary layer up to Re_{crit} and turbulent thereafter can be calculated from

$$\overline{C}_f = \frac{0.455}{(\log \mathrm{Re}_L)^{2.584}} - \frac{A}{\mathrm{Re}_L} \qquad \mathrm{Re}_L < 10^9 \tag{5-79}$$

where the constant A depends on Re_{crit} in accordance with Table 5-1. A somewhat simpler formula can be obtained for lower Reynolds numbers as

$$\overline{C}_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{A}{\mathrm{Re}_L} \qquad \mathrm{Re}_L < 10^7 \tag{5-80}$$

Table 5-1

Re_{crit}	3×10^5	5×10^5	10^6	3×10^6
A	1055	1742	3340	8940

Equations (5-79) and (5-80) are in agreement within their common range of applicability, and the one to be used in practice will depend on computational convenience.

Applying the fluid-friction analogy $\mathrm{St}\,\mathrm{Pr}^{2/3} = C_f/2$, we obtain the local turbulent heat transfer as:

$$\mathrm{St}_x\,\mathrm{Pr}^{2/3} = 0.0296\,\mathrm{Re}_x^{-1/5} \qquad 5 \times 10^5 < \mathrm{Re}_x < 10^7 \tag{5-81}$$

or

$$\mathrm{St}_x\,\mathrm{Pr}^{2/3} = 0.185(\log \mathrm{Re}_x)^{-2.584} \qquad 10^7 < \mathrm{Re}_x < 10^9 \tag{5-82}$$

The average heat transfer over the entire laminar-turbulent boundary layer is

$$\overline{\mathrm{St}}\,\mathrm{Pr}^{2/3} = \frac{\overline{C}_f}{2} \tag{5-83}$$

For $\mathrm{Re}_{crit} = 5 \times 10^5$ and $\mathrm{Re}_L < 10^7$, Equation (5-80) can be used to obtain

$$\overline{\mathrm{St}}\,\mathrm{Pr}^{2/3} = 0.037\,\mathrm{Re}_L^{-1/5} - 871\,\mathrm{Re}_L^{-1} \tag{5-84}$$

Recalling that $\overline{\mathrm{St}} = \overline{\mathrm{Nu}}/(\mathrm{Re}_L\,\mathrm{Pr})$, we can rewrite Equation (5-84) as

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}L}{k} = \mathrm{Pr}^{1/3}(0.037\,\mathrm{Re}_L^{0.8} - 871) \tag{5-85}$$

The average heat-transfer coefficient can also be obtained by integrating the local values over the entire length of the plate. Thus,

$$\bar{h} = \frac{1}{L}\left(\int_0^{x_{crit}} h_{lam}\,dx + \int_{x_{crit}}^{L} h_{turb}\,dx\right)$$

Using Equation (5-55) for the laminar portion, $\mathrm{Re}_{crit} = 5 \times 10^5$, and Equation (5-81) for the turbulent portion gives the same result as Equation (5-85). For higher Reynolds numbers

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the friction coefficient from Equation (5-79) may be used, so that

$$\mathrm{Nu}_L = \frac{\bar{h}L}{k} = [0.228\mathrm{Re}_L(\log \mathrm{Re}_L)^{-2.584} - 871]\mathrm{Pr}^{1/3} \qquad \textbf{[5-85a]}$$

for $10^7 < \mathrm{Re}_L < 10^9$ and $\mathrm{Re}_{crit} = 5 \times 10^5$.

The reader should note that if a transition Reynolds number different from 500,000 is chosen, then Equations (5-84) and (5-85) must be changed accordingly. An alternative equation is suggested by Whitaker [10] that may give better results with some liquids because of the viscosity-ratio term:

$$\overline{\mathrm{Nu}}_L = 0.036\,\mathrm{Pr}^{0.43}(\mathrm{Re}_L^{0.8} - 9200)\left(\frac{\mu_\infty}{\mu_w}\right)^{1/4} \qquad \textbf{[5-86]}$$

for

$$0.7 < \mathrm{Pr} < 380$$
$$2 \times 10^5 < \mathrm{Re}_L < 5.5 \times 10^6$$
$$0.26 < \frac{\mu_\infty}{\mu_w} < 3.5$$

All properties except μ_w are evaluated at the free-stream temperature. For gases the viscosity ratio is dropped and the properties are evaluated at the film temperature.

Constant Heat Flux

For constant-wall-heat flux in turbulent flow it is shown in Reference 11 that the local Nusselt number is only about 4 percent higher than for the isothermal surface; that is,

$$\mathrm{Nu}_x = 1.04\,\mathrm{Nu}_x\Big]_{T_w=\mathrm{const}} \qquad \textbf{[5-87]}$$

Some more comprehensive methods of correlating turbulent-boundary-layer heat transfer are given by Churchill [11].

Turbulent Heat Transfer from Isothermal Flat Plate

EXAMPLE 5-9

Air at 20°C and 1 atm flows over a flat plate at 35 m/s. The plate is 75 cm long and is maintained at 60°C. Assuming unit depth in the z direction, calculate the heat transfer from the plate.

■ **Solution**

We evaluate properties at the film temperature:

$$T_f = \frac{20+60}{2} = 40°\mathrm{C} = 313\ \mathrm{K}$$

$$\rho = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(313)} = 1.128\ \mathrm{kg/m^3}$$

$$\mu = 1.906 \times 10^{-5}\ \mathrm{kg/m \cdot s}$$

$$\mathrm{Pr} = 0.7 \qquad k = 0.02723\ \mathrm{W/m \cdot °C} \qquad c_p = 1.007\ \mathrm{kJ/kg \cdot °C}$$

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The Reynolds number is

$$\mathrm{Re}_L = \frac{\rho u_\infty L}{\mu} = \frac{(1.128)(35)(0.75)}{1.906 \times 10^{-5}} = 1.553 \times 10^6$$

and the boundary layer is turbulent because the Reynolds number is greater than 5×10^5. Therefore, we use Equation (5-85) to calculate the average heat transfer over the plate:

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}L}{k} = \mathrm{Pr}^{1/3}(0.037\,\mathrm{Re}_L^{0.8} - 871)$$

$$= (0.7)^{1/3}[(0.037)(1.553 \times 10^6)^{0.8} - 871] = 2180$$

$$\bar{h} = \overline{\mathrm{Nu}}_L \frac{k}{L} = \frac{(2180)(0.02723)}{0.75} = 79.1\ \mathrm{W/m^2 \cdot {}^\circ C} \quad [13.9\ \mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}]$$

$$q = \bar{h}A(T_w - T_\infty) = (79.1)(0.75)(60 - 20) = 2373\ \mathrm{W} \quad [8150\ \mathrm{Btu/h}]$$

5-9 | TURBULENT-BOUNDARY-LAYER THICKNESS

A number of experimental investigations have shown that the velocity profile in a turbulent boundary layer, outside the laminar sublayer, can be described by a one-seventh-power relation

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/7} \qquad \textbf{[5-88]}$$

where δ is the boundary-layer thickness as before. For purposes of an integral analysis the momentum integral can be evaluated with Equation (5-88) because the laminar sublayer is so thin. However, the wall shear stress cannot be calculated from Equation (5-88) because it yields an infinite value at $y = 0$.

To determine the turbulent-boundary-layer thickness we employ Equation (5-17) for the integral momentum relation and evaluate the wall shear stress from the empirical relations for skin friction presented previously. According to Equation (5-52),

$$\tau_w = \frac{C_f \rho u_\infty^2}{2}$$

and so for $\mathrm{Re}_x < 10^7$ we obtain from Equation (5-77)

$$\tau_w = 0.0296\left(\frac{\nu}{u_\infty x}\right)^{1/5} \rho u_\infty^2 \qquad \textbf{[5-89]}$$

Now, using the integral momentum equation for zero pressure gradient [Equation (5-17)] along with the velocity profile and wall shear stress, we obtain

$$\frac{d}{dx}\int_0^\delta \left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right]\left(\frac{y}{\delta}\right)^{1/7} dy = 0.0296\left(\frac{\nu}{u_\infty x}\right)^{1/5}$$

Integrating and clearing terms gives

$$\frac{d\delta}{dx} = \frac{72}{7}(0.0296)\left(\frac{\nu}{u_\infty}\right)^{1/5} x^{-1/5} \qquad \textbf{[5-90]}$$

We shall integrate this equation for two physical situations:

1. The boundary layer is fully turbulent from the leading edge of the plate.

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2. The boundary layer follows a laminar growth pattern up to $\mathrm{Re}_{\mathrm{crit}} = 5 \times 10^5$ and a turbulent growth thereafter.

For the first case, we integrate Equation (5-89) with the condition that $\delta = 0$ at $x = 0$ to obtain

$$\frac{\delta}{x} = 0.381\ \mathrm{Re}_x^{-1/5} \qquad \textbf{[5-91]}$$

For case 2 we have the condition

$$\delta = \delta_{\mathrm{lam}} \qquad \text{at } x_{\mathrm{crit}} = 5 \times 10^5 \frac{\nu}{u_\infty} \qquad \textbf{[5-92]}$$

Now, δ_{lam} is calculated from the exact relation of Equation (5-21*a*):

$$\delta_{\mathrm{lam}} = 5.0 x_{\mathrm{crit}} (5 \times 10^5)^{-1/2} \qquad \textbf{[5-93]}$$

Integrating Equation (5-89) gives

$$\delta - \delta_{\mathrm{lam}} = \frac{72}{7}(0.0296)\left(\frac{\nu}{u_\infty}\right)^{1/5} \frac{5}{4}\left(x^{4/5} - x_{\mathrm{crit}}^{4/5}\right) \qquad \textbf{[5-94]}$$

Combining the various relations above gives

$$\frac{\delta}{x} = 0.381\ \mathrm{Re}_x^{-1/5} - 10{,}256\ \mathrm{Re}_x^{-1} \qquad \textbf{[5-95]}$$

This relation applies only for the region $5 \times 10^5 < \mathrm{Re}_x < 10^7$.

Turbulent-Boundary-Layer Thickness — EXAMPLE 5-10

Calculate the turbulent-boundary-layer thickness at the end of the plate for Example 5-9, assuming that it develops (*a*) from the leading edge of the plate and (*b*) from the transition point at $\mathrm{Re}_{\mathrm{crit}} = 5 \times 10^5$.

Solution

Since we have already calculated the Reynolds number as $\mathrm{Re}_L = 1.553 \times 10^6$, it is a simple matter to insert this value in Equations (5-91) and (5-95) along with $x = L = 0.75$ m to give

$$(a)\ \delta = (0.75)(0.381)(1.553 \times 10^6)^{-0.2} = 0.0165\ \mathrm{m} = 16.5\ \mathrm{mm}\ [0.65\ \mathrm{in}]$$

$$(b)\ \delta = (0.75)[(0.381)(1.553 \times 10^6)^{-0.2} - 10{,}256(1.553 \times 10^6)^{-1}]$$
$$= 0.0099\ \mathrm{m} = 9.9\ \mathrm{mm}\ [0.39\ \mathrm{in}]$$

The two values differ by 40 percent.

An overall perspective of the behavior of the local and average heat-transfer coefficients is indicated in Figure 5-13. The fluid is atmospheric air flowing across an isothermal flat plate at $u_\infty = 30$ m/s, and the calculations were made with Equations (5-55), (5-81), and (5-85), which assume a value of $\mathrm{Re}_{\mathrm{crit}} = 5 \times 10^5$. The corresponding value of x_{crit} is 0.2615 m and the plate length is 5.23 m at $\mathrm{Re} = 10^7$. The corresponding boundary-layer thickness is plotted in Figure 5-14. As we have noted before, the heat-transfer coefficient varies inversely with the boundary-layer thickness, and an increase in heat transfer is experienced when turbulence begins.

Figure 5-13 | Local and average heat-transfer coefficient for atmospheric airflow over isothermal flat plate at $u_\infty = 30$ m/s (*a*) semilog scale (*b*) log scale.

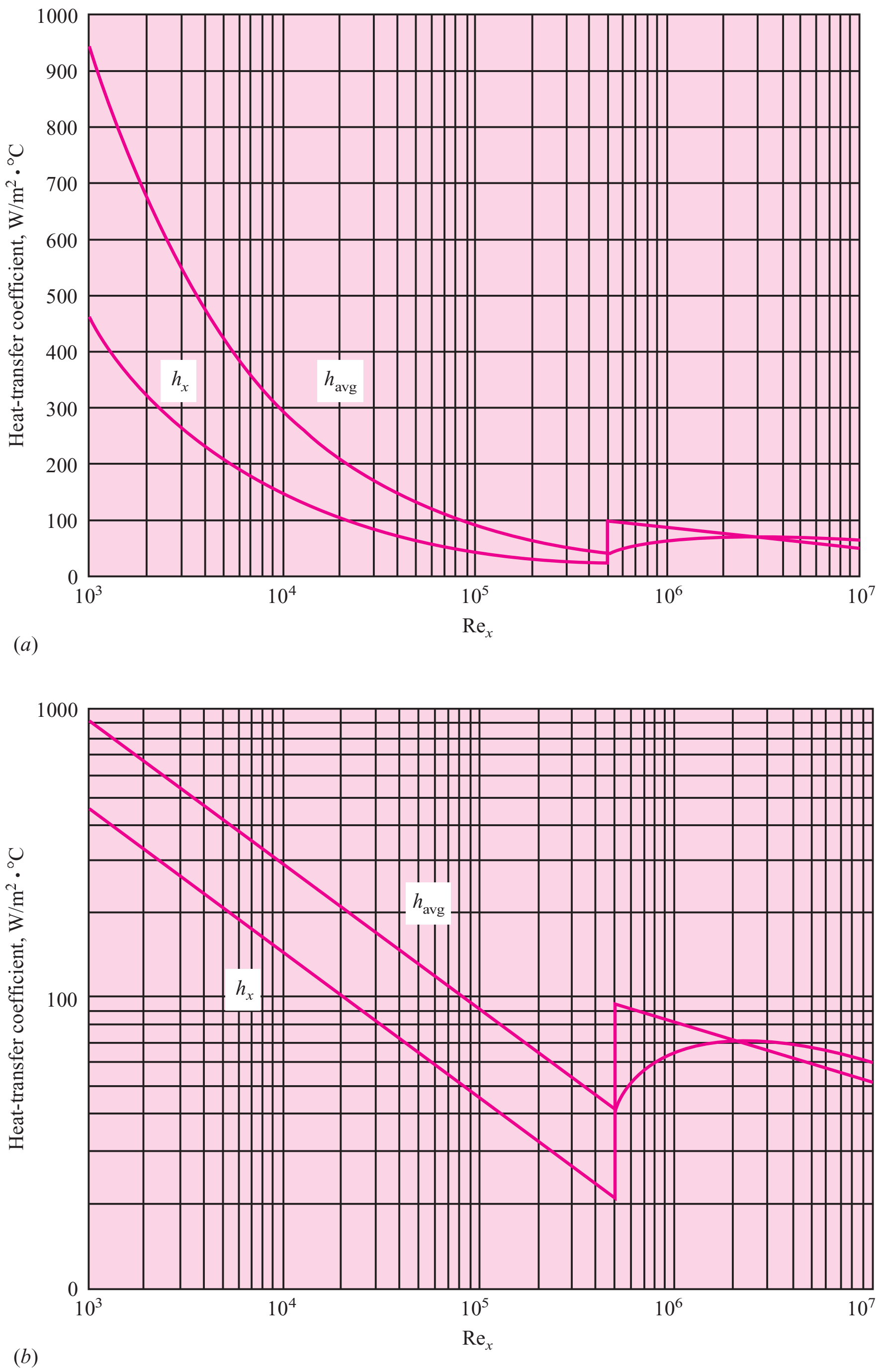
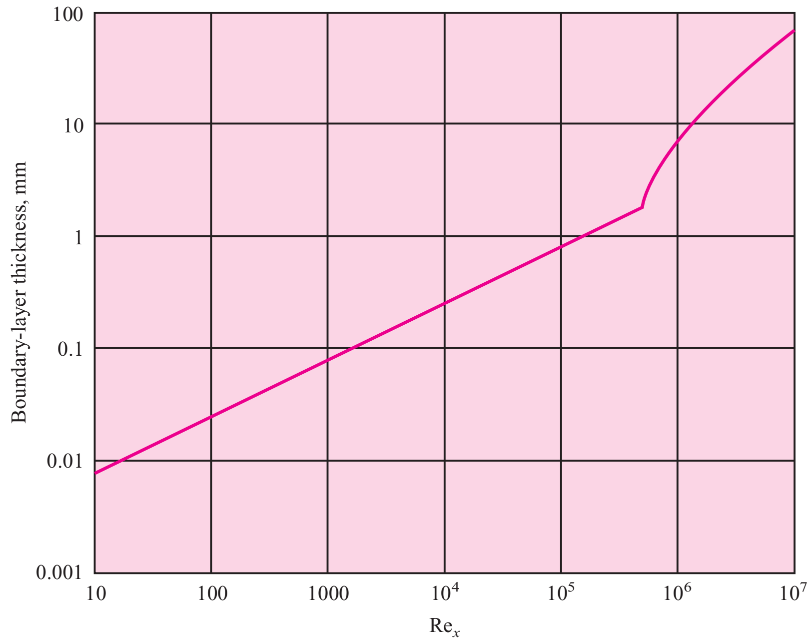


Figure 5-14 | Boundary-layer thickness for atmospheric air at $u_\infty = 30$ m/s.



5-10 | HEAT TRANSFER IN LAMINAR TUBE FLOW

Consider the tube-flow system in Figure 5-15. We wish to calculate the heat transfer under developed flow conditions when the flow remains laminar. The wall temperature is T_w, the radius of the tube is r_o, and the velocity at the center of the tube is u_0. It is assumed that the pressure is uniform at any cross section. The velocity distribution may be derived by considering the fluid element shown in Figure 5-16. The pressure forces are balanced by the viscous-shear forces so that

$$\pi r^2 \, dp = \tau 2\pi r \, dx = 2\pi r \mu \, dx \, \frac{du}{dr}$$

or

$$du = \frac{1}{2\mu} r \frac{dp}{dx} \, dr$$

Figure 5-15 | Control volume for energy analysis in tube flow.

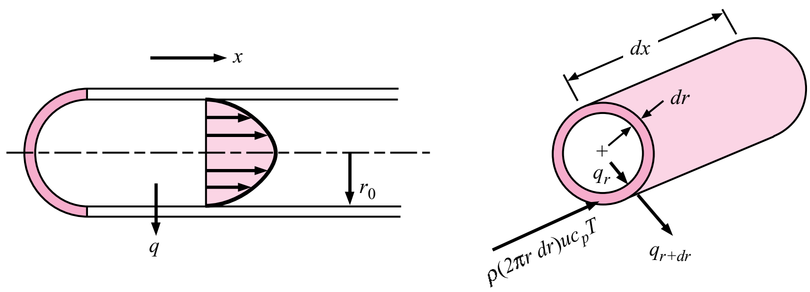


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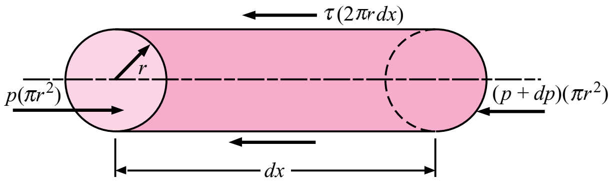
Figure 5-16 | Force balance on fluid element in tube flow.



and

$$u = \frac{1}{4\mu}\frac{dp}{dx}r^2 + \text{const} \qquad \textbf{[5-96]}$$

With the boundary condition

$$u = 0 \qquad \text{at } r = r_o$$

$$u = \frac{1}{4\mu}\frac{dp}{dx}(r^2 - r_o^2)$$

the velocity at the center of the tube is given by

$$u_0 = -\frac{r_o^2}{4\mu}\frac{dp}{dx} \qquad \textbf{[5-97]}$$

so that the velocity distribution may be written

$$\frac{u}{u_0} = 1 - \frac{r^2}{r_o^2} \qquad \textbf{[5-98]}$$

which is the familiar parabolic distribution for laminar tube flow. Now consider the heat-transfer process for such a flow system. To simplify the analysis, we assume that there is a constant heat flux at the tube wall; that is,

$$\frac{dq_w}{dx} = 0$$

The heat flow conducted into the annular element is

$$dq_r = -k2\pi r\,dx\,\frac{\partial T}{\partial r}$$

and the heat conducted out is

$$dq_{r+dr} = -k2\pi(r + dr)\,dx\left(\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\,dr\right)$$

The net heat convected out of the element is

$$2\pi r\,dr\,\rho c_p u\frac{\partial T}{\partial x}\,dx$$

The energy balance is

Net energy convected out = net heat conducted in

or, neglecting second-order differentials,

$$r\rho c_p u\frac{\partial T}{\partial x}\,dx\,dr = k\left(\frac{\partial T}{\partial r} + r\frac{\partial^2 T}{\partial r^2}\right)dx\,dr$$

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which may be rewritten

$$\frac{1}{ur}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial x} \qquad \textbf{[5-99]}$$

We assume that the heat flux at the wall is constant, so that the average fluid temperature must increase linearly with x, or

$$\frac{\partial T}{\partial x}=\text{const}$$

This means that the temperature profiles will be similar at various x distances along the tube. The boundary conditions on Equation (5-98) are

$$\frac{\partial T}{\partial r}=0 \qquad \text{at } r=0$$

$$k\left.\frac{\partial T}{\partial r}\right]_{r=r_o}=q_w=\text{const}$$

To obtain the solution to Equation (5-99), the velocity distribution given by Equation (5-98) must be inserted. It is assumed that the temperature and velocity fields are independent; that is, a temperature gradient does not affect the calculation of the velocity profile. This is equivalent to specifying that the properties remain constant in the flow. With the substitution of the velocity profile, Equation (5-99) becomes

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial x}u_0\left(1-\frac{r^2}{r_o^2}\right)r$$

Integration yields

$$r\frac{\partial T}{\partial r}=\frac{1}{\alpha}\frac{\partial T}{\partial x}u_0\left(\frac{r^2}{2}-\frac{r^4}{4r_o^2}\right)+C_1$$

and a second integration gives

$$T=\frac{1}{\alpha}\frac{\partial T}{\partial x}u_0\left(\frac{r^2}{4}-\frac{r^4}{16r_o^2}\right)+C_1\ln r+C_2$$

Applying the first boundary condition, we find that

$$C_1=0$$

The second boundary condition has been satisfied by noting that the axial temperature gradient $\partial T/\partial x$ is constant. The temperature distribution may finally be written in terms of the temperature at the center of the tube:

$$T=T_c \qquad \text{at } r=0 \qquad \text{so that} \qquad C_2=T_c$$

$$T-T_c=\frac{1}{\alpha}\frac{\partial T}{\partial x}\frac{u_0 r_o^2}{4}\left[\left(\frac{r}{r_o}\right)^2-\frac{1}{4}\left(\frac{r}{r_o}\right)^4\right] \qquad \textbf{[5-100]}$$

The Bulk Temperature

In tube flow the convection heat-transfer coefficient is usually defined by

$$\text{Local heat flux}=q''=h(T_w-T_b) \qquad \textbf{[5-101]}$$

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where T_w is the wall temperature and T_b is the so-called *bulk temperature,* or energy-average fluid temperature across the tube, which may be calculated from

$$T_b = \overline{T} = \frac{\int_0^{r_o} \rho 2\pi r\, dr\, u c_p T}{\int_0^{r_o} \rho 2\pi r\, dr\, u c_p} \tag{5-102}$$

The reason for using the bulk temperature in the definition of heat-transfer coefficients for tube flow may be explained as follows. In a tube flow there is no easily discernible free-stream condition as is present in the flow over a flat plate. Even the centerline temperature T_c is not easily expressed in terms of the inlet flow variables and the heat transfer. For most tube- or channel-flow heat-transfer problems, the topic of central interest is the total energy transferred to the fluid in either an elemental length of the tube or over the entire length of the channel. At any x position, the temperature that is indicative of the total energy of the flow is an integrated mass-energy average temperature over the entire flow area. The numerator of Equation (5-102) represents the total energy flow through the tube, and the denominator represents the product of mass flow and specific heat integrated over the flow area. The bulk temperature is thus representative of the total energy of the flow at the particular location. For this reason, the bulk temperature is sometimes referred to as the "mixing cup" temperature, since it is the temperature the fluid would assume if placed in a mixing chamber and allowed to come to equilibrium. For the temperature distribution given in Equation (5-100), the bulk temperature is a linear function of x because the heat flux at the tube wall is constant. Calculating the bulk temperature from Equation (5-102), we have

$$T_b = T_c + \frac{7}{96}\frac{u_0 r_o^2}{\alpha}\frac{\partial T}{\partial x} \tag{5-103}$$

and for the wall temperature

$$T_w = T_c + \frac{3}{16}\frac{u_0 r_o^2}{\alpha}\frac{\partial T}{\partial x} \tag{5-104}$$

The heat-transfer coefficient is calculated from

$$q = hA(T_w - T_b) = kA\left(\frac{\partial T}{\partial r}\right)_{r=r_o} \tag{5-105}$$

$$h = \frac{k(\partial T/\partial r)_{r=r_o}}{T_w - T_b}$$

The temperature gradient is given by

$$\left.\frac{\partial T}{\partial r}\right]_{r=r_o} = \frac{u_0}{\alpha}\frac{\partial T}{\partial x}\left(\frac{r}{2} - \frac{r^3}{4r_o^2}\right)_{r=r_o} = \frac{u_0 r_o}{4\alpha}\frac{\partial T}{\partial x} \tag{5-106}$$

Substituting Equations (5-103), (5-104), and (5-106) in Equation (5-105) gives

$$h = \frac{24}{11}\frac{k}{r_o} = \frac{48}{11}\frac{k}{d_o}$$

Expressed in terms of the Nusselt number, the result is

$$\mathrm{Nu}_d = \frac{hd_o}{k} = 4.364 \tag{5-107}$$

which is in agreement with an exact calculation by Sellars, Tribus, and Klein [3], that considers the temperature profile as it develops. Some empirical relations for calculating heat transfer in laminar tube flow will be presented in Chapter 6.

We may remark at this time that when the statement is made that a fluid enters a tube at a certain temperature, it is the bulk temperature to which we refer. The bulk temperature is used for overall energy balances on systems.

5-11 | TURBULENT FLOW IN A TUBE

The developed velocity profile for turbulent flow in a tube will appear as shown in Figure 5-17. A laminar sublayer, or "film," occupies the space near the surface, while the central core of the flow is turbulent. To determine the heat transfer analytically for this situation, we require, as usual, a knowledge of the temperature distribution in the flow. To obtain this temperature distribution, the analysis must take into consideration the effect of the turbulent eddies in the transfer of heat and momentum. We shall use an approximate analysis that relates the conduction and transport of heat to the transport of momentum in the flow (i.e., viscous effects).

The heat flow across a fluid element in laminar flow may be expressed by

$$\frac{q}{A} = -k\frac{dT}{dy}$$

Dividing both sides of the equation by ρc_p,

$$\frac{q}{\rho c_p A} = -\alpha\frac{dT}{dy}$$

It will be recalled that α is the molecular diffusivity of heat. In turbulent flow one might assume that the heat transport could be represented by

$$\frac{q}{\rho c_p A} = -(\alpha + \epsilon_H)\frac{dT}{dy} \qquad \text{[5-108]}$$

where ϵ_H is an eddy diffusivity of heat.

Equation (5-108) expresses the total heat conduction as a sum of the molecular conduction and the macroscopic eddy conduction. In a similar fashion, the shear stress in turbulent flow could be written

$$\frac{\tau}{\rho} = \left(\frac{\mu}{\rho} + \epsilon_M\right)\frac{du}{dy} = (\nu + \epsilon_M)\frac{du}{dy} \qquad \text{[5-109]}$$

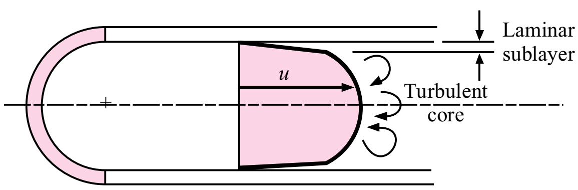
where ϵ_M is the eddy diffusivity for momentum. We now assume that the heat and momentum are transported at the same rate; that is, $\epsilon_M = \epsilon_H$ and $\nu = \alpha$, or $\Pr = 1$.

Dividing Equation (5-108) by Equation (5-109) gives

$$\frac{q}{c_p A \tau}\, du = -dT$$

An additional assumption is that the ratio of the heat transfer per unit area to the shear stress is constant across the flow field. This is consistent with the assumption that heat and

Figure 5-17 | Velocity profile in turbulent tube flow.



momentum are transported at the same rate. Thus

$$\frac{q}{A\tau} = \text{const} = \frac{q_w}{A_w \tau_w} \qquad \textbf{[5-110]}$$

Then, integrating Equation (5-109) between wall conditions and mean bulk conditions gives

$$\frac{q_w}{A_w \tau_w c_p} \int_{u=0}^{u=u_m} du = \int_{T_w}^{T_b} -dT$$

$$\frac{q_w u_m}{A_w \tau_w c_p} = T_w - T_b \qquad \textbf{[5-111]}$$

But the heat transfer at the wall may be expressed by

$$q_w = h A_w (T_w - T_b)$$

and the shear stress may be calculated from

$$\tau_w = \frac{\Delta p(\pi d_o^2)}{4\pi d_o L} = \frac{\Delta p}{4} \frac{d_o}{L}$$

The pressure drop may be expressed in terms of a friction factor f by

$$\Delta p = f \frac{L}{d_o} \rho \frac{u_m^2}{2} \qquad \textbf{[5-112]}$$

so that

$$\tau_w = \frac{f}{8} \rho u_m^2 \qquad \textbf{[5-113]}$$

Substituting the expressions for τ_w and q_w in Equation (5-111) gives

$$\text{St} = \frac{h}{\rho c_p u_m} = \frac{\text{Nu}_d}{\text{Re}_d \ \text{Pr}} = \frac{f}{8} \qquad \textbf{[5-114]}$$

Equation (5-114) is called the Reynolds analogy for tube flow. It relates the heat-transfer rate to the frictional loss in tube flow and is in fair agreement with experiments when used with gases whose Prandtl numbers are close to unity. (Recall that $\text{Pr} = 1$ was one of the assumptions in the analysis.)

An empirical formula for the turbulent-friction factor up to Reynolds numbers of about 2×10^5 for the flow in smooth tubes is

$$f = \frac{0.316}{\text{Re}_d^{1/4}} \qquad \textbf{[5-115]}$$

Inserting this expression in Equation (5-113) gives

$$\frac{\text{Nu}_d}{\text{Re}_d \ \text{Pr}} = 0.0395 \ \text{Re}_d^{-1/4}$$

or

$$\text{Nu}_d = 0.0395 \ \text{Re}_d^{3/4} \qquad \textbf{[5-116]}$$

since we assumed the Prandtl number to be unity. This derivation of the relation for turbulent heat transfer in smooth tubes is highly restrictive because of the $\text{Pr} \approx 1.0$ assumption. The heat-transfer–fluid-friction analogy of Section 5-7 indicated a Prandtl-number dependence

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of $\mathrm{Pr}^{2/3}$ for the flat-plate problem and, as it turns out, this dependence works fairly well for turbulent tube flow. Equations (5-114) and (5-116) may be modified by this factor to yield

$$\mathrm{St}\,\mathrm{Pr}^{2/3} = \frac{f}{8} \qquad \textbf{[5-114}a\textbf{]}$$

$$\mathrm{Nu}_d = 0.0395\,\mathrm{Re}_d^{3/4}\,\mathrm{Pr}^{1/3} \qquad \textbf{[5-116}a\textbf{]}$$

As we shall see in Chapter 6, Equation (5-116*a*) predicts heat-transfer coefficients that are somewhat higher than those observed in experiments. The purpose of the discussion at this point has been to show that one may arrive at a relation for turbulent heat transfer in a fairly simple analytical fashion. As we have indicated earlier, a rigorous development of the Reynolds analogy between heat transfer and fluid friction involves considerations beyond the scope of our discussion, and the simple path of reasoning chosen here is offered for the purpose of indicating the general nature of the physical processes.

For calculation purposes, a more correct relation to use for turbulent flow in a smooth tube is Equation (6-4a), which we list here for comparison:

$$\mathrm{Nu}_d = 0.023\,\mathrm{Re}_d^{0.8}\,\mathrm{Pr}^{0.4} \qquad \textbf{[6-4}a\textbf{]}$$

All properties in Equation (6-4*a*) are evaluated at the bulk temperature.

5-12 | HEAT TRANSFER IN HIGH-SPEED FLOW

Our previous analysis of boundary-layer heat transfer (Section 5-6) neglected the effects of viscous dissipation within the boundary layer. When the free-stream velocity is very high, as in high-speed aircraft, these dissipation effects must be considered. We begin our analysis by considering the adiabatic case, i.e., a perfectly insulated wall. In this case the wall temperature may be considerably higher than the free-stream temperature even though no heat transfer takes place. This high temperature results from two situations: (1) the increase in temperature of the fluid as it is brought to rest at the plate surface while the kinetic energy of the flow is converted to internal thermal energy, and (2) the heating effect due to viscous dissipation. Consider the first situation. The kinetic energy of the gas is converted to thermal energy as the gas is brought to rest, and this process is described by the steady-flow energy equation for an adiabatic process:

$$i_0 = i_\infty + \frac{1}{2g_c}u_\infty^2 \qquad \textbf{[5-117]}$$

where i_0 is the stagnation enthalpy of the gas. This equation may be written in terms of temperature as

$$c_p(T_0 - T_\infty) = \frac{1}{2g_c}u_\infty^2$$

where T_0 is the stagnation temperature and T_∞ is the static free-stream temperature. Expressed in terms of the free-stream Mach number, it is

$$\frac{T_0}{T_\infty} = 1 + \frac{\gamma - 1}{2}M_\infty^2 \qquad \textbf{[5-118]}$$

where M_∞ is the Mach number, defined as $M_\infty = u_\infty/a$, and a is the acoustic velocity, which for an ideal gas may be calculated with

$$a = \sqrt{\gamma g_c RT} \qquad \textbf{[5-119]}$$

where R is the gas constant for the particular gas.

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In the actual case of a boundary-layer flow problem, the fluid is not brought to rest reversibly because the viscous action is basically an irreversible process in a thermodynamic sense. In addition, not all the free-stream kinetic energy is converted to thermal energy—part is lost as heat, and part is dissipated in the form of viscous work. To take into account the irreversibilities in the boundary-layer flow system, a *recovery factor* is defined by

$$r = \frac{T_{aw} - T_\infty}{T_0 - T_\infty} \qquad \textbf{[5-120]}$$

where T_{aw} is the actual adiabatic wall temperature and T_∞ is the static temperature of the free stream. The recovery factor may be determined experimentally, or, for some flow systems, analytical calculations may be made.

The boundary-layer energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2$$

has been solved for the high-speed-flow situation, taking into account the viscous-heating term. Although the complete solution is somewhat tedious, the final results are remarkably simple. For our purposes we present only the results and indicate how they may be applied. The reader is referred to Appendix B for an exact solution to Equation (5-22). An excellent synopsis of the high-speed heat-transfer problem is given in a report by Eckert [4]. Some typical boundary-layer temperature profiles for an adiabatic wall in high-speed flow are given in Figure B-3.

The essential result of the high-speed heat-transfer analysis is that heat-transfer rates may generally be calculated with the same relations used for low-speed incompressible flow when the average heat-transfer coefficient is redefined with the relation

$$q = \bar{h}A(T_w - T_{aw}) \qquad \textbf{[5-121]}$$

Notice that the difference between the adiabatic wall temperature and the actual wall temperature is used in the definition so that the expression will yield a value of zero heat flow when the wall is at the adiabatic wall temperature. For gases with Prandtl numbers near unity, the following relations for the recovery factor have been derived:

Laminar flow: $$r = \mathrm{Pr}^{1/2} \qquad \textbf{[5-122]}$$

Turbulent flow: $$r = \mathrm{Pr}^{1/3} \qquad \textbf{[5-123]}$$

These recovery factors may be used in conjunction with Equation (5-120) to obtain the adiabatic wall temperature.

In high-velocity boundary layers substantial temperature gradients may occur, and there will be correspondingly large property variations across the boundary layer. The constant-property heat-transfer equations may still be used if the properties are introduced at a reference temperature T^* as recommended by Eckert:

$$T^* = T_\infty + 0.50(T_w - T_\infty) + 0.22(T_{aw} - T_\infty) \qquad \textbf{[5-124]}$$

The analogy between heat transfer and fluid friction [Equation (5-56)] may also be used when the friction coefficient is known. Summarizing the relations used for high-speed heat-transfer calculations:

Laminar boundary layer ($\mathrm{Re}_x < 5 \times 10^5$):

$$\mathrm{St}_x^* \, \mathrm{Pr}^{*2/3} = 0.332\,\mathrm{Re}_x^{*-1/2} \qquad \textbf{[5-125]}$$

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Turbulent boundary layer $(5 \times 10^5 < \mathrm{Re}_x < 10^7)$:

$$\mathrm{St}_x^* \, \mathrm{Pr}^{*2/3} = 0.0296 \, \mathrm{Re}_x^{*-1/5} \qquad \textbf{[5-126]}$$

Turbulent boundary layer $(10^7 < \mathrm{Re}_x < 10^9)$:

$$\mathrm{St}_x^* \, \mathrm{Pr}^{*2/3} = 0.185(\log \mathrm{Re}_x^*)^{-2.584} \qquad \textbf{[5-127]}$$

The superscript * in the above equations indicates that the properties are evaluated at the reference temperature given by Equation (5-124).

To obtain an average heat-transfer coefficient, the above expressions must be integrated over the length of the plate. If the Reynolds number falls in a range such that Equation (5-127) must be used, the integration cannot be expressed in closed form, and a numerical integration must be performed. Care must be taken in performing the integration for the high-speed heat-transfer problem since the reference temperature is different for the laminar and turbulent portions of the boundary layer. This results from the different value of the recovery factor used for laminar and turbulent flow as given by Equations (5-122) and (5-123).

When very high flow velocities are encountered, the adiabatic wall temperature may become so high that dissociation of the gas will take place and there will be a very wide variation of the properties in the boundary layer. Eckert [4] recommends that these problems be treated on the basis of a heat-transfer coefficient defined in terms of *enthalpy* difference:

$$q = h_i A(i_w - i_{aw}) \qquad \textbf{[5-128]}$$

The enthalpy recovery factor is then defined as

$$r_i = \frac{i_{aw} - i_\infty}{i_0 - i_\infty} \qquad \textbf{[5-129]}$$

where i_{aw} is the enthalpy at the adiabatic wall conditions. The same relations as before are used to calculate the recovery factor and heat-transfer except that all properties are evaluated at a reference enthalpy i^* given by

$$i^* = i_\infty + 0.5(i_w - i_\infty) + 0.22(i_{aw} - i_\infty) \qquad \textbf{[5-130]}$$

The Stanton number is redefined as

$$\mathrm{St}_i = \frac{h_i}{\rho u_\infty} \qquad \textbf{[5-131]}$$

This Stanton number is then used in Equation (5-125), (5-126), or (5-127) to calculate the heat-transfer coefficient. When calculating the enthalpies for use in the above relations, the *total* enthalpy must be used; that is chemical energy of dissociation as well as internal thermal energy must be included. The reference-enthalpy method has proved successful for calculating high-speed heat-transfer with an accuracy of better than 10 percent.

High-Speed Heat Transfer for a Flat Plate — EXAMPLE 5-11

A flat plate 70 cm long and 1.0 m wide is placed in a wind tunnel where the flow conditions are $M = 3$, $p = \frac{1}{20}$ atm, and $T = -40°\mathrm{C}$. How much cooling must be used to maintain the plate temperature at 35°C?

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Solution

We must consider the laminar and turbulent portions of the boundary layer separately because the recovery factors, and hence the adiabatic wall temperatures, used to establish the heat flow will be different for each flow regime. It turns out that the difference is rather small in this problem, but we shall follow a procedure that would be used if the difference were appreciable, so that the general method of solution may be indicated. The free-stream acoustic velocity is calculated from

$$a = \sqrt{\gamma g_c R T_\infty} = [(1.4)(1.0)(287)(233)]^{1/2} = 306 \text{ m/s} \quad [1003 \text{ ft/s}]$$

so that the free-stream velocity is

$$u_\infty = (3)(306) = 918 \text{ m/s} \quad [3012 \text{ ft/s}]$$

The maximum Reynolds number is estimated by making a computation based on properties evaluated at free-stream conditions:

$$\rho_\infty = \frac{(1.0132 \times 10^5)(\frac{1}{20})}{(287)(233)} = 0.0758 \text{ kg/m}^3 \quad [4.73 \times 10^{-3} \text{ lb}_m/\text{ft}^3]$$

$$\mu_\infty = 1.434 \times 10^{-5} \text{ kg/m}\cdot\text{s} \quad [0.0347 \text{ lb}_m/\text{h}\cdot\text{ft}]$$

$$\text{Re}_{L,\infty} = \frac{(0.0758)(918)(0.70)}{1.434 \times 10^{-5}} = 3.395 \times 10^6$$

Thus we conclude that both laminar and turbulent-boundary-layer heat transfer must be considered. We first determine the reference temperatures for the two regimes and then evaluate properties at these temperatures.

Laminar portion

$$T_0 = T_\infty \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right) = (233)[1 + (0.2)(3)^2] = 652 \text{ K}$$

Assuming a Prandtl number of about 0.7, we have

$$r = \text{Pr}^{1/2} = (0.7)^{1/2} = 0.837$$

$$r = \frac{T_{aw} - T_\infty}{T_0 - T_\infty} = \frac{T_{aw} - 233}{652 - 233}$$

and $T_{aw} = 584$ K $= 311°$C [592°F]. Then the reference temperature from Equation (5-123) is

$$T^* = 233 + (0.5)(308 - 233) + (0.22)(584 - 233) = 347.8 \text{ K}$$

Checking the Prandtl number at this temperature, we have

$$\text{Pr}^* = 0.697$$

so that the calculation is valid. If there were an appreciable difference between the value of Pr* and the value used to determine the recovery factor, the calculation would have to be repeated until agreement was reached.

The other properties to be used in the laminar heat-transfer analysis are

$$\rho^* = \frac{(1.0132 \times 10^5)(1/20)}{(287)(347.8)} = 0.0508 \text{ kg/m}^3$$

$$\mu^* = 2.07 \times 10^{-5} \text{ kg/m}\cdot\text{s}$$

$$k^* = 0.03 \text{ W/m}\cdot°\text{C} \quad [0.0173 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}]$$

$$c_p^* = 1.009 \text{ kJ/kg}\cdot°\text{C}$$

Turbulent portion

Assuming Pr = 0.7 gives

$$r = \text{Pr}^{1/3} = 0.888 = \frac{T_{aw} - T_\infty}{T_0 - T_\infty} = \frac{T_{aw} - 233}{652 - 233}$$

$$T_{aw} = 605\ \text{K} = 332^\circ\text{C}$$

$$T^* = 233 + (0.5)(308 - 233) + (0.22)(605 - 233) = 352.3\ \text{K}$$

$$\text{Pr}^* = 0.695$$

The agreement between Pr* and the assumed value is sufficiently close. The other properties to be used in the turbulent heat-transfer analysis are

$$\rho^* = \frac{(1.0132 \times 10^5)(1/20)}{(287)(352.3)} = 0.0501\ \text{kg/m}^3$$

$$\mu^* = 2.09 \times 10^{-5}\ \text{kg/m} \cdot \text{s}$$

$$k^* = 0.0302\ \text{W/m} \cdot {}^\circ\text{C} \qquad c_p{}^* = 1.009\ \text{kJ/kg} \cdot {}^\circ\text{C}$$

Laminar heat transfer

We assume

$$\text{Re}^*_{\text{crit}} = 5 \times 10^5 = \frac{\rho^* u_\infty x_c}{\mu^*}$$

$$x_c = \frac{(5 \times 10^5)(2.07 \times 10^{-5})}{(0.0508)(918)} = 0.222\ \text{m}$$

$$\overline{\text{Nu}^*} = \frac{\bar{h} x_c}{k^*} = 0.664 \left(\text{Re}^*_{\text{crit}}\right)^{1/2} \text{Pr}^{*1/3}$$

$$= (0.664)(5 \times 10^5)^{1/2}(0.697)^{1/3} = 416.3$$

$$\bar{h} = \frac{(416.3)(0.03)}{0.222} = 56.25\ \text{W/m}^2 \cdot {}^\circ\text{C} \quad [9.91\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}]$$

This is the average heat-transfer coefficient for the laminar portion of the boundary layer, and the heat transfer is calculated from

$$q = \bar{h} A (T_w - T_{aw})$$

$$= (56.26)(0.222)(308 - 584)$$

$$= -3445\ \text{W} \quad [-11{,}750\ \text{Btu/h}]$$

so that 3445 W of cooling is required in the laminar region of the plate per meter of depth in the z direction.

Turbulent heat transfer

To determine the turbulent heat transfer we must obtain an expression for the local heat-transfer coefficient from

$$\text{St}^*_x\ \text{Pr}^{*2/3} = 0.0296\ \text{Re}_x^{*-1/5}$$

and then integrate from $x = 0.222$ m to $x = 0.7$ m to determine the total heat transfer:

$$h_x = \text{Pr}^{*-2/3} \rho^* u_\infty c_p (0.0296) \left(\frac{\rho^* u_\infty x}{\mu^*}\right)^{-1/5}$$

Inserting the numerical values for the properties gives

$$h_x = 94.34 x^{-1/5}$$

The average heat-transfer coefficient in the turbulent region is determined from

$$\bar{h} = \frac{\int_{0.222}^{0.7} h_x\,dx}{\int_{0.222}^{0.7} dx} = 111.46\ \text{W/m}^2 \cdot {}^\circ\text{C} \quad [19.6\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}]$$

Using this value we may calculate the heat transfer in the turbulent region of the flat plate:

$$\begin{aligned} q &= \bar{h}A(T_w - T_{aw}) \\ &= (111.46)(0.7 - 0.222)(308 - 605) \\ &= -15{,}823\ \text{W} \quad [-54{,}006\ \text{Btu/h}] \end{aligned}$$

The total amount of cooling required is the sum of the heat transfers for the laminar and turbulent portions:

$$\text{Total cooling} = 3445 + 15{,}823 = 19{,}268\ \text{W} \quad [65{,}761\ \text{Btu/h}]$$

These calculations assume unit depth of 1 m in the z direction.

5-13 | SUMMARY

Most of this chapter has been concerned with flow over flat plates and the associated heat transfer. For convenience of the reader we have summarized the heat-transfer, boundary-layer thickness, and friction-coefficient equations in Table 5-2 along with the restrictions that apply. Our presentation of convection heat transfer is incomplete at this time and will be developed further in Chapters 6 and 7. Even so, we begin to see the structure of a procedure for solution of convection problems:

1. Establish the geometry of the situation; for now we are mainly restricted to flow over flat plates.
2. Determine the fluid involved and evaluate the fluid properties. This will usually be at the film temperature.
3. Establish the boundary conditions (i.e., constant temperature or constant heat flux).
4. Establish the flow regime as determined by the Reynolds number.
5. Select the appropriate equation, taking into account the flow regime and any fluid property restrictions which may apply.
6. Calculate the value(s) of the convection heat-transfer coefficient and/or heat transfer.

At the conclusion of Chapter 7 we shall give a general procedure for all convection problems and the information contained in Table 5-2 will comprise one ingredient in the overall recipe. The interested reader may wish to consult Section 7-14 and Figure 7-15 for a preview of this information and some perspective of the way the material in the present chapter fits in.

REVIEW QUESTIONS

1. What is meant by a hydrodynamic boundary level?
2. Define the Reynolds number. Why is it important?
3. What is the physical mechanism of viscous action?
4. Distinguish between laminar and turbulent flow in a physical sense.

Table 5-2 | Summary of equations for flow over flat plates. Properties evaluated at $T_f = (T_w + T_\infty)/2$ unless otherwise noted.

Flow regime	Restrictions	Equation	Equation number
		Heat transfer	
Laminar, local	$T_w = \text{const}$, $\text{Re}_x < 5 \times 10^5$, $0.6 < \text{Pr} < 50$	$\text{Nu}_x = 0.332\ \text{Pr}^{1/3}\text{Re}_x^{1/2}$	(5-44)
Laminar, local	$T_w = \text{const}$, $\text{Re}_x < 5 \times 10^5$, $\text{Re}_x\ \text{Pr} > 100$	$\text{Nu}_x = \dfrac{0.3387\ \text{Re}_x^{1/2}\ \text{Pr}^{1/3}}{\left[1 + \left(\dfrac{0.0468}{\text{Pr}}\right)^{2/3}\right]^{1/4}}$	(5-51)
Laminar, local	$q_w = \text{const}$, $\text{Re}_x < 5 \times 10^5$, $0.6 < \text{Pr} < 50$	$\text{Nu}_x = 0.453\ \text{Re}_x^{1/2}\ \text{Pr}^{1/3}$	(5-48)
Laminar, local	$q_w = \text{const}$, $\text{Re}_x < 5 \times 10^5$	$\text{Nu}_x = \dfrac{0.4637\ \text{Re}_x^{1/2}\ \text{Pr}^{1/3}}{\left[1 + \left(\dfrac{0.0207}{\text{Pr}}\right)^{2/3}\right]^{1/4}}$	(5-51)
Laminar, average	$\text{Re}_L < 5 \times 10^5$, $T_w = \text{const}$	$\overline{\text{Nu}}_L = 2\ \text{Nu}_{x=L} = 0.664\ \text{Re}_L^{1/2}\ \text{Pr}^{1/3}$	(5-46)
Laminar, local	$T_w = \text{const}$, $\text{Re}_x < 5 \times 10^5$, $\text{Pr} \ll 1$ (liquid metals)	$\text{Nu}_x = 0.564(\text{Re}_x\ \text{Pr})^{1/2}$	
Laminar, local	$T_w = \text{const}$, starting at $x = x_0$, $\text{Re}_x < 5 \times 10^5$, $0.6 < \text{Pr} < 50$	$\text{Nu}_x = 0.332\ \text{Pr}^{1/3}\ \text{Re}_x^{1/2}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$	(5-43)
Turbulent, local	$T_w = \text{const}$, $5 \times 10^5 < \text{Re}_x < 10^7$	$\text{St}_x\ \text{Pr}^{2/3} = 0.0296\ \text{Re}_x^{-0.2}$	(5-81)
Turbulent, local	$T_w = \text{const}$, $10^7 < \text{Re}_x < 10^9$	$\text{St}_x\ \text{Pr}^{2/3} = 0.185(\log \text{Re}_x)^{-2.584}$	(5-82)
Turbulent, local	$q_w = \text{const}$, $5 \times 10^5 < \text{Re}_x < 10^7$	$\text{Nu}_x = 1.04\ \text{Nu}_{x\,T_w=\text{const}}$	(5-87)
Laminar-turbulent, average	$T_w = \text{const}$, $\text{Re}_x < 10^7$, $\text{Re}_{\text{crit}} = 5 \times 10^5$	$\overline{\text{St}}\ \text{Pr}^{2/3} = 0.037\ \text{Re}_L^{-0.2} - 871\ \text{Re}_L^{-1}$ $\overline{\text{Nu}}_L = \text{Pr}^{1/3}(0.037\ \text{Re}_L^{0.8} - 871)$	(5-84) (5-85)
Laminar-turbulent, average	$T_w = \text{const}$, $\text{Re}_x < 10^7$, liquids, μ at T_∞, μ_w at T_w	$\overline{\text{Nu}}_L = 0.036\ \text{Pr}^{0.43}(\text{Re}_L^{0.8} - 9200)\left(\frac{\mu_\infty}{\mu_w}\right)^{1/4}$	(5-86)
High-speed flow	$T_w = \text{const}$, $q = hA(T_w - T_{aw})$ $r = (T_{aw} - T_\infty)/(T_o - T_\infty)$ = recovery factor = $\text{Pr}^{1/2}$ (laminar) = $\text{Pr}^{1/3}$ (turbulent)	Same as for low-speed flow with properties evaluated at $T^* = T_\infty + 0.5(T_w - T_\infty) + 0.22(T_{aw} - T_\infty)$	(5-124)
		Boundary-layer thickness	
Laminar	$\text{Re}_x < 5 \times 10^5$	$\frac{\delta}{x} = 5.0\ \text{Re}_x^{-1/2}$	(5-21a)
Turbulent	$\text{Re}_x < 10^7$, $\delta = 0$ at $x = 0$	$\frac{\delta}{x} = 0.381\ \text{Re}_x^{-1/5}$	(5-91)
Turbulent	$5 \times 10^5 < \text{Re}_x < 10^7$, $\text{Re}_{\text{crit}} = 5 \times 10^5$, $\delta = \delta_{\text{lam}}$ at Re_{crit}	$\frac{\delta}{x} = 0.381\ \text{Re}_x^{-1/5} - 10{,}256\ \text{Re}_x^{-1}$	(5-95)
		Friction coefficients	
Laminar, local	$\text{Re}_x < 5 \times 10^5$	$C_{fx} = 0.332\ \text{Re}_x^{-1/2}$	(5-54)
Turbulent, local	$5 \times 10^5 < \text{Re}_x < 10^7$	$C_{fx} = 0.0592\ \text{Re}_x^{-1/5}$	(5-77)
Turbulent, local	$10^7 < \text{Re}_x < 10^9$	$C_{fx} = 0.37(\log \text{Re}_x)^{-2.584}$	(5-78)
Turbulent, average	$\text{Re}_{\text{crit}} < \text{Re}_x < 10^9$	$\overline{C}_f = \frac{0.455}{(\log \text{Re}_L)^{2.584}} - \frac{A}{\text{Re}_L}$ A from Table 5-1	(5-79)

5. What is the momentum equation for the laminar boundary layer on a flat plate? What assumptions are involved in the derivation of this equation?
6. How is the boundary-layer thickness defined?
7. What is the energy equation for the laminar boundary layer on a flat plate? What assumptions are involved in the derivation of this equation?
8. What is meant by a thermal boundary layer?
9. Define the Prandtl number. Why is it important?
10. Describe the physical mechanism of convection. How is the convection heat-transfer coefficient related to this mechanism?
11. Describe the relation between fluid friction and heat transfer.
12. Define the bulk temperature. How is it used?
13. How is the heat-transfer coefficient defined for high-speed heat-transfer calculations?

LIST OF WORKED EXAMPLES

5-1 Water flow in a diffuser
5-2 Isentropic expansion of air
5-3 Mass flow and boundary-layer thickness
5-4 Isothermal flat plate heated over entire length
5-5 Flat plate with constant heat flux
5-6 Plate with unheated starting length
5-7 Oil flow over heated flat plate
5-8 Drag force on a flat plate
5-9 Turbulent heat transfer from isothermal flat plate
5-10 Turbulent-boundary-layer thickness
5-11 High-speed heat transfer for a flat plate

PROBLEMS

5-1 A certain nozzle is designed to expand air from stagnation conditions of 1.38 MPa and 200°C to 0.138 MPa. The mass rate of flow is designed to be 4.5 kg/s. Suppose this nozzle is used in conjunction with a blowdown wind-tunnel facility so that the nozzle is suddenly allowed to discharge into a perfectly evacuated tank. What will the temperature of the air in the tank be when the pressure in the tank equals 0.138 MPa? Assume that the tank is perfectly insulated and that air behaves as a perfect gas. Assume that the expansion in the nozzle is isentropic.

5-2 Using a linear velocity profile

$$\frac{u}{u_\infty} = \frac{y}{\delta}$$

for a flow over a flat plate, obtain an expression for the boundary-layer thickness as a function of x.

5-3 Using the continuity relation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

along with the velocity distribution

$$\frac{u}{u_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

and the expression for the boundary-layer thickness

$$\frac{\delta}{x} = \frac{4.64}{\sqrt{\mathrm{Re}_x}}$$

derive an expression for the y component of velocity v as a function of x and y. Calculate the value of v at the outer edge of the boundary layer at distances of 6 and 12 in from the leading edge for the conditions of Example 5-3.

5-4 Repeat Problem 5-3 for the linear velocity profile of Problem 5-2.

5-5 Using the linear-velocity profile in Problem 5-2 and a cubic-parabola temperature distribution [Equation (5-30)], obtain an expression for heat-transfer coefficient as a function of the Reynolds number for a laminar boundary layer on a flat plate.

5-6 Air at 20 kPa and 5°C enters a 2.5-cm-diameter tube at a velocity of 1.5 m/s. Using a flat-plate analysis, estimate the distance from the entrance at which the flow becomes fully developed.

5-7 Oxygen at a pressure of 2 atm and 27°C blows across a 50-cm-square plate at a velocity of 30 m/s. The plate temperature is maintained constant at 127°C. Calculate the total heat lost by the plate.

5-8 A fluid flows between two large parallel plates. Develop an expression for the velocity distribution as a function of distance from the centerline between the two plates under developed flow conditions.

5-9 Using the energy equation given by Equation (5-32), determine an expression for heat-transfer coefficient under the conditions

$$u = u_\infty = \text{const} \qquad \frac{T - T_w}{T_\infty - T_w} = \frac{y}{\delta_t}$$

where δ_t is the thermal-boundary-layer thickness.

5-10 Derive an expression for the heat transfer in a laminar boundary layer on a flat plate under the condition $u = u_\infty = \text{constant}$. Assume that the temperature distribution is given by the cubic-parabola relation in Equation (5-30). This solution approximates the condition observed in the flow of a liquid metal over a flat plate.

5-11 Show that $\partial^3 u/\partial y^3 = 0$ at $y = 0$ for an incompressible laminar boundary layer on a flat plate with zero-pressure gradient.

5-12 Review the analytical developments of this chapter and list the restrictions that apply to the following equations: (5-25), (5-26), (5-44), (5-46), (5-85), and (5-107).

5-13 Calculate the ratio of thermal-boundary-layer thickness to hydrodynamic-boundary-layer thickness for the following fluids: air at 1 atm and 20°C, water at 20°C, helium at 1 atm and 20°C, liquid ammonia at 20°C, glycerine at 20°C.

5-14 For water flowing over a flat plate at 15°C and 3 m/s, calculate the mass flow through the boundary layer at a distance of 5 cm from the leading edge of the plate.

5-15 Air at 90°C and 1 atm flows over a flat plate at a velocity of 30 m/s. How thick is the boundary layer at a distance of 2.5 cm from the leading edge of the plate?

5-16 Air flows over a flat plate at a constant velocity of 20 m/s and ambient conditions of 20 kPa and 20°C. The plate is heated to a constant temperature of 75°C, starting at a distance of 7.5 cm from the leading edge. What is the total heat transfer from the leading edge to a point 35 cm from the leading edge?

5-17 Water at 15°C flows between two large parallel plates at a velocity of 1.5 m/s. The plates are separated by a distance of 15 mm. Estimate the distance from the leading edge where the flow becomes fully developed.

5-18 Air at standard conditions of 1 atm and 27°C flows over a flat plate at 20 m/s. The plate is 60 cm square and is maintained at 97°C. Calculate the heat transfer from the plate.

5-19 Air at 7 kPa and 35°C flows across a 30-cm-square flat plate at 7.5 m/s. The plate is maintained at 65°C. Estimate the heat lost from the plate.

5-20 Air at 90°C and atmospheric pressure flows over a horizontal flat plate at 60 m/s. The plate is 60 cm square and is maintained at a uniform temperature of 10°C. What is the total heat transfer?

5-21 Nitrogen at 2 atm and 500 K flows across a 40-cm-square plate at a velocity of 25 m/s. Calculate the cooling required to maintain the plate surface at a constant temperature of 300 K.

5-22 Plot the heat-transfer coefficient versus length for flow over a 1-m-long flat plate under the following conditions: (*a*) helium at 1 lb/in^2 abs, 80°F, $u_\infty = 10$ ft/s [3.048 m/s]; (*b*) hydrogen at 1 lb/in^2 abs, 80°F, $u_\infty = 10$ ft/s; (*c*) air at 1 lb/in^2 abs, 80°F, $u_\infty = 10$ ft/s; (*d*) water at 80°F, $u_\infty = 10$ ft/s; (*e*) helium at 20 lb/in^2 abs, 80°F, $u_\infty = 10$ ft/s.

5-23 Calculate the heat transfer from a 20-cm-square plate over which air flows at 35°C and 14 kPa. The plate temperature is 250°C, and the free-stream velocity is 6 m/s.

5-24 Air at 20 kPa and 20°C flows across a flat plate 60 cm long. The free-stream velocity is 30 m/s, and the plate is heated over its total length to a temperature of 55°C. For $x = 30$ cm, calculate the value of y for which u will equal 22.5 m/s.

5-25 For the flow system in Problem 5-24, calculate the value of the friction coefficient at a distance of 15 cm from the leading edge.

5-26 Air at a pressure of 200 kPa and free-stream temperature of 27°C flows over a square flat plate at a velocity of 30 m/s. The Reynolds number is 10^6 at the edge of the plate. Calculate the heat transfer for an isothermal plate maintained at 57°C.

5-27 Calculate the boundary layer thickness at the edge of the plate for the flow system in Problem 5-26. State the assumptions.

5-28 Air at 5°C and 70 kPa flows over a flat plate at 6 m/s. A heater strip 2.5 cm long is placed on the plate at a distance of 15 cm from the leading edge. Calculate the heat lost from the strip per unit depth of plate for a heater surface temperature of 65°C.

5-29 Air at 1 atm and 27°C blows across a large concrete surface 15 m wide maintained at 55°C. The flow velocity is 4.5 m/s. Calculate the convection heat loss from the surface.

5-30 Air at 300 K and 75 kPa flows over a 1-m-square plate at a velocity of 45 m/s. The plate is maintained at a constant temperature of 400 K. Calculate the heat lost by the plate.

5-31 A horizontal flat plate is maintained at 50°C and has dimensions of 50 cm by 50 cm. Air at 50 kPa and 10°C is blown across the plate at 20 m/s. Calculate the heat lost from the plate.

5-32 Air flows across a 20-cm-square plate with a velocity of 5 m/s. Free-stream conditions are 10°C and 0.2 atm. A heater in the plate surface furnishes a constant heat-flux

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condition at the wall so that the average wall temperature is 100°C. Calculate the surface heat flux and the value of h at an x position of 10 cm.

5-33 Calculate the flow velocity necessary to produce a Reynolds number of 10^7 for flow across a 1-m-square plate with the following fluids: (*a*) water at 20°C, (*b*) air at 1 atm and 20°C, (*c*) Freon 12 at 20°C, (*d*) ammonia at 20°C, and (*e*) helium at 20°C.

5-34 Calculate the average heat-transfer coefficient for each of the cases in Problem 5-31 assuming all properties are evaluated at 20°C.

5-35 Calculate the boundary-layer thickness at the end of the plate for each case in Problem 5-33.

5-36 A blackened plate is exposed to the sun so that a constant heat flux of 800 W/m^2 is absorbed. The back side of the plate is insulated so that all the energy absorbed is dissipated to an airstream that blows across the plate at conditions of 25°C, 1 atm, and 3 m/s. The plate is 25 cm square. Estimate the average temperature of the plate. What is the plate temperature at the trailing edge?

5-37 Air at 0.5 atm pressure and 27°C flows across a 34-cm-square plate at a velocity of 20 m/s. The plate temperature is maintained at 127°C. Calculate the heat lost by the plate.

5-38 Helium at 3 atm and 73°C flows across a 35-cm-square plate that is maintained at a surface temperature of 113°C. The free-stream velocity is 50 m/s. Calculate the convection heat lost by the plate.

5-39 Air at 1 atm and 300 K blows across a 50-cm-square flat plate at a velocity such that the Reynolds number at the downstream edge of the plate is 1.1×10^5. Heating does not begin until halfway along the plate and then the surface temperature is 400 K. Calculate the heat transfer from the plate.

5-40 Air at 20°C and 14 kPa flows at a velocity of 150 m/s past a flat plate 1 m long that is maintained at a constant temperature of 150°C. What is the average heat-transfer rate per unit area of plate?

5-41 Derive equations equivalent to Equation (5-85) for critical Reynolds numbers of 3×10^5, 10^6, and 3×10^6.

5-42 Assuming that the local heat-transfer coefficient for flow on a flat plate can be represented by Equation (5-81) and that the boundary layer starts at the leading edge of the plate, determine an expression for the average heat-transfer coefficient.

5-43 A 10-cm-square plate has an electric heater installed that produces a constant heat flux. Water at 10°C flows across the plate at a velocity of 3 m/s. What is the total heat which can be dissipated if the plate temperature is not to exceed 80°C?

5-44 Repeat Problem 5-41 for air at 1 atm and 300 K.

5-45 Helium at 1 atm and 300 K is used to cool a l-m-square plate maintained at 500 K. The flow velocity is 50 m/s. Calculate the total heat loss from the plate. What is the boundary-layer thickness as the flow leaves the plate?

5-46 A light breeze at 10 mi/h blows across a metal building in the summer. The height of the building wall is 3.7 m, and the width is 6.1 m. A net energy flux of 347 W/m^2 from the sun is absorbed in the wall and subsequently dissipated to the surrounding air by convection. Assuming that the air is 27°C and 1 atm and blows across the wall as on a flat plate, estimate the average temperature the wall will attain for equilibrium conditions.

5-47 The bottom of a corn-chip fryer is 10 ft long by 3 ft wide and is maintained at a temperature of 420°F. Cooking oil flows across this surface at a velocity of 1 ft/s and has a free-stream temperature of 400°F. Calculate the heat transfer to the oil and estimate the maximum boundary-layer thickness. Properties of the oil may be taken as $\nu = 2 \times 10^{-6}$ m^2/s, $k = 0.12$ W/m · °C, and Pr = 40.

5-48 Air at 27°C and 1 atm blows over a 4.0-m-square flat plate at a velocity of 40 m/s. The plate temperature is 77°C. Calculate the total heat transfer.

5-49 The roof of a building is 30 m by 60 m, and because of heat loading by the sun it attains a temperature of 300 K when the ambient air temperature is 0°C. Calculate the heat loss from the roof for a mild breeze blowing at 5 mi/h across the roof ($L = 30$ m).

5-50 Air at 1 atm and 30°C flows over a 15-cm-square plate at a velocity of 10 m/s. Calculate the maximum boundary layer thickness.

5-51 Nitrogen at 1 atm and 300 K blows across a horizontal flat plate at a velocity of 33 m/s. The plate has a constant surface temperature of 400 K. Calculate the heat lost by the plate if the plate dimensions are 60 cm by 30 cm with the longer dimension in the direction of flow. Express in watts.

5-52 Helium at atmospheric pressure and 30°C flows across a square plate at a free-stream velocity of 15 m/s. Calculate the boundary layer thickness at a position where $Re_x = 250,000$.

5-53 Suppose the Reynolds number in problem 5-52 is attained at the edge of the plate, and the plate is maintained at a constant temperature of 60°C. Calculate the heat lost by the plate.

5-54 Air at 0.2 MPa and 25°C flows over a square flat plate at a velocity of 60 m/s. The plate is 0.5 m on a side and is maintained at a constant temperature of 150°C. Calculate the heat lost from the plate.

5-55 Helium at a pressure of 150 kPa and a temperature of 20°C flows across a l-m-square plate at a velocity of 50 m/s. The plate is maintained at a constant temperature of 100°C. Calculate the heat lost by the plate.

5-56 Air at 50 kPa and 250 K flows across a 2-m-square plate at a velocity of 20 m/s. The plate is maintained at a constant temperature of 350 K. Calculate the heat lost by the plate.

5-57 Nitrogen at 50 kPa and 300 K flows over a flat plate at a velocity of 100 m/s. The length of the plate is 1.2 m and the plate is maintained at a constant temperature of 400 K. Calculate the heat lost by the plate.

5-58 Hydrogen at 2 atm and 15°C flows across a l-m-square flat plate at a velocity of 6 m/s. The plate is maintained at a constant temperature of 139°C. Calculate the heat lost by the plate.

5-59 Liquid ammonia at 10°C is forced across a square plate 40 cm on a side at a velocity of 5 m/s. The plate is maintained at 50°C. Calculate the heat lost by the plate.

5-60 Helium flows across a 1.0-m-square plate at a velocity of 50 m/s. The helium is at a pressure of 45 kPa and a temperature of 50°C. The plate is maintained at a constant temperature of 136°C. Calculate the heat lost by the plate.

5-61 Air at 0.1 atm flows over a flat plate at a velocity of 300 m/s. The plate temperature is maintained constant at 100°C and the free-stream air temperature is 10°C. Calculate the heat transfer for a plate that is 80 cm square.

5-62 Water at 21°C flows across a 30-cm-square flat plate at a velocity of 6 m/s. The plate is maintained at a constant temperature of 54°F. Calculate the heat lost by the plate.

5-63 Plot h_x versus x for air at 1 atm and 300 K flowing at a velocity of 30 m/s across a flat plate. Take $Re_{crit} = 5 \times 10^5$ and use semilog plotting paper. Extend the plot to an x value equivalent to $Re = 10^9$. Also plot the average heat-transfer coefficient over this same range.

5-64 Air flows over a flat plate at 1 atm and 350 K with a velocity of 30 m/s. Calculate the mass flow through the boundary layer at x locations where $\mathrm{Re}_x = 10^6$ and 10^7.

5-65 Air flows with a velocity of 6 m/s across a 20-cm-square plate at 50 kPa and 300 K. An electrical heater is installed in the plate such that it produces a constant heat flux. What is the total heat that can be dissipated if the plate temperature cannot exceed 600 K?

5-66 "Slug" flow in a tube may be described as that flow in which the velocity is constant across the entire flow area of the tube. Obtain an expression for the heat-transfer coefficient in this type of flow with a constant-heat-flux condition maintained at the wall. Compare the results with those of Section 5-10. Explain the reason for the difference in answers on a physical basis.

5-67 Compare the average heat transfer coefficients for the following three situations: (*a*) airflow at 1 atm and 300 K across a flat plate such that $\mathrm{Re}_L = 10^5$; (*b*) helium flow at 1 atm and 300 K across a flat plate with the same values of ρu_∞ and L as in (*a*); (*c*) Flow of liquid water at 300 K across a flat plate with the same values of ρu_∞ and L as in (*a*).

Evaluate all properties at $T = 300$ K. What do you conclude from this comparison?

5-68 Air at 1.2 atm and 27°C flows across a 60-cm-square plate at a free-stream velocity of 40 m/s. The plate is maintained at a constant temperature of 177°C. Calculate the heat lost by one side of the plate.

5-69 Air at 50.66 kPa and −23°C blows across a square flat plate that is maintained at a constant temperature of 77°C. The free-stream velocity is 30 m/s and the plate is 50 cm on each side. Calculate the heat lost from the plate, expressed in watts.

5-70 Helium at a pressure of 200 kPa and −18°C flows over a flate plate at a velocity of 20 m/s. The plate is maintained at a constant temperature of 93°C. If the plate is 30 cm square, calculate the heat loss expressed in watts.

5-71 Assume that the velocity distribution in the turbulent core for tube flow may be represented by

$$\frac{u}{u_c} = \left(1 - \frac{r}{r_o}\right)^{1/7}$$

where u_c is the velocity at the center of the tube and r_o is the tube radius. The velocity in the laminar sublayer may be assumed to vary linearly with the radius. Using the friction factor given by Equation (5-115), derive an equation for the thickness of the laminar sublayer. For this problem the average flow velocity may be calculated using only the turbulent velocity distribution.

5-72 Using the velocity profile in Problem 5-71, obtain an expression for the eddy diffusivity of momentum as a function of radius.

5-73 In heat-exchanger applications, it is frequently important to match heat-transfer requirements with pressure-drop limitations. Assuming a fixed total heat-transfer requirement and a fixed temperature difference between wall and bulk conditions as well as a fixed pressure drop through the tube, derive expressions for the length and diameter of the tube, assuming turbulent flow of a gas with the Prandtl number near unity.

5-74 Water flows in a 2.5-cm-diameter pipe so that the Reynolds number based on diameter is 1500 (laminar flow is assumed). The average bulk temperature is 35°C. Calculate the maximum water velocity in the tube. (Recall that $u_m = 0.5u_0$.) What would the heat-transfer coefficient be for such a system if the tube wall was subjected to a constant heat

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flux and the velocity and temperature profiles were completely developed? Evaluate properties at bulk temperature.

5-75 A slug flow is encountered in an annular-flow system that is subjected to a constant heat flux at both the inner and outer surfaces. The temperature is the same at both inner and outer surfaces at identical x locations. Derive an expression for the temperature distribution in such a flow system, assuming constant properties and laminar flow.

5-76 Air at Mach 4 and 3 lb/in^2 abs, 0°F, flows past a flat plate. The plate is to be maintained at a constant temperature of 200°F. If the plate is 18 in long, how much cooling will be required to maintain this temperature?

5-77 Air flows over an isothermal flat plate maintained at a constant temperature of 65°C. The velocity of the air is 600 m/s at static properties of 15°C and 7 kPa. Calculate the average heat-transfer coefficient for a plate 1 m long.

5-78 Air at 7 kPa and −40°C flows over a flat plate at Mach 4. The plate temperature is 35°C, and the plate length is 60 cm. Calculate the adiabatic wall temperature for the laminar portion of the boundary layer.

5-79 A wind tunnel is to be constructed to produce flow conditions of Mach 2.8 at $T_\infty = -40°\text{C}$ and $p = 0.05$ atm. What is the stagnation temperature for these conditions? What would be the adiabatic wall temperature for the laminar and turbulent portions of a boundary layer on a flat plate? If a flat plate were installed in the tunnel such that $\text{Re}_L = 10^7$, what would the heat transfer be for a constant wall temperature of 0°C?

5-80 Compute the drag force exerted on the plate by each of the systems in Problem 5-22.

5-81 Glycerin at 30°C flows past a 30-cm-square flat plate at a velocity of 1.5 m/s. The drag force is measured as 8.9 N (both sides of the plate). Calculate the heat-transfer coefficient for such a flow system.

5-82 Calculate the drag (viscous-friction) force on the plate in Problem 5-23 under the conditions of no heat transfer. Do not use the analogy between fluid friction and heat transfer for this calculation; that is, calculate the drag directly by evaluating the viscous-shear stress at the wall.

5-83 Nitrogen at 1 atm and 20°C is blown across a 130-cm-square flat plate at a velocity of 3.0 m/s. The plate is maintained at a constant temperature of 100°C. Calculate the average-friction coefficient and the heat transfer from the plate.

5-84 Using the velocity distribution for developed laminar flow in a tube, derive an expression for the friction factor as defined by Equation 5-112.

5-85 Engine oil at 10°C flows across a 15-cm-square plate upon which is imposed a constant heat flux of 10 kW/m^2. Determine (*a*) the average temperature difference, (*b*) the temperature difference at the trailing edge, and (*c*) the average heat-transfer coefficient. Use the Churchill relation [Equation (5-51)]. $u_\infty = 0.5$ m/s.

5-86 Work Problem 5-85 for a constant plate surface temperature equal to that at the trailing edge, and determine the total heat transfer.

5-87 For air at 25°C and 1 atm, with a free-stream velocity of 45 m/s, calculate the length of a flat plate to produce Reynolds numbers of 5×10^5 and 10^8. What are the boundary-layer thicknesses at these Reynolds numbers?

5-88 Determine the boundary-layer thickness at $\text{Re} = 5 \times 10^5$ for the following fluids flowing over a flat plate at 20 m/s: (*a*) air at 1 atm and 10°C, (*b*) saturated liquid water at 10°C, (*c*) hydrogen at 1 atm and 10°C, (*d*) saturated liquid ammonia at 10°C, and (*e*) saturated liquid Freon 12 at 10°C.

5-89 Many of the heat-transfer relations for flow over a flat plate are of the form

$$\mathrm{Nu}_x = \frac{h_x x}{k} = C\ \mathrm{Re}_x^n\, f(\mathrm{Pr})$$

Obtain an expression for $\bar{h}_L / h_{x=L}$ in terms of the constants C and n.

5-90 Compare Equations (5-51) and (5-44) for engine oil at 20°C and a Reynolds number of 10,000.

5-91 Air at 1 atm and 300 K blows across a square plate 75 cm on a side that is maintained at 350 K. The free-stream velocity is 45 m/s. Calculate the heat transfer and drag force on one side of the plate. Also calculate the heat transfer for just the laminar portion of the boundary layer.

5-92 Taking the critical Reynolds number as 5×10^5 for Problem 5-87, calculate the boundary-layer thickness at this point and at the trailing edge of the plate assuming (*a*) laminar flow to $\mathrm{Re}_{\mathrm{crit}}$ and turbulent thereafter and (*b*) turbulent flow from the leading edge.

5-93 If the plate temperature in Problem 5-91 is raised to 500 K while keeping the free-stream conditions the same, calculate the total heat transfer evaluating properties at (*a*) free-stream conditions, (*b*) film temperature, and (*c*) wall temperature. Comment on the results.

5-94 Air at 250 K and 1 atm blows across a 30-cm-square plate at a velocity of 10 m/s. The plate maintains a constant heat flux of 700 $\mathrm{W/m^2}$. Determine the plate temperatures at x locations of 1, 5, 10, 20, and 30 cm.

5-95 Engine oil at 20°C is forced across a 20-cm-square plate at 10 m/s. The plate surface is maintained at 40°C. Calculate the heat lost by the plate and the drag force for one side of an unheated plate.

5-96 A large flat plate 4.0 m long and 1.0 m wide is exposed to an atmospheric air at 27°C with a velocity of 30 mi/h in a direction parallel to the 4.0-m dimension. If the plate is maintained at 77°C, calculate the total heat loss. Also calculate the heat flux in watts per square meter at x locations of 3 cm, 50 cm, 1.0 m, and 4.0 m.

5-97 Air at 1 atm and 300 K blows across a 10-cm-square plate at 30 m/s. Heating does not begin until $x = 5.0$ cm, after which the plate surface is maintained at 400 K. Calculate the total heat lost by the plate.

5-98 For the plate and flow conditions of Problem 5-97, only a 0.5-cm strip centered at $x = 5.0$ cm is heated to 400 K. Calculate the heat lost by this strip.

5-99 Two 20-cm-square plates are separated by a distance of 3.0 cm. Air at 1 atm, 300 K, and 15 m/s enters the space separating the plates. Will there be interference between the two boundary layers?

5-100 Water at 15.6°C flows across a 20-cm-square plate with a velocity of 2 m/s. A thin strip, 5 mm wide, is placed on the plate at a distance of 10 cm from the leading edge. If the strip is heated to a temperature of 37.8°C, calculate the heat lost from the strip.

5-101 Air at 300 K and 4 atm blows across a 10-cm-square plate at a velocity of 35 m/s. The plate is maintained at a constant temperature of 400 K. Calculate the heat lost from the plate.

5-102 An electric heater is installed on the plate of Problem 5-97 that will produce a constant heat flux of 1000 $\mathrm{W/m^2}$ for the same airflow conditions across the plate. What is the maximum temperature that will be experienced by the plate surface?

5-103 In a certain application the critical Reynolds number for flow over a flat plate is 10^6. Air at 1 atm, 300 K, and 10 m/s flows across an isothermal plate with this critical

Reynolds number, and with a plate temperature of 400 K. The Reynolds number at the end of the plate is 5×10^6. What will the average heat transfer coefficient be for this system? How long is the plate? What is the heat lost from the plate?

5-104 Calculate the average heat transfer coefficient for the flow conditions of Problem 5-103, but with a critical Reynolds number of 5×10^5. What is the heat lost by the plate in this circumstance?

5-105 Glycerin at 10°C flows across a 30-cm-square plate with a velocity of 2 m/s. The plate surface is isothermal at 30°C. Calculate the heat lost by the plate.

5-106 Ethylene glycol at 20°C flows across an isothermal plate maintained at 0°C. The plate is 20 cm square and the Reynolds number at the end of the plate is 100,000. Calculate the heat gained by the plate.

Design-Oriented Problems

5-107 A low-speed wind tunnel is to be designed to study boundary layers up to $\mathrm{Re}_x = 10^7$ with air at 1 atm and 25°C. The maximum flow velocity that can be expected from an existing fan system is 30 m/s. How long must the flat-plate test-section be to produce the required Reynolds numbers? What will the maximum boundary-layer thickness be under these conditions? What would the maximum boundary-layer thicknesses be for flow velocities at 7 m/s and 12 m/s?

5-108 Using Equations (5-55), (5-81), and (5-82) for the local heat transfer in their respective ranges, obtain an expression for the average heat transfer coefficient, or Nusselt number, over the range $5 \times 10^5 < \mathrm{Re}_L < 10^9$ with $\mathrm{Re}_{\mathrm{crit}} = 5 \times 10^5$. Use a numerical technique to perform the necessary integration and a curve fit to simplify the results.

5-109 An experiment is to be deligned to demonstrate measurement of heat loss for water flowing over a flat plate. The plate is 30 cm square and it will be maintained nearly constant in temperature at 50°C while the water temperature will be about 10°C. (*a*) Calculate the flow velocities necessary to study a range of Reynolds numbers from 10^4 to 10^7. (*b*) Estimate the heat-transfer coefficients and heat-transfer rates for several points in the specified range.

5-110 Consider the flow of air over a flat plate under laminar flow conditions at 1 atm. Investigate the influence of temperature on the heat transfer coefficient by examining five cases with constant free-stream temperature of 20°C, constant free-stream velocity, and surface temperatures of 50, 100, 150, 250, and 350°C. What do you conclude from this analysis? From the results, determine an approximate variation of the heat-transfer coefficient with absolute temperature for air at 1 atm.

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CHAPTER

6 Empirical and Practical Relations for Forced-Convection Heat Transfer

6-1 | INTRODUCTION

The discussion and analyses of Chapter 5 have shown how forced-convection heat transfer may be calculated for several cases of practical interest; the problems considered, however, were those that could be solved in an analytical fashion. In this way, the principles of the convection process and their relation to fluid dynamics were demonstrated, with primary emphasis being devoted to a clear understanding of physical mechanism. Regrettably, it is not always possible to obtain analytical solutions to convection problems, and the individual is forced to resort to experimental methods to obtain design information, as well as to secure the more elusive data that increase the physical understanding of the heat-transfer processes.

Results of experimental data are usually expressed in the form of either empirical formulas or graphical charts so that they may be utilized with a maximum of generality. It is in the process of trying to generalize the results of one's experiments, in the form of some empirical correlation, that difficulty is encountered. If an analytical solution is available for a similar problem, the correlation of data is much easier, since one may guess at the functional form of the results, and hence use the experimental data to obtain values of constants or exponents for certain significant parameters such as the Reynolds or Prandtl numbers. If an analytical solution for a similar problem is not available, the individual must resort to intuition based on physical understanding of the problem, or shrewd inferences that one may be able to draw from the differential equations of the flow processes based upon dimensional or order-of-magnitude estimates. In any event, there is no substitute for physical insight and understanding.

To show how one might proceed to analyze a new problem to obtain an important functional relationship from the differential equations, consider the problem of determining the hydrodynamic-boundary-layer thickness for flow over a flat plate. This problem was solved in Chapter 5, but we now wish to make an order-of-magnitude analysis of the differential equations to obtain the functional form of the solution. The momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}$$

must be solved in conjunction with the continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

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Within the boundary layer we may say that the velocity u is of the order of the free-stream velocity u_∞. Similarly, the y dimension is of the order of the boundary-layer thickness δ. Thus

$$u \sim u_\infty$$
$$y \sim \delta$$

and we might write the continuity equation in an approximate form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
$$\frac{u_\infty}{x} + \frac{v}{\delta} \approx 0$$

or

$$v \sim \frac{u_\infty \delta}{x}$$

Then, by using this order of magnitude for v, the analysis of the momentum equation would yield

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$
$$u_\infty \frac{u_\infty}{x} + \frac{u_\infty \delta}{x}\frac{u_\infty}{\delta} \approx \nu\frac{u_\infty}{\delta^2}$$

or

$$\delta^2 \sim \frac{\nu x}{u_\infty}$$
$$\delta \sim \sqrt{\frac{\nu x}{u_\infty}}$$

Dividing by x to express the result in dimensionless form gives

$$\frac{\delta}{x} \sim \sqrt{\frac{\nu}{u_\infty x}} = \frac{1}{\sqrt{\mathrm{Re}_x}}$$

This functional variation of the boundary-layer thickness with the Reynolds number and x position is precisely that which was obtained in Section 5-4. Although this analysis is rather straightforward and does indeed yield correct results, the order-of-magnitude analysis may not always be so fortunate when applied to more complex problems, particularly those involving turbulent- or separated-flow regions. Nevertheless, one may often obtain valuable information and physical insight by examining the order of magnitude of various terms in a governing differential equation for the particular problem at hand.

A conventional technique used in correlation of experimental data is that of dimensional analysis, in which appropriate dimensionless groups such as the Reynolds and Prandtl numbers are derived from purely dimensional and functional considerations. There is, of course, the assumption of flow-field and temperature-profile similarity for geometrically similar heating surfaces. Generally speaking, the application of dimensional analysis to any new problem is extremely difficult when a previous analytical solution of some sort is not available. It is usually best to attempt an order-of-magnitude analysis such as the one above if the governing differential equations are known. In this way it may be possible to determine the significant dimensionless variables for correlating experimental data. In some complex flow and heat-transfer problems a clear physical model of the processes may not be available, and the engineer must first try to establish this model before the experimental data can be correlated.

Schlichting [6], Giedt [7], and Kline [28] discuss similarity considerations and their use in boundary-layer and heat-transfer problems.

The purpose of the foregoing discussion has not been to emphasize or even to imply any new method for solving problems, but rather to indicate the necessity of applying intuitive physical reasoning to a difficult problem and to point out the obvious advantage of using any and all information that may be available. When the problem of correlation of experimental data for a previously unsolved situation is encountered, one must frequently adopt devious methods to accomplish the task.

6-2 | EMPIRICAL RELATIONS FOR PIPE AND TUBE FLOW

The analysis of Section 5-10 has shown how one might analytically attack the problem of heat transfer in fully developed laminar tube flow. The cases of undeveloped laminar flow, flow systems where the fluid properties vary widely with temperature, and turbulent-flow systems are considerably more complicated but are of very important practical interest in the design of heat exchangers and associated heat-transfer equipment. These more complicated problems may sometimes be solved analytically, but the solutions, when possible, are very tedious. For design and engineering purposes, empirical correlations are usually of greatest practical utility. In this section we present some of the more important and useful empirical relations and point out their limitations.

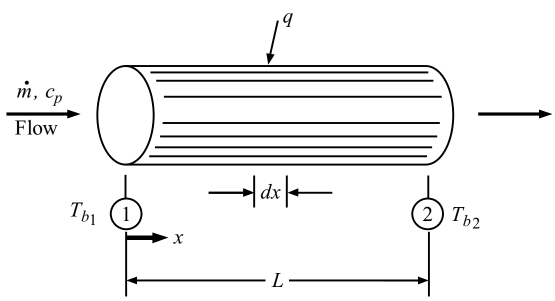
The Bulk Temperature

First let us give some further consideration to the bulk-temperature concept that is important in all heat-transfer problems involving flow inside closed channels. In Chapter 5 we noted that the bulk temperature represents energy average or “mixing cup” conditions. Thus, for the tube flow depicted in Figure 6-1 the total energy added can be expressed in terms of a bulk-temperature difference by

$$q = \dot{m} c_p (T_{b_2} - T_{b_1}) \qquad \text{[6-1]}$$

provided c_p is reasonably constant over the length. In some differential length dx the heat added dq can be expressed either in terms of a bulk-temperature difference or in terms of

Figure 6-1 | Total heat transfer in terms of bulk-temperature difference.



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the heat-transfer coefficient

$$dq = \dot{m} c_p dT_b = h(2\pi r)dx(T_w - T_b) \qquad \textbf{[6-2]}$$

where T_w and T_b are the wall and bulk temperatures at the particular x location. The total heat transfer can also be expressed as

$$q = hA(T_w - T_b)_{\text{av}} \qquad \textbf{[6-3]}$$

where A is the total surface area for heat transfer. Because both T_w and T_b can vary along the length of the tube, a suitable averaging process must be adopted for use with Equation (6-3). In this chapter most of our attention will be focused on methods for determining h, the convection heat-transfer coefficient. Chapter 10 will discuss different methods for taking proper account of temperature variations in heat exchangers.

A traditional expression for calculation of heat transfer in fully developed turbulent flow in smooth tubes is that recommended by Dittus and Boelter [1]:*

$$\text{Nu}_d = 0.023\,\text{Re}_d^{0.8}\text{Pr}^n \qquad \textbf{[6-4}a\textbf{]}$$

The properties in this equation are evaluated at the average fluid bulk temperature, and the exponent n has the following values:

$$n = \begin{cases} 0.4 & \text{for heating of the fluid} \\ 0.3 & \text{for cooling of the fluid} \end{cases}$$

Equation (6-4) is valid for fully developed turbulent flow in smooth tubes for fluids with Prandtl numbers ranging from about 0.6 to 100 and with moderate temperature differences between wall and fluid conditions. More recent information by Gnielinski [45] suggests that better results for turbulent flow in smooth tubes may be obtained from the following:

$$\text{Nu} = 0.0214\left(\text{Re}^{0.8} - 100\right)\text{Pr}^{0.4} \qquad \textbf{[6-4}b\textbf{]}$$

for $0.5 < \text{Pr} < 1.5$ and $10^4 < \text{Re} < 5 \times 10^6$ or

$$\text{Nu} = 0.012\left(\text{Re}^{0.87} - 280\right)\text{Pr}^{0.4} \qquad \textbf{[6-4}c\textbf{]}$$

for $1.5 < \text{Pr} < 500$ and $3000 < \text{Re} < 10^6$.

One may ask the reason for the functional form of Equation (6-4). Physical reasoning, based on the experience gained with the analyses of Chapter 5, would certainly indicate a dependence of the heat-transfer process on the flow field, and hence on the Reynolds number. The relative rates of diffusion of heat and momentum are related by the Prandtl number, so that the Prandtl number is expected to be a significant parameter in the final solution. We can be rather confident of the dependence of the heat transfer on the Reynolds and Prandtl numbers. But the question arises as to the correct functional form of the relation; that is, would one necessarily expect a product of two power functions of the Reynolds and Prandtl numbers? The answer is that one might expect this functional form since it appears in the flat-plate analytical solutions of Chapter 5, as well as the Reynolds analogy for turbulent flow in tubes. In addition, this type of functional relation is convenient to use in correlating experimental data, as described below.

*Equation (6-4*a*) is a rather famous equation in the annals of convection heat transfer, but it appears [47] that the constant 0.023 and exponents 0.4 and 0.3 were actually recommended by McAdams [10, 2nd ed., 1942] as a meld between the values given in Reference 1 and those suggested by Colburn [15].

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Suppose a number of experiments are conducted with measurements taken of heat-transfer rates of various fluids in turbulent flow inside smooth tubes under different temperature conditions. Different-diameter tubes may be used to vary the range of the Reynolds number in addition to variations in the mass-flow rate. We wish to generalize the results of these experiments by arriving at one empirical equation that represents all the data. As described above, we may anticipate that the heat-transfer data will be dependent on the Reynolds and Prandtl numbers. A power function for each of these parameters is a simple type of relation to use, so we assume

$$\mathrm{Nu}_d = C\,\mathrm{Re}_d{}^m\,\mathrm{Pr}^n$$

where C, m, and n are constants to be determined from the experimental data.

A log-log plot of Nu_d versus Re_d is first made for one fluid to estimate the dependence of the heat transfer on the Reynolds number (i.e., to find an approximate value of the exponent m). This plot is made for one fluid at a constant temperature, so that the influence of the Prandtl number will be small, since the Prandtl number will be approximately constant for the one fluid. By using this first estimate for the exponent m, the data for all fluids are plotted as $\log(\mathrm{Nu}_d/\mathrm{Re}_d{}^m)$ versus $\log \mathrm{Pr}$, and a value for the exponent n is determined. Then, by using this value of n, all the data are plotted again as $\log(\mathrm{Nu}_d/\mathrm{Pr}^n)$ versus $\log \mathrm{Re}_d$, and a final value of the exponent m is determined as well as a value for the constant C. An example of this final type of data plot is shown in Figure 6-2. The final correlation equation may represent the data within ±25 percent.

Readers should recognize that obtaining empirical correlations for convection heat transfer phenomena is not as simple as the preceding discussion might imply. The "data

Figure 6-2 | Typical data correlation for forced convection in smooth tubes, turbulent flow.

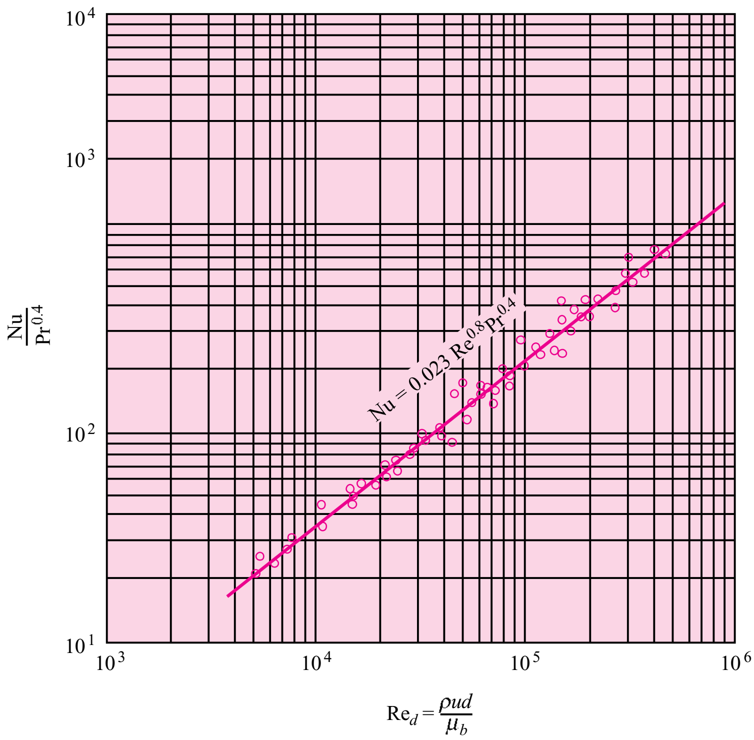
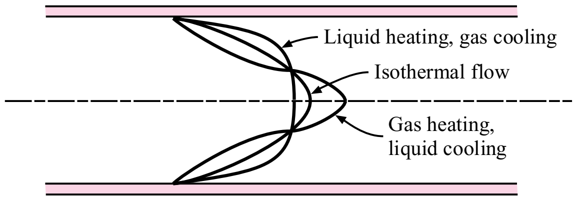


Figure 6-3 | Influence of heating on velocity profile in laminar tube flow.



points" shown in Figure 6-2 are entirely fictitious and more consistent than normally might be encountered. Careful attention must be given to experiment design to minimize experimental uncertainties that can creep into the final data correlation. A very complete discussion of the design of an experiment for measurement of convection heat transfer in smooth tubes is given in Reference 51, along with an extensive discussion of techniques for estimating the propagation of experimental uncertainties and the methods for obtaining the best correlation equation for the available data.

If wide temperature differences are present in the flow, there may be an appreciable change in the fluid properties between the wall of the tube and the central flow. These property variations may be evidenced by a change in the velocity profile as indicated in Figure 6-3. The deviations from the velocity profile for isothermal flow as shown in this figure are a result of the fact that the viscosity of gases increases with an increase in temperature, while the viscosities of liquids decrease with an increase in temperature.

To take into account the property variations, Sieder and Tate [2] recommend the following relation:

$$\mathrm{Nu}_d = 0.027\,\mathrm{Re}_d^{0.8}\mathrm{Pr}^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} \qquad \textbf{[6-5]}$$

All properties are evaluated at bulk-temperature conditions, except μ_w, which is evaluated at the wall temperature.

Equations (6-4) and (6-5) apply to fully developed turbulent flow in tubes. In the entrance region the flow is not developed, and Nusselt [3] recommended the following equation:

$$\mathrm{Nu}_d = 0.036\,\mathrm{Re}_d^{0.8}\mathrm{Pr}^{1/3}\left(\frac{d}{L}\right)^{0.055} \qquad \text{for } 10 < \frac{L}{d} < 400 \qquad \textbf{[6-6]}$$

where L is the length of the tube and d is the tube diameter. The properties in Equation (6-6) are evaluated at the mean bulk temperature. Hartnett [24] has given experimental data on the thermal entrance region for water and oils. Definitive studies of turbulent transfer with water in smooth tubes and at uniform heat flux have been presented by Allen and Eckert [25].

The above equations offer simplicity in computation, but uncertainties on the order of ± 25 percent are not uncommon. Petukhov [42] has developed a more accurate, although more complicated, expression for fully developed turbulent flow in smooth tubes:

$$\mathrm{Nu}_d = \frac{(f/8)\,\mathrm{Re}_d\mathrm{Pr}}{1.07 + 12.7(f/8)^{1/2}(\mathrm{Pr}^{2/3} - 1)}\left(\frac{\mu_b}{\mu_w}\right)^n \qquad \textbf{[6-7]}$$

where $n = 0.11$ for $T_w > T_b$, $n = 0.25$ for $T_w < T_b$, and $n = 0$ for constant heat flux or for gases. All properties are evaluated at $T_f = (T_w + T_b)/2$ except for μ_b and μ_w. The friction factor may be obtained either from Figure 6-4 or from the following for smooth tubes:

$$f = (1.82\ \log_{10}\mathrm{Re}_d - 1.64)^{-2} \qquad \textbf{[6-8]}$$

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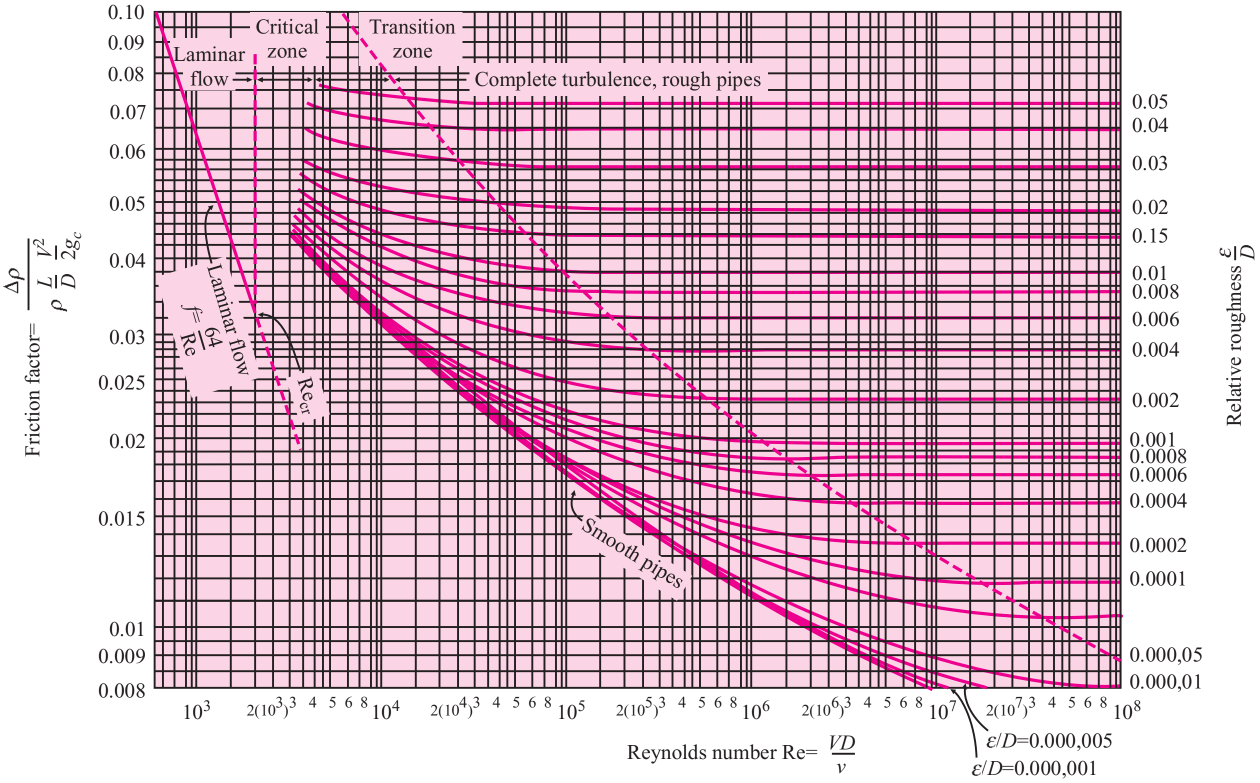
Figure 6-4 | Friction factors for pipes, from Reference 5.



Equation (6-7) is applicable for the following ranges:

$0.5 < \mathrm{Pr} < 200$	for 6 percent accuracy
$0.5 < \mathrm{Pr} < 2000$	for 10 percent accuracy
$10^4 < \mathrm{Re}_d < 5 \times 10^6$	
$0.8 < \mu_b/\mu_w < 40$	

Hausen [4] presents the following empirical relation for fully developed laminar flow in tubes at constant wall temperature:

$$\overline{\mathrm{Nu}}_d = 3.66 + \frac{0.0668(d/L)\,\mathrm{Re}_d\mathrm{Pr}}{1 + 0.04[(d/L)\,\mathrm{Re}_d\mathrm{Pr}]^{2/3}} \qquad \textbf{[6-9]}$$

The heat-transfer coefficient calculated from this relation is the average value over the entire length of tube. Note that the Nusselt number approaches a constant value of 3.66 when the tube is sufficiently long. This situation is similar to that encountered in the constant-heat-flux problem analyzed in Chapter 5 [Equation (5-107)], except that in this case we have a constant wall temperature instead of a linear variation with length. The temperature profile is fully developed when the Nusselt number approaches a constant value.

A somewhat simpler empirical relation was proposed by Sieder and Tate [2] for laminar heat transfer in tubes:

$$\overline{\mathrm{Nu}}_d = 1.86(\mathrm{Re}_d\mathrm{Pr})^{1/3}\left(\frac{d}{L}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} \qquad \textbf{[6-10]}$$

In this formula the average heat-transfer coefficient is based on the arithmetic average of the inlet and outlet temperature differences, and all fluid properties are evaluated at the mean bulk temperature of the fluid, except μ_w, which is evaluated at the wall temperature. Equation (6-10) obviously cannot be used for extremely long tubes since it would yield a zero heat-transfer coefficient. A comparison by Knudsen and Katz [9, p. 377] of Equation (6-10) with other relationships indicates that it is valid for

$$\mathrm{Re}_d \mathrm{Pr} \frac{d}{L} > 10$$

The product of the Reynolds and Prandtl numbers that occurs in the laminar-flow correlations is called the Peclet number.

$$\mathrm{Pe} = \frac{du\rho c_p}{k} = \mathrm{Re}_d \mathrm{Pr} \qquad \textbf{[6-11]}$$

The calculation of laminar heat-transfer coefficients is frequently complicated by the presence of natural-convection effects that are superimposed on the forced-convection effects. The treatment of combined forced- and free-convection problems is discussed in Chapter 7.

The empirical correlations presented above, with the exception of Equation (6-7), apply to smooth tubes. Correlations are, in general, rather sparse where rough tubes are concerned, and it is sometimes appropriate that the Reynolds analogy between fluid friction and heat transfer be used to effect a solution under these circumstances. Expressed in terms of the Stanton number,

$$\mathrm{St}_b \,\mathrm{Pr}_f^{2/3} = \frac{f}{8} \qquad \textbf{[6-12]}$$

The friction coefficient f is defined by

$$\Delta p = f \frac{L}{d} \rho \frac{u_m{}^2}{2g_c} \qquad \textbf{[6-13]}$$

where u_m is the mean flow velocity. Values of the friction coefficient for different roughness conditions are shown in Figure 6-4. An empirical relation for the friction factor for rough tubes is given in References [49, 50] as

$$f = 1.325 / \left[\ln(\varepsilon/3.7d) + 5.74/\mathrm{Re}_d^{0.9} \right]^2 \qquad \textbf{[6-13}\textbf{\textit{a}}\textbf{]}$$

for $10^{-6} < \varepsilon/d < 10^{-3}$ and $5000 < \mathrm{Re}_d < 10^8$.

Note that the relation in Equation (6-12) is the same as Equation (5-114), except that the Stanton number has been multiplied by $\mathrm{Pr}^{2/3}$ to take into account the variation of the thermal properties of different fluids. This correction follows the recommendation of Colburn [15], and is based on the reasoning that fluid friction and heat transfer in tube flow are related to the Prandtl number in the same way as they are related in flat-plate flow [Equation (5-56)]. In Equation (6-12) the Stanton number is based on bulk temperature, while the Prandtl number and friction factor are based on properties evaluated at the film temperature. Further information on the effects of tube roughness on heat transfer is given in References 27, 29, 30, and 31.

If the channel through which the fluid flows is not of circular cross section, it is recommended that the heat-transfer correlations be based on the hydraulic diameter D_H, defined by

$$D_H = \frac{4A}{P} \qquad \textbf{[6-14]}$$

where A is the cross-sectional area of the flow and P is the wetted perimeter. This particular grouping of terms is used because it yields the value of the physical diameter when applied to a circular cross section. The hydraulic diameter should be used in calculating the Nusselt and Reynolds numbers, and in establishing the friction coefficient for use with the Reynolds analogy.

Although the hydraulic-diameter concept frequently yields satisfactory relations for fluid friction and heat transfer in many practical problems, there are some notable exceptions where the method does not work. Some of the problems involved in heat transfer in noncircular channels have been summarized by Irvine [20] and Knudsen and Katz [9]. The interested reader should consult these discussions for additional information.

Shah and London [40] have compiled the heat-transfer and fluid-friction information for fully developed laminar flow in ducts with a variety of flow cross sections, and some of the resulting relations are shown in Table 6-1. In this table the following nomenclature applies, with the Nusselt and Reynolds numbers based on the hydraulic diameter of the flow cross-section area:

$\overline{\mathrm{Nu}}_H$ = average Nusselt number for uniform heat flux in flow direction and uniform wall temperature at particular flow cross section

$\overline{\mathrm{Nu}}_T$ = average Nusselt number for uniform wall temperature

$f\mathrm{Re}_{D_H}/4$ = product of friction factor and Reynolds number based on hydraulic diameter

Table 6-1 | Heat transfer and fluid friction for fully developed laminar flow in ducts of various cross sections. Average Nusselt numbers based on hydraulic diameters of cross sections.

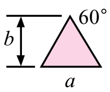
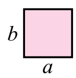
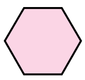
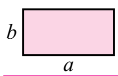
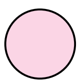
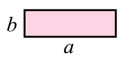
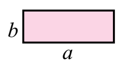
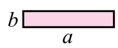
Geometry ($L/D_h > 100$)	Nu_H Constant axial wall heat flux	Nu_T Constant axial wall temperature	$f\,\mathrm{Re}_{D_H}/4$
 $\frac{b}{a} = \frac{\sqrt{3}}{2}$	3.111	2.47	13.333
 $\frac{b}{a} = 1$	3.608	2.976	14.227
	4.002	3.34	15.054
 $\frac{b}{a} = \frac{1}{2}$	4.123	3.391	15.548
	4.364	3.657	16.000
 $\frac{b}{a} = \frac{1}{4}$	5.331	4.44	18.23
 $\frac{b}{a} = \frac{1}{3}$	4.79	3.96	17.25
 $\frac{b}{a} = \frac{1}{8}$	6.490	5.597	20.585
 $\frac{b}{a} = 0$	8.235	7.541	24.000
Heated  Insulated $\frac{b}{a} = 0$	5.385	4.861	24.000

Figure 6-5 | Local and average Nusselt numbers for circular tube thermal entrance regions in fully developed laminar flow.

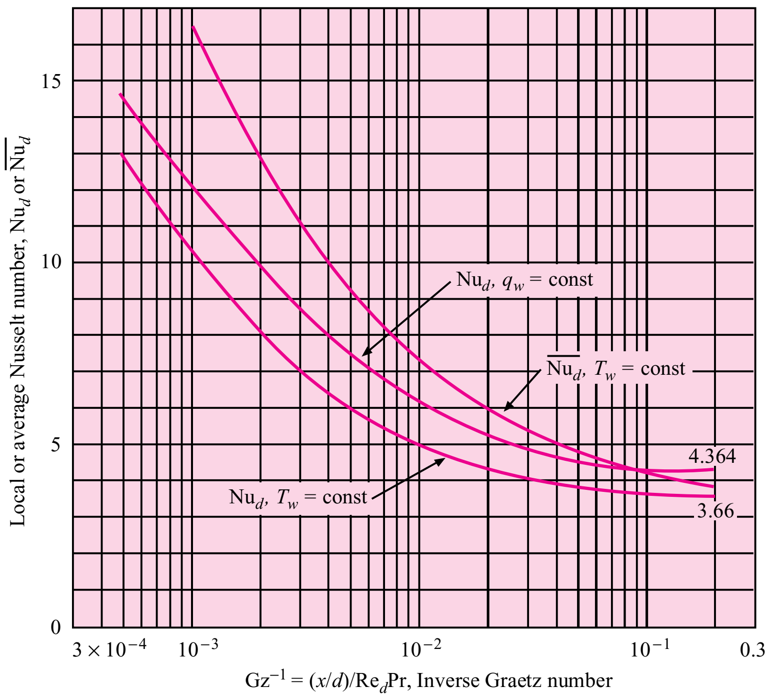
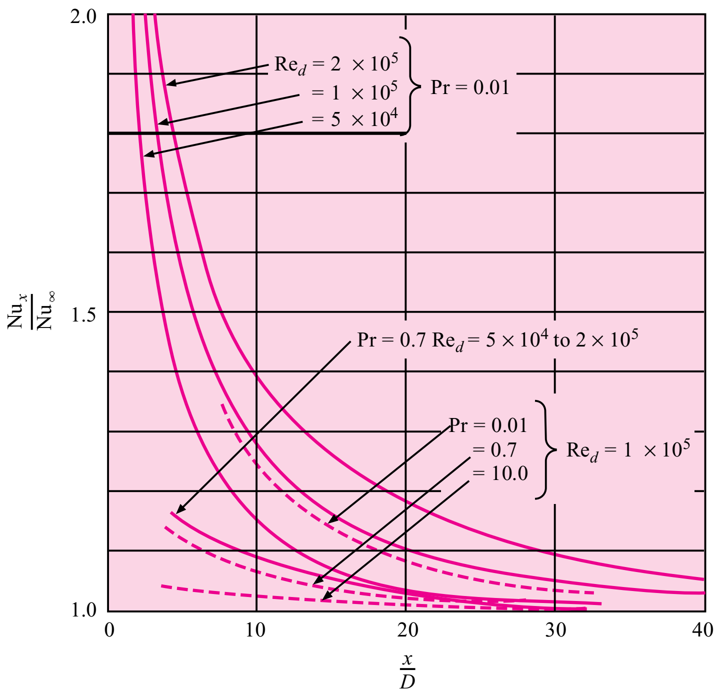


Figure 6-6 | Turbulent thermal entry Nusselt numbers for circular tubes with q_w = constant.



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Kays [36] and Sellars, Tribus, and Klein (Reference 3, Chapter 5) have calculated the local and average Nusselt numbers for laminar entrance regions of circular tubes for the case of a fully developed velocity profile. Results of these analyses are shown in Figure 6-5 in terms of the inverse Graetz number, where

$$\text{Graetz number} = \text{Gz} = \text{Re Pr}\frac{d}{x} \qquad \textbf{[6-15]}$$

Entrance Effects in Turbulent Flow

Entrance effects for turbulent flow in tubes are more complicated than for laminar flow and cannot be expressed in terms of a simple function of the Graetz number. Kays [36] has computed the influence for several values of Re and Pr with the results summarized in Figure 6-6. The ordinate is the ratio of the local Nusselt number to that a long distance from the inlet, or for fully developed thermal conditions. In general, the higher the Prandtl number, the shorter the entry length. We can see that the thermal entry lengths are much shorter for turbulent flow than for the laminar counterpart.

A very complete survey of the many heat-transfer correlations that are available for tube and channel flow is given by Kakac, Shah, and Aung [46].

Turbulent Heat Transfer in a Tube — EXAMPLE 6-1

Air at 2 atm and 200°C is heated as it flows through a tube with a diameter of 1 in (2.54 cm) at a velocity of 10 m/s. Calculate the heat transfer per unit length of tube if a constant-heat-flux condition is maintained at the wall and the wall temperature is 20°C above the air temperature, all along the length of the tube. How much would the bulk temperature increase over a 3-m length of the tube?

Solution

We first calculate the Reynolds number to determine if the flow is laminar or turbulent, and then select the appropriate empirical correlation to calculate the heat transfer. The properties of air at a bulk temperature of 200°C are

$$\rho = \frac{p}{RT} = \frac{(2)(1.0132 \times 10^5)}{(287)(473)} = 1.493 \text{ kg/m}^3 \qquad [0.0932 \text{ lb}_m/\text{ft}^3]$$

$$\text{Pr} = 0.681$$

$$\mu = 2.57 \times 10^{-5} \text{ kg/m} \cdot \text{s} \qquad [0.0622 \text{ lb}_m/\text{h} \cdot \text{ft}]$$

$$k = 0.0386 \text{ W/m} \cdot {}^\circ\text{C} \qquad [0.0223 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}]$$

$$c_p = 1.025 \text{ kJ/kg} \cdot {}^\circ\text{C}$$

$$\text{Re}_d = \frac{\rho u_m d}{\mu} = \frac{(1.493)(10)(0.0254)}{2.57 \times 10^{-5}} = 14{,}756$$

so that the flow is turbulent. We therefore use Equation (6-4a) to calculate the heat-transfer coefficient.

$$\text{Nu}_d = \frac{hd}{k} = 0.023\,\text{Re}_d^{0.8}\,\text{Pr}^{0.4} = (0.023)(14{,}756)^{0.8}(0.681)^{0.4} = 42.67$$

$$h = \frac{k}{d}\text{Nu}_d = \frac{(0.0386)(42.67)}{0.0254} = 64.85 \text{ W/m}^2 \cdot {}^\circ\text{C} \qquad [11.42 \text{ Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}]$$

The heat flow per unit length is then

$$\frac{q}{L} = h\pi d(T_w - T_b) = (64.85)\pi(0.0254)(20) = 103.5 \text{ W/m} \qquad [107.7 \text{ Btu/ft}]$$

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We can now make an energy balance to calculate the increase in bulk temperature in a 3.0-m length of tube:

$$q = \dot{m} c_p \Delta T_b = L\left(\frac{q}{L}\right)$$

We also have

$$\dot{m} = \rho u_m \frac{\pi d^2}{4} = (1.493)(10)\pi \frac{(0.0254)^2}{4}$$
$$= 7.565 \times 10^{-3} \text{ kg/s} \qquad [0.0167 \text{ lb}_m/\text{s}]$$

so that we insert the numerical values in the energy balance to obtain

$$(7.565 \times 10^{-3})(1025)\Delta T_b = (3.0)(103.5)$$

and

$$\Delta T_b = 40.04°\text{C} \qquad [104.07°\text{F}]$$

EXAMPLE 6-2 Heating of Water in Laminar Tube Flow

Water at 60°C enters a tube of 1-in (2.54-cm) diameter at a mean flow velocity of 2 cm/s. Calculate the exit water temperature if the tube is 3.0 m long and the wall temperature is constant at 80°C.

Solution

We first evaluate the Reynolds number at the inlet bulk temperature to determine the flow regime. The properties of water at 60°C are

$$\rho = 985 \text{ kg/m}^3 \qquad c_p = 4.18 \text{ kJ/kg} \cdot °\text{C}$$
$$\mu = 4.71 \times 10^{-4} \text{ kg/m} \cdot \text{s} \qquad [1.139 \text{ lb}_m/\text{h} \cdot \text{ft}]$$
$$k = 0.651 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 3.02$$
$$\text{Re}_d = \frac{\rho u_m d}{\mu} = \frac{(985)(0.02)(0.0254)}{4.71 \times 10^{-4}} = 1062$$

so the flow is laminar. Calculating the additional parameter, we have

$$\text{Re}_d \text{Pr} \frac{d}{L} = \frac{(1062)(3.02)(0.0254)}{3} = 27.15 > 10$$

so Equation (6-10) is applicable. We do not yet know the mean bulk temperature to evaluate properties so we first make the calculation on the basis of 60°C, determine an exit bulk temperature, and then make a second iteration to obtain a more precise value. When inlet and outlet conditions are designated with the subscripts 1 and 2, respectively, the energy balance becomes

$$q = h\pi dL\left(T_w - \frac{T_{b_1} + T_{b_2}}{2}\right) = \dot{m} c_p (T_{b_2} - T_{b_1}) \qquad \textbf{[a]}$$

At the wall temperature of 80°C we have

$$\mu_w = 3.55 \times 10^{-4} \text{ kg/m} \cdot \text{s}$$

From Equation (6-10)

$$\text{Nu}_d = (1.86)\left[\frac{(1062)(3.02)(0.0254)}{3}\right]^{1/3}\left(\frac{4.71}{3.55}\right)^{0.14} = 5.816$$

$$h = \frac{k\text{Nu}_d}{d} = \frac{(0.651)(5.816)}{0.0254} = 149.1 \text{ W/m}^2 \cdot °\text{C} \qquad [26.26 \text{ Btu/h} \cdot \text{ft}^2 \cdot °\text{F}]$$

The mass flow rate is

$$\dot{m} = \rho \frac{\pi d^2}{4} u_m = \frac{(985)\pi(0.0254)^2(0.02)}{4} = 9.982 \times 10^{-3} \text{ kg/s}$$

Inserting the value for h into Equation (a) along with $\dot{m}$ and $T_{b_1} = 60°\text{C}$ and $T_w = 80°\text{C}$ gives

$$(149.1)\pi(0.0254)(3.0)\left(80 - \frac{T_{b_2} + 60}{2}\right) = (9.982 \times 10^{-3})(4180)(T_{b_2} - 60) \qquad [b]$$

This equation can be solved to give

$$T_{b_2} = 71.98°\text{C}$$

Thus, we should go back and evaluate properties at

$$T_{b,\text{mean}} = \frac{71.98 + 60}{2} = 66°\text{C}$$

We obtain

$$\rho = 982 \text{ kg/m}^3 \qquad c_p = 4185 \text{ J/kg} \cdot °\text{C} \qquad \mu = 4.36 \times 10^{-4} \text{ kg/m} \cdot \text{s}$$

$$k = 0.656 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 2.78$$

$$\text{Re}_d = \frac{(1062)(4.71)}{4.36} = 1147$$

$$\text{Re Pr} \frac{d}{L} = \frac{(1147)(2.78)(0.0254)}{3} = 27.00$$

$$\text{Nu}_d = (1.86)(27.00)^{1/3} \left(\frac{4.36}{3.55}\right)^{0.14} = 5.743$$

$$h = \frac{(0.656)(5.743)}{0.0254} = 148.3 \text{ W/m}^2 \cdot °\text{C}$$

We insert this value of h back into Equation (a) to obtain

$$T_{b_2} = 71.88°\text{C} \qquad [161.4°\text{F}]$$

The iteration makes very little difference in this problem. If a large bulk-temperature difference had been encountered, the change in properties could have had a larger effect.

Heating of Air in Laminar Tube Flow for Constant Heat Flux

EXAMPLE 6-3

Air at 1 atm and 27°C enters a 5.0-mm-diameter smooth tube with a velocity of 3.0 m/s. The length of the tube is 10 cm. A constant heat flux is imposed on the tube wall. Calculate the heat transfer if the exit bulk temperature is 77°C. Also calculate the exit wall temperature and the value of h at exit.

■ Solution

We first must evaluate the flow regime and do so by taking properties at the average bulk temperature

$$\overline{T}_b = \frac{27 + 77}{2} = 52°\text{C} = 325 \text{ K}$$

$$\nu = 18.22 \times 10^{-6} \text{ m}^2/\text{s} \qquad \text{Pr} = 0.703 \qquad k = 0.02814 \text{ W/m} \cdot °\text{C}$$

$$\text{Re}_d = \frac{ud}{\nu} = \frac{(3)(0.005)}{18.22 \times 10^{-6}} = 823 \qquad [a]$$

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so that the flow is laminar. The tube length is rather short, so we expect a thermal entrance effect and shall consult Figure 6-5. The inverse Graetz number is computed as

$$\mathrm{Gz}^{-1} = \frac{1}{\mathrm{Re}_d \mathrm{Pr}} \frac{x}{d} = \frac{0.1}{(823)(0.703)(0.005)} = 0.0346$$

Therefore, for $q_w =$ constant, we obtain the Nusselt number at exit from Figure 6-5 as

$$\mathrm{Nu} = \frac{hd}{k} = 4.7 = \frac{q_w d}{(T_w - T_b)k} \qquad [b]$$

The total heat transfer is obtained in terms of the overall energy balance:

$$q = \dot{m} c_p \left(T_{b_2} - T_{b_1}\right)$$

At entrance $\rho = 1.1774$ kg/m^3, so the mass flow is

$$\dot{m} = (1.1774)\pi(0.0025)^2(3.0) = 6.94 \times 10^{-5}\ \mathrm{kg/s}$$

and

$$q = (6.94 \times 10^{-5})(1006)(77 - 27) = 3.49\ \mathrm{W}$$

Thus we may find the heat transfer without actually determining wall temperatures or values of h. However, to determine T_w we must compute q_w for insertion in Equation (b). We have

$$q = q_w \pi d L = 3.49\ \mathrm{W}$$

and

$$q_w = 2222\ \mathrm{W/m^2}$$

Now, from Equation (b)

$$(T_w - T_b)_{x=L} = \frac{(2222)(0.005)}{(4.7)(0.02814)} = 84^\circ\mathrm{C}$$

The wall temperature at exit is thus

$$T_w]_{x=L} = 84 + 77 = 161^\circ\mathrm{C}$$

and the heat-transfer coefficient is

$$h_{x=L} = \frac{q_w}{(T_w - T_b)_{x=L}} = \frac{2222}{84} = 26.45\ \mathrm{W/m^2 \cdot {}^\circ C}$$

EXAMPLE 6-4 Heating of Air with Isothermal Tube Wall

Repeat Example 6-3 for the case of constant wall temperature.

■ Solution

We evaluate properties as before and now enter Figure 6-5 to determine $\overline{\mathrm{Nu}}_d$ for $T_w =$ constant. For $\mathrm{Gz}^{-1} = 0.0346$ we read

$$\overline{\mathrm{Nu}}_d = 5.15$$

We thus calculate the average heat-transfer coefficient as

$$\bar{h} = (5.15)\left(\frac{k}{d}\right) = \frac{(5.15)(0.02814)}{0.005} = 29.98\ \mathrm{W/m^2 \cdot {}^\circ C}$$

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We base the heat transfer on a mean bulk temperature of 52°C, so that

$$q = \bar{h}\pi d L(T_w - \overline{T_b}) = 3.49 \text{ W}$$

and

$$T_w = 76.67 + 52 = 128.67°\text{C}$$

Heat Transfer in a Rough Tube

EXAMPLE 6-5

A 2.0-cm-diameter tube having a relative roughness of 0.001 is maintained at a constant wall temperature of 90°C. Water enters the tube at 40°C and leaves at 60°C. If the entering velocity is 3 m/s, calculate the length of tube necessary to accomplish the heating.

Solution

We first calculate the heat transfer from

$$q = \dot{m} c_p \Delta T_b = (989)(3.0)\pi(0.01)^2(4174)(60 - 40) = 77{,}812 \text{ W}$$

For the rough-tube condition, we may employ the Petukhov relation, Equation (6-7). The mean film temperature is

$$T_f = \frac{90 + 50}{2} = 70°\text{C}$$

and the fluid properties are

$$\rho = 978 \text{ kg/m}^3 \qquad \mu = 4.0 \times 10^{-4} \text{ kg/m} \cdot \text{s}$$
$$k = 0.664 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 2.54$$

Also,

$$\mu_b = 5.55 \times 10^{-4} \text{ kg/m} \cdot \text{s}$$
$$\mu_w = 2.81 \times 10^{-4} \text{ kg/m} \cdot \text{s}$$

The Reynolds number is thus

$$\text{Re}_d = \frac{(978)(3)(0.02)}{4 \times 10^{-4}} = 146{,}700$$

Consulting Figure 6-4, we find the friction factor as

$$f = 0.0218 \qquad f/8 = 0.002725$$

Because $T_w > T_b$, we take $n = 0.11$ and obtain

$$\text{Nu}_d = \frac{(0.002725)(146{,}700)(2.54)}{1.07 + (12.7)(0.002725)^{1/2}(2.54^{2/3} - 1)} \left(\frac{5.55}{2.81}\right)^{0.11}$$
$$= 666.8$$

$$h = \frac{(666.8)(0.664)}{0.02} = 22138 \text{ W/m}^2 \cdot °\text{C}$$

The tube length is then obtained from the energy balance

$$q = \bar{h}\pi d L(T_w - \overline{T_b}) = 77{,}812 \text{ W}$$
$$L = 1.40 \text{ m}$$

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EXAMPLE 6-6 Turbulent Heat Transfer in a Short Tube

Air at 300 K and 1 atm enters a smooth tube having a diameter of 2 cm and length of 10 cm. The air velocity is 40 m/s. What constant heat flux must be applied at the tube surface to result in an air temperature rise of 5°C? What average wall temperature would be necessary for this case?

Solution

Because of the relatively small value of $L/d = 10/2 = 5$ we may anticipate that thermal entrance effects will be present in the flow. First, we determine the air properties at 300 K as

$$\nu = 15.69 \times 10^{-6}\,\text{m}^2/\text{s} \qquad k = 0.02624\ \text{W/m}\cdot{}^\circ\text{C} \qquad \text{Pr} = 0.7$$
$$c_p = 1006\ \text{J/kg}\cdot{}^\circ\text{C} \qquad \rho = 1.18\ \text{kg/m}^3$$

We calculate the Reynolds number as

$$\text{Re}_d = ud/\nu = (40)(0.02)/15.69 \times 10^{-6} = 50{,}988$$

so the flow is turbulent. Consulting Figure 6-6 for this value of Re_d, Pr = 0.7, and $L/d = 5$ we find

$$\text{Nu}_x/\text{Nu}_\infty \cong 1.15$$

or the heat-transfer coefficient is about 15 percent higher that it would be for thermally developed flow. We calculate the heat-transfer coefficient for developed flow using

$$\text{Nu}_d = 0.023\,\text{Re}_d^{0.8}\text{Pr}^{0.4}$$
$$= 0.023(50988)^{0.8}(0.7)^{0.4} = 116.3$$

and

$$h = k\text{Nu}_d/d = (0.02624)(116.3)/0.02 = 152.6\ \text{W/m}^2\cdot{}^\circ\text{C}$$

Increasing this value by 15 percent,

$$h = (1.15)(152.6) = 175.5\ \text{W/m}^2\cdot{}^\circ\text{C}$$

The mass flow is

$$\dot{m} = \rho u A_c = (1.18)(40)\pi(0.02)^2/4 = 0.0148\ \text{kg/s}$$

so the total heat transfer is

$$q = \dot{m} c_p \Delta T_b = (0.0148)(1006)(5) = 74.4\ \text{W}$$

This heat flow is convected from a tube surface area of

$$A = \pi\,dL = \pi(0.02)(0.1) = 0.0628\ \text{m}^2$$

so the heat flux is

$$q/A = 74.4/0.0628 = 11841\ \text{W/m}^2 = h(T_w - T_b)$$

We have

$$\overline{T}_b = (300 + 305)/2 = 302.5\ \text{K}$$

so that

$$\overline{T}_w = \overline{T}_b + 11841/175.5 = 302.5 + 67.5 = 370\ \text{K}$$

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6-3 | FLOW ACROSS CYLINDERS AND SPHERES

While the engineer may frequently be interested in the heat-transfer characteristics of flow systems inside tubes or over flat plates, equal importance must be placed on the heat transfer that may be achieved by a cylinder in cross flow, as shown in Figure 6-7. As would be expected, the boundary-layer development on the cylinder determines the heat-transfer characteristics. As long as the boundary layer remains laminar and well behaved, it is possible to compute the heat transfer by a method similar to the boundary-layer analysis of Chapter 5. It is necessary, however, to include the pressure gradient in the analysis because this influences the boundary-layer velocity profile to an appreciable extent. In fact, it is this pressure gradient that causes a separated flow region to develop on the back side of the cylinder when the free-stream velocity is sufficiently large.

The phenomenon of boundary-layer separation is indicated in Figure 6-8. The physical reasoning that explains the phenomenon in a qualitative way is as follows: Consistent with boundary-layer theory, the pressure through the boundary layer is essentially constant at any x position on the body. In the case of the cylinder, one might measure x distance from the front stagnation point of the cylinder. Thus the pressure in the boundary layer should follow that of the free stream for potential flow around a cylinder, provided this behavior would not contradict some basic principle that must apply in the boundary layer. As the flow progresses along the front side of the cylinder, the pressure would decrease and then increase along the back side of the cylinder, resulting in an increase in free-stream velocity on the front side of the cylinder and a decrease on the back side. The transverse velocity (that velocity parallel to the surface) would decrease from a value of u_∞ at the outer edge of the boundary layer to zero at the surface. As the flow proceeds to the back side of the cylinder, the pressure increase causes a reduction in velocity in the free stream and throughout the

Figure 6-7 | Cylinder in cross flow.

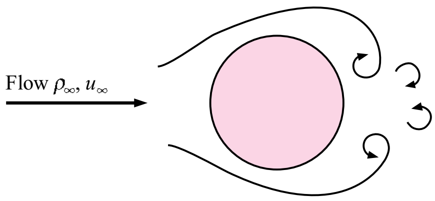
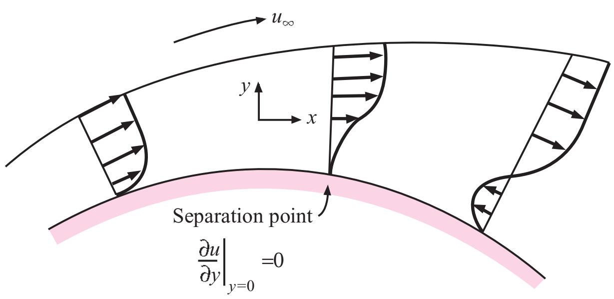


Figure 6-8 | Velocity distributions indicating flow separation on a cylinder in cross flow.



boundary layer. The pressure increase and reduction in velocity are related through the Bernoulli equation written along a streamline:

$$\frac{dp}{\rho} = -d\left(\frac{u^2}{2g_c}\right)$$

Since the pressure is assumed constant throughout the boundary layer, we note that reverse flow may begin in the boundary layer near the surface; that is, the momentum of the fluid layers near the surface is not sufficiently high to overcome the increase in pressure. When the velocity gradient at the surface becomes zero, the flow is said to have reached a separation point:

$$\text{Separation point at } \left.\frac{\partial u}{\partial y}\right]_{y=0} = 0$$

This separation point is indicated in Figure 6-8. As the flow proceeds past the separation point, reverse-flow phenomena may occur, as also shown in Figure 6-8. Eventually, the separated-flow region on the back side of the cylinder becomes turbulent and random in motion.

The drag coefficient for bluff bodies is defined by

$$\text{Drag force} = F_D = C_D A \frac{\rho u_\infty{}^2}{2g_c} \qquad \textbf{[6-16]}$$

where C_D is the drag coefficient and A is the *frontal area* of the body exposed to the flow, which, for a cylinder, is the product of diameter and length. The values of the drag coefficient for cylinders and spheres are given as a function of the Reynolds number in Figures 6-9 and 6-10.

The drag force on the cylinder is a result of a combination of frictional resistance and so-called form, or pressure drag, resulting from a low-pressure region on the rear of the cylinder created by the flow-separation process. At low Reynolds numbers of the order

Figure 6-9 | Drag coefficient for circular cylinders as a function of the Reynolds number, from Reference 6.

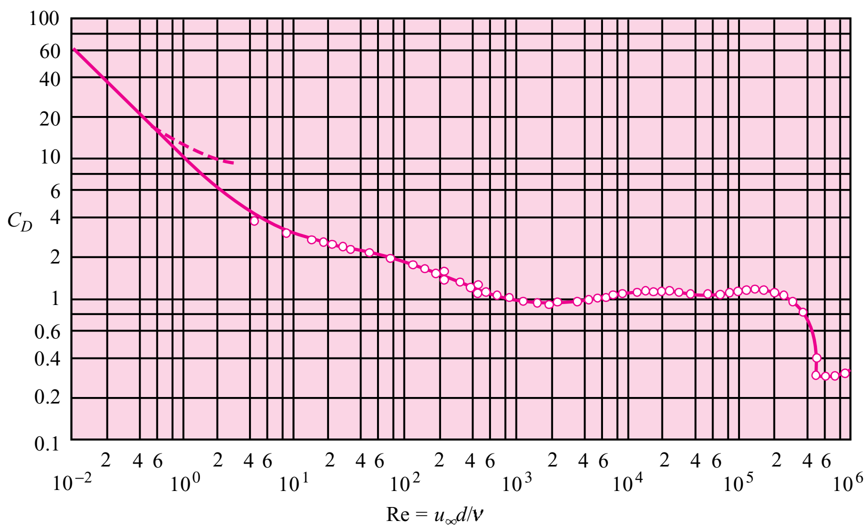
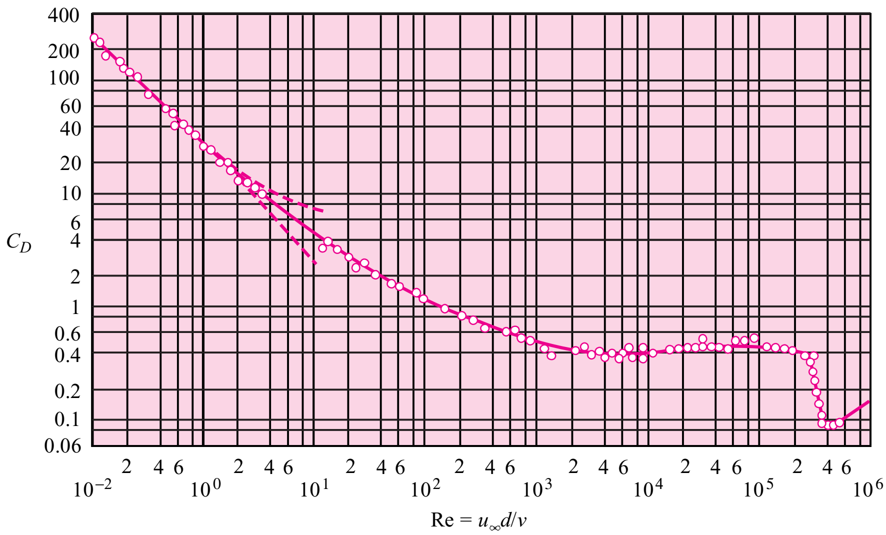


Figure 6-10 | Drag coefficient for spheres as a function of the Reynolds number, from Reference 6.



of unity, there is no flow separation, and all the drag results from viscous friction. At Reynolds numbers of the order of 10, the friction and form drag are of the same order, while the form drag resulting from the turbulent separated-flow region predominates at Reynolds numbers greater than 1000. At Reynolds numbers of approximately 10^5, based on diameter, the boundary-layer flow may become turbulent, resulting in a steeper velocity profile and extremely late flow separation. Consequently, the form drag is reduced, and this is represented by the break in the drag-coefficient curve at about $\mathrm{Re} = 3 \times 10^5$. The same reasoning applies to the sphere as to the circular cylinder. Similar behavior is observed with other bluff bodies, such as elliptic cylinders and airfoils.

The flow processes discussed above obviously influence the heat transfer from a heated cylinder to a fluid stream. The detailed behavior of the heat transfer from a heated cylinder to air has been investigated by Giedt [7], and the results are summarized in Figure 6-11. At the lower Reynolds numbers (70,800 and 101,300) a minimum point in the heat-transfer coefficient occurs at approximately the point of separation. There is a subsequent increase in the heat-transfer coefficient on the rear side of the cylinder, resulting from the turbulent eddy motion in the separated flow. At the higher Reynolds numbers two minimum points are observed. The first occurs at the point of transition from laminar to turbulent boundary layer, and the second minimum occurs when the turbulent boundary layer separates. There is a rapid increase in heat transfer when the boundary layer becomes turbulent and another when the increased eddy motion at separation is encountered.

Because of the complicated nature of the flow-separation processes, it is not possible to calculate analytically the average heat-transfer coefficients in cross flow; however, McAdams [10] was able to correlate the data of a number of investigators for heating and cooling of air as shown in the plot of Figure 6-12. The data points have been omitted, but scatter of ±20 percent is not uncommon. The Prandtl number was not included in the original correlation plot because it is essentially constant at about 0.72 for all the data. Following the reasoning of Prandtl number dependence indicated by Equation (5-85), Knudsen and Katz [9] suggested that the correlation be extended to liquids by inclusion of $\mathrm{Pr}^{1/3}$.

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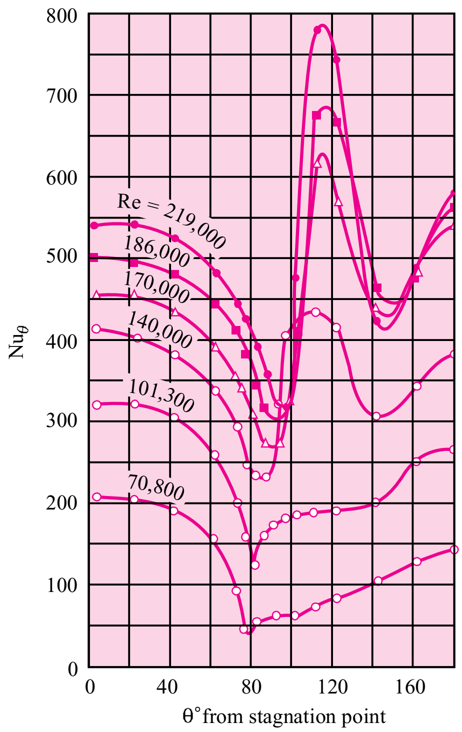
Figure 6-11 | Local Nusselt number for heat transfer from a cylinder in cross flow, from Reference 7.



The resulting correlation for average heat-transfer coefficients in cross flow over circular cylinders is

$$\mathrm{Nu}_{df} = \frac{hd}{k_f} = C\left(\frac{u_\infty d}{\nu_f}\right)^n \mathrm{Pr}_f^{1/3} \qquad \textbf{[6-17]}$$

where the constants C and n are tabulated in Table 6-2. Properties for use with Equation (6-17) are evaluated at the film temperature as indicated by the subscript f.

In obtaining the correlation constants for Table 6-2, the original calculations were based on air data alone, fitting straight-line segments to a log–log plot like that of Figure 6-12. For such data the Prandtl number is very nearly constant at about 0.72. It was reasoned in Reference 9 that the same correlation might be employed for liquids by introducing the factor $\mathrm{Pr}^{1/3}$ and dividing out $(0.72)^{1/3}$, or multiplying by 1.11. This reasoning has been borne out in practice.

Figure 6-13 shows the temperature field around heated cylinders placed in a transverse airstream. The dark lines are lines of constant temperature, made visible through the use of an interferometer. Note the separated-flow region that develops on the back side of the cylinder at the higher Reynolds numbers and the turbulent field that is present in that region.

Note also the behavior at the lowest Reynolds number of 23. The wake rises because of thermal buoyancy effects. At this point, we are observing a behavior resulting from the superposition of free-convection currents of the same order as the forced-convection-flow

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Figure 6-12 | Correlation for heating and cooling in cross flow over circular cylinders.

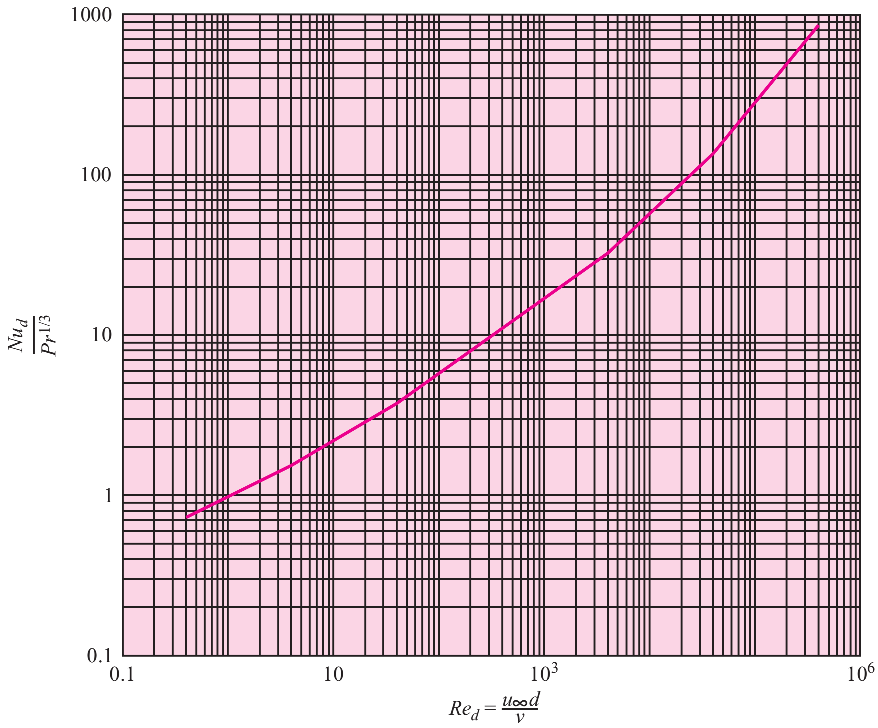

Table 6-2 | Constants for use with Equation (6-17), based on References 8 and 9.

Re_{df}	C	n
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.0266	0.805

velocities. In this regime the heat transfer is also dependent on a parameter called the Grashof number, which we shall describe in detail in Chapter 7. For higher Reynolds numbers the heat transfer is predominately by forced convection.

Fand [21] has shown that the heat-transfer coefficients from liquids to cylinders in cross flow may be better represented by the relation

$$\mathrm{Nu}_f = (0.35 + 0.56\,\mathrm{Re}_f^{0.52})\mathrm{Pr}_f^{0.3} \qquad \textbf{[6-18]}$$

This relation is valid for $10^{-1} < \mathrm{Re}_f < 10^5$ provided excessive free-stream turbulence is not encountered.

In some instances, particularly those involving calculations on a computer, it may be more convenient to utilize a more complicated expression than Equation (6-17) if it can be applied over a wider range of Reynolds numbers. Eckert and Drake [34] recommend the

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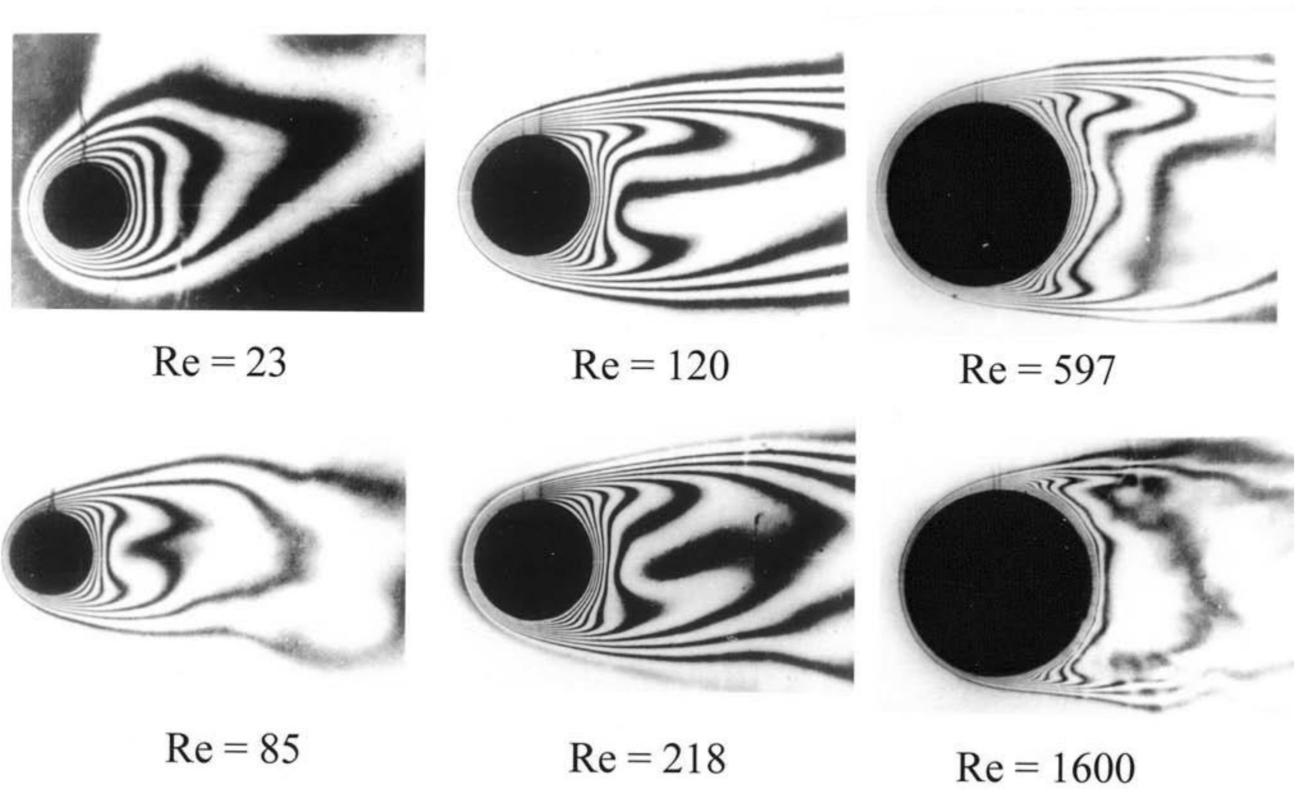
Figure 6-13 | Interferometer photograph showing isotherms around heated horizontal cylinders placed in a transverse airstream. $\text{Re} = \rho u_\infty d/\mu$. (*Photograph courtesy E. Soehngen.*)



following relations for heat transfer from tubes in cross flow, based on the extensive study of References 33 and 39:

$$\text{Nu} = (0.43 + 0.50\,\text{Re}^{0.5})\text{Pr}^{0.38}\left(\frac{\text{Pr}_f}{\text{Pr}_w}\right)^{0.25} \quad \text{for } 1 < \text{Re} < 10^3 \qquad \textbf{[6-19]}$$

$$\text{Nu} = 0.25\,\text{Re}^{0.6}\text{Pr}^{0.38}\left(\frac{\text{Pr}_f}{\text{Pr}_w}\right)^{0.25} \quad \text{for } 10^3 < \text{Re} < 2\times 10^5 \qquad \textbf{[6-20]}$$

For gases the Prandtl number ratio may be dropped, and fluid properties are evaluated at the film temperature. For liquids the ratio is retained, and fluid properties are evaluated at the free-stream temperature. Equations (6-19) and (6-20) are in agreement with results obtained using Equation (6-17) within 5 to 10 percent.

Still a more comprehensive relation is given by Churchill and Bernstein [37] that is applicable over the complete range of available data:

$$\text{Nu}_d = 0.3 + \frac{0.62\,\text{Re}^{1/2}\text{Pr}^{1/3}}{[1 + (0.4/\text{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$\text{for } 10^2 < \text{Re}_d < 10^7;\ \text{Pe}_d > 0.2 \qquad \textbf{[6-21]}$$

This relation underpredicts the data somewhat in the midrange of Reynolds numbers between 20,000 and 400,000, and it is suggested that the following be employed for this range:

$$\mathrm{Nu}_d = 0.3 + \frac{0.62\,\mathrm{Re}_d^{1/2}\mathrm{Pr}^{1/3}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_d}{282{,}000}\right)^{1/2}\right]$$
$$\text{for } 20{,}000 < \mathrm{Re}_d < 400{,}000;\ \mathrm{Pe}_d > 0.2 \qquad \textbf{[6-22]}$$

The heat-transfer data that were used to arrive at Equations (6-21) and (6-22) include fluids of air, water, and liquid sodium. Still another correlation equation is given by Whitaker [35] as

$$\mathrm{Nu} = \frac{\bar{h}d}{k} = (0.4\,\mathrm{Re}^{0.5} + 0.06\,\mathrm{Re}^{2/3})\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_w}\right)^{0.25} \qquad \textbf{[6-23]}$$

for $40 < \mathrm{Re} < 10^5$, $0.65 < \mathrm{Pr} < 300$, and $0.25 < \mu_\infty/\mu_w < 5.2$. All properties are evaluated at the free-stream temperature except that μ_w is at the wall temperature.

Below $\mathrm{Pe}_d = 0.2$, Nakai and Okazaki [38] present the following relation:

$$\mathrm{Nu}_d = \left[0.8237 - \ln\left(\mathrm{Pe}_d^{1/2}\right)\right]^{-1} \quad \text{for } \mathrm{Pe}_d < 0.2 \qquad \textbf{[6-24]}$$

Properties in Equations (6-21), (6-22), and (6-24) are evaluated at the film temperature.

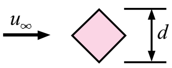
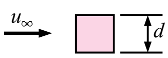
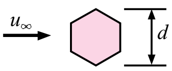
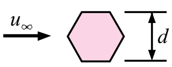
Choice of Equation for Cross Flow Over Cylinders

The choice of equation to use for cross flow over cylinders is subject to some conjecture. Clearly, Equation (6-17) is easiest to use from a computational standpoint, and Equation (6-21) is the most comprehensive. The more comprehensive relations are preferable for computer setups because of the wide range of fluids and Reynolds numbers covered. For example, Equation (6-21) has been successful in correlating data for fluids ranging from air to liquid sodium. Equation (6-17) could not be used for liquid metals. If one were making calculations for air, either relation would be satisfactory.

Noncircular Cylinders

Jakob [22] has summarized the results of experiments with heat transfer from noncircular cylinders. Equation (6-17) is employed in order to obtain an empirical correlation for gases, and the constants for use with this equation are summarized in Table 6-3. The data upon

Table 6-3 | Constants for heat transfer from noncircular cylinders according to Reference 22, for use with Equation (6-17).

Geometry	Re_{df}	C	n
	$5\times10^3 - 10^5$	0.246	0.588
	$5\times10^3 - 10^5$	0.102	0.675
	$5\times10^3 - 1.95\times10^4$ $1.95\times10^4 - 10^5$	0.160 0.0385	0.638 0.782
	$5\times10^3 - 10^5$	0.153	0.638
	$4\times10^3 - 1.5\times10^4$	0.228	0.731

which Table 6-3 is based were for gases with $\mathrm{Pr} \sim 0.7$ and were modified by the same $1.11\,\mathrm{Pr}^{1/3}$ factor employed for the information presented in Table 6-2.

Spheres

McAdams [10] recommends the following relation for heat transfer from spheres to a flowing gas:

$$\frac{hd}{k_f} = 0.37\left(\frac{u_\infty d}{\nu_f}\right)^{0.6} \quad \text{for } 17 < \mathrm{Re}_d < 70{,}000 \qquad \textbf{[6-25]}$$

Achenbach [43] has obtained relations applicable over a still wider range of Reynolds numbers for air with $\mathrm{Pr} = 0.71$:

$$\mathrm{Nu} = 2 + (0.25\,\mathrm{Re} + 3 \times 10^{-4}\mathrm{Re}^{1.6})^{1/2} \quad \text{for } 100 < \mathrm{Re} < 3 \times 10^5 \qquad \textbf{[6-26]}$$

$$\mathrm{Nu} = 430 + a\,\mathrm{Re} + b\,\mathrm{Re}^2 + c\mathrm{Re}^3 \quad \text{for } 3 \times 10^5 < \mathrm{Re} < 5 \times 10^6 \qquad \textbf{[6-27]}$$

with

$$a = 0.5 \times 10^{-3} \qquad b = 0.25 \times 10^{-9} \qquad c = -3.1 \times 10^{-17}$$

For flow of liquids past spheres, the data of Kramers [11] may be used to obtain the correlation

$$\frac{hd}{k_f}\mathrm{Pr}_f^{-0.3} = 0.97 + 0.68\left(\frac{u_\infty d}{\nu_f}\right)^{0.5} \quad \text{for } 1 < \mathrm{Re}_d < 2000 \qquad \textbf{[6-28]}$$

Vliet and Leppert [19] recommend the following expression for heat transfer from spheres to oil and water over a more extended range of Reynolds numbers from 1 to 200,000:

$$\mathrm{Nu}\,\mathrm{Pr}^{-0.3}\left(\frac{\mu_w}{\mu}\right)^{0.25} = 1.2 + 0.53\,\mathrm{Re}_d^{0.54} \qquad \textbf{[6-29]}$$

where all properties are evaluated at free-stream conditions, except μ_w, which is evaluated at the surface temperature of the sphere. Equation (6-26) represents the data of Reference 11, as well as the more recent data of Reference 19.

All the above data have been brought together by Whitaker [35] to develop a single equation for gases and liquids flowing past spheres:

$$\mathrm{Nu} = 2 + \left(0.4\,\mathrm{Re}_d^{1/2} + 0.06\,\mathrm{Re}_d^{2/3}\right)\mathrm{Pr}^{0.4}\,(\mu_\infty/\mu_w)^{1/4} \qquad \textbf{[6-30]}$$

which is valid for the range $3.5 < \mathrm{Re}_d < 8 \times 10^4$ and $0.7 < \mathrm{Pr} < 380$. Properties in Equation (6-30) are evaluated at the free-stream temperature.

EXAMPLE 6-7 Airflow Across Isothermal Cylinder

Air at 1 atm and 35°C flows across a 5.0-cm-diameter cylinder at a velocity of 50 m/s. The cylinder surface is maintained at a temperature of 150°C. Calculate the heat loss per unit length of the cylinder.

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Solution

We first determine the Reynolds number and then find the applicable constants from Table 6-2 for use with Equation (6-17). The properties of air are evaluated at the film temperature:

$$T_f = \frac{T_w + T_\infty}{2} = \frac{150 + 35}{2} = 92.5°\text{C} = 365.5 \text{ K}$$

$$\rho_f = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(365.5)} = 0.966 \text{ kg/m}^3 \quad [0.0603 \text{ lb}_m/\text{ft}^3]$$

$$\mu_f = 2.14 \times 10^{-5} \text{ kg/m} \cdot \text{s} \quad [0.0486 \text{ lb}_m/\text{h} \cdot \text{ft}]$$

$$k_f = 0.0312 \text{ W/m} \cdot °\text{C} \quad [0.018 \text{ Btu/h} \cdot \text{ft} \cdot °\text{F}]$$

$$\text{Pr}_f = 0.695$$

$$\text{Re}_f = \frac{\rho u_\infty d}{\mu} = \frac{(0.966)(50)(0.05)}{2.14 \times 10^{-5}} = 1.129 \times 10^5$$

From Table 6-2

$$C = 0.0266 \qquad n = 0.805$$

so from Equation (6-17)

$$\frac{hd}{k_f} = (0.0266)(1.129 \times 10^5)^{0.805}(0.695)^{1/3} = 275.1$$

$$h = \frac{(275.1)(0.0312)}{0.05} = 171.7 \text{ W/m}^2 \cdot °\text{C} \qquad [30.2 \text{ Btu/h} \cdot \text{ft}^2 \cdot °\text{F}]$$

The heat transfer per unit length is therefore

$$\frac{q}{L} = h\pi d(T_w - T_\infty)$$
$$= (171.7)\pi(0.05)(150 - 35)$$
$$= 3100 \text{ W/m} \quad [3226 \text{ Btu/ft}]$$

Heat Transfer from Electrically Heated Wire

EXAMPLE 6-8

A fine wire having a diameter of 3.94×10^{-5} m is placed in a 1-atm airstream at 25°C having a flow velocity of 50 m/s perpendicular to the wire. An electric current is passed through the wire, raising its surface temperature to 50°C. Calculate the heat loss per unit length.

Solution

We first obtain the properties at the film temperature:

$$T_f = (25 + 50)/2 = 37.5°\text{C} = 310 \text{ K}$$

$$\nu_f = 16.7 \times 10^{-6} \text{ m}^2/\text{s} \qquad k = 0.02704 \text{ W/m} \cdot °\text{C}$$

$$\text{Pr}_f = 0.706$$

The Reynolds number is

$$\text{Re}_d = \frac{u_\infty d}{\nu_f} = \frac{(50)(3.94 \times 10^{-5})}{16.7 \times 10^{-6}} = 118$$

The Peclet number is Pe = Re Pr = 83.3, and we find that Equations (6-17), (6-21), or (6-19) apply. Let us make the calculation with both the simplest expression, (6-17), and the most complex, (6-21), and compare results.

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Using Equation (6-17) with $C = 0.683$ and $n = 0.466$, we have

$$\text{Nu}_d = (0.683)(118)^{0.466}(0.705)^{1/3} = 5.615$$

and the value of the heat-transfer coefficient is

$$h = \text{Nu}_d\left(\frac{k}{d}\right) = 5.615\frac{0.02704}{3.94\times10^{-5}} = 3854\ \text{W/m}^2\cdot{}^\circ\text{C}$$

The heat transfer per unit length is then

$$q/L = \pi\, dh(T_w - T_\infty) = \pi(3.94\times10^{-5})(3854)(50-25)$$
$$= 11.93\ \text{W/m}$$

Using Equation (6-21), we calculate the Nusselt number as

$$\text{Nu}_d = 0.3 + \frac{(0.62)(118)^{1/2}(0.705)^{1/3}}{[1+(0.4/0.705)^{2/3}]^{1/4}}[1+(118/282{,}000)^{5/8}]^{4/5}$$
$$= 5.593$$

and

$$h = \frac{(5.593)(0.02704)}{3.94\times10^{-5}} = 3838\ \text{W/m}^2\cdot{}^\circ\text{C}$$

and

$$q/L = (3838)\pi(3.94\times10^{-5})(50-25) = 11.88\ \text{W/m}$$

Here, we find the two correlations differing by 0.4 percent if the value from Equation (6-21) is taken as correct, or 0.2 percent from the mean value. Data scatter of ±15 percent is not unusual for the original experiments.

EXAMPLE 6-9 Heat Transfer from Sphere

Air at 1 atm and 27°C blows across a 12-mm-diameter sphere at a free-stream velocity of 4 m/s. A small heater inside the sphere maintains the surface temperature at 77°C. Calculate the heat lost by the sphere.

Solution

Consulting Equation (6-30) we find that the Reynolds number is evaluated at the free-stream temperature. We therefore need the following properties: at $T_\infty = 27^\circ\text{C} = 300\ \text{K}$,

$$\nu = 15.69\times10^{-6}\ \text{m}^2/\text{s} \qquad k = 0.02624\ \text{W/m}\cdot{}^\circ\text{C},$$
$$\text{Pr} = 0.708 \qquad \mu_\infty = 1.8462\times10^{-5}\ \text{kg/m}\cdot\text{s}$$

At $T_w = 77^\circ\text{C} = 350\ \text{K}$,

$$\mu_w = 2.075\times10^{-5}$$

The Reynolds number is thus

$$\text{Re}_d = \frac{(4)(0.012)}{15.69\times10^{-6}} = 3059$$

From Equation (6-30),

$$\overline{\text{Nu}} = 2 + [(0.4)(3059)^{1/2} + (0.06)(3059)^{2/3}](0.708)^{0.4}\left(\frac{1.8462}{2.075}\right)^{1/4}$$
$$= 31.40$$

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and

$$\bar{h} = \overline{\text{Nu}}\left(\frac{k}{d}\right) = \frac{(31.4)(0.02624)}{0.012} = 68.66\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

The heat transfer is then

$$q = \bar{h}A(T_w - T_\infty) = (68.66)(4\pi)(0.006)^2(77 - 27) = 1.553\ \text{W}$$

For comparison purposes let us also calculate the heat-transfer coefficient using Equation (6-25). The film temperature is $T_f = (350 + 300)/2 = 325$ K so that

$$\nu_f = 18.23 \times 10^{-6}\ \text{m}^2/\text{s} \qquad k_f = 0.02814\ \text{W/m} \cdot {}^\circ\text{C}$$

and the Reynolds number is

$$\text{Re}_d = \frac{(4)(0.012)}{18.23 \times 10^{-6}} = 2633$$

From Equation (6-25)

$$\text{Nu}_f = (0.37)(2633)^{0.6} = 41.73$$

and $\bar{h}$ is calculated as

$$\bar{h} = \text{Nu}\left(\frac{k_f}{d}\right) = \frac{(41.73)(0.02814)}{0.012} = 97.9\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

or about 42 percent higher than the value calculated before.

6-4 | FLOW ACROSS TUBE BANKS

Because many heat-exchanger arrangements involve multiple rows of tubes, the heat-transfer characteristics for tube banks are of important practical interest. The heat-transfer characteristics of staggered and in-line tube banks were studied by Grimson [12], and on the basis of a correlation of the results of various investigators, he was able to represent data in the form of Equation (6-17). The original data were for gases with $\text{Pr} \sim 0.7$. To extend the use to liquids, the present writer has modified the constants by the same $1.11\text{Pr}^{1/3}$ factor employed in Tables 6-2 and 6-3. The values of the constant C and the exponent n are given in Table 6-4 in terms of the geometric parameters used to describe the tube-bundle arrangement. The Reynolds number is based on the maximum velocity occurring in the tube bank; that is, the velocity through the minimum-flow area. This area will depend on the geometric tube arrangement. The nomenclature for use with Table 6-4 is shown in Figure 6-14. The data of Table 6-4 pertain to tube banks having 10 or more rows of tubes in the direction of flow. For fewer rows the ratio of h for N rows deep to that for 10 rows is given in Table 6-5.

Determination of Maximum Flow Velocity

For flows normal to in-line tube banks the maximum flow velocity will occur through the minimum frontal area $(S_n - d)$ presented to the incoming free stream velocity u_∞. Thus,

$$u_{\max} = u_\infty[S_n/(S_n - d)] \qquad \text{(in-line arrangement)}$$

for this configuration. For the staggered arrangement the same maximum flow velocity will be experienced *if the normal area at the entrance to the tube bank is the minimum flow area.* This may not be the case for close spacing in the parallel direction, as when S_p is small. For the staggered case, the flow enters the tube bank through the area $S_n - d$ and then splits

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Table 6-4 | Modified correlation of Grimson for heat transfer in tube banks of 10 rows or more, from Reference 12, for use with Equation (6-17).

	$\frac{S_n}{d}$							
	1.25		**1.5**		**2.0**		**3.0**	
$\frac{S_p}{d}$	**C**	**n**	**C**	**n**	**C**	**n**	**C**	**n**
In line								
1.25	0.386	0.592	0.305	0.608	0.111	0.704	0.0703	0.752
1.5	0.407	0.586	0.278	0.620	0.112	0.702	0.0753	0.744
2.0	0.464	0.570	0.332	0.602	0.254	0.632	0.220	0.648
3.0	0.322	0.601	0.396	0.584	0.415	0.581	0.317	0.608
Staggered								
0.6	—	—	—	—	—	—	0.236	0.636
0.9	—	—	—	—	0.495	0.571	0.445	0.581
1.0	—	—	0.552	0.558	—	—	—	—
1.125	—	—	—	—	0.531	0.565	0.575	0.560
1.25	0.575	0.556	0.561	0.554	0.576	0.556	0.579	0.562
1.5	0.501	0.568	0.511	0.562	0.502	0.568	0.542	0.568
2.0	0.448	0.572	0.462	0.568	0.535	0.556	0.498	0.570
3.0	0.344	0.592	0.395	0.580	0.488	0.562	0.467	0.574

Figure 6-14 | Nomenclature for use with Table 6-4: (*a*) in-line tube rows; (*b*) staggered tube rows.

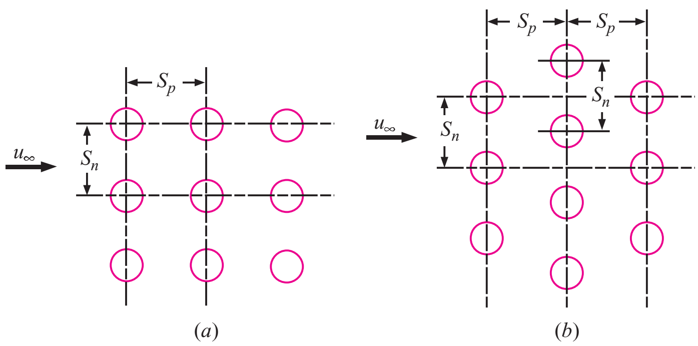

Table 6-5 | Ratio of h for N rows deep to that for 10 rows deep, for use with Equation (6-17).

N	1	2	3	4	5	6	7	8	9	10
Ratio for staggered tubes	0.68	0.75	0.83	0.89	0.92	0.95	0.97	0.98	0.99	1.0
Ratio for in-line tubes	0.64	0.80	0.87	0.90	0.92	0.94	0.96	0.98	0.99	1.0

From Reference 17.

into the two areas $[(S_n/2)^2 + S_p^2]^{1/2} - d$. If the sum of these two areas is less than $S_n - d$, then they will represent the minimum flow area and the maximum velocity in the tube bank will be

$$u_{\max} = \frac{u_\infty(S_n/2)}{\left[(S_n/2)^2 + S_p^2\right]^{1/2} - d}$$

where, again, u_∞ is the free-stream velocity entering the tube bank.

Pressure drop for flow of gases over a bank of tubes may be calculated with Equation (6-31), expressed in pascals:

$$\Delta p = \frac{2f' G_{\max}^2 N}{\rho} \left(\frac{\mu_w}{\mu_b}\right)^{0.14} \qquad \textbf{[6-31]}$$

where

$G_{\max}$ = mass velocity at minimum flow area, $kg/m^2 \cdot s$
ρ = density evaluated at free-stream conditions, kg/m^3
N = number of transverse rows
μ_b = average free-stream viscosity, $N \cdot s/m^2$

The empirical friction factor f' is given by Jakob [18] as

$$f' = \left\{0.25 + \frac{0.118}{[(S_n - d)/d]^{1.08}}\right\} \mathrm{Re}_{\max}^{-0.16} \qquad \textbf{[6-32]}$$

for staggered tube arrangements, and

$$f' = \left\{0.044 + \frac{0.08 S_p/d}{[(S_n - d)/d]^{0.43 + 1.13d/S_p}}\right\} \mathrm{Re}_{\max}^{-0.15} \qquad \textbf{[6-33]}$$

for in-line arrangements.

Zukauskas [39] has presented additional information for tube bundles that takes into account wide ranges of Reynolds numbers and property variations. The correlating equation takes the form

$$\mathrm{Nu} = \frac{\bar{h}d}{k} = C \mathrm{Re}_{d,\max}^{n} \mathrm{Pr}^{0.36} \left(\frac{\mathrm{Pr}}{\mathrm{Pr}_w}\right)^{1/4} \qquad \textbf{[6-34]}$$

where all properties except Pr_w are evaluated at T_∞ and the values of the constants are given in Table 6-6 for greater than 20 rows of tubes. This equation is applicable for $0.7 < \mathrm{Pr} < 500$ and $10 < \mathrm{Re}_{d,\max} < 10^6$. For gases the Prandtl number ratio has little influence and is dropped. Once again, note that the Reynolds number is based on the maximum velocity in the tube bundle. For less than 20 rows in the direction of flow the correction factor in Table 6-7 should be applied. It is essentially the same as for the Grimson correlation.

Table 6-6 | Constants for Zukauskas correlation [Equation (6-34)] for heat transfer in tube banks of 20 rows or more.

Geometry	$\mathrm{Re}_{d,\max}$	C	n
In-line	10–100	0.8	0.4
	$100–10^3$	Treat as individual tubes	
	$10^3 - 2 \times 10^5$	0.27	0.63
	$> 2 \times 10^5$	0.21	0.84
Staggered	10–100	0.9	0.4
	$100–10^3$	Treat as individual tubes	
	$10^3 - 2 \times 10^5$	$0.35 \left(\frac{S_n}{S_L}\right)^{0.2}$ for $\frac{S_n}{S_L} < 2$	0.60
	$10^3 - 2 \times 10^5$	0.40 for $\frac{S_n}{S_L} > 2$	0.60
	$> 2 \times 10^5$	0.022	0.84

From Reference 39.

Table 6-7 | Ratio of h for N rows deep to that for 20 rows deep according to Reference 39 and for use with Equation (6-34).

N	2	3	4	5	6	8	10	16	20
Staggered	0.77	0.84	0.89	0.92	0.94	0.97	0.98	0.99	1.0
In-line	0.70	0.80	0.90	0.92	0.94	0.97	0.98	0.99	1.0

Additional information is given by Morgan [44]. Further information on pressure drop is given in Reference 39.

The reader should keep in mind that these relations correlate experimental data with an uncertainty of about ± 25 percent.

EXAMPLE 6-10 Heating of Air with In-Line Tube Bank

Air at 1 atm and 10°C flows across a bank of tubes 15 rows high and 5 rows deep at a velocity of 7 m/s measured at a point in the flow before the air enters the tube bank. The surfaces of the tubes are maintained at 65°C. The diameter of the tubes is 1 in [2.54 cm]; they are arranged in an in-line manner so that the spacing in both the normal and parallel directions to the flow is 1.5 in [3.81 cm]. Calculate the total heat transfer per unit length for the tube bank and the exit air temperature.

Solution

The constants for use with Equation (6-17) may be obtained from Table 6-4, using

$$\frac{S_p}{d} = \frac{3.81}{2.54} = 1.5 \qquad \frac{S_n}{d} = \frac{3.81}{2.54} = 1.5$$

so that

$$C = 0.278 \qquad n = 0.620$$

The properties of air are evaluated at the film temperature, which at entrance to the tube bank is

$$T_{f_1} = \frac{T_w + T_\infty}{2} = \frac{65 + 10}{2} = 37.5°\text{C} = 310.5 \text{ K} \quad [558.9°\text{R}]$$

Then

$$\rho_f = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(310.5)} = 1.137 \text{ kg/m}^3$$
$$\mu_f = 1.894 \times 10^{-5} \text{ kg/m} \cdot \text{s}$$
$$k_f = 0.027 \text{ W/m} \cdot °\text{C} \quad [0.0156 \text{ Btu/h} \cdot \text{ft} \cdot °\text{F}]$$
$$c_p = 1007 \text{ J/kg} \cdot °\text{C} \quad [0.24 \text{ Btu/lb}_m \cdot °\text{F}]$$
$$\text{Pr} = 0.706$$

To calculate the maximum velocity, we must determine the minimum flow area. From Figure 6-14 we find that the ratio of the minimum flow area to the total frontal area is $(S_n - d)/S_n$. The maximum velocity is thus

$$u_{\max} = u_\infty \frac{S_n}{S_n - d} = \frac{(7)(3.81)}{3.81 - 2.54} = 21 \text{ m/s} \quad [68.9 \text{ ft/s}] \qquad [a]$$

where u_∞ is the incoming velocity before entrance to the tube bank. The Reynolds number is computed by using the maximum velocity.

$$\text{Re} = \frac{\rho u_{\max} d}{\mu} = \frac{(1.137)(21)(0.0254)}{1.894 \times 10^{-5}} = 32{,}020 \qquad [b]$$

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The heat-transfer coefficient is then calculated with Equation (6-17):

$$\frac{hd}{k_f} = (0.278)(32{,}020)^{0.62}(0.706)^{1/3} = 153.8 \qquad \textbf{[c]}$$

$$h = \frac{(153.8)(0.027)}{0.0254} = 164\ \mathrm{W/m^2 \cdot {}^\circ C} \quad [28.8\ \mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}] \qquad \textbf{[d]}$$

This is the heat-transfer coefficient that would be obtained if there were 10 rows of tubes in the direction of the flow. Because there are only 5 rows, this value must be multiplied by the factor 0.92, as determined from Table 6-5.

The total surface area for heat transfer, considering unit length of tubes, is

$$A = N\pi d(1) = (15)(5)\pi(0.0254) = 5.985\ \mathrm{m^2/m}$$

where N is the total number of tubes.

Before calculating the heat transfer, we must recognize that the air temperature increases as the air flows through the tube bank. Therefore, this must be taken into account when using

$$q = hA(T_w - T_\infty) \qquad \textbf{[e]}$$

As a good approximation, we can use an arithmetic average value of T_∞ and write for the energy balance*

$$q = hA\left(T_w - \frac{T_{\infty,1} + T_{\infty,2}}{2}\right) = \dot{m}c_p(T_{\infty,2} - T_{\infty,1}) \qquad \textbf{[f]}$$

where now the subscripts 1 and 2 designate entrance and exit to the tube bank. The mass flow at entrance to the 15 tubes is

$$\dot{m} = \rho_\infty u_\infty (15) S_n$$

$$\rho_\infty = \frac{p}{RT_\infty} = \frac{1.0132 \times 10^5}{(287)(283)} = 1.246\ \mathrm{kg/m^3} \qquad \textbf{[g]}$$

$$\dot{m} = (1.246)(7)(15)(0.0381) = 4.99\ \mathrm{kg/s} \quad [11.0\ \mathrm{lb}_m/\mathrm{s}]$$

so that Equation (f) becomes

$$(0.92)(164)(5.985)\left(65 - \frac{10 + T_{\infty,2}}{2}\right) = (4.99)(1006)(T_{\infty,2} - 10)$$

that may be solved to give

$$T_{\infty,2} = 19.08^\circ\mathrm{C}$$

The heat transfer is then obtained from the right side of Equation (f):

$$q = (4.99)(1006)(19.08 - 10) = 45.6\ \mathrm{kW/m}$$

This answer could be improved somewhat by recalculating the air properties based on a mean value of T_∞, but the improvement would be small and well within the accuracy of the empirical heat-transfer correlation of Equation (6-17).

* A better approach may be to base the heat transfer on a so-called log mean temperature difference (LMTD). This method is discussed in detail in Section 10-5 in connection with heat exchangers. In the present problem, the arithmetic temperature difference is satisfactory.

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EXAMPLE 6-11 Alternate Calculation Method

Compare the heat-transfer coefficient calculated with Equation (6-34) with the value obtained in Example 6-10.

Solution

Properties for use in Equation (6-34) are evaluated at free-stream conditions of 10°C, so we have

$$\nu = 14.2 \times 10^{-6} \qquad \mathrm{Pr} = 0.712 \qquad k = 0.0249 \qquad \mathrm{Pr}_w = 0.70$$

The Reynolds number is

$$\mathrm{Re}_{d,\max} = \frac{(21)(0.0254)}{14.2 \times 10^{-6}} = 37{,}563$$

so that the constants for Equation (6-34) are $C = 0.27$ and $n = 0.63$.

Inserting values, we obtain

$$\frac{\bar{h}d}{k} = (0.27)(37{,}563)^{0.63}(0.712/0.7)^{1/4} = 206.5$$

and

$$h = \frac{(206.5)(0.0249)}{0.0254} = 202.4 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

Multiplying by a factor of 0.92 from Table 6-7 to correct for only 5 tube rows gives

$$h = (0.92)(202.4) = 186.3 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

or a value about 13 percent higher than in Example 6-10. Both values are within the accuracies of the correlations.

6-5 | LIQUID-METAL HEAT TRANSFER

Considerable interest has been placed on liquid-metal heat transfer because of the high heat-transfer rates that may be achieved with these media. These high heat-transfer rates result from the high thermal conductivities of liquid metals as compared with other fluids; as a consequence, they are particularly applicable to situations where large energy quantities must be removed from a relatively small space, as in a nuclear reactor. In addition, the liquid metals remain in the liquid state at higher temperatures than conventional fluids like water and various organic coolants. This also makes more compact heat-exchanger design possible. Liquid metals are difficult to handle because of their corrosive nature and the violent action that may result when they come into contact with water or air; even so, their advantages in certain heat-transfer applications have overshadowed their shortcomings, and suitable techniques for handling them have been developed.

Let us first consider the simple flat plate with a liquid metal flowing across it. The Prandtl number for liquid metals is very low, of the order of 0.01, so that the thermal-boundary-layer thickness should be substantially larger than the hydrodynamic-boundary-layer thickness. The situation results from the high values of thermal conductivity for liquid metals and is depicted in Figure 6-15. Since the ratio of δ/δ_t is small, the velocity profile has a very blunt shape over most of the thermal boundary layer. As a first approximation, then, we might assume a slug-flow model for calculation of the heat transfer; that is we take

$$u = u_\infty \tag{6-35}$$

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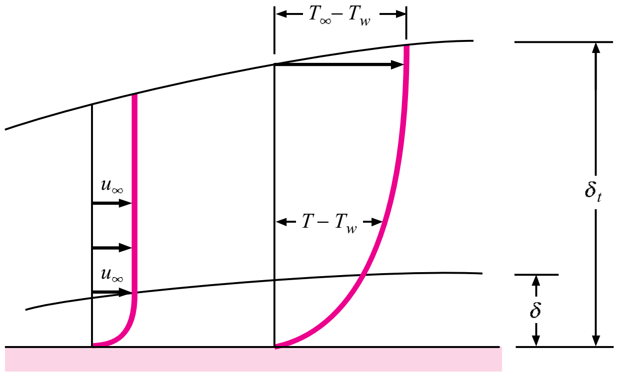
Figure 6-15 | Boundary-layer regimes for analysis of liquid-metal heat transfer.



throughout the thermal boundary layer for purposes of computing the energy-transport term in the integral energy equation (Section 5-6):

$$\frac{d}{dx}\left[\int_0^{\delta_t}(T_\infty - T)u\,dy\right] = \alpha\frac{\partial T}{\partial y}\bigg]_w \qquad [6\text{-}36]$$

The conditions on the temperature profile are the same as those in Section 5-6, so that we use the cubic parabola as before:

$$\frac{\theta}{\theta_\infty} = \frac{T - T_w}{T_\infty - T_w} = \frac{3}{2}\frac{y}{\delta_t} - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3 \qquad [6\text{-}37]$$

Inserting Equations (6-35) and (6-37) in (6-36) gives

$$\theta_\infty u_\infty \frac{d}{dx}\left\{\int_0^{\delta_t}\left[1 - \frac{3}{2}\frac{y}{\delta_t} + \frac{1}{2}\left(\frac{u}{\delta_t}\right)^3\right]dy\right\} = \frac{3\alpha\theta_\infty}{2\delta_t} \qquad [6\text{-}38]$$

that may be integrated to give

$$2\delta_t d\delta_t = \frac{8\alpha}{u_\infty}dx \qquad [6\text{-}39]$$

The solution to this differential equation is

$$\delta_t = \sqrt{\frac{8\alpha x}{u_\infty}} \qquad [6\text{-}40]$$

for a plate heated over its entire length.

The heat-transfer coefficient may be expressed by

$$h_x = \frac{-k(\partial T/\partial y)_w}{T_w - T_\infty} = \frac{3k}{2\delta_t} = \frac{3\sqrt{2}}{8}k\sqrt{\frac{u_\infty}{\alpha x}} \qquad [6\text{-}41]$$

This relationship may be put in dimensionless form as

$$\text{Nu}_x = \frac{h_x x}{k} = 0.530(\text{Re}_x\text{Pr})^{1/2} = 0.530\text{Pe}^{1/2} \qquad [6\text{-}42]$$

Using Equation (5-21) for the hydrodynamic-boundary-layer thickness,

$$\frac{\delta}{x} = \frac{4.64}{\mathrm{Re}_x^{1/2}} \tag{6-43}$$

we may compute the ratio δ/δ_t:

$$\frac{\delta}{\delta_t} = \frac{4.64}{\sqrt{8}}\sqrt{\mathrm{Pr}} = 1.64\sqrt{\mathrm{Pr}} \tag{6-44}$$

Using $\mathrm{Pr} \sim 0.01$, we obtain

$$\frac{\delta}{\delta_t} \sim 0.16$$

which is in reasonable agreement with our slug-flow model.

The flow model for liquid metals discussed above illustrates the general nature of liquid-metal heat transfer, and it is important to note that the heat transfer is dependent on the Peclet number. Empirical correlations are usually expressed in terms of this parameter, four of which we present below.

Extensive data on liquid metals are given in Reference 13, and the heat-transfer characteristics are summarized in Reference 23. Lubarsky and Kaufman [14] recommended the following relation for calculation of heat-transfer coefficients in fully developed turbulent flow of liquid metals in smooth tubes with uniform heat flux at the wall:

$$\mathrm{Nu}_d = \frac{hd}{k} = 0.625(\mathrm{Re}_d \mathrm{Pr})^{0.4} \tag{6-45}$$

All properties for use in Equation (6-45) are evaluated at the bulk temperature. Equation (6-45) is valid for $10^2 < \mathrm{Pe} < 10^4$ and for $L/d > 60$. Seban and Shimazaki [16] propose the following relation for calculation of heat transfer to liquid metals in tubes with constant wall temperature:

$$\mathrm{Nu}_d = 5.0 + 0.025(\mathrm{Re}_d \mathrm{Pr})^{0.8} \tag{6-46}$$

where all properties are evaluated at the bulk temperature. Equation (6-42) is valid for $\mathrm{Pe} > 10^2$ and $L/d > 60$.

More recent data by Skupinshi, Tortel, and Vautrey [26] with sodium-potassium mixtures indicate that the following relation may be preferable to that of Equation (6-45) for constant-heat-flux conditions:

$$\mathrm{Nu} = 4.82 + 0.0185\,\mathrm{Pe}^{0.827} \tag{6-47}$$

This relation is valid for $3.6 \times 10^3 < \mathrm{Re} < 9.05 \times 10^5$ and $10^2 < \mathrm{Pe} < 10^4$. Additional correlations are given by Sleicher and Rouse [48].

Witte [32] has measured the heat transfer from a sphere to liquid sodium during forced convection, with the data being correlated by

$$\mathrm{Nu} = 2 + 0.386(\mathrm{Re\,Pr})^{0.5} \tag{6-48}$$

for the Reynolds number range $3.56 \times 10^4 < \mathrm{Re} < 1.525 \times 10^5$.

Kalish and Dwyer [41] have presented information on liquid-metal heat transfer in tube bundles.

In general, there are many open questions concerning liquid-metal heat transfer, and the reader is referred to References 13 and 23 for more information.

Heating of Liquid Bismuth in Tube

EXAMPLE 6-12

Liquid bismuth flows at a rate of 4.5 kg/s through a 5.0-cm-diameter stainless-steel tube. The bismuth enters at 415°C and is heated to 440°C as it passes through the tube. If a constant heat flux is maintained along the tube and the tube wall is at a temperature 20°C higher than the bismuth bulk temperature, calculate the length of tube required to effect the heat transfer.

Solution

Because a constant heat flux is maintained, we may use Equation (6-47) to calculate the heat-transfer coefficient. The properties of bismuth are evaluated at the average bulk temperature of $(415 + 440)/2 = 427.5°\text{C}$

$$\mu = 1.34 \times 10^{-3}\ \text{kg/m} \cdot \text{s} \quad [3.242\ \text{lb}_m/\text{h} \cdot \text{ft}]$$
$$c_p = 0.149\ \text{kJ/kg} \cdot °\text{C} \quad [0.0356\ \text{Btu/lb}_m \cdot °\text{F}]$$
$$k = 15.6\ \text{W/m} \cdot °\text{C} \quad [9.014\ \text{Btu/h} \cdot \text{ft} \cdot °\text{F}]$$
$$\text{Pr} = 0.013$$

The total transfer is calculated from

$$q = \dot{m} c_p \Delta T_b = (4.5)(149)(440 - 415) = 16.76\ \text{kW} \quad [57{,}186\ \text{Btu/h}] \qquad [a]$$

We calculate the Reynolds and Peclet numbers as

$$\text{Re}_d = \frac{dG}{\mu} = \frac{(0.05)(4.5)}{[\pi(0.05)^2/4](1.34 \times 10^{-3})} = 85{,}520 \qquad [b]$$
$$\text{Pe} = \text{Re}\,\text{Pr} = (85{,}520)(0.013) = 1111$$

The heat-transfer coefficient is then calculated from Equation 6-47

$$\text{Nu}_d = 4.82 + (0.0185)(1111)^{0.827} = 10.93 \qquad [c]$$
$$h = \frac{(10.93)(15.6)}{0.05} = 3410\ \text{W/m}^2 \cdot °\text{C} \quad [600\ \text{Btu/h} \cdot \text{ft}^2 \cdot °\text{F}]$$

The total required surface area of the tube may now be computed from

$$q = hA(T_w - T_b) \qquad [d]$$

where we use the temperature difference of 20°C;

$$A = \frac{16{,}760}{(3410)(20)} = 0.246\ \text{m}^2 \quad [2.65\ \text{ft}^2]$$

This area in turn can be expressed in terms of the tube length

$$A = \pi d L \qquad \text{and} \qquad L = \frac{0.246}{\pi(0.05)} = 1.57\ \text{m} \quad [5.15\ \text{ft}]$$

6-6 | SUMMARY

In contrast to Chapter 5, which was mainly analytical in character, this chapter has dealt almost entirely with empirical correlations that may be used to calculate convection heat transfer. The general calculation procedure is as follows:

1. Establish the geometry of the situation.
2. Make a preliminary determination of appropriate fluid properties.
3. Establish the flow regime by calculating the Reynolds or Peclet number.

Table 6-8 | Summary of forced-convection relations. (See text for property evaluation.)

Subscripts: b = bulk temperature, f = film temperature, ∞ = free stream temperature, w = wall temperature

Geometry	Equation	Restrictions	Equation number
Tube flow	$\mathrm{Nu}_d = 0.023\,\mathrm{Re}_d^{0.8}\mathrm{Pr}^n$	Fully developed turbulent flow, $n = 0.4$ for heating, $n = 0.3$ for cooling, $0.6 < \mathrm{Pr} < 100$, $2500 < \mathrm{Re}_d < 1.25 \times 10^5$	(6-4a)
Tube flow	$\mathrm{Nu}_d = 0.0214(\mathrm{Re}_d^{0.8} - 100)\mathrm{Pr}^{0.4}$	$0.5 < \mathrm{Pr} < 1.5$, $10^4 < \mathrm{Re}_d < 5 \times 10^6$	(6-4b)
	$\mathrm{Nu}_d = 0.012(\mathrm{Re}_d^{0.87} - 280)\mathrm{Pr}^{0.4}$	$1.5 < \mathrm{Pr} < 500$, $3000 < \mathrm{Re}_d < 10^6$	(6-4c)
Tube flow	$\mathrm{Nu}_d = 0.027\,\mathrm{Re}_d^{0.8}\mathrm{Pr}^{1/3}\left(\dfrac{\mu}{\mu_w}\right)^{0.14}$	Fully developed turbulent flow	(6-5)
Tube flow, entrance region	$\mathrm{Nu}_d = 0.036\,\mathrm{Re}_d^{0.8}\mathrm{Pr}^{1/3}\left(\dfrac{d}{L}\right)^{0.055}$ See also Figures 6-5 and 6-6	Turbulent flow $10 < \dfrac{L}{d} < 400$	(6-6)
Tube flow	Petukov relation	Fully developed turbulent flow, $0.5 < \mathrm{Pr} < 2000$, $10^4 < \mathrm{Re}_d < 5 \times 10^6$, $0 < \dfrac{\mu_b}{\mu_w} < 40$	(6-7)
Tube flow	$\mathrm{Nu}_d = 3.66 + \dfrac{0.0668(d/L)\,\mathrm{Re}_d\mathrm{Pr}}{1 + 0.04[(d/L)\,\mathrm{Re}_d\mathrm{Pr}]^{2/3}}$	Laminar, T_w = const.	(6-9)
Tube flow	$\mathrm{Nu}_d = 1.86(\mathrm{Re}_d\mathrm{Pr})^{1/3}\left(\dfrac{d}{L}\right)^{1/3}\left(\dfrac{\mu}{\mu_w}\right)^{0.14}$	Fully developed laminar flow, T_w = const. $\mathrm{Re}_d\mathrm{Pr}\dfrac{d}{L} > 10$	(6-10)
Rough tubes	$\mathrm{St}_b\mathrm{Pr}_f^{2/3} = \dfrac{f}{8}$ or Equation (6-7)	Fully developed turbulent flow	(6-12)
Noncircular ducts	Reynolds number evaluated on basis of hydraulic diameter $D_H = \dfrac{4A}{P}$ A = flow cross-section area, P = wetted perimeter	Same as particular equation for tube flow	(6-14)
Flow across cylinders	$\mathrm{Nu}_f = C\,\mathrm{Re}_{df}^n\,\mathrm{Pr}^{1/3}$ C and n from Table 6-2	$0.4 < \mathrm{Re}_{df} < 400{,}000$	(6-17)
Flow across cylinders	$\mathrm{Nu}_{df} = 0.3 + \dfrac{0.62\,\mathrm{Re}_f^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1 + \left(\dfrac{0.4}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4}}\left[1 + \left(\dfrac{\mathrm{Re}_f}{282{,}000}\right)^{5/8}\right]^{4/5}$	$10^2 < \mathrm{Re}_f < 10^7$, $\mathrm{Pe} > 0.2$	(6-21)
Flow across cylinders		See text	(6-18) to (6-20) (6-22) to (6-24)
Flow across noncircular cylinders	$\mathrm{Nu} = C\,\mathrm{Re}_{df}^n\,\mathrm{Pr}^{1/3}$ See Table 6-3 for values of C and n.		(6-17)

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Table 6-8 | (*Continued*).

Subscripts: b = bulk temperature, f = film temperature, ∞ = free stream temperature, w = wall temperature

Geometry	Equation	Restrictions	Equation number
Flow across spheres	$\mathrm{Nu}_{df} = 0.37\,\mathrm{Re}_{df}^{0.6}$	$\mathrm{Pr} \sim 0.7$ (gases), $17 < \mathrm{Re} < 70{,}000$	(6-25)
	$\mathrm{Nu}_d \mathrm{Pr}^{-0.3} (\mu_w/\mu)^{0.25} = 1.2 + 0.53\,\mathrm{Re}_d^{0.54}$	Water and oils $1 < \mathrm{Re} < 200{,}000$ Properties at T_∞	(6-29)
	$\mathrm{Nu}_d = 2 + \left(0.4\,\mathrm{Re}_d^{1/2} + 0.06\,\mathrm{Re}_d^{2/3}\right) \mathrm{Pr}^{0.4} (\mu_\infty/\mu_w)^{1/4}$	$0.7 < \mathrm{Pr} < 380$, $3.5 < \mathrm{Re}_d < 80{,}000$, Properties at T_∞	(6-30)
Flow across tube banks	$\mathrm{Nu}_f = C\mathrm{Re}_{f,\max}^n\, \mathrm{Pr}_f^{1/3}$ C and n from Table 6-4	See text	(6-17)
Flow across tube banks	$\mathrm{Nu}_d = C\mathrm{Re}_{d,\max}^n\, \mathrm{Pr}^{0.36} \left(\frac{\mathrm{Pr}}{\mathrm{Pr}_w}\right)^{1/4}$	$0.7 < \mathrm{Pr} < 500$, $10 < \mathrm{Re}_{d,\max} < 10^6$	(6-34)
Liquid metals		See text	(6-37) to (6-48)
Friction factor	$\Delta p = f(L/d)\rho u_m^2/2g_c$, $u_m = \dot{m}/\rho A_c$		(6-13)

4. Select an equation that fits the geometry and flow regime and reevaluate properties, if necessary, in accordance with stipulations and the equation.
5. Proceed to calculate the value of h and/or the heat-transfer rate.

We should note that the data upon which the empirical equations are based are most often taken under laboratory conditions where it is possible to exert careful control over temperature and flow variables. In a practical application such careful control may not be present and there may be deviations from heat-transfer rates calculated from the equations given here. Our purpose is not to discourage the reader by this remark, but rather to indicate that sometimes it will be quite satisfactory to use a simple correlation over a more elaborate expression even if the simple relation has a larger scatter in its data representation. Our purpose has been to present a variety of expressions (where available) so that some choices can be made.

Finally, the most important relations of this chapter are listed in Table 6-8 for quick reference purposes.

Our presentation of convection is not yet complete. Chapter 7 will discuss the relations that are used for calculation of free convection heat transfer as well as combined free and forced convection. At the conclusion of that chapter we will present a general procedure to follow in *all* convection problems that will extend the outline given in the five steps above. This procedure will make use of the correlation summary Tables 5-2 and 6-8 along with a counterpart presented in Table 7-5 for free convection systems.

REVIEW QUESTIONS

1. What is the Dittus-Boelter equation? When does it apply?
2. How may heat-transfer coefficients be calculated for flow in rough pipes?
3. What is the hydraulic diameter? When is it used?
4. What is the form of equation used to calculate heat transfer for flow over cylinders and bluff bodies?

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5. Why does a slug-flow model yield reasonable results when applied to liquid-metal heat transfer?
6. What is the Peclet number?
7. What is the Graetz number?

LIST OF WORKED EXAMPLES

6-1 Turbulent heat transfer in a tube
6-2 Heating of water in laminar tube flow
6-3 Heating of air in laminar tube flow for constant heat flux
6-4 Heating of air with isothermal tube wall
6-5 Heat transfer in a rough tube
6-6 Turbulent heat transfer in a short tube
6-7 Airflow across isothermal cylinder
6-8 Heat transfer from electrically heated wire
6-9 Heat transfer from sphere
6-10 Heating of air with in-line tube bank
6-11 Alternate calculation method
6-12 Heating of liquid bismuth in tube

PROBLEMS

6-1 Engine oil enters a 5.0-mm-diameter tube at 120°C. The tube wall is maintained at 50°C, and the inlet Reynolds number is 1000. Calculate the heat transfer, average heat-transfer coefficient, and exit oil temperature for tube lengths of 10, 20, and 50 cm.

6-2 Water at an average bulk temperature of 80°F flows inside a horizontal smooth tube with wall temperature maintained at 180°F. The tube length is 2 m, and diameter is 3 mm. The flow velocity is 0.04 m/s. Calculate the heat-transfer rate.

6-3 Calculate the flow rate necessary to produce a Reynolds number of 15,000 for the flow of air at 1 atm and 300 K in a 2.5-cm-diameter tube. Repeat for liquid water at 300 K.

6-4 Liquid ammonia flows in a duct having a cross section of an equilateral triangle 1.0 cm on a side. The average bulk temperature is 20°C, and the duct wall temperature is 50°C. Fully developed laminar flow is experienced with a Reynolds number of 1000. Calculate the heat transfer per unit length of duct.

6-5 Water flows in a duct having a cross section 5 × 10 mm with a mean bulk temperature of 20°C. If the duct wall temperature is constant at 60°C and fully developed laminar flow is experienced, calculate the heat transfer per unit length.

6-6 Water at the rate of 3 kg/s is heated from 5 to 15°C by passing it through a 5-cm-ID copper tube. The tube wall temperature is maintained at 90°C. What is the length of the tube?

6-7 Water at the rate of 0.8 kg/s is heated from 35 to 40°C in a 2.5-cm-diameter tube whose surface is at 90°C. How long must the tube be to accomplish this heating?

6-8 Water flows through a 2.5-cm-ID pipe 1.5 m long at a rate of 1.0 kg/s. The pressure drop is 7 kPa through the 1.5-m length. The pipe wall temperature is maintained at a

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constant temperature of 50°C by a condensing vapor, and the inlet water temperature is 20°C. Estimate the exit water temperature.

6-9 Water at the rate of 1.3 kg/s is to be heated from 60°F to 100°F in a 2.5-cm-diameter tube. The tube wall is maintained at a constant temperature of 40°C. Calculate the length of tube required for the heating process.

6-10 Water at the rate of 1 kg/s is forced through a tube with a 2.5-cm ID. The inlet water temperature is 15°C, and the outlet water temperature is 50°C. The tube wall temperature is 14°C higher than the water temperature all along the length of the tube. What is the length of the tube?

6-11 Engine oil enters a 1.25-cm-diameter tube 3 m long at a temperature of 38°C. The tube wall temperature is maintained at 65°C, and the flow velocity is 30 cm/s. Estimate the total heat transfer to the oil and the exit temperature of the oil.

6-12 Air at 1 atm and 15°C flows through a long rectangular duct 7.5 cm by 15 cm. A 1.8-m section of the duct is maintained at 120°C, and the average air temperature at exit from this section is 65°C. Calculate the airflow rate and the total heat transfer.

6-13 Water at the rate of 0.5 kg/s is forced through a smooth 2.5-cm-ID tube 15 m long. The inlet water temperature is 10°C, and the tube wall temperature is 15°C higher than the water temperature all along the length of the tube. What is the exit water temperature?

6-14 Water at an average temperature of 300 K flows at 0.7 kg/s in a 2.5-cm-diameter tube 6 m long. The pressure drop is measured as 2 kPa. A constant heat flux is imposed, and the average wall temperature is 55°C. Estimate the exit temperature of the water.

6-15 An oil with $\Pr = 1960$, $\rho = 860\ \text{kg/m}^3$, $\nu = 1.6 \times 10^{-4}\ \text{m}^2/\text{s}$, and $k = 0.14\ \text{W/m} \cdot {}^\circ\text{C}$ enters a 2.5-mm-diameter tube 60 cm long. The oil entrance temperature is 20°C, the mean flow velocity is 30 cm/s, and the tube wall temperature is 120°C. Calculate the heat-transfer rate.

6-16 Liquid ammonia flows through a 2.5-cm-diameter smooth tube 2.5 m long at a rate of 0.4 kg/s. The ammonia enters at 10°C and leaves at 38°C, and a constant heat flux is imposed on the tube wall. Calculate the average wall temperature necessary to effect the indicated heat transfer.

6-17 Liquid Freon 12 (CCl_2F_2) flows inside a 1.25-cm-diameter tube at a velocity of 3 m/s. Calculate the heat-transfer coefficient for a bulk temperature of 10°C. How does this compare with water at the same conditions?

6-18 Water at an average temperature of 10°C flows in a 2.5-cm-diameter tube 6 m long at a rate of 0.4 kg/s. The pressure drop is measured as 3 kPa. A constant heat flux is imposed, and the average wall temperature is 50°C. Estimate the exit temperature of the water.

6-19 Water at the rate of 0.4 kg/s is to be cooled from 71 to 32°C. Which would result in less pressure drop—to run the water through a 12.5-mm-diameter pipe at a constant temperature of 4°C or through a constant-temperature 25-mm-diameter pipe at 20°C?

6-20 Air at 1400 kPa enters a duct 7.5 cm in diameter and 6 m long at a rate of 0.5 kg/s. The duct wall is maintained at an average temperature of 500 K. The average air temperature in the duct is 550 K. Estimate the decrease in temperature of the air as it passes through the duct.

6-21 Air flows at 100°C and 300 kPa in a 1.2-cm-(inside)-diameter tube at a velocity such that a Reynolds number of 15,000 is obtained. The outside of the tube is subjected

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to a crossflow of air at 100 kPa, 30°C, and a free-stream velocity of 20 m/s. The tube wall thickness is 1.0 mm. Calculate the overall heat transfer coefficient for this system. What would be the temperature drop of the air inside the tube per centimeter of length.

6-22 Liquid water is to be heated from 60°F to 120°F in a smooth tube. The tube has an electric heat supplied that provides a constant heat flux such that the tube wall temperature is always 30°F above the water bulk temperature. The Reynolds number used for calculating the heat-transfer coefficient is 100,000. Calculate the length of tube required for heating, expressed in meters, if the tube has a diameter of 0.5 cm.

6-23 An annulus consists of the region between two concentric tubes having diameters of 4 cm and 5 cm. Ethylene glycol flows in this space at a velocity of 6.9 m/s. The entrance temperature is 20°C, and the exit temperature is 40°C. Only the inner tube is a heating surface, and it is maintained constant at 80°C. Calculate the length of annulus necessary to effect the heat transfer.

6-24 An air-conditioning duct has a cross section of 45 cm by 90 cm. Air flows in the duct at a velocity of 7.5 m/s at conditions of 1 atm and 300 K. Calculate the heat-transfer coefficient for this system and the pressure drop per unit length.

6-25 Water flows in a 3.0-cm-diameter tube having a relative roughness of 0.002 with a constant wall temperature of 80°C. If the water enters at 20°C, estimate the convection coefficient for a Reynolds number of 10^5.

6-26 Liquid Freon 12 (CCl_2F_2) enters a 3.5-mm-diameter tube at 0°C and with a flow rate such that the Reynolds number is 700 at entrance conditions. Calculate the length of tube necessary to raise the fluid temperature to 20°C if the tube wall temperature is constant at 40°C.

6-27 Air enters a small duct having a cross section of an equilateral triangle, 3.0 mm on a side. The entering temperature is 27°C and the exit temperature is 77°C. If the flow rate is 5×10^{-5} kg/s and the tube length is 30 cm, calculate the tube wall temperature necessary to effect the heat transfer. Also calculate the pressure drop. The pressure is 1 atm.

6-28 Air at 90 kPa and 27°C enters a 4.0-mm-diameter tube with a mass flow rate of 7×10^{-5} kg/s. A constant heat flux is imposed at the tube surface so that the tube wall temperature is 70°C above the fluid bulk temperature. Calculate the exit air temperature for a tube length of 12 cm.

6-29 Air at 110 kPa and 40°C enters a 6.0-mm-diameter tube with a mass flow rate of 8×10^{-5} kg/s. The tube wall temperature is maintained constant at 140°C. Calculate the exit air temperature for a tube length of 14 cm.

6-30 Engine oil at 40°C enters a 1-cm-diameter tube at a flow rate such that the Reynolds number at entrance is 50. Calculate the exit oil temperature for a tube length of 8 cm and a constant tube wall temperature of 80°C.

6-31 Water flows in a 2-cm-diameter tube at an average flow velocity of 8 m/s. If the water enters at 20°C and leaves at 30°C and the tube length is 10 m, estimate the average wall temperature necessary to effect the required heat transfer.

6-32 Engine oil at 20°C enters a 2.0-mm-diameter tube at a velocity of 1.2 m/s. The tube wall temperature is constant at 60°C and the tube is 1.0 m long. Calculate the exit oil temperature.

6-33 Water flows inside a smooth tube at a mean flow velocity of 3.0 m/s. The tube diameter is 25 mm and a constant heat flux condition is maintained at the tube

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wall such that the tube temperature is always 20°C above the water temperature. The water enters the tube at 30°C and leaves at 50°C. Calculate the tube length necessary to accomplish the indicated heating.

6-34 Liquid ammonia at 10°C and 1 atm flows across a horizontal cylinder at a velocity of 5 m/s. The cylinder has a diameter of 2.5 cm and length of 125 cm and is maintained at a temperature of 30°C. Calculate the heat lost by the cylinder.

6-35 Water enters a 3-mm-diameter tube at 21°C and leaves at 32°C. The flow rate is such that the Reynolds number is 600. The tube length is 10 cm and is maintained at a constant temperature of 60°C. Calculate the water flow rate.

6-36 Water enters a 3.0-cm-diameter tube at 15°C and leaves at 38°C. The flow rate is 1.0 kg/s and the tube wall temperature is 60°C. Calculate the length of the tube.

6-37 Glycerin flows in a 5-mm-diameter tube at such a rate that the Reynolds number is 10. The glycerine enters at 10°C and leaves at 30°C. The tube wall is maintained constant at 40°C. Calculate the length of the tube.

6-38 A 5-cm-diameter cylinder maintained at 80°C is placed in a nitrogen flow stream at 2 atm pressure and 10°C. The nitrogen flows across the cylinder with a velocity of 5 m/s. Calculate the heat lost by the cylinder per meter of length.

6-39 Air at 1 atm and 10°C blows across a 4-cm-diameter cylinder maintained at a surface temperature of 54°C. The air velocity is 25 m/s. Calculate the heat loss from the cylinder per unit length.

6-40 Air at 200 kPa blows across a 20-cm-diameter cylinder at a velocity of 25 m/s and temperature of 10°C. The cylinder is maintained at a constant temperature of 80°C. Calculate the heat transfer and drag force per unit length.

6-41 Water at 43°C enters a 5-cm-ID pipe having a relative roughness of 0.002 at a rate of 6 kg/s. If the pipe is 9 m long and is maintained at 71°C, calculate the exit water temperature and the total heat transfer.

6-42 A short tube is 6.4 mm in diameter and 15 cm long. Water enters the tube at 1.5 m/s and 38°C, and a constant-heat-flux condition is maintained such that the tube wall temperature remains 28°C above the water bulk temperature. Calculate the heat-transfer rate and exit water temperature.

6-43 Ethylene glycol is to be cooled from 65 to 40°C in a 3.0-cm-diameter tube. The tube wall temperature is maintained constant at 20°C. The glycol enters the tube with a velocity of 10 m/s. Calculate the length of tube necessary to accomplish this cooling.

6-44 Air at 70 kPa and 20°C flows across a 5-cm-diameter cylinder at a velocity of 15 m/s. Compute the drag force exerted on the cylinder.

6-45 A heated cylinder at 450 K and 2.5 cm in diameter is placed in an atmospheric airstream at 1 atm and 325 K. The air velocity is 30 m/s. Calculate the heat loss per meter of length for the cylinder.

6-46 Assuming that a human can be approximated by a cylinder 30 cm in diameter and 1.1 m high with a surface temperature of 24°C, calculate the heat the person would lose while standing in a 30-mi/h wind whose temperature is 0°C.

6-47 Assume that one-half the heat transfer from a cylinder in cross flow occurs on the front half of the cylinder. On this assumption, compare the heat transfer from a cylinder in cross flow with the heat transfer from a flat plate having a length equal to the distance from the stagnation point on the cylinder. Discuss this comparison.

6-48 Water at 15.56°C is to be heated in a 2-mm-ID tube until the exit temperature reaches 26.67°C. The tube wall temperature is maintained at 48.99°C and the inlet

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flow velocity is 0.3 m/s. Calculate the length of tube required in meters to accomplish this heating. Also calculate the total heating required, expressed in watts.

6-49 An isothermal cylinder having a diameter of 2.0 cm and maintained at 50°C is placed in a helium flow system having free-stream conditions of 200 kPa, 20°C, and $u_\infty = 25$ m/s. Calculate the heat lost for a cylinder length of 50 cm.

6-50 A 0.13-mm-diameter wire is exposed to an airstream at −30°C and 54 kPa. The flow velocity is 230 m/s. The wire is electrically heated and is 12.5 mm long. Calculate the electric power necessary to maintain the wire surface temperature at 175°C.

6-51 Air at 90°C and 1 atm flows past a heated 1.5-mm-diameter wire at a velocity of 6 m/s. The wire is heated to a temperature of 150°C. Calculate the heat transfer per unit length of wire.

6-52 A fine wire 0.025 mm in diameter and 15 cm long is to be used to sense flow velocity by measuring the electrical heat that can be dissipated from the wire when placed in an airflow stream. The resistivity of the wire is 70 $\mu\Omega \cdot$ cm. The temperature of the wire is determined by measuring its electric resistance relative to some reference temperature T_o so that

$$R = R_o[1 + a(T - T_o)]$$

For this particular wire the value of the temperature coefficient a is 0.006°C^{-1}. The resistance can be determined from measurements of the current and voltage impressed on the wire, and

$$R = \frac{E}{I}$$

Suppose a measurement is made for air at 20°C with a flow velocity of 10 m/s and the wire temperature is 40°C. What values of voltage and current would be measured for these conditions if R_o is evaluated at $T_o = 20$°C? What values of voltage and current would be measured for the same wire temperature but flow velocities of 15 m/s and 20 m/s?

6-53 Helium at 1 atm and 325 K flows across a 3-mm-diameter cylinder that is heated to 425 K. The flow velocity is 9 m/s. Calculate the heat transfer per unit length of wire. How does this compare with the heat transfer for air under the same conditions?

6-54 Calculate the heat-transfer rate per unit length for flow over a 0.025-mm-diameter cylinder maintained at 65°C. Perform the calculation for (*a*) air at 20°C and 1 atm and (*b*) water at 20°C; $u_\infty = 6$ m/s.

6-55 Compare the heat-transfer results of Equations (6-17) and (6-18) for water at Reynolds numbers of 10^3, 10^4, and 10^5 and a film temperature of 90°C.

6-56 A pipeline in the Arctic carries hot oil at 50°C. A strong arctic wind blows across the 50-cm-diameter pipe at a velocity of 13 m/s and a temperature of −35°C. Estimate the heat loss per meter of pipe length.

6-57 Two tubes are available, a 4.0-cm-diameter tube and a 4.0-cm-square tube. Air at 1 atm and 27°C is blown across the tubes with a velocity of 20 m/s. Calculate the heat transfer in each case if the tube wall temperature is maintained at 50°C.

6-58 A 3.0-cm-diameter cylinder is subjected to a cross flow of carbon dioxide at 200°C and a pressure of 1 atm. The cylinder is maintained at a constant temperature of 50°C and the carbon dioxide velocity is 40 m/s. Calculate the heat transfer to the cylinder per meter of length.

6-59 Water having an average bulk temperature of 100°F flows in a smooth tube with a diameter of 1.25 cm. The flow rate is such that a Reynolds number of 100,000 is experienced, and the tube wall is maintained at an average temperature of 160°F.

If the tube length is 1.5 m calculate the exit bulk temperature of the water. Express in °C.

6-60 Using a suitable computer software package, integrate the local heat transfer coefficient results of Figure 6-11 to obtain average values of h for each Reynolds number shown. Subsequently, compare the results with values calculated from the information in Table 6-2. If needed, consult Reference 7 for additional information.

6-61 Helium at 150 kPa and 20°C is forced at 50 m/s across a horizontal cylinder having a diameter of 30 cm and a length of 6 m. Calculate the heat lost by the cylinder if its surface temperature is maintained constant at 100°C.

6-62 A 0.25-inch-diameter cylinder is maintained at a constant temperature of 300°C and placed in a cross flow of CO_2 at $p = 100$ kPa and $T = 30$°C. Calculate the heat loss for a 4.5-m length of the cylinder if the CO_2 velocity is 35 m/s.

6-63 A 20-cm-diameter cylinder is placed in a cross-flow CO_2 stream at 1 atm and 300 K. The cylinder is maintained at a constant temperature of 400 K and the CO_2 velocity is 50 m/s. Calculate the heat lost by the cylinder per meter of length.

6-64 Air flows across a 4-cm-square cylinder at a velocity of 12 m/s. The surface temperature is maintained at 85°C. Free-stream air conditions are 20°C and 0.6 atm. Calculate the heat loss from the cylinder per meter of length.

6-65 Water flows over a 3-mm-diameter sphere at 5 m/s. The free-stream temperature is 38°C, and the sphere is maintained at 93°C. Calculate the heat-transfer rate.

6-66 A spherical water droplet having a diameter of 1.3 mm is allowed to fall from rest in atmospheric air at 1 atm and 20°C. Estimate the velocities the droplet will attain after a drop of 30, 60, and 300 m.

6-67 A spherical tank having a diameter of 4.0 m is maintained at a surface temperature of 40°C. Air at 1 atm and 20°C blows across the tank at 6 m/s. Calculate the heat loss.

6-68 A heated sphere having a diameter of 3 cm is maintained at a constant temperature of 90°C and placed in a water flow stream at 20°C. The water flow velocity is 3.5 m/s. Calculate the heat lost by the sphere.

6-69 A small sphere having a diameter of 6 mm has an electric heating coil inside, which maintains the outer surface temperature at 220°C. The sphere is exposed to an airstream at 1 atm and 20°C with a velocity of 20 m/s. Calculate the heating rate which must be supplied to the sphere.

6-70 Air at a pressure of 3 atm blows over a flat plate at a velocity of 75 m/s. The plate is maintained at 200°C and the free-stream temperature is 30°C. Calculate the heat loss for a plate which is 1 m square.

6-71 Air at 3.5 MPa and 38°C flows across a tube bank consisting of 400 tubes of 1.25-cm OD arranged in a staggered manner 20 rows high; $S_p = 3.75$ cm and $S_n = 2.5$ cm. The incoming-flow velocity is 9 m/s, and the tube-wall temperatures are maintained constant at 20°C by a condensing vapor on the inside of the tubes. The length of the tubes is 1.5 m. Estimate the exit air temperature as it leaves the tube bank.

6-72 A tube bank uses an in-line arrangement with $S_n = S_p = 1.9$ cm and 6.33-mm-diameter tubes. Six rows of tubes are employed with a stack 50 tubes high. The surface temperature of the tubes is constant at 90°C, and atmospheric air at 20°C is forced across them at an inlet velocity of 4.5 m/s before the flow enters the tube bank. Calculate the total heat transfer per unit length for the tube bank. Estimate the pressure drop for this arrangement.

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6-73 Air at 1 atm and 300 K flows across an in-line tube bank having 10 vertical and 10 horizontal rows. The tube diameter is 2 cm and the center-to-center spacing is 4 cm in both the normal and parallel directions. Calculate the convection heat-transfer coefficient for this situation if the entering free-stream velocity is 10 m/s and properties may be evaluated at free-stream conditions.

6-74 Repeat Problem 6-73 for a staggered-tube arrangement with the same values of S_p and S_n.

6-75 Condensing steam at 150°C is used on the inside of a bank of tubes to heat a cross-flow stream of CO_2 that enters at 3 atm, 35°C, and 5 m/s. The tube bank consists of 100 tubes of 1.25-cm OD in a square in-line array with $S_n = S_p = 1.875$ cm. The tubes are 60 cm long. Assuming the outside-tube-wall temperature is constant at 150°C, calculate the overall heat transfer to the CO_2 and its exit temperature.

6-76 An in-line tube bank is constructed of 2.5-cm-diameter tubes with 15 rows high and 7 rows deep. The tubes are maintained at 90°C, and atmospheric air is blown across them at 20°C and $u_\infty = 12$ m/s. The arrangement has $S_p = 3.75$ and $S_n = 5.0$ cm. Calculate the heat transfer from the tube bank per meter of length. Also calculate the pressure drop.

6-77 Air at 300 K and 1 atm enters an in-line tube bank consisting of five rows of 10 tubes each. The tube diameter is 2.5 cm and $S_n = S_p = 5.0$ cm. The incoming velocity is 10 m/s and the tube wall temperatures are constant at 350 K. Calculate the exit air temperature.

6-78 Atmospheric air at 20°C flows across a 5-cm-square rod at a velocity of 15 m/s. The velocity is normal to one of the faces of the rod. Calculate the heat transfer per unit length for a surface temperature of 90°C.

6-79 A certain home electric heater uses thin metal strips to dissipate heat. The strips are 6 mm wide and are oriented normal to the airstream, which is produced by a small fan. The air velocity is 2 m/s, and seven 35-cm strips are employed. If the strips are heated to 870°C, estimate the total convection heat transfer to the room air at 20°C. (Note that in such a heater, much of the *total* transfer will be by thermal radiation.)

6-80 A square duct, 30 cm by 30 cm, is maintained at a constant temperature of 30°C and an airstream of 50°C and 1 atm is forced across it with a velocity of 6 m/s. Calculate the heat gained by the duct. How much would the heat flow be reduced if the flow velocity were reduced in half?

6-81 Using the slug-flow model, show that the boundary-layer energy equation reduces to the same form as the transient-conduction equation for the semi-infinite solid of Section 4-3. Solve this equation and compare the solution with the integral analysis of Section 6-5.

6-82 Liquid bismuth enters a 2.5-cm-diameter stainless-steel pipe at 400°C at a rate of 1 kg/s. The tube wall temperature is maintained constant at 450°C. Calculate the bismuth exit temperature if the tube is 60 cm long.

6-83 Liquid sodium is to be heated from 120 to 149°C at a rate of 2.3 kg/s. A 2.5-cm-diameter electrically heated tube is available (constant heat flux). If the tube wall temperature is not to exceed 200°C, calculate the minimum length required.

6-84 Determine an expression for the average Nusselt number for liquid metals flowing over a flat plate. Use Equation (6-42) as a starting point.

6-85 Water at the rate of 0.8 kg/s at 93°C is forced through a 5-cm-ID copper tube at a suitable velocity. The wall thickness is 0.8 mm. Air at 15°C and atmospheric pressure is forced over the outside of the tube at a velocity of 15 m/s in a direction normal to the axis of the tube. What is the heat loss per meter of length of the tube?

6-86 Air at 1 atm and 350 K enters a 1.25-cm-diameter tube with a flow rate of 35 g/s. The surface temperature of the tube is 300 K, and its length is 12 m. Calculate the heat lost by the air and the exit air temperature.

6-87 Air flows across a 5.0-cm-diameter smooth tube with free-stream conditions of 20°C, 1 atm, and $u_\infty = 25$ m/s. If the tube surface temperature is 120°C, calculate the heat loss per unit length.

6-88 Engine oil enters an 8-m-long tube at 20°C. The tube diameter is 20 mm, and the flow rate is 0.4 kg/s. Calculate the outlet temperature of the oil if the tube surface temperature is maintained at 80°C.

6-89 Air at 1 atm and 300 K with a flow rate of 0.2 kg/s enters a rectangular 10-by-20-cm duct that is 250 cm long. If the duct surface temperature is maintained constant at 400 K, calculate the heat transfer to the air and the exit air temperature.

6-90 Air at 1 atm and 300 K flows inside a 1.5-mm-diameter smooth tube such that the Reynolds number is 1200. Calculate the heat-transfer coefficients for tube lengths of 1, 10, 20, and 100 cm.

6-91 Water at an average bulk temperature of 10°C flows inside a channel shaped like an equilateral triangle 2.5 cm on a side. The flow rate is such that a Reynolds number of 50,000 is obtained. If the tube-wall temperature is maintained 15°C higher than the water bulk temperature, calculate the length of tube needed to effect a 10°C increase in bulk temperature. What is the total heat transfer under this condition?

6-92 Air at 1 atm and 300 K flows normal to a square noncircular cylinder such that the Reynolds number is 10^4. Compare the heat transfer for this system with that for a circular cylinder having diameter equal to a side of the square. Repeat the calculation for the first, third, and fourth entries of Table 6-3.

6-93 Air at 1 atm and 300 K flows across a sphere such that the Reynolds number is 50,000. Compare Equations (6-25) and (6-26) for these conditions. Also compare with Equation (6-30).

6-94 Water at 10°C flows across a 2.5-cm-diameter sphere at a free-stream velocity of 4 m/s. If the surface temperature of the sphere is 60°C, calculate the heat loss.

6-95 A tube bank consists of a square array of 144 tubes arranged in an in-line position. The tubes have a diameter of 1.5 cm and length of 1.0 m; the center-to-center tube spacing is 2.0 cm. If the surface temperature of the tubes is maintained at 350 K and air enters the tube bank at 1 atm, 300 K, and $u_\infty = 6$ m/s, calculate the total heat lost by the tubes.

6-96 Though it may be classified as a rather simple mistake, a frequent cause for substantial error in convection calculations is failure to select the correct geometry for the problem. Consider the following three geometries for flow of air at 1 atm, 300 K, and a Reynolds number of 50,000: (*a*) flow across a cylinder with diameter of 10 cm, (*b*) flow inside a tube with diameter of 10 cm, and (*c*) flow along a flat plate of length 10 cm. Calculate the average heat-transfer coefficient for each of these geometries and comment on the results.

6-97 Water flows at an average flow velocity of 10 ft/s in a smooth tube at an average temperature of 60°F. The tube diameter is 2.5 cm. Calculate the length of tube required to cause the bulk temperature of the water to rise 10°C if the tube wall temperature is maintained at 150°F.

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6-98 It has been noted that convection heat transfer is dependent on fluid properties, which in turn are dependent on temperature. Consider flow of atmospheric air at 0.012 kg/s in a smooth 2.5-cm-diameter tube. Assuming that the Dittus-Boelter relation [Equation (6-4*a*)] applies, calculate the average heat-transfer coefficient for properties evaluated at 300, 400, 500, and 800 K. Comment on the results.

6-99 Repeat Problem 6-98 for the same mass flow of atmospheric helium with properties evaluated at 255, 477, and 700 K and comment on the results.

6-100 Air at 300 K flows in a 5-mm-diameter tube at a flow rate such that the Reynolds number is 50,000. The tube length is 50 mm. Estimate the average heat-transfer coefficient for a constant heat flux at the wall.

6-101 Water at 15.6°C flows in a 5-mm-diameter tube having a length of 50 mm. The flow rate is such that the Peclet number is 1000. If the tube wall temperature is constant at 49°C, what temperature increase will be experienced by the water?

6-102 Air at 1 atm flows in a rectangular duct having dimensions of 30 cm by 60 cm. The mean flow velocity is 7.5 m/s at a mean bulk temperature of 300 K. If the duct wall temperature is constant at 325 K, estimate the air temperature increase over a duct length of 30 m.

6-103 Glycerin at 10°C flows in a rectangular duct 1 cm by 8 cm and 1 m long. The flow rate is such that the Reynolds number is 250. Estimate the average heat transfer coefficient for an isothermal wall condition.

6-104 Air at 300 K blows normal to a 6-mm heated strip maintained at 600 K. The air velocity is such that the Reynolds number is 15,000. Calculate the heat loss for a 50-cm-long strip.

6-105 Repeat Problem 6-104 for flow normal to a square rod 6 mm on a side.

6-106 Repeat Problem 6-104 for flow parallel to a 6-mm strip. (Calculate heat transfer for both sides of the strip.)

6-107 Air at 1 atm flows normal to a square in-line bank of 400 tubes having diameters of 6 mm and lengths of 50 cm. $S_n = S_d = 9$ mm. The air enters the tube bank at 300 K and at a velocity such that the Reynolds number based on inlet properties and the maximum velocity at inlet is 50,000. If the outside wall temperature of the tubes is 400 K, calculate the air temperature rise as it flows through the tube bank.

6-108 Repeat Problem 6-107 for a tube bank with a staggered arrangement, the same dimensions, and the same free-stream inlet velocity to the tube bank.

6-109 Compare the Nusselt number results for heating air in a smooth tube at 300 K and Reynolds numbers of 50,000 and 100,000, as calculated from Equation (6-4*a*), (6-4*b*), and (6-4*c*). What do you conclude from these results?

6-110 Repeat Problem 6-109 for heating water at 21°C.

6-111 Compare the results obtained from Equations (6-17), (6-21), (6-22), and (6-23) for air at 1 atm and 300 K flowing across a cylinder maintained at 400 K, with Reynolds numbers of 50,000 and 100,000. What do you conclude from these results?

6-112 Repeat Problem 6-111 for flow or water at 21°C across a cylinder maintained at 32.2°C. What do you conclude from the results?

Design-Oriented Problems

6-113 Using the values of the local Nusselt number given in Figure 6-11, obtain values for the average Nusselt number as a function of the Reynolds number. Plot the results as

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log Nu versus log Re, and obtain an equation that represents all the data. Compare this correlation with that given by Equation (6-17) and Table 6-2.

6-114 A heat exchanger is constructed so that hot flue gases at 700 K flow inside a 2.5-cm-ID copper tube with 1.6-mm wall thickness. A 5.0-cm tube is placed around the 2.5-cm-diameter tube, and high-pressure water at 150°C flows in the annular space between the tubes. If the flow rate of water is 1.5 kg/s and the total heat transfer is 17.5 kW, estimate the length of the heat exchanger for a gas mass flow of 0.8 kg/s. Assume that the properties of the flue gas are the same as those of air at atmospheric pressure and 700 K.

6-115 Compare Equations (6-19), (6-20), and (6-21) with Equation (6-17) for a gas with $\mathrm{Pr} = 0.7$ at the following Reynolds numbers: (*a*) 500, (*b*) 1000, (*c*) 2000, (*d*) 10,000, (*e*) 100,000.

6-116 A more compact version of the tube bank in Problem 6-72 can be achieved by reducing the S_p and S_n dimensions while still retaining the same number of tubes. Investigate the effect of reducing S_p and S_n in half, that is, $S_p = S_n = 0.95$ cm. Calculate the heat transfer and pressure drop for this new arrangement.

6-117 The drag coefficient for a sphere at Reynolds numbers less than 100 may be approximated by $C_D = b\mathrm{Re}^{-1}$, where b is a constant. Assuming that the Colburn analogy between heat transfer and fluid friction applies, derive an expression for the heat loss from a sphere of diameter d and temperature T_s, released from rest and allowed to fall in a fluid of temperature T_∞. (Obtain an expression for the heat lost between the time the sphere is released and the time it reaches some velocity v. Assume that the Reynolds number is less than 100 during this time and that the sphere remains at a constant temperature.)

6-118 Consider the application of the Dittus-Boelter relation [Equation (6-4*a*)] to turbulent flow of air in a smooth tube under developed turbulent flow conditions. For a fixed mass flow rate and tube diameter (selected at your discretion) investigate the effect of bulk temperature on the heat-transfer coefficient by calculating values of h for average bulk temperatures of 20, 50, 100, 200, and 300°C. What do you conclude from this calculation? From the results, estimate the dependence of the heat-transfer coefficient for air on absolute temperature.

6-119 A *convection* electric oven is one that employs a fan to force air across the food in addition to radiant heat from electric heating elements. Consider two oven temperature settings at 175°C and 230°C. Make assumptions regarding airflow velocities in order to estimate oven heating performance with and without convection under these two temperature conditions. Make your own assumptions as to the type of food to be cooked. Enthusiasts claim that the convection oven will cook in half the time of the all-radiant model. How do you evaluate this claim? What would you recommend as a prudent claim for the manufacturer of the oven to make? As a concrete example consider cooking a 25-pound turkey at Thanksgiving. Consult whatever sources (cookbooks) you think appropriate to check your calculations. Make recommendations that you feel would be acceptable to a typical home chef who is fussy about such matters.

6-120 A smooth glass plate is coated with a special electrically conductive film that may be used to produce a constant heat flux on the plate. Estimate the airflow velocity that must be used to remove 850 W from a 0.5-m-square plate, maintained at an average temperature of 65°C and dissipating heat to air at 1 atm and 20°C. Suppose the plate also radiates like a black surface to the surroundings at 20°C. What flow velocity would be necessary to dissipate the 850 W under these conditions?

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