Chapter 2

Numerical Solutions for Nonlinear Equations

There are lots of real problems, can be solved by mathematical forms, and these forms has nonlinear equations. Mostly, it is difficult to calculate the exact solutions for these equations; therefore, we study some numerical methods in order to be able to find the approximate solutions for these equations.

Examples

$f(x) = e^{x^2}\ cosx,\ f(x) = x^2 + x - 1$ nonlinear equations for one variable

$f(x, y) = (x + y)^2, \quad f(x, y) = 1/x + y + lnxy$
nonlinear equations for two variables

Roots of nonlinear equations of one variable

Finding the solution for a nonlinear equation of one variable, $f(x) = 0$ means, finding a value α, such that $f(\alpha) = 0$, where α is called the root of $f(x) = 0$.

Remark: Some equations have more than one root.

Example:

1- $f(x) = x^2 + 3x + 2 = (\mathrm{x} + 2)(\mathrm{x} + 1) = 0$

Since $f(-1) = f(-2) = 0$, it follows that both of $-1, -2$ are roots for the nonlinear equation above.

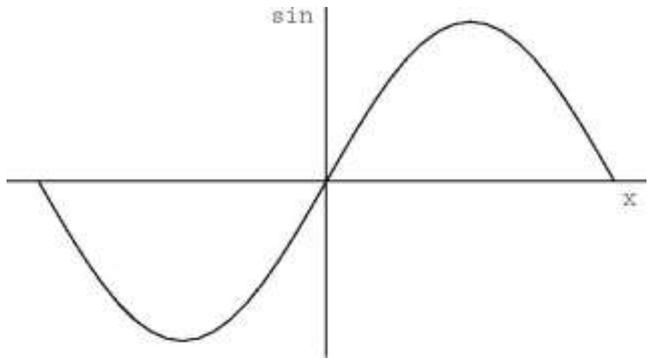
2- $f(x) = sin\ x,$ we see that $x = n\pi, n = 0,1,2, \ldots,$ are roots for f

3- $f(x) = (x - 1)^2$, we see that $x = 1$, is the double root for f.

The approximate roots for non-linear equations

In order to ensure that, there exist a root α, for the equation $f(x) = 0$ on the interval $[a, b]$, we have to make sure that $f(a)f(b) < 0$, see the following figure:

$f(b)$



$f(a)$

Remark:- Let α be the exact root of the equation $f(x) = 0$, and α_n is a subsequence of approximate roots, that can be got from using a numerical method, for large n the following condition has to satisfy:

$$|f(\alpha_n)| < \in, \quad \text{or} \quad |\alpha - \alpha_n| < \in$$